PROCEEDINGS

OF THE

ROYAL SOCIETY OF LONDON.

From November 19, 1863, *to December* 22, 1864, *inclusive.*

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CONTENTS.

VOL. XIII.

Page

A General Catalogue of Nebulæ and Clusters of Stars for the Year 1860·0, with Precessions for 1880·0. By Sir J. F. W. Herschel, Bart., F.R.S. .. 2

Note on Kinone. By A. W. Hofmann, LL.D., F.R.S. 4

Researches on the Colouring-matters derived from Coal-tar.—I. On Aniline-yellow. By A. W. Hofmann, LL.D., F.R.S. 6

Researches on the Colouring-matters derived from Coal-tar.—II. On Aniline-blue. By A. W. Hofmann, LL.D., F.R.S. 9

Account of Magnetic Observations made between the years 1858 and 1861 inclusive, in British Columbia, Washington Territory, and Vancouver Island. By Captain R. W. Haig, R.A. 15

On Plane Water-Lines. By W. J. Macquorn Rankine, C.E., LL.D., F.R.SS.L. & E., Assoc. Inst. N.A. 15

On the degree of uncertainty which Local Attraction, if not allowed for, occasions in the Map of a Country, and in the Mean Figure of the Earth as determined by Geodesy: a Method of obtaining the Mean Figure free from ambiguity, from a comparison of the Anglo-Gallic, Russian, and Indian Arcs: and Speculations on the Constitution of the Earth's Crust. By the Venerable J. H. Pratt, Archdeacon of Calcutta..... 18

On the Meteorological Results shown by the Self-registering Instruments at Greenwich during the extraordinary Storm of October 30, 1863. By James Glaisher, F.R.S., F.R.A.S. 19

Anniversary Meeting:—
- Report of Auditors..... 21
- List of Fellows deceased, &c. 21
- ——— elected since last Anniversary 22
- Address of the President 22
- Presentation of the Medals 31
- Election of Council and Officers 39
- Financial Statements..... 40 & 41
- Changes and present state of the number of Fellows..... 42

On the Spectra of some of the Chemical Elements. By William Huggins, F.R.A.S. 43

On the Acids derivable from the Cyanides of the Oxy-radicals of the Di- and Tri-atomic Alcohols. By Maxwell Simpson, A.B., M.B., F.R.S. 44

42488

Inst. of Science

Page

First Analysis of 177 Magnetic Storms, registered by the Magnetic Instruments in the Royal Observatory, Greenwich, from 1841 to 1857. By George Biddell Airy, F.R.S., Astronomer Royal 48

On the Sudden Squalls of 30th October and 21st November 1863. By Balfour Stewart, M.A., F.R.S., Superintendent of the Kew Observatory. (Plate I.) 51

On the Equations of Rotation of a Solid Body about a Fixed Point. By William Spottiswoode, M.A., F.R.S. 52

Experiments, made at Watford, on the Vibrations occasioned by Railway Trains passing through a Tunnel. By Sir James South, LL.D., F.R.S., one of the Visitors of the Royal Observatory of Greenwich 65

Extract of a Letter to General Sabine from Dr. Otto Torell, dated from Copenhagen, Dec. 12, 1863 83

Results of hourly Observations of the Magnetic Declination made by Sir Francis Leopold M'Clintock, R.N., and the Officers of the Yacht 'Fox,' at Port Kennedy, in the Arctic Sea, in the Winter of 1858–59; and a Comparison of these Results with those obtained by Captain Maguire, R.N., and the Officers of H.M.S. 'Plover,' in 1852, 1853, and 1854, at Point Barrow. By Major-General Sabine, R.A., President 84

Examination of *Rubia munjista*, the East-Indian Madder, or Munjeet of Commerce. By John Stenhouse, LL.D., F.R.S. 86

On the Magnetic Variations observed at Greenwich. By Professor Wolf, of Zurich 87

A Description of the Pneumogastric and Great Sympathetic Nerves in an Acephalous Fœtus. By Robert James Lee, B.A. Cantab. 90

On the Conditions, Extent, and Realization of a Perfect Musical Scale on Instruments with Fixed Tones. By Alexander J. Ellis, B.A., F.C.P.S... 93

On the Osteology of the genus *Glyptodon*. By Thomas Henry Huxley, F.R.S. 108

On the Great Storm of December 3, 1863, as recorded by the Self-registering Instruments at the Liverpool Observatory. By John Hartnup, F.R.A.S., Director of the Observatory 109

On the Criterion of Resolubility in Integral Numbers of the Indeterminate Equation $f=ax^2+a'x'^2+a''x''^2+2bx'x''+2b'xx''+2b''x'x=0$. By H. J. Stephen Smith, M.A., F.R.S., Savilian Professor of Geometry in the University of Oxford 110

Results of a Comparison of certain Traces produced simultaneously by the Self-recording Magnetographs at Kew and at Lisbon; especially of those which record the Magnetic Disturbance of July 15, 1863. By Senhor Capello, of the Lisbon Observatory, and Balfour Stewart, M.A., F.R.S. (Plate II.) 111

Experiments to determine the effects of impact, vibratory action, and a long-continued change of Load on Wrought-iron Girders. By William Fairbairn, LL.D., F.R.S. 121

On the Calculus of Symbols.—Fourth Memoir. With Applications to the Theory of Non-Linear Differential Equations. By W. H. L. Russell, A.B. 126

On Molecular Mechanics. By the Rev. Joseph Bayma, of Stonyhurst College, Lancashire 126

Page

On some further Evidence bearing on the Excavation of the Valley of the Somme by River-action, as exhibited in a Section at Drucat near Abbeville. By Joseph Prestwich, F.R.S. 135

A Contribution to the Minute Anatomy of the Retina of Amphibia and Reptiles. By J. W. Hulke, F.R.C.S., Assistant-Surgeon to the Middlesex and the Royal London Ophthalmic Hospitals 138

Notes of Researches on the Acids of the Lactic Series.—No. I. Action of Zinc upon a mixture of the Iodide and Oxalate of Methyl. By E. Frankland, F.R.S., Professor of Chemistry, Royal Institution, and B. F. Duppa..... 140

On the Joint Systems of Ireland and Cornwall, and their Mechanical Origin. By the Rev. Samuel Haughton, M.D., F.R.S., Fellow of Trinity College, Dublin 142

On the supposed Identity of Biliverdin with Chlorophyll, with remarks on the Constitution of Chlorophyll. By G. G. Stokes, M.A., Sec. R.S. 144

Continuation of an Examination of *Rubia munjista*, the East-Indian Madder, or Munjeet of Commerce. By John Stenhouse, LL.D., F.R.S. 145

On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance. By Dr. Julius Plücker, of Bonn, For. Mem. R.S., and Dr. J. W. Hittorf, of Münster 153

On the Influence of Physical and Chemical Agents upon Blood; with special reference to the mutual action of the Blood and the Respiratory Gases. By George Harley, M.D., Professor of Medical Jurisprudence in University College, London 157

Researches on Radiant Heat.—Fifth Memoir. Contributions to Molecular Physics. By J. Tyndall, F.R.S. 160

Remarks on Sun Spots. By Balfour Stewart, M.A., F.R.S., Superintendent of the Kew Observatory 168

Description of an Improved Mercurial Barometer. By James Hicks..... 169

On Mauve or Aniline-purple. By W. H. Perkin, F.C.S. 170

On the Functions of the Cerebellum. By William Howship Dickinson, M.D. Cantab. 177

An Inquiry into Newton's Rule for the Discovery of Imaginary Roots. By J. J. Sylvester, F.R.S. 179

Description of a Train of Eleven Sulphide-of-Carbon Prisms arranged for Spectrum Analysis. By J. P. Gassiot, F.R.S. 183

The Croonian Lecture.—On the Normal Motions of the Human Eye in relation to Binocular Vision. By Professor Hermann Helmholtz, For. Mem. R.S. 186

On the Orders and Genera of Quadratic Forms containing more than three Indeterminates. By H. T. Stephen Smith, M.A., F.R.S. 199

On some Phenomena exhibited by Gun-cotton and Gunpowder under special conditions of Exposure to Heat. By F. A. Abel, F.R.S..... 204

On Magnesium. By Dr. T. L. Phipson, F.C.S. 217

Page

On the Magnetic Elements and their Secular Variations at Berlin, as observed by A. Erman 218

On the Action of Chlorine upon Methyl. By C. Schorlemmer, Assistant in the Laboratory of Owens College, Manchester 225

On the Calculus of Symbols (Fifth Memoir), with Applications to Linear Partial Differential Equations, and the Calculus of Functions. By W. H. L. Russell, A.B. 227

Second Part of the Supplement to the two Papers on Mortality published in the Philosophical Transactions in 1820 and 1825. By Benjamin Gompertz, F.R.S. 228

Investigations of the Specific Heat of Solid and Liquid Bodies. By Hermann Kopp, Ph.D. 229

On some Foraminifera from the North Atlantic and Arctic Oceans, including Davis Strait and Baffin Bay. By W. Kitchen Parker, F.Z.S., and Professor T. Rupert Jones, F.G.S. 239

Note on the Variations of Density produced by Heat in Mineral Substances. By Dr. T. L. Phipson, F.C.S. 240

On the Spectra of some of the Fixed Stars. By W. Huggins, F.R.A.S., and William A. Miller, M.D., LL.D., Treasurer & V.P.R.S. 242

A Second Memoir on Skew Surfaces, otherwise Scrolls. By A. Cayley, F.R.S. 244

On the Differential Equations which determine the form of the Roots of Algebraic Equations. By George Boole, F.R.S., Professor of Mathematics in Queen's College, Cork 245

A Comparison of the most notable Disturbances of the Magnetic Declination in 1858 and 1859 at Kew and Nertschinsk, preceded by a brief Retrospective View of the Progress of the Investigation into the Laws and Causes of the Magnetic Disturbances. By Major-General Edward Sabine, R.A., President of the Royal Society 247

On the degree of uncertainty which Local Attraction, if not allowed for, occasions in the Map of a Country, and in the Mean Figure of the Earth as determined by Geodesy: a Method of obtaining the Mean Figure free from ambiguity by a comparison of the Anglo-Gallic, Russian, and Indian Arcs: and Speculations on the Constitution of the Earth's Crust. By the Venerable J. H. Pratt, Archdeacon of Calcutta 253

Annual Meeting for the Election of Fellows 276

Description of the Cavern of Bruniquel, and its Organic Contents.—Part I. Human Remains. By Professor Richard Owen, F.R.S. 277

On Complex Binary Quadratic Forms. By H. J. Stephen Smith, M.A., F.R.S. 278

Inquiries into the National Dietary. By Dr. E. Smith, F.R.S. 298

On some Varieties in Human Myology. By John Wood, F.R.C.S. 299

Researches on Isomeric Alkaloids. By C. Greville Williams, F.R.S. 303

On the Synchronous Distribution of Temperature over the Earth's Surface. By Henry G. Hennessy, F.R.S. 312

Page		Page
218	Experimental Researches on Spontaneous Generation. By Gilbert W. Child, M.D. Oxon.	313
225	On a Colloid Acid, a Normal Constituent of Human Urine. By William Marcet, M.D., F.R.S.	314
227	Further observations on the Amyloid Substance met with in the Animal Economy. By Robert M'Donnell, M.D.	317
228	Description of a New Mercurial Gasometer and Air-pump. By T. R. Robinson, D.D., LL.D., F.R.S.	321
229	On the Distal Communication of the Blood-vessels with the Lymphatics; and on a Diaplasmatic System of Vessels. By Thomas Albert Carter, M.D., M.R.C.P.	327
239	Aërial Tides. By Pliny Earle Chase, A.M., S.P.A.S.	329
240	On the Microscopical Structure of Meteorites. By H. C. Sorby, F.R.S.	333
242	On the Functions of the Cerebellum. By W. H. Dickinson, M.D.	334
244	On the Properties of Silicic Acid and other analogous Colloidal Substances. By Thomas Graham, F.R.S.	335
245	Researches on the Colouring-matters derived from Coal-tar.—III. Diphenylamine. By A. W. Hofmann, LL.D., F.R.S.	341
	A Table of the Mean Declination of the Magnet in each Decade from January 1858 to December 1863, derived from the Observations made at the Magnetic Observatory at Lisbon; showing the Annual Variation, or Semiannual Inequality to which that element is subject. Drawn up by the Superintendent of the Lisbon Observatory, Senhor da Silveira	347
247	On Organic Substances artificially formed from Albumen. By Alfred H. Smee, F.C.S.	350
253	On the Reduction and Oxidation of the Colouring-matter of the Blood. By G. G. Stokes, M.A., Sec. R.S.	355
276	Further Inquiries concerning the Laws and Operation of Electrical Force. By Sir W. Snow Harris, F.R.S.	364
277	On a New Class of Compounds in which Nitrogen is substituted for Hydrogen. By Peter Griess	375
278	New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue. By Lionel S. Beale, M.B., F.R.S., F.R.C.P.	384
298	Indications of the Paths taken by the Nerve-currents as they traverse the caudate Nerve-cells of the Spinal Cord and Encephalon. By Lionel S. Beale, M.B., F.R.S., F.R.C.P. (Plate III.)	386
299	On the Physical Constitution and Relations of Musical Chords. By Alexander J. Ellis, F.R.S., F.C.P.S.	392
303	On the Temperament of Musical Instruments with Fixed Tones. By Alexander J. Ellis, F.R.S., F.C.P.S.	404
312	On the Calculus of Symbols.—Fourth Memoir. With Applications to the Theory of Non-Linear Differential Equations. By W. H. L. Russell, A.B.	423
	On the Calculus of Symbols.—Fifth Memoir. With Application to Linear Partial Differential Equations, and the Calculus of Functions. By W. H. L. Russell, A.B.	432

Page

Comparison of Mr. De la Rue's and Padre Secchi's Eclipse Photographs. By Warren De la Rue, F.R.S. 442

On Drops. By Frederick Guthrie, Professor of Chemistry and Physics at the Royal College, Mauritius 444

On Drops.—Part II. By Frederick Guthrie, Professor of Chemistry and Physics at the Royal College, Mauritius. (Plates IV. & V.) 457

On the Chemical Constitution of Reichenbach's Creosote.—Preliminary Notice. By Hugo Müller, Ph.D. 484

Remarks on the Colouring-matters derived from Coal-tar.—No. IV. Phenyltolylamine. By A. W. Hofmann, LL.D., F.R.S. 485

On the Spectra of some of the Nebulæ. By W. Huggins, F.R.A.S.;—a Supplement to the Paper "On the Spectra of some of the Fixed Stars," by W. Huggins and W. A. Miller, M.D., Treas. and V.P.R.S. 492

On the Composition of Sea Water in different Parts of the Ocean. By Dr. George Forchhammer, Professor in the University of Copenhagen. . 493 & 494

Anniversary Meeting:—
- Report of Auditors 494
- List of Fellows deceased, &c. 495
- ———— elected since last Anniversary 495
- First Report of the Scientific Relief Committee 495
- Address of the President 497
- Presentation of the Medals 505
- Election of Council and Officers 517
- Financial Statement ..... 518 & 519
- Changes and present state of the number of Fellows 520

Researches on certain Ethylphosphates. By Arthur Herbert Church, M.A. Oxon., Professor of Chemistry, Royal Agricultural College, Cirencester. . 520

A Dynamical Theory of the Electromagnetic Field. By Professor J. Clerk Maxwell, F.R.S. 531

On the production of Diabetes artificially in Animals by the external use of Cold. By Henry Bence Jones, M.D., F.R.S. 537

On the Action of Chloride of Iodine upon Organic Bodies. By Maxwell Simpson, M.B., F.R.S. 540

On Fermat's Theorem of the Polygonal Numbers, with Supplement. By the Right Hon. Sir Frederick Pollock, F.R.S. 542

On the Structure and Affinities of *Eozoon Canadense*. In a Letter to the President. By W. B. Carpenter, M.D., F.R.S. 545

On the Functions of the Fœtal Liver and Intestines. By Robert James Lee, B.A. Cantab., Fellow of the Cambridge Philosophical Society 549

Completion of the Preliminary Survey of Spitzbergen, undertaken by the Swedish Government with the view of ascertaining the practicability of the Measurement of an Arc of the Meridian. In a Letter addressed to Major-General Sabine by Captain C. Skogman, of the Royal Swedish Navy: dated Stockholm, Nov. 21, 1864. (Plate VI.) 551

On the Sextactic Points of a Plane Curve. By A. Cayley, F.R.S. 553

Page

On a Method of Meteorological Registration of the Chemical Action of Total Daylight. By Henry E. Roscoe, B.A., F.R.S. 555

Obituary Notices of Deceased Fellows:—

Arthur Connell i
Edward Joshua Cooper i
Joshua Field iii
Richard Fowler, M.D. iii
Peter Hardy v
John Taylor v
William Tooke vi
Rear-Admiral John Washington vii
César Mansuète Despretz viii
Eilhardt Mitscherlich ix
Carl Ludwig Christian Rümker xvi

INDEX 561

ERRATA.

Page 153, *for* S. W. Hittorf *read* J. W. Hittorf.

Pages 201 & 202. The words "If a quadratic form . . . . . Phil. Trans. vol. cxliii. p. 481)" should have been printed as a foot-note in explanation of the term "index of inertia."

Page 330, *for* Flangergues *read* Flaugergues.

ERRATA IN OBITUARY (VOL. XII.).

Page xxxvi, line 8 from bottom, *for* Poisson *read* Brisson.

„ xxxviii, line 4 from top, *for* son *read* husband.

NOTICE TO THE BINDER.

In this Volume the following pages are to be cancelled:—Pages 83, 227 & 228, 275 & 276, 457, 491, 519 & 520.

The Plate to Dr. Beale's Paper, p. 386, is Plate III.

BURLINGTON HOUSE,
30th November, 1864.

Sir,

Five years having elapsed since a Committee was appointed to administer the Royal Society Relief Fund, the first Report of the Committee has been prepared, a copy of which I have been requested to hand you, with a list of subscriptions received to *this* date.

You will observe by the Report that the subscriptions to the Fund are invested without any deductions, and that consequently the total sum applicable at any time for relief is limited to the balance in hand arising from the amount of Dividends previously received.

No application for relief is entertained except on the recommendation of the President of one of the following Chartered Scientific Societies:—The Astronomical, Chemical, Geographical, Geological, Linnean, or Royal Society; it being understood that the several Presidents will consult their respective Councils as to the persons whom they intend to recommend for relief.

Subscriptions in aid of the Fund will be received by Messrs. Robarts, Lubbock, & Co., Bankers, Lombard Street, or by

Your obedient Servant,
WALTER WHITE,
Ass.-Sec. Royal Society.

FIRST REPORT OF THE ROYAL SOCIETY RELIEF FUND COMMITTEE.

In presenting this, the first Report of the proceedings of the Royal Society Relief Fund Committee, it may be advisable very briefly to explain the circumstances under which the Fund originated, as well as the distribution of the Income arising therefrom.

On the 26th of May, 1859, a statement was presented to the Council, intimating a desire on the part of several of the Fellows of the Royal Society, to promote the establishment of a permanent fund for the aid of such scientific men, or their families, as may from time to time require and deserve assistance. A scheme having been suggested, it was after some modification approved, and adopted by the Council of the Royal Society, and copies thereof were forwarded to the Fellows of the Chartered Societies.

It formed no part of the scheme to attempt the grant of annuities; it was rather intended to afford *prompt* relief of the immediate wants of those upon whom sudden affliction had fallen; although at

the same time, it in no way debarred a continuation of such relief being given should the funds admit thereof.

The Committee, as originally named by Council Minutes of 22nd December 1859, consisted of Mr. Busk, Dr. W. A. Miller, Mr. Wheatstone, Dr. Tyndall, and Mr. Gassiot. Subsequently Dr. Carpenter, Mr. Sylvester, and Mr. Huxley were appointed to succeed those Members who retired by rotation; and this year Dr. Miller and Mr. Gassiot will also retire; it being a regulation that no member can remain for a longer period than five consecutive years on the Committee.

The first Meeting took place on the 19th March 1860, Mr. Wheatstone in the Chair. At this time £3204 14*s.* had been received, and invested in £3351 7*s.* 6*d.* New 3 per Cents. In 1860 there was only one applicant for relief; a sum was voted, but in consequence of the applicant's sudden decease, the cheque was subsequently returned to the Treasurer and cancelled.

In 1861 there were six applicants, to five of whom relief was afforded.

In 1862 there were four applicants, of whom three were relieved.

In 1863 there were five applicants, of whom four were relieved.

In 1864 there were two applicants—both were relieved.

The total amount thus expended has been £460; and there still remains £233 1*s.* 11*d.* applicable for relief, exclusive of £17 2*s.*, subscriptions to be invested.

The amount of the Fund, as invested, consists of £5300 New 3 per cent. Consols, every shilling (with the above exception of £17 2*s.*) which has been received having been invested, without any deduction for expenses; the economical principle on which this Fund was originally founded has been thus strictly carried out. The Council of the Royal Society grants the use of a room, as well as the little stationery that is from time to time required, and permits the Assistant-Secretary to summon a meeting of the Members as soon as any application for relief is received. One of the members of the Committee undertakes the duties of a Secretary, and in this simple manner *all* expense is avoided, while the relief, if granted, is immediate. The amount at disposal for distribution is necessarily limited to the accumulation of interest, the gross amount of subscriptions being invested.

The peculiar and distinctive character of the Royal Society Relief Fund is, that although relief has invariably been granted with the greatest promptitude, the claims have nevertheless been most carefully examined by those whose pursuits in life enable them to form the most correct conclusion as to the character of the claimant. The Committee can most conscientiouly state that, in no instance, has any applicant having the slightest claim on the Fund been refused assistance. Sometimes the payments to recipients have been the first information they have received of such a fund being in existence; the Committee may have on some occasions regretted that it had not a larger sum at command, but the relief, whatever its amount, has always been received with gratitude for the substantial aid it

afforded, and with pleasure as a recognition on the part of the Council of the Royal Society of good service rendered to science.

The Committee is naturally desirous to retain sufficient funds in hand to meet any sudden emergency that may arise; at this time there appears a balance somewhat larger than usual, but this arises from there having been only two applicants this year instead of five, as in 1863. It is very probable that if the *existence* of such a fund were more generally known, especially among the members of the Chartered Scientific Societies, further subscriptions would be tendered.

(Signed) W. A. Miller.
W. B. Carpenter.
J. J. Sylvester.
T. H. Huxley.
J. P. Gassiot.

Burlington House,
24th November, 1864.

THE ROYAL SOCIETY SCIENTIFIC RELIEF FUND.

Subscriptions received to November 30th, 1864.

	£	s.	d.
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R. E. Arden, Esq.	5	5	0
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Earl of Ellesmere	50	0	0
F.	10	10	0
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Joshua Field, Esq.	50	0	0
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Sir J. W. Lubbock, Bart.	100	0	0
John Lubbock, Esq. ..	30	0	0
Robert M^c^Andrew, Esq.	25	0	0
Dr. W. A. Miller.....	21	0	0
Professor W. H. Miller	10	0	0
Major J. A. Moore	10	10	0
Sir R. Murchison	100	0	0
W. C. Mylne, Esq.....	10	10	0
Dr. Odling	5	0	0
A. F. Osler, Esq.	25	0	0
Bishop of Oxford	10	0	0
Sir C. Pasley	50	0	0
J. Penn, Esq.	100	0	0
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Brought forward ..	3439	19	0
Joseph Prestwich, Esq.	10	10	0
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John Rogers, Esq.	10	0	0
Dr. Roget	10	0	0
Francis Ronalds	10	0	0
Lord Rosse	100	0	0
E. J. Rudge, Esq.	5	5	0
J. W. Russell, Esq. ..	20	0	0
General Sabine	25	0	0
Bishop of St. David's..	10	10	0
W. W. Saunders, Esq..	50	0	0
Dr. Sibson	10	0	0
John Simon, Esq.	10	0	0
Sir A. Smith	15	0	0
H. Smith, Esq.	21	0	0
W. H. Smithe, Esq. ..	25	0	0
Admiral Smyth	10	10	0
W. W. Smyth, Esq. ..	10	10	0
S. R. Solly, Esq.....	20	0	0
William Spence, Esq...	100	0	0
Per Spooner & Co.....	10	0	0
H. T. Stainton, Esq. ..	50	0	0
Benjamin Standring ..	5	5	0
Robert Stephenson, Esq., per Executors	100	0	0
G. J. Stoney, Esq.	5	5	0
Professor J. J. Sylvester	5	0	0
Lord Talbot de Malahide	10	0	0
William Tite, Esq.....	100	0	0
Sir W. Trevelyan	10	0	0
F. Tuckett, Esq.....	10	0	0
James Tulloch, Esq. ..	50	0	0
T. S. Turnbull, Esq. ..	10	0	0
Dr. Tyndall	5	0	0
N. B. Ward, Esq.	21	0	0
R. Warminston, Esq...	10	10	0
C. Wheatstone, Esq. ..	20	0	0
S. C. Whitbread, Esq. .	100	0	0
J. Whitmore, Esq.	5	0	0
J. Whitworth, Esq. ..	100	0	0
Erasmus Wilson, Esq...	100	0	0
George Wilson, Esq. ..	10	0	0
William Winsor, Esq. .	20	0	0
Sir W. P. Wood	100	0	0
Lord Wrottesley.....	50	0	0
J. Yates, Esq.	21	0	0
James Young, Esq.....	100	0	0
	£5001	14	0

PROCEEDINGS

OF

THE ROYAL SOCIETY.

November 19, 1863.

Major-General SABINE, President, in the Chair.

In accordance with the Statutes, notice of the ensuing Anniversary Meeting for the election of Council and Officers was given from the Chair.

The President stated that Captain L. L. B. Ibbetson, who by reason of non-payment of his annual contribution ceased to be a Fellow of the Society at the last Anniversary, had applied for readmission; and an extract from his letter to the Council was read, explaining the circumstances under which, during his absence on the Continent, the omission of payment had taken place. Notice was accordingly given that the question of Captain Ibbetson's readmission would be put to the ballot at the next Meeting.

William Crookes, Esq.; Frederick Field, Esq.; and John Russell Hind, Esq., were admitted into the Society.

Dr. Alderson, Mr. Busk, Dr. Gladstone, Professor Pole, and Mr. Archibald Smith, having been nominated by the President, were elected by ballot Auditors of the Treasurer's accounts on the part of the Society.

The following communications were read:—

I. "On Mauve or Aniline-Purple." By W. H. PERKIN, Esq. (See vol. xii. p. 713.)

II. "Notes of Researches on the Intimate Structure of the Brain." —Third Series. By J. LOCKHART CLARKE, Esq., F.R.S. (See vol. xii. p. 716.)

III. "A General Catalogue of Nebulæ and Clusters of Stars for the Year 1860·0, with Precessions for 1880·0." By Sir J. F. W. Herschel, Bart., F.R.S. Received Oct. 16, 1863.

(Abstract.)

This Catalogue contains all the nebulæ and clusters of stars which its compiler has been able to find anywhere described, and identified in position sufficiently to warrant their inclusion, with exception of a few which, having been observed by Lacaille or others with telescopes of very small power, have been obviously nothing but insignificant groups of small stars indistinctly seen. The number of objects comprised in it is 5078, including —1st. 2508 nebulæ and clusters described by the late Sir Wm. Herschel in his Catalogues of Nebulæ communicated to the Royal Society. 2ndly. Those comprised in the lists published by Messier, discovered by himself, Mairan, Oriani, and others, to the number of 102. 3rdly. Those contained in M. Auwers's list of "New Nebulæ" (Verzeichniss neuer Nebelflecke) at the end of his Catalogue of Sir Wm. Herschel's nebulæ (about 50 in number), and those few of Lacaille's nebulæ which seem entitled to be regarded as such from the description given of them. 4thly. A great many nebulæ pointed out by Lord Rosse in his paper in Phil. Trans. 1861, their places being indicated with sufficient probable precision to allow of their being re-observed and identified. 5thly. 125 new nebulæ obligingly communicated by M. D'Arrest, of his own discovery, for inclusion in this Catalogue; and some few others (some very remarkable ones) collected from various sources, as announced from time to time by their respective discoverers. And 6thly. 15 nebulæ not before described, communcated by Professor Bond, which are included in a small supplementary list. The remainder will be found described and their places (reduced to 1830) given in the Catalogue of Nebulæ and Clusters communicated to the Royal Society by the compiler in 1833, and in his 'Results of Astronomical Observations at the Cape of Good Hope,' published in 1847.

The places of the objects contained in the present Catalogue were in the first instance brought up by its compiler to the common epoch (1830), availing himself, so far as respects the nebulæ of Sir Wm. Herschel's catalogues, of a reduction to 1800 of all the individual observations of each nebula, by his sister the late Miss Caroline Herschel, which reduction, arranged in the form of a catalogue in zones, together with the originals of all the "sweeps" in which the observations are contained, and a synoptic register of those of each nebula in separate sheets for reference, with other original papers elucidatory of the above-mentioned documents, as well as the whole series of Sir Wm. Herschel's observations of Messier's nebulæ, accompany this communication for future reference.

In order, however, to render the catalogue so compiled available for future observation, it was considered desirable to bring the whole up to a later epoch. The computations necessary for this purpose being very

extensive and of a nature to be safely entrusted to other hands, the Royal Society, on the application of the compiler, readily and most liberally consented to supply the funds for defraying the necessary expense of this operation. On consultation with the Astronomer Royal, it was resolved that the places having been first roughly brought up to 1860, the places so obtained should be used to compute the precessions for 1880, *by the application of which* to the original places the final and exact places for 1860 should be obtained and entered up. This will secure the availability for the use of observations, of the present Catalogue, without fear of material error up to the year 1930 at least. The actual computation was executed by Mr. Kerschner, one of the computists employed at the Royal Observatory, the Astronomer Royal kindly undertaking the arrangement and supervision of the work. The computations were made on printed forms, and are preserved for reference.

The Catalogue is arranged in general order of right ascension—in columns, containing a current general number, four columns of synonyms and references to the original authorities; the right ascension, precession in R.A., and the number of observations on which this element relies; a similar set of columns for the North Polar distance, and a brief description, in abbreviated language, of the object, deduced from a careful comparison *inter se* of all the descriptions given in the original observations. Lastly, are appended two columns,—the one containing the total number of times the object has been *seen* by Sir Wm. Herschel and by the author of the present paper; the other, references to a series of notes annexed at the end of the Catalogue, and to a general list of places where engraved figures of the objects will be found.

The notes so appended contain remarks on every particular brought under discussion as affecting the evidence on which the adopted places rest, and whatever else may be considered requiring explanation in reference to each object. In particular they give the results of a very careful comparison of the present Catalogue with the elaborate catalogue (for 1830) of M. Auwers, already mentioned, of the existence of which the compiler was not aware till the whole of the computations had been completed and the present Catalogue arranged and copied out. This comparison has led to the detection (as might very reasonably be expected) of several instances of mistaken identification of stars of comparison, and some few of numerical error, and has so far resulted in the expurgation and improvement of both catalogues.

A general list of figured nebulæ, with references to the works in which the figures are to be found, and lists of errata and corrections discovered in the various works consulted, concludes the work.

IV. "Note on Kinone." By A. W. Hofmann, LL.D., F.R.S.

Received June 23, 1863.

The easy and perfect transformation of beta-phenylene-diamine into kinone, which I have pointed out in a former communication, has induced me to examine the action of oxidizing agents upon other derivatives of the phenyl-series.

Aniline, when submitted to the action of a mixture of peroxide of manganese and sulphuric acid, furnishes very appreciable quantities of kinone, which sublimes, the residue containing the sulphates of ammonium and manganese.

$$\underbrace{C_6H_7N}_{\text{Aniline.}}+O_2=\underbrace{C_6H_4O_2}_{\text{Kinone.}}+H_3N.$$

This equation represents, however, only one phase of the reaction. The result, in a measure, depends upon the mode of experimenting: one part of aniline, four parts of peroxide of manganese, and four parts of sulphuric acid diluted with its own bulk of water were found to be appropriate proportions. But the amount of kinone is always limited, the greater portion of the aniline undergoing further alterations.

The experiment succeeds much better with *benzidine*. On heating the mixture of this base with the oxidizing agents, torrents of kinone are instantaneously evolved, which condense in the receiver into magnificent yellow needles. The quantity of kinone thus obtained corresponds to the amount of benzidine employed.

$$\underbrace{C_{12}H_{12}N_2}_{\text{Benzidine.}}+H_2O+O_3=2\,\underbrace{C_6H_4O_2}_{\text{Kinone.}}+2\,H_3N.$$

The transformation of aniline into kinone, very naturally suggested the idea of examining the behaviour of these two bodies with one another.

The reddish-brown liquid obtained by dissolving kinone in aniline very rapidly solidifies into a crystalline mass. The crystalline product of the reaction proved to be insoluble in water, alcohol, and ether, and several solvents which I tried, so that purification by crystallization became impossible; it was therefore found to be convenient to accomplish the reaction in the presence of a large quantity of boiling alcohol. The brown liquid deposits on cooling reddish brown almost metal-lustrous scales, which by washing with cold alcohol become perfectly pure.

The analysis of this substance shows that it has the following composition:—

$$C_{18}H_{14}N_2O_2=\left.\begin{matrix}(C_6H_5)_2\\(C_6H_2O_2)''\\H_2\end{matrix}\right\}N_2.$$

The complementary product of the reaction was discovered without difficulty in the mother-liquor of the reddish-brown crystals. The saline

residue which is left on evaporating this liquid with hydrochloric acid, is a mixture of hydrochlorate of aniline and *hydrokinone*. They are easily separated by treatment with ether, which dissolves the hydrokinone, leaving the aniline-salt as an insoluble residue. The ethereal solution, when evaporated, yields colourless needles of hydrokinone possessing all the characteristic properties which distinguish this remarkable body. Addition of ferric chloride to their aqueous solution produces at once the green prisms, with golden lustre of the intermediate hydrokinone.

The action of kinone upon aniline is therefore represented by the following equation:—

$$\underbrace{2C_6H_7N}_{\text{Aniline.}} + \underbrace{3\,C_6H_4O_2}_{\text{Kinone.}} = \underbrace{C_{18}H_{14}N_2O_2}_{\text{Brown crystals.}} + \underbrace{2C_6H_6O_2}_{\text{Hydrokinone.}}.$$

The study of this reaction has induced me to repeat an experiment mentioned by M. Hesse in his beautiful researches on the kinone group*.

By submitting aniline to the action of *chloranile* (tetrachlorkinone), M. Hesse has obtained a compound crystallizing in reddish-brown scales, the general properties of which resemble those of the kinone derivative above described. The composition of the compound formed with chloranile M. Hesse represents by the formula

$$C_{42}H_{31}Cl_4O_4N_5 = \left.\begin{matrix}(C_6H_5)_5\\(C_6Cl_2O_2)''_2\\H_6\end{matrix}\right\}N_5.$$

I cannot confirm this somewhat complicated expression. In studying the action of chloranile upon aniline, I have observed all the phenomena described by M. Hesse: the compound formed had all the properties which he assigns to it, but was found on analysis to contain about 2 per cent. of carbon less than he had observed. The substance examined by me contained

$$C_{18}H_{12}Cl_2N_2O_2 = \left.\begin{matrix}(C_6H_5)_2\\(C_6Cl_2O_2)''\\H_2\end{matrix}\right\}N_2.$$

This is the formula of the kinone derivative with two atoms of hydrogen replaced by chlorine. The action of chloranile on aniline is therefore, in a measure, analogous to that of kinone.

$$\underbrace{4\,C_6H_7N}_{\text{Aniline.}} + \underbrace{C_6Cl_4O_2}_{\text{Chloranile.}} = C_{18}H_{12}Cl_2N_2O_2 + 2\underbrace{C_6H_7N,HCl}_{\text{Hydrochlorate of aniline.}}$$

The formula which I propose to substitute for that of M. Hesse is moreover supported by the result obtained in studying the deportment of chloranile under the influence of ammonia. This gives rise to the formation of *chloranilamide* discovered by Laurent†, and represented by the formula

$$C_6H_4Cl_2N_2O_2 = \left.\begin{matrix}(C_6Cl_2O_2)''\\H_2\\H_2\end{matrix}\right\}N_2.$$

* Ann. Chem. Pharm. cxiv. p. 307.

† Laurent, xix. 323.

I have ascertained that toluidine furnishes, both with kinone and chloranile, analogous compounds. The higher percentage of carbon observed by M. Hesse may possibly find a satisfactory explanation in the contamination with toluidine of the aniline which has served for his experiments. Commercial aniline invariably contains more or less toluidine.

V. "Researches on Colouring Matters derived from Coal-tar.—I. On Aniline-yellow." By A. W. Hofmann, LL.D., F.R.S. Received June 29, 1863.

In a short paper submitted to the Royal Society in the commencement of last year, I have described a few experiments on the remarkable new colouring matters derived from aniline, which of late have attracted such general attention. This paper had more particularly reference to aniline-crimson, the industrial production of which, in the hands of Mr. E. Nicholson, has reached so high a degree of perfection that the analysis of this compound and of its numerous salts presented no serious difficulty. But the problem was not solved by establishing the formula of rosaniline and its salts: by far the more important obstacles remained to be conquered; the molecular constitution of rosaniline, on which at that time I had not even been able to offer an hypothesis, and the genesis of this well-defined triamine from aniline, had still to be traced. Since that time considerable progress has been made towards the solution of this problem. Some of the latest observations which I have had the honour of submitting to the Royal Society will doubtless help to untie this knot. Nevertheless many doubtful points still remain to be cleared up, and I found it desirable for the better elucidation of the subject to investigate simultaneously several of the other artificial organic colouring matters, in order to trace if possible analogies of composition and constitution in these substances, which, it was reasonable to hope, would throw some light upon the principal subject of the inquiry.

The present moment appeared to be particularly appropriate for an investigation of this kind. The International Exhibition has brought together a collection of these new bodies, such as no other occasion could possibly have assembled in one place and at one time, displaying in a remarkable manner the rapidity with which the industry of our time assimilates and, in many cases, anticipates the results of pure science.

I have commenced the study of a few of the new colouring matters which several of the distinguished exhibitors of these compounds have placed at my disposal—a study which has been greatly facilitated by the zeal and experimental skill of a young chemist, Dr. A. Geyger, who has assisted me in these experiments. Owing to the number of these substances, and in some cases the difficulties of the reactions to be disentangled, some time must elapse before their investigation can be finished,

and I therefore beg leave to submit to the Royal Society the results of these researches as they present themselves. These Notes must necessarily be of a somewhat fragmentary character; but I hope to collect the results thus gradually accumulating, and to lay them before the Royal Society in more logical order and a more elaborated form. I begin the account of this series of experiments with the description of a yellow colouring matter which is obtained as a secondary product in the manufacture of rosaniline.

Chrysaniline.—It is well known that even in the most successful operation, and whatever the process of preparation may be, the rosaniline produced is only a small percentage of the aniline employed. Together with the crimson-colour a large proportion of a resinous substance of feebly basic properties is formed, the generally ill-defined characters of which have hitherto baffled all attempts at a thorough investigation. This mixture contains nevertheless several individual compounds, which may be extracted with boiling water, and subsequently separated by treatment with reagents. Mr. E. C. Nicholson has thus isolated a magnificent yellow colouring matter. Considerable quantities of this interesting body, Mr. Nicholson with his usual liberality has placed at my disposal, for which my best thanks are due to him.

The yellow colouring matter, for which, on account of the splendid golden-yellow tint it imparts to wool and silk, and in order to record its origin, I propose the name of *chrysaniline*, presents itself in the form of a finely divided yellow powder, closely resembling freshly precipitated chromate of lead, perfectly uncrystalline, scarcely soluble in water, which it just colours, easily soluble in alcohol and in ether. This compound is a well-defined organic base, which forms with the acids two series of crystallized saline compounds. The most characteristic salts of chrysaniline are the nitrates, more especially the mononitrate, which is difficultly soluble in water, and crystallizes with facility. It was from this compound, purified by half a dozen crystallizations, that I prepared the chrysaniline for analysis. An aqueous solution of the pure nitrate decomposed with ammonia yields the chrysaniline in a state of perfect purity. The analysis of this substance, dried at 100°, has furnished results which may be translated into the formula

$$C_{20}H_{17}N_3.$$

This expression is corroborated by the examination of several salts, more especially the beautiful compound which this base produces with hydrochloric acid.

Hydrochlorate of Chrysaniline.—On adding concentrated hydrochloric acid to a solution of chrysaniline in the dilute acid, a scarlet crystalline precipitate is produced, consisting of minute scales very soluble in water, less soluble in alcohol, almost insoluble in ether. These crystals constitute the diacid chloride of chrysaniline,

$$C_{20}H_{17}N_3, 2HCl.$$

Under conditions not yet sufficiently defined, this substance is precipitated with water of crystallization as

$$C_{20}H_{17}N_3, 2HCl + H_2O.$$

For analysis these salts were dried at 100° or 120°, at which temperature they remain quite unchanged. When heated more strongly they lose hydrochloric acid. When the diacid chloride is maintained for a fortnight between 160° and 180°, the weight of the salt again becomes constant. The residuary yellow crystalline powder, differing from the original hydrochlorate only by its somewhat diminished solubility in water, was by analysis found to be the pure monacid hydrochlorate of chrysaniline,

$$C_{20}H_{17}N_3, HCl.$$

The crystalline compounds which chrysaniline forms with hydrobromic acid and hydriodic acid are perfectly analogous to the salts produced by hydrochloric acid. I have not analyzed them.

The *nitrates of chrysaniline* are the finest salts of this base; these compounds crystallize with the utmost facility in ruby-red needles, which are remarkably insoluble in water. A dilute solution of nitric acid (1 grm. of HNO_3 in a litre of water), when mixed with moderately dilute solutions of the chloride, yields immediately a crystalline precipitate, so that soluble chrysaniline salts might be used as a test for nitric acid. For the same reason nitric acid is conveniently employed in separating chrysaniline from the crude liquid obtained by boiling out the secondary products of the manufacture of rosaniline. Nevertheless the preparation of the nitrates presents unusual difficulties, and I have lost much time in endeavouring to fix the conditions under which the monacid and the diacid salts may be separately produced. On boiling an excess of free chrysaniline with dilute nitric acid, a solution is obtained depositing, on cooling, needles which are the mononitrate,

$$C_{20}H_{17}N_3, HNO_3,$$

in a state approaching purity. On pouring the solution of this salt into cold concentrated nitric acid, a salt is at once precipitated which crystallizes in ruby-red prisms very similar to ferricyanide of potassium, and constitutes the nearly pure dinitrate,

$$C_{20}H_{17}N_3 (HNO_3)_2.$$

But here also analysis exhibits slight discrepancies, indicating the presence of traces of the former compound. By treatment with water the dinitrate gradually loses its nitric acid, and after two or three crystallizations it is converted into the mononitrate.

The *sulphate* is very soluble, scarcely crystalline.

The platinum-salt is a splendid scarlet crystalline precipitate, which, from hot and rather dilute solutions containing much free hydrochloric acid, is often deposited in very fine and large plates. All my attempts to obtain this substance in a state of purity have failed. The platinum percentages vary with every new preparation, indicating the formation of a

monochloroplatinate and a dichloroplatinate, combining with more or less water of crystallization.

The composition of chrysaniline places this substance in immediate juxtaposition with rosaniline and leucaniline. These three triamines simply differ by the amount of hydrogen which they contain.

Chrysaniline.....	$C_{20} H_{17} N_3$
Rosaniline	$C_{20} H_{19} N_3$
Leucaniline	$C_{20} H_{21} N_3$.

Chrysaniline is monacid or diacid; rosaniline monacid or triacid, but with essentially monacid predilections; leucaniline forms exclusively triatomic compounds.

The formula of chrysaniline suggests the possibility of transforming this substance into rosaniline and leucaniline, or of producing chrysaniline from rosaniline or leucaniline. Up to the present moment this transformation has not been experimentally accomplished. The constitution and genesis of chrysaniline remain to be made out.

VI. "Researches on the Colouring Matters derived from Coal-tar.—II. On Aniline-blue." By A. W. Hofmann, LL.D., F.R.S. Received June 30, 1863.

Among the several stages which mark the development of the industry of coal-tar colours, the discovery of the transformation of aniline-red into aniline-blue will always hold a prominent position. This transition, for the first time observed by MM. Girard and De Laire*, two young French chemists of M. Pelouze's Laboratory, and subsequently matured by M. Persoz, De Laynes, and Salvétat†, has become the foundation of an enormous industrial production, which, having received a powerful impulse by MM. Renard Brothers and Franc in France, and more recently by Messrs. Simpson, Maule, and Nicholson in this country, has rapidly attained to proportions of colossal magnitude.

The transformation of aniline-red into aniline-blue is accomplished by a process of great simplicity, and consists, briefly expressed, in the treatment at a high temperature of rosaniline with an excess of aniline. The mode of this treatment is by no means indifferent. Rosaniline itself cannot in this manner conveniently be converted into the blue colouring matter; the transformation is, however, easily accomplished by heating rosaniline salts with aniline, or, *vice versâ*, rosaniline with salts of aniline. Again, the nature of the acids with which the bases are combined is by no means without influence upon the result of the operation; manufacturers give a decided preference to organic acids, such as acetic or benzoic acids.

The production of the new colouring matter on a very large scale has already elicited a good deal of most valuable information regarding the

* French patent, January 1861.

† Comptes Rendus, March and April 8th, 1861.

phenomena which characterize the transition of rosaniline into its blue derivative; again, the several processes of purification to which the crude product is submitted have thrown much light upon the chemical character of the compound.

MM. Girard and De Laire, whose names are so intimately associated with the development of the new colour-industry, have pointed out that the passage from red to blue is attended by an evolution of torrents of ammonia; and Mr. Nicholson, who combines the genius of the manufacturer with the habits of the scientific inquirer, has ascertained that the blue colouring matter is invariably a *saline* compound of a base itself colourless, like rosaniline. But the relations between the two colourless bases, and consequently the nature of the reaction by which rosaniline is converted into the blue colouring matter, had hitherto remained unknown.

It was therefore with great pleasure that I accepted the kind offer of my friend Mr. Nicholson to supply me with the necessary materials for the elucidation of this question.

The salt transmitted to me, and which Mr. Nicholson had prepared himself, was the chloride.

Hydrochlorate.—This compound is an indistinctly crystalline powder of a bluish-brown colour, which at 100° becomes pure brown. It is perfectly insoluble in water, cold or boiling—so much so, indeed, that it imparts no colour to the water with which it is washed. It is likewise insoluble in ether, but dissolves, although with difficulty, in alcohol, which assumes the magnificent deep-blue tint characteristic of this colouring matter. The boiling saturated alcoholic solution deposits the chloride on cooling in the form of imperfect crystalline granules. The alcoholic solution, when evaporated, leaves the compound as a thin film, which reflects the light with a peculiar metallic, half-golden, half-coppery lustre.

The hydrochlorate has the same composition whether dried *in vacuo* or at 100°. Several analyses made with specimens of different preparations lead unmistakably to the expression

$$C_{38}\,H_{32}\,N_3\,Cl.$$

This formula contains the history of aniline-blue, pointing out as it does not only its chemical character and the relation in which it stands to rosaniline, but explaining also, in the most satisfactory manner, the reaction by which the passage from red to blue is accomplished.

The simple and natural interpretation of the formula which I have given, exhibits the new compound in the light of the hydrochlorate of triphenylic rosaniline*,

$$C_{38}\,H_{32}\,N_3\,Cl = C_{20}\,H_{16}\,(C_6\,H_5)_3\,N_3,\,HCl,$$

when the process of transformation becomes represented by the equation

$$\underbrace{C_{20}\,H_{19}\,N_3,\,HCl}_{\text{Rosaniline-salt.}} + 3\,\underbrace{C_6\,H_7\,N}_{\text{Aniline.}} = \underbrace{C_{20}\,H_{16}\,(C_6\,H_5)_3\,N_3,\,HCl}_{\text{Salt of triphenylic rosaniline.}} + \underbrace{3H_3\,N}_{\text{Ammonia.}}.$$

* The relation between aniline-red and aniline-blue is already pointed out in a short note submitted to the Royal Society a few weeks ago.—A. W. H.

Free Base.—The separation of the base from the hydrochlorate presents no difficulty. This salt dissolves in alcoholic ammonia, giving rise to a fine-yellow solution. This liquid contains the base in the free state, together with chloride of ammonium. On ebullition the blue colour reappears, the salt being reproduced with evolution of ammonia. Addition of water, on the other hand, produces a white or greyish precipitate, consisting of *triphenylic rosaniline*. The best mode of procuring this compound in a state fit for analysis is to pour the concentrated solution of the hydrochlorate in ammoniacal alcohol into water, when the base separates as a curdy mass which soon collects upon the surface of the liquid. During the process of washing, and especially of drying, even *in vacuo*, the greyish powder gradually assumes a blue tint. The vacuum-dry substance, when exposed to 100°, assumes a deep brown colour, which it retains on cooling; at 100° it slightly fuses, but does not change weight.

Triphenylic rosaniline shows a tendency to crystallize, but hitherto I have not been able to obtain it in distinct crystals. The solution in alcohol and also in ether (which dissolves the base with the greatest facility) have, even on spontaneous evaporation, deposited the base in the form of an almost amorphous residue.

Analysis assigns to this base the composition which corresponds to that of the hydrochlorate previously examined, namely

$$C_{38}H_{33}N_3O = C_{20}H_{16}(C_6H_5)_3N_3, H_2O.$$

Triphenylic rosaniline is thus seen to separate from its saline combinations in the state of hydrate, exactly like rosaniline itself.

I have endeavoured to obtain further confirmation of these results by the analysis of several salts of triphenylic rosaniline. These salts were invariably prepared by treatment of the free base with the free acids. They resemble in their properties the hydrochlorate—so much so, indeed, that they could not possibly be distinguished without analysis. The nitrate is perhaps a little more, the sulphate a little less soluble in alcohol than the hydrochlorate. The following salts were submitted to analysis:—

Hydrobromate.

$$C_{38}H_{32}N_3Br = C_{20}H_{16}(C_6H_5)_3N_3, HBr.$$

Hydriodate.

$$C_{38}H_{32}N_3I = C_{20}H_{16}(C_6H_5)_3N_3, HI.$$

Nitrate.

$$C_{38}H_{32}N_4O_3 = C_{20}H_{16}(C_6H_5)_3N_3, HNO_3.$$

Sulphate.

$$C_{76}H_{64}N_6SO_4 = \left.\begin{matrix} C_{20}H_{16}(C_6H_5)_3N_3 \\ C_{20}H_{16}(C_6H_5)_3N_3 \end{matrix}\right\} \left.\begin{matrix} H \\ H \end{matrix}\right\} SO_4.$$

Rosaniline, it will be remembered, forms, in addition to its ordinary monatomic compounds, a series of triatomic salts, which are more soluble and comparatively colourless. I have vainly endeavoured to prepare similar compounds with the triphenylic derivative of rosaniline.

Action of reducing agents upon Triphenylic Rosaniline.—Remembering the facility with which rosaniline is attacked by reducing agents, and the valuable help which the examination of the leucaniline thus produced afforded in establishing the formula of rosaniline, I was led to study the deportment of the triphenylic derivative under similar circumstances. This substance indeed is readily reduced both by nascent hydrogen and by sulphide of ammonium.

The alcoholic solution of the chloride, when left in contact with zinc and hydrochloric acid, is rapidly decolorized. The clear liquid when mixed with water yields a white, scarcely crystalline precipitate, which may be freed from chloride of zinc by washing, and separated from accidental impurities by solution in ether, in which it is easily soluble.

If the reduction be effected by sulphide of ammonium, the product is apt to be contaminated with sulphur and secondary products. In this case the separation has to be accomplished by treating the crude mass obtained in the reaction with bisulphide of carbon, which dissolves both the sulphur and the product of the reduction, leaving behind a brown resinous substance, the nature of which is not yet investigated. The mixture remaining after the evaporation of the bisulphide of carbon is repeatedly boiled with soda, which dissolves the sulphur; the residuary compound is then finally purified by solution in ether, from which it is deposited on spontaneous evaporation in the form of a friable resin.

Unfortunately this compound is no longer basic, so that it was impossible to combine it with acids; but its combustion has furnished numbers agreeing exactly with the composition assigned to it by theory, namely

$$C_{38}H_{33}N_3 = C_{20}H_{18}(C_6H_5)_3N_3.$$

The compound accordingly is *triphenylic leucaniline*. It will be observed that the triphenylic derivative, like leucaniline itself, is anhydrous—a constancy of behaviour in the normal and derived compounds which has already been pointed out in the case of rosaniline and its phenylic derivative. Under the influence of oxidizing agents, the hydrogenetted body is rapidly reconverted into the compound from which it has been obtained. The experiment succeeds best with platinum-chloride. The colourless solution of triphenylic leucaniline, when boiled with a few drops of dichloride of platinum, immediately assumes the splendid blue colour which distinguishes the salts of the nor-hydrogenetted base.

The transformation of aniline-red into aniline-blue possesses a variety of interests. A lively imagination might feel tempted to speculate on the relation between colour and composition; but there are other questions claiming more immediately the attention of the experimentalist.

Up to the present moment chemists were unacquainted with a method of *phenylation*. The chloride, bromide, and iodide of the phenyl-series have been but imperfectly studied; but we are sufficiently acquainted with them to know that they are far from possessing the plastic character of the corresponding compounds of the methyl- and ethyl-series, which

confers such value upon these substances as agents of research. We are unable to substitute phenyl for hydrogen by processes borrowed from the experience gathered in experimenting with the ordinary alcohols. Diphenylamine and triphenylamine are substances existing at present only in the conception of the chemist. It was reserved for the peculiar, I might almost say instinctive mode of experimenting belonging to industry to fill up this blank.

The transformation of rosaniline into aniline-blue suggests some other questions which must not altogether remain unnoticed here, although I hope to enter more fully into this subject elsewhere. Does this transformation simply involve an interchange between the hydrogen and phenyl atoms, or does the rosaniline molecule lose ammonia, which is replaced by aniline?

I do not pretend to answer this question; but I beg leave to record some experiments as materials towards the solution of the problem.

Methylic, Ethylic, and Amylic Derivatives of Rosaniline.

The interpretation of the results delineated in the previous pages legitimately suggested the study of the behaviour of rosaniline under ordinary processes of substitution—in other words, the treatment of this base with the iodides of methyl, ethyl, and amyl. I will not describe the pleasure with which I observed the intense blue colour of the mixture of rosaniline with these iodides when, after a day's digestion, I took the sealed glass tubes from the boiler. The action of iodide of methyl and ethyl is readily accomplished at 100° C.; iodide of amyl requires a temperature of from 150° to 160°. The presence of alcohol facilitates the reaction.

Up to the present moment I have only examined in detail the action of iodide of ethyl. The product of this action is an iodide which dissolves with a magnificent blue colour in alcohol. The tinctorial powers of this solution are scarcely inferior to that of rosaniline itself; and industry will probably not disdain to utilize this latest indication of science.

The blue ethylated derivative of rosaniline, as might have been expected, presents in its properties greater analogies with rosaniline itself than the triphenylic compound. This analogy suggested difficulties in the separation of the two substances which it appeared better to avoid. The iodide produced by the reaction was therefore at once decomposed by soda, and the ethylic derivative, together with the unaltered rosaniline, again submitted to the action of iodide of ethyl. After this process had been once more repeated, the alcoholic solution of the final product was precipitated by water, which separated a soft resin-like substance, solidifying on cooling with crystalline structure, and exhibiting a very peculiar metallic lustre intermediate between those presented by the salts of rosaniline and of its phenylic derivative. Crystallization from dilute spirit furnished the iodide in the pure state. The results obtained in the combustion and iodine determination of this substance agree with the formula

$$C_{28}H_{36}N_3I = C_{20}H_{16}(C_2H_5)_3N_3C_2H_5I,$$

showing that the frequent repetition of the process of ethylation had produced, not the *hydriodate* of *triethylic rosaniline*, but the *ethyliodate* of this substance,—a result which appeared particularly welcome, inasmuch as it threw at the same time considerable light upon the degree of substitution which belongs to rosaniline itself.

The facts elicited by the study of the action of iodide of ethyl upon rosaniline open a new field of research, which promises a harvest of results. The question very naturally suggests itself, Whether the substitution for hydrogen in rosaniline of radicals other than methyl, ethyl, and amyl, may not possibly give rise to colours differing from blue; and whether chemistry may not ultimately teach us systematically to build up colouring molecules, the particular tint of which we may predict with the same certainty with which we at present anticipate the boiling-point and other physical properties of the compounds of our theoretical conceptions?

This idea appears to have floated in the mind of M. E. Kopp when, with remarkable sagacity, he concluded his paper on Aniline-red* in the following terms:—"The hydrogen of this substance being replaceable also by methyl, ethyl, and amyl, &c., we may anticipate the existence of a numerous series of compounds, all belonging to the same type, and which might constitute colouring matters either red, or violet, or blue."

Conceptions which only two years ago appeared little more than a scientific dream, are now in the very act of accomplishment.

I propose to continue these researches, and intend in a later communication to submit to the Royal Society the results obtained in the study of two other colouring matters derived from rosaniline, viz. *aniline-green* and *aniline-violet*.

November 26, 1863.

Major-General SABINE, President, in the Chair.

In accordance with the Statutes, notice was given from the Chair of the ensuing Anniversary Meeting, and the list of Officers and Council proposed for election was read as follows:—

President.—Major-General Edward Sabine, R.A., D.C.L., LL.D.

Treasurer.—William Allen Miller, M.D., LL.D.

Secretaries.—{ William Sharpey, M.D., LL.D.
George Gabriel Stokes, Esq., M.A., D.C.L.

Foreign Secretary.—Prof. William Hallows Miller, M.A.

Other Members of the Council.—James Alderson, M.D.; George Busk, Esq., Sec. L.S.; Col. Sir George Everest, C.B.; Hugh Falconer, M.A., M.D.;

* Ann. de Chim. et de Phys. [3] lxii. 230.

John Hall Gladstone, Esq., Ph.D.; Joseph Dalton Hooker, M.D.; Henry Bence Jones, M.A., M.D.; Prof. James Clerk Maxwell, M.A.; Prof. William Pole, C.E.; Archibald Smith, Esq., M.A.; Prof. Henry J. Stephen Smith, M.A.; The Earl Stanhope, P.S.A., D.C.L.; Prof. James Joseph Sylvester, M.A.; Thomas Watson, M.D., D.C.L.; Prof. Charles Wheatstone, D.C.L.; Rev. Prof. Robert Willis, M.A.

The question of Captain Ibbetson's readmission into the Society was put to the ballot, and, the ballot having been taken, Captain Ibbetson was declared to be readmitted.

The following communications were read:—

I. "Account of Magnetic Observations made between the years 1858 and 1861 inclusive, in British Columbia, Washington Territory, and Vancouver Island." By Captain R. W. Haig, R.A. Communicated by the President. Received November 4, 1863.

(Abstract.)

This paper contains the results of magnetic observations made between the years 1858 and 1861 inclusive, in British Columbia, Washington Territory, and Vancouver Island. The results are tabulated; and from them the direction and position of the lines of equal dip, total force, and declination or variation are determined.

Three maps at the end show the position of these lines, the stations of observation, and the observed values of the three magnetic elements at each station.

II. "On Plane Water-Lines." By W. J. Macquorn Rankine, C.E., LL.D., F.R.SS.L. & E., Assoc. Inst. N.A., &c. Received July 28, 1863.

(Abstract.)

1. By the term "Plane Water-Line" is meant one of those curves which a particle of a liquid describes in flowing past a solid body when such flow takes place in plane layers. Such curves are suitable for the water-lines of a ship; for during the motion of a well-formed ship, the vertical displacements of the particles of water are small, compared with the dimensions of the ship; so that the assumption that the flow takes place in plane layers, though not absolutely true, is sufficiently near the truth for practical purposes*.

2. The author refers to the researches of Professor Stokes (Camb. Trans. 1842), "On the Steady Motion of an Incompressible Fluid," and of Pro-

* As water-line curves have at present no single word to designate them in mathematical language, it is proposed to call them *Neoids*, from νηός, the Ionic genitive of ναῦς.

fessor William Thomson (made in 1858, but not yet published), as containing the demonstration of the general principles of the flow of a liquid past a solid body.

3. Every figure of a solid, past which a liquid is capable of flowing smoothly, generates an endless series of water-lines, which become sharper in their forms as they are more distant from the primitive water-line of the solid. The only exact water-lines whose forms have hitherto been completely investigated, are those generated by the cylinder in two dimensions, and by the sphere in three dimensions. In addition to what is already known of those lines, the author points out that, when a cylinder moves through still water, the orbit of each particle of water is one loop of an elastic curve.

4. The profiles of waves have been used with success in practice as water-lines for ships, first by Mr. Scott Russell (for the explanation of whose system the author refers to the Transactions of the Institution of Naval Architects for 1860–62), and afterwards by others. As to the frictional resistance of vessels having such lines, the author refers to his own papers —one read to the British Association in 1861, and printed in various engineering journals, and another read to the Royal Society in 1862, and printed in the Philosophical Transactions. Viewed as plane water-lines, however, the profiles of waves are not exact, but approximate; for the "solitary wave of translation," investigated experimentally by Mr. Scott Russell (Reports of the British Association, 1844), and mathematically by Mr. Earnshaw (Camb. Trans. 1845), is strictly applicable to a channel of limited dimensions only, and the trochoidal form belongs properly to an endless series of waves, whereas a ship is a solitary body.

5. The author proceeds to investigate and explain the properties of a class of water-lines comprising an endless variety of forms and proportions. In each series of such lines, the primitive water-line is a particular sort of oval, characterized by this property, that the ordinate at any point of the oval is proportional to the angle between two lines drawn from that point to two foci. Ovals of this class differ from ellipses in being considerably fuller at the ends and flatter at the sides.

6. The length of the oval may bear any proportion to its breadth, from equality (when the oval becomes a circle) to infinity.

7. Each oval generates an endless series of water-lines, which become sharper in figure as they are further from the oval*. In each of those derived lines, the excess of the ordinate at a given point above a certain minimum value is proportional to the angle between a pair of lines drawn from that point to the two foci.

8. There is thus an endless series of ovals, each generating an endless series of water-lines; and amongst those figures, a continuous or "fair" curve can always be found combining any proportion of length to breadth,

* As a convenient and significant name for these water-lines, the term "Oögenous Neoïds" is proposed (from Ὠογενής, generated from an egg, or oval).

from equality to infinity, with any degree of fullness or fineness of entrance, from absolute bluffness to a knife-edge.

9. The lines thus obtained present striking likenesses to those at which naval architects have arrived through practical experience; and every successful model in existing vessels can be closely imitated by means of them.

10. Any series of water-lines, including the primitive oval, are easily and quickly constructed with the ruler and compasses.

11. The author shows how to construct two algebraic curves traversing certain important points in the water-lines, which are exactly similar for all water-lines of this class. One is a rectangular hyperbola, having its vertex at the end of the oval. It traverses all the points at which the motion of the particles, in still water, is at right angles to the water-lines. The other is a curve of the fourth order, having two branches, one of which traverses a series of points, at each of which the velocity of gliding of the particles of water along the water-line is less than at any other point on the same water-line; while the other branch traverses a series of points, at each of which the velocity of gliding is greater than at any other point on the same water-line.

12. A certain point in the second branch of that curve divides each series of water-lines into two classes,—those which lie within that point having three points of minimum and two of maximum velocity of gliding, while every water-line which passes through or beyond the same point has only two points of minimum and one of maximum velocity of gliding. Hence the latter class of lines cause less commotion in the water than the former.

13. On the water-line which traverses the point of division itself, the velocity of gliding changes more gradually than on any other water-line having the same proportion of length to breadth. Water-lines possessing this character can be constructed with any proportion of length to breadth, from $\sqrt{3}$ (which gives an oval) to infinity. The finer of those lines are found to be nearly approximated to by wave-lines, but are less hollow at the bow than wave-lines are.

14. The author shows how horizontal water-lines at the bow, drawn according to this system, may be combined with vertical plane lines of motion for the water at the stern, if desired by the naval architect.

15. In this, as in every system of water-lines, a certain relation (according to a principle first pointed out by Mr. Scott Russell) must be preserved between the form and dimensions of the bow and the maximum speed of the ship, in order that the appreciable resistance may be wholly frictional and proportional to the square of the velocity (as the experimental researches of Mr. J. R. Napier and the author have shown it to be in well-formed ships), and may not be augmented by terms increasing as the fourth and higher powers of the velocity, through the action of vertical disturbances of the water.

III. "On the degree of uncertainty which Local Attraction, if not allowed for, occasions in the Map of a Country, and in the mean figure of the Earth as determined by Geodesy: a method of obtaining the mean figure free from ambiguity, from a comparison of the Anglo-Gallic, Russian, and Indian Arcs: and speculations on the Constitution of the Earth's Crust." By the Venerable J. H. Pratt, Archdeacon of Calcutta. Communicated by Professor Stokes, Sec. R.S. Received Oct. 5, 1863.

(Abstract.)

After referring to a former paper in which he had shown that, in the Great Indian Arc of meridian, deflections of the plumb-line amounting to as much as 20″ or 30″ would be produced if there were no sources of compensation in variations of density beneath the surface of the earth, and after alluding to a remarkable local deflection which M. Otto Struve had discovered in the neighbourhood of Moscow, the author proceeds to consider, in the first instance, the effect of local attraction in mapping a country according to the method followed by geodesists, in which differences of latitude and longitude are determined by means of the measured lengths of arcs, by substituting these lengths and the observed middle latitudes in the known trigonometrical formulæ, using the *mean* figure of the earth, although the actual level surface may differ from that belonging to the mean figure in consequence of local attraction. He concludes that no sensible error is thus introduced, either in latitude or longitude, if the arc do not exceed $12\frac{1}{2}^\circ$ of latitude or 15° of longitude in extent, but that the position of the map thus formed on the terrestrial spheroid will be uncertain to the extent of the deflection due to local attraction at the station used for fixing that position. In the Great Indian Arc this displacement might amount to half a mile if the deflections were as great as those calculated from the attraction of the mountains and the defect of attraction of the ocean, irrespective of subjacent variations of density; but the author shows in the next two sections that some cause of compensation exists which would rarely allow the actual uncertainty to be of any considerable amount, unless the station used for fixing the map were obviously situated in a most disadvantageous position.

The author then proceeds to examine the effect of local attraction on the mean figure of the earth, considering more particularly the eight arcs which have been employed for the purpose in the volume of the British Ordnance Survey. He supposes the reference station of each arc to be affected to an unknown extent by local attraction, and obtains formulæ giving the elements of the mean figure obtained by combining the eight arcs, these formulæ involving eight unknown constants expressing the deviations due to local attraction at each of the selected stations. By substituting reasonable values for the unknown deflections, he shows that local attraction s competent to affect the deduced mean figure to a very sensible extent.

He then institutes a comparison between the results afforded by those three of the eight arcs which are of considerable extent, namely, the Anglo-Gallic, Russian, and Indian Arcs. For each arc in particular he deduces values of the principal semiaxes of the earth, involving an unknown constant expressing the effect of local attraction at the reference station of the arc. In order that the three pairs of semiaxes should agree, there are four equations to be satisfied by means of three disposable quantities (namely, the three unknown attractions). On combining these four equations by the method of least squares, the unknown deflections come out extremely small, and the values of each semiaxis deduced for the three arcs separately come out very nearly equal to one another, and therefore to their mean. These mean values the author ventures to assume are the mean semiaxes of the earth. They are as follows:—

$$a = 20926180,\ b = 20855316 \text{ feet, giving } \epsilon = \frac{1}{295 \cdot 3},$$

where a is the equatorial, and b the polar semiaxis, and ϵ the ellipticity.

The author concludes with certain speculations respecting the constitution of the earth's crust. On adopting the mean figure determined as above explained, the errors of latitude to be attributed to local attraction at each of the fifty-five stations of the eight arcs, which will be found at p. 766 of the Ordnance Survey volume, come out very small. With respect to the Great Indian Arc, it is especially remarkable that the residual deflections are insignificant, while those calculated from the action of the causes visibly at work are considerable. It would seem as if some general cause were at work to increase the density under the ocean, and diminish the density under mountainous tracts of country. The author conceives that, as the earth cooled down from a state of fusion sufficiently to allow a permanent crust to be formed, those regions where the crust contracted became basins into which the waters ran, while regions where expansion accompanied solidification became elevated without any consequent increase in the total quantity of matter in a vertical column extending from the surface down to a given surface of equal pressure in the yet viscous mass below. The author considers that the deviations of latitude at the other principal stations of the measured arcs, if not positively confirmatory of, are at least not opposed to this view.

IV. "On the Meteorological Results shown by the Self-registering Instruments at Greenwich during the extraordinary Storm of October 30, 1863." By James Glaisher, F.R.S., F.R.A.S., &c. Received November 23, 1863.

In the year 1841 Osler's anemometer was erected at the Royal Observatory, Greenwich, and from that time, up to the year 1860, the greatest pressure on the square foot recorded was 25 lbs. In February 1860 one of 28 lbs. was registered, which was the greatest up to October 30 of the present year; on that day a pressure of no less than 29½ lbs. took place

during a heavy squall of wind and rain, which passed over the observatory at 3h. 30m. P.M. At this time, moreover, the readings of the several other self-registering meteorological instruments at the Royal Observatory, Greenwich, exhibited very large changes, and of so remarkable a character, that the Astronomer Royal expressed a wish that I should bring them under the notice of the Royal Society. The following are extracts from the several registers of the day mentioned:—

At 6h. A.M., on October 30, the barometer read 29·32 in., and it commenced falling slowly after this time, reaching 29·30 in. at 8h. A.M. The decrease then became more decided, and a steady fall was experienced; 29·10 in. was reached by 0½h. P.M., and 28·96 in. by 2h. P.M.; from 2h. P.M. to 3½h. P.M. the decline was very rapid; and the minimum reading, 28·80 in., was reached at the latter time.

After 3h. 30m. P.M. the barometer turned to increase rapidly; at 3h. 39m. P.M. it read 28·85 in.; at 4h. P.M., 28·92 in.; 4h. 20m. P.M., 29·00 in.; at 5h. P.M., 29·07 in.; and afterwards a gradual increase took place to 29·30 in. by 11h. P.M.

At 8h. A.M., with the first indications of decided barometric fall, the wind commenced to blow strongly from S.W.; at 8h. 20m. A.M. it had reached a force of 1 lb. on the square foot; shortly after, 2 lbs., and at 8h. 30m. A.M. 3 lbs. A force of 1½ lb. to 3 lbs generally prevailed, till 9h. 25m. A.M.; at 9h. 30m. A.M. a gust of short duration was experienced of 15 lbs., which produced a decline of temperature of 2°. From 9h. 35m. A.M. to 9h. 50m. A.M. the pressure of the wind varied between 3 lbs. and 5lbs.; from 1½ lb. to 3 lbs. from 9h. 59m. A.M. to 0h. 45m. P.M.; there was no pressure for two or three minutes about 0h. 50m. P.M.; the wind then again commenced blowing strongly, reaching 4 lbs. at 0h. 55m. P.M., and from 3 lbs. to 5 lbs. from 1h. P.M. to 1h. 15m. P.M., the pressure was generally 2 lbs. to 4 lbs. from 1h. 15m. P.M. to 2h. P.M.; from 2h. 0m. P.M. to 2h. 45m. P.M. it varied between ¼ lb. and 2 lbs.; the wind again commenced blowing strongly, reached 3½ lbs. at 2h. 50m. P.M., 4 lbs. at 3h. P.M., 5 lbs. at 3h. 10m. P.M., 7 lbs. at 3h. 16m. P.M., 12 lbs. at 3h. 20m. P.M., 13 lbs. at 3h. 23m. P.M., 11 lbs. at 3h. 26m. P.M., 17 lbs. at 3h. 29m. P.M., and 29½ lbs. at 3h. 30m. P.M.; then declined suddenly, pressing with forces varying between 6 lbs. and 9 lbs. from 3h. 35m. P.M. to 3h. 45m. P.M., and 4 lbs. to 6 lbs. from 3h. 45m. P.M. to 4h. P.M.; another gust at 4h. 10m. P.M. reached 8 lbs.; again declined to 4 lbs. at 4h. 15m. P.M.; after this time, till 5h. P.M., the pressure varied between 2 lbs. and 4 lbs., between 2 lbs. and 3 lbs. from 5h. P.M. to 6h. P.M., from ½ lb. to 2 lbs. (with occasional lulls) from 6h. P.M. to 7h. P.M., from 2 lbs. to 4 lbs. from 7h. P.M. to 9h. P.M.; scarcely any pressure was recorded between 9h. P.M. and 10h. P.M., and from 10h. P.M. to 11h. P.M. the amount varied between 1½ lb. to 3 lbs.

At the time of the great gust, viz. 3h. 30m. P.M., the barometer reached its minimum, 28·80 in.; the temperature declined rapidly (from

53½° at 3h. 15m. P.M. to 46° by 4h. P.M., and to 43° by 5h. P.M.); and the direction of the wind immediately changed to the amount of 90°, following the direction of the sun, or from S.S.W. to W.N.W.

At the Radcliffe Observatory, Oxford, the barometer-reading at 6h. A.M. was 29·18 in., and decreased to 28·80 in. at 2h. 30m.; it then suddenly increased to 28·85 in. at 2h. 35m., and to 29·25 in. by 11h. P.M. At 2h. the direction of the wind was S.; at 3h. 30m. it was W., and continued W. till 4h. 30m., and then returned to S.W. by 5h. The temperature at 2h. was 51°, declined to 43° at 2h. 30m., and to 41° by 5h.

The general changes of temperature agree very closely with those at Greenwich; but, as in the case of the barometer, those at Oxford preceded those at Greenwich by one hour nearly.

The general fact frequently noticed of a change in the direction of the wind simultaneously with a sudden and great pressure, and for the most part in one direction (that is to say, in the direction of the sun's motion, or N. to E. to S.), is very remarkable, and not easily accounted for.

November 30, 1863.

ANNIVERSARY MEETING.

Major-General SABINE, President, in the Chair.

Dr. Alderson, on the part of the Auditors of the Treasurer's Accounts, reported that the total receipts during the past year, including a balance of £635 7*s*. 3*d*. carried from the preceding year, amounted to £5133 10*s*. 3*d*., and that the total expenditure in the same period amounted to £4475 10*s*. 2*d*., leaving a balance at the Bank of £641 14*s*., and in the hands of the Treasurer of £16 6*s*. 1*d*.

On the motion of Sir Andrew S. Waugh, seconded by Mr. Hudson, the thanks of the Society were voted to the Treasurer and Auditors.

The Secretary read the following Lists:—

Fellows deceased since the last Anniversary.

On the Home List.

Beriah Botfield, Esq.
Loftus Longueville Clarke, Esq.
Prof. Arthur Connell.
Edward J. Cooper, Esq.
Walter Ewer, Esq.
Joshua Field, Esq.
Richard Fowler, M.D.
James William Gilbart, Esq.
Joseph Glynn, Esq.
Peter Hardy, Esq.
John Jesse, Esq.
Henry, Marquis of Lansdowne.
John Singleton Copley, Lord Lyndhurst.
John Gorham Maitland, Esq.
Richard Penn, Esq.
Clement Tudway Swanston, Esq.
John Taylor, Esq.
William Tooke, Esq.
James Tulloch, Esq.
Rev. William Walton.
Rear-Admiral John Washington.
Alexander Wilson, Esq.

On the Foreign List.

César Mansuète Despretz. | Eilhard Mitscherlich.

Carl Rümker.

Withdrawn.

Edward John Littleton, Baron Hatherton.
Lieut.-General William Monteith.

Fellows elected since the last Anniversary.

On the Home List.

The Right Hon. Edward Pleydell Bouverie.
Edward William Cooke, Esq., A.R.A.
William Crookes, Esq.
Col. Frederick M. Eardley-Wilmot, R.A.
James Fergusson, Esq.
Frederick Field, Esq.
Rev. Robert Harley.
The Right Hon. Sir Edmund Walker Head, Bart.
John Russell Hind, Esq.
Charles Watkins Merrifield, Esq.
Professor Daniel Oliver.
Frederick William Pavy, M.D.
William Pengelly, Esq.
Henry Enfield Roscoe, B.A.
Rev. George Salmon, D.D.
Samuel James Augustus Salter, M.B.
Rev. Arthur Penrhyn Stanley, D.D.
William Thomson, D.D., His Grace The Archbishop of York.

Readmitted.

Capt. L. L. Boscawen Ibbetson.

On the Foreign List.

Ernst Eduard Kummer. | Heinrich Gustav Magnus.

Johannes Japetus Smith Steenstrup.

The PRESIDENT then addressed the Society as follows:—

GENTLEMEN,

WHEN I had last the honour of addressing you at the Anniversary Meeting in 1862, I acquainted you that a communication had been received by your President and Council from the Duke of Newcastle, Her Majesty's principal Secretary of State for the Colonies, requesting the opinion of the Royal Society on the scientific importance of the results to be expected from the establishment of a Telescope of great optical power at Melbourne, in the Colony of Victoria, for the observation of the nebulæ and multiple stars of the Southern Hemisphere. The communication was founded on a despatch from Sir Henry Barkly, K.C.B., Governor of Victoria, soliciting on his own part and on that of the Visitors of the Melbourne

Observatory, the opinion of the Royal Society on this subject, and also on the most suitable construction of the telescope, both as to the optical part and the mounting, its probable cost, and the time required for its completion. It had happened that in 1853 the Royal Society and the British Association had united in an earnest representation to Her Majesty's Government of the scientific importance of establishing in some convenient locality in Her Majesty's dominions, from whence the southern nebulæ and multiple stars could be observed, a telescope of the requisite optical power; and in a preparatory correspondence, which was printed at the time, and in which the principal persons interested in such researches had participated, the best form of telescope, its probable cost, and all particulars relating to it, had been largely discussed. The representation thus concurred in by the two principal scientific bodies of the United Kingdom was not successful in securing the desired object; but the correspondence then printed was still fitted to supply in great measure in 1862 the information on which the President and Council could ground their reply. The discussion in 1853 had terminated in the appointment of a committee, consisting of the Earl of Rosse, Dr. Robinson, and Messrs. Lassell and Warren de la Rue, to superintend the construction of the telescope, in the event of the recommendation of the two Societies being favourably received. But as it was possible that the opinions previously entertained might have been in some degree modified by subsequent consideration or by more recent experience, the correspondence with those gentlemen was reopened, and their replies have formed a second correspondence, which, like the first, has been printed for the information of those Fellows of the Society who take a special interest in the subject. Availing themselves of these valuable communications, the President and Council replied to the Colonial Office by a report dated December 18, 1862. They have been since informed that copies of the report and of the correspondence have been sent to Melbourne for the information of the gentlemen with whom the proposition originated.

It is quite possible that the thoughtful discussions embodied in the correspondence referred to may be found to have a prospective value not limited to the occasion which has given rise to them. The considerations which apply to a telescope for the observation of the Southern Nebulæ at Melbourne are no less applicable to one which might be established on a site from whence a great part of the Southern Nebulæ could also be observed (as well as those of our own hemisphere), but enjoying the immense advantage conferred by elevation into the higher and less dense strata of the atmosphere. Such sites are to be found in the Nílgiris at elevations of several thousand feet, combining also convenient accessibility and proximity to the resources of civilized life. It may be hoped that at some not distant day the subject will receive the consideration which it deserves from those who are entrusted with the government of that now integral part of the British empire.

Having learnt that a series of pendulum experiments at the principal stations of the Great Russian Arc were in contemplation, I availed myself of an opportunity of informing M. Savitsch, by whom the operations were to be conducted, that the Invariable Pendulums which had been employed in the English experiments were now in the possession of the Royal Society, and, being unemployed, would, I was persuaded, be most readily lent by the Society on an application to that effect being made. The constants of these instruments, including the coefficient in the reduction to a vacuum, having been most carefully determined, they were ready, with the clocks and stands belonging to them, for immediate use, and would have the further advantage, that experiments made with them in Russia would be at once brought into direct connexion with the British series extending from 79° 50′ N. to 62° 56′ S. latitude. The communication was most courteously received and replied to. It appeared, however, that a detached invariable pendulum had been already ordered by the Russian Government from M. Repsold, of Hamburg, shorter than the English pendulums for convenience in land transport, and with two knife-edges and two fixed lenses, symmetrical in size and shape but one light and the other heavy, and so arranged that the times of vibration should be the same on either knife-edge in air of the same temperature and density. M. Savitsch expressed his desire to bring this pendulum in the first instance to Kew, and to secure thereby the connexion of his own with the English series, where also he would have the opportunity of testing the exactness of the correction for buoyancy by vibrating his pendulum on both its knife-edges in the vacuum-apparatus which is now established at Kew.

It is much to be desired that a similar series of pendulum experiments to those about to be undertaken in Russia should be made at the principal points of the Great Indian Arc; and the steps which are understood to be in progress in providimg new instruments for the verification of the astronomical and geodesical operations of the Trigonometrical Survey of India, and to give them a still greater extension, would seem to present a most favourable opportunity for the combination of pendulum experiments. In such case the pendulums of the Royal Society might be made available with excellent effect.

The large size of our printed volumes in the present year gives no unfavourable and, I think, no unfair idea of the present scientific activity of the Society; for I believe it may be safely said that our Council has not been less vigilant and cautious than heretofore in the selection of the papers to be printed. Although much care has been given to keeping the expenses of illustration within reasonable bounds, the cost of the Society's publications has been this year unusually high; yet I am glad to be able to state that our whole expenditure within the year has fallen within our income. With your permission, I will briefly advert to a few of the subjects which have occupied the Society's attention in the past year.

The researches of Kirchhoff and Bunsen have rendered it in a high degree probable that we shall be able to obtain much insight into the chemical nature of the atmospheres of the brighter fixed stars, by observing the dark lines in their spectra and comparing them with the bright lines in the spectra of elementary, and perhaps also of compound, bodies in the state of incandescent gas or vapour. The interest of such an inquiry is obvious; but the difficulties involved in it are very great. The quantity of light coming from even such a star as Sirius is so small, that without the use of a powerful telescope the spectrum obtained would be too faint to bear sufficient enlargement to show properly the fixed lines. The apparent diurnal motion of the stars causes much embarrassment, unless the instrument be mounted equatoreally, and furnished with a clock movement. The control of the experiments on incandescent bodies requires a thorough knowledge of chemistry, so as to avoid being misled by impurities in the substances examined, and to be prepared to interpret decompositions or combinations which may take place under unusual circumstances, and which may be manifested only by their effects. Nor can the astronomical and physical parts of the inquiry be well dissociated, so as to be separately undertaken by different individuals; for the most elaborate drawings can hardly convey a faithful idea of the various aspects of the different dark and bright lines, which yet must be borne in mind in instituting a comparison in cases of apparent coincidence. It is fortunate, therefore, that the inquiry has been taken up by two gentlemen working in concert. In a short paper read to the Society on the 26th of last February, and published in the Proceedings, Mr. Huggins and Dr. Miller have described and figured the spectra of three of the brighter stars; and this part of the inquiry will doubtless be continued. In a paper since presented to the Society, Mr. Huggins describes the means employed for practically determining with accuracy the positions of any stellar lines which may be observed, with reference to known points of the spectrum, and has given beautiful maps of the spectra of twenty-four of the elementary bodies under the action of the inductive discharge, reserving others for a future communication. When the inquiry is completed, it is possible that we may obtain an amount of knowledge, respecting the constitution of those distant heavenly bodies, of which we have at present little conception.

Professor Tyndall has given us the fourth of a series of papers upon the relation of Gases and Vapours to Radiant Heat. In the course of these inquiries, he has shown that the different aëriform bodies, even though colourless, exert very different degrees of absorptive action on the rays of heat,—and that certain portions of these heat-rays are more powerfully absorbed than others—rays from objects at a low temperature being more easily absorbed than those from objects at an elevated temperature. He has also proved that gases radiate as well as absorb, and, in conformity

42488

with what is known in the case of solids, that in gaseous media also there is equality in the powers of radiation and absorption. Bodies which exert an absorbent effect in the liquid form preserve it in the gaseous state. If further experiments should confirm Prof. Tyndall's views upon the absorptive action of aqueous vapour upon radiant heat of low intensity, these results must materially modify some of the views hitherto held upon the meteorological relations of aqueous vapour.

The Bakerian Lecture, by Mr. Sorby, is entitled by him "On the Direct Correlation of Mechanical and Chemical Forces." In this paper are embodied a series of observations upon the influence of pressure upon the solubility of salts, in which he has obtained results analogous to the changes observed in the freezing-point of liquids under pressure. He finds, in cases where, as is usual, the volume of the water and the salt is *less* than the volume of the water and the salt separately, that the solubility is *increased* by pressure, but that, in cases where (as when sal-ammoniac is dissolved in water) the bulk of the solution is *greater* than that of the water and salt taken separately, the solubility is *lessened* by a small but measurable amount. On the contrary, salts which expand in crystallizing from solution must, under pressure, overcome mechanical resistance in that change; and as this resistance is opposed to the force of crystallization, the salt is rendered more soluble. The extent of the influence of pressure, and the mechanical value of the force of crystalline polarity, were found to vary in different salts. Mr. Sorby also indicates the results of the action of salts upon certain carbonates under pressure, and purposes pursuing his researches upon chemical action under pressure. This paper may therefore be regarded as the first of a series upon a highly interesting and important branch of investigation, for which Mr. Sorby appears to be specially fitted, from his combining the needful geological knowledge with the skill in manipulation required in the physical and chemical part of the inquiry.

The examination of the bright lines in the spectra of electric discharges passing through various gases, and between electrodes of various metals, has of late years attracted very general attention. Each elementary gas and each metal shows certain well-marked characteristic lines, from the presence or absence of which it is commonly assumed that the presence or absence of the element in question may be inferred. But the question may fairly be asked, Has it been established that these lines depend so absolutely on chemical character that none of them can be common to two or more different bodies? Has it been ascertained that, while the *chemical nature* of the bodies remains unchanged, the lines never vary if the circumstances of mass, density, &c. are changed? What evidence have we that spectra are superposed, so that we observe the full sum of the spectra which the electrodes and the medium would produce separately?

To examine these and similar questions in the only unimpeachable way

(that of actual experiment) formed the object of a long and laborious research by Dr. Robinson, the results of which are contained in a paper in our Transactions. In the course of this research, Dr. Robinson had occasion to take careful measures of the positions of all the bright lines visible (and not too weak to measure) in a great number of spectra—those, namely, of the induction discharge passing between electrodes of twenty different metals, as well as graphite, most of which were observed in each of five different gases (including air), and for each gas separately at the atmospheric pressure and at the low pressure obtained by a good air-pump.

On taking an impartial survey of this great assemblage of experimental facts, Dr. Robinson inclines to the opinion that the origin of the lines is to be referred to some yet undiscovered relation between *matter in general* and the transfer of electric action; and that while the *places* of the lines are thus determined independently of particular circumstances, the *brightness* of the lines is modified, according to the special properties of the molecules which are present, through a range from great intensity down to a faintness which may elude our most powerful means of observation.

By a discussion of the results of the magnetic observations maintained for several years past at the Kew Observatory with an accuracy previously unattained, and by combining these with the earlier results of the observations at the British colonial observatories, I have been enabled to trace and, as I believe, satisfactorily to establish the existence of an annual variation in the three elements of the earth's magnetism, which has every appearance of being dependent upon the earth's position in her orbit relatively to the sun. Substantiated by the concurrent testimony of observations in both hemispheres, and in parts of the globe most widely distant from each other, this conclusion furnishes an additional evidence of a cosmical magnetic relation subsisting between the earth and other bodies of the solar system, and thus extends the scope and widens the basis of sound induction upon which the permanent relations of magnetical science must rest.

To Dr. Otto Torell, Professor of Zoology in the University of Lund, we are indebted for a communication of much interest, informing us of the progress made by an expedition appointed by the Swedish Government at the recommendation of the Royal Academy of Sciences at Stockholm, to execute a survey preliminary to the measurement of an arc of the meridian at Spitzbergen. The objects of the preliminary survey were to ascertain whether suitable angular points for a triangulation could be found from Ross Island at the extreme north, to Hope Island at the extreme south of Spitzbergen, and to determine on a favourable locality for the measurement of a base-line. The result of the first year's exploration has been the selection of stations, on hills of moderate height and easy access from the coast, for nine triangles shown in the sketch accompanying Dr. Torell's paper, including Ross Island in the extreme north, and extending

over about 1° 50′ of the proposed arc of 4½ degrees. A convenient locality has also been found for the base-line. The continuation of the preliminary survey to the extreme southern limit is to be the work of the summer of 1864. The report of the Geodesical Surveyors has shown that the northern portion presents no impediments which may not be surmounted by courage and perseverance; and with regard to the southern portion, the knowledge already acquired is considered to justify the expectation that the result of the second year's exploration will be no less favourable. Should such be the case, it is anticipated that the necessary steps will be taken for carrying into execution the measurement of the arc itself.

I may perhaps be permitted to allude for a moment to the peculiar interest with which I must naturally regard the proposed undertaking. The measurement of an arc of the meridian at Spitzbergen is an enterprise which nearly forty years ago was a cherished project of my own, which I had planned the means of executing, and which I ardently desired to be permitted to carry out personally. I may well therefore feel a peculiar pleasure in now seeing it renewed under what I regard as yet more promising auspices,—whilst I cannot but be sensible of how little I could have anticipated that I should have had the opportunity, at this distance of time, and from this honourable chair, of congratulating the Swedish Government and Academy upon their undertaking, and of thanking Dr. Torell for having traced its origination to my early proposition.

It is well remarked by Dr. Torell, that the triangulation, should it be proceeded with, will not be the only result of the years of scientific labour at Spitzbergen. There are, indeed, many important investigations for which the geographical circumstances would be eminently favourable. Two such may be specified, for which we may reasonably anticipate that full opportunity would be afforded, and for which the requisite instruments of precision are neither costly nor cumbersome. One is a more exact determination of the data on which our Tables of Astronomical Refraction are founded. The other is the employment of Cagnoli's method for determining the figure of the earth by occultations of the fixed stars*. This last would be tried under circumstances far more favourable than those contemplated by its original proposer, by reason of the high latitude of the northern observer—the greater number of stars in the moon's path, now included in our catalogues, of which a special ephemeris might be made —and the much greater amount of concerted corresponding observations which might now be secured. The advantage peculiar to this mode of determination is, that it is exempt from the influence of local irregularities in the direction and force of gravity which embarrass the results of

* Antonio Cagnoli, "Nuovo e sicuro mezzo per riconoscere la Figura della Terra," Memorie della Società Italiana, Verona, vol. vi. 1792.

An English translation, with Notes and an Appendix, was printed for private circulation in 1819, by Mr. Francis Baily.

the measurements of degrees and of pendulum experiments. As a third and thoroughly distinct method of investigation, it seems at least well deserving of a trial.

Swedish naturalists are not likely to undervalue the interest attaching to careful examinations of the constancy or variation of the elevation of land above the sea-level; and I may therefore venture to refer them to a paper in the Phil. Trans. for 1824 (Art. xvi.), written from Spitzbergen itself in July 1823, containing the particulars of a barometrical and trigonometrical determination of the height (approximately 1644 English feet) of the well-defined summit of a conspicuous hill in the vicinity of Fairhaven. The barometrical comparison was repeated on several days, the barometer on the summit of the hill being stationary, and the observation of the two barometers strictly simultaneous, the stations being visible from each other by a telescope. The height as given by the two methods, barometrical and trigonometrical, was in excellent accord. The hill may be identified with certainty by the plan which accompanies the paper referred to: it is of easy access, and may be remeasured with little difficulty.

It will be remembered that a few years ago the attention of the Royal Society was called by the Foreign Office to the circumstance of several glass bottles with closed necks having been found on the shores of the west coast of Nova Zembla, leading to a conjecture that they might afford some clue to the discovery of the missing ships of Sir John Franklin's Expedition. The inquiries instituted by the Royal Society traced the bottles in question to a recent manufacture in Norway, where they are used as floats to the fishing-nets employed on that coast. These floats, accidentally separated from the nets, had been carried by the stream-current which sets along the Norwegian coast round the North Cape, and thus afforded evidence of the prolongation of the current to Nova Zembla. The Swedish Expedition, in the course of its summer exploration, found on the northern shore of Spitzbergen several more of these bottle-floats, some of which even bore Norwegian marks and names, supplying evidence, of considerable geographical interest, of the extension of the Norwegian stream-current to Spitzbergen, either by a circuitous course past the shores of Nova Zembla, or by a more direct offshoot of which no previous knowledge existed. It is thus that step by step we improve our knowledge of the currents which convey the waters of the more temperate regions to the Polar seas and produce effects which are traceable in many departments of physical geography.

The application of gun-cotton to warlike purposes and engineering operations, and the recent improvements in its manufacture, have been the subject of a Report prepared by a joint Committee of the Chemical and Mechanical Sections of the British Association, consisting chiefly of Fellows of the Royal Society. The Report was presented at the Meeting in New-

castle in September last, and is now in the press. The Committee had the advantage of personal communication with General von Lenk, of the Imperial Austrian Artillery, the inventor of the system of preparation and adaptation by which gun-cotton has been made practically available for warlike purposes in the Austrian service. On the invitation of the Committee, and with the very liberal permission of the Emperor of Austria, General von Lenk visited England for the purpose of thoroughly explaining his system; and we have in the Report of the Committee the information, thus gained directly from the fountain-head, of the results of his experience in the course of trials extending over many years, together with additional investigations by individual members of the Committee.

The advantages which are claimed for gun-cotton over gunpowder for ordnance-purposes and mining-operations are so many and so important as to call imperatively for the fullest investigation. Such an inquiry, however, in its complete sense, is both beyond and beside the scope and purposes of a purely scientific body; and the British Association have done well (whilst reappointing the Committee to complete certain experiments which they had devised with the view of clearing up some scientific points which are still more or less obscure) in pressing on the attention of Her Majesty's Government the expediency of instituting under its own auspices a full and searching inquiry into the possible applications of gun-cotton in the public service.

The absence of smoke, and the entire freedom from the fouling of the gun, are points of great moment in promoting the rapidity of fire and the accuracy of aim of guns employed in casemates or in the between decks of ships of war; to these we must add the innocuous character of the products of combustion in comparison with those of gunpowder, and the far inferior heat imparted to the gun itself by repeated and rapid discharges. With equal projectile effects, the weight of the charge of gun-cotton is but one-third of that of gunpowder; the recoil is stated to be reduced in the proportion of 2 to 3, and the length of the gun itself to admit of a diminution of nearly one-third. These conclusions are based on the evidence of long and apparently very carefully conducted courses of experiment in the Imperial Factory in the neighbourhood of Vienna. The results appear to be especially deserving the attention of those who are engaged in the important problems of facilitating the employment of guns of large calibre and of great projectile force in the broadsides of our line-of-battle ships, and in reducing, as far as may be possible, the dimensions of the ports.

In the varied applications of explosive force in military or civil engineering, the details of many experiments which bear on this branch of the inquiry are stated in the Report of the Committee, and appear to be highly worthy of consideration and of further experiment.

It cannot be said that the advantages now claimed for gun-cotton are altogether a novel subject of discussion in this country. When the

material was first introduced by Schönbein in 1846, its distinctive qualities in comparison with gunpowder were recognized, although at that period they were far less well ascertained by experiment than they are at present. To the employment of gun-cotton as then known there was, however, a fatal drawback in its liability to spontaneous combustion. The elaborate experiments of General von Lenk have shown that this liability was due to imperfection in its preparation, and ceases altogether when suitable processes are adopted in its manufacture. Perfect gun-cotton is a definite chemical compound; and certain processes for the removal of all extraneous matter and of every trace of free acid are absolutely indispensable. But when thus prepared it appears to be no longer liable to spontaneous combustion, it can be transported from place to place with perfect security, or be stored for any length of time without danger of deterioration. It is not impaired by damp—and may be submerged without injury, its original qualities returning unchanged on its being dried in the open air and at ordinary temperatures.

A scarcely less important point towards the utilization of gun-cotton and the safety with which it may be employed in gunnery is the power of modifying and regulating its explosive energy at pleasure, by means of variations in the mechanical structure of the cartridge, and in the relative size of the chamber in which it is fired.

The experiments made by the Austrian Artillery Commission, as well as those for blasting and mining, were conducted on a very large scale; with small arms the trials appear to have been comparatively few.

There can be no hesitation in assenting to and accepting the concluding sentence of the Committee's report. "The subject has neither chemically nor mechanically received that thorough investigation that it deserves. There remain many exact measures still to be made, and many important data to be obtained. The phenomena attending the explosion of both gun-cotton and gunpowder have to be investigated, both as to the temperatures generated in the act of explosion and the nature of the compounds which result from them, under circumstances strictly analogous to those which occur in artillery practice."

I proceed to announce the awards which the Council has made of the Medals in the present year; and to state the grounds on which those awards have been made.

The Copley Medal has been awarded to the Reverend Adam Sedgwick, for his observations and discoveries in the Geology of the Palæozoic Series of Rocks, and more especially for his determination of the characters of the Devonian System, by observations of the order and superposition of the Killas Rocks and their Fossils, in Devonshire.

Mr. Sedgwick was appointed Woodwardian Professor of Geology in the University of Cambridge in the year 1818, since which time, up to a recent period, comprising an interval of upwards of forty years, he has

devoted himself to geological researches with an ability, a persistent zea and untiring perseverance which place him amongst the foremost of thos eminent men by whose genius, sagacity, and labours the science of Geolog has attained its present high position. To duly appreciate his earlie work as a geological observer and reasoner, we must recall to recollection the comparative ignorance which prevailed forty or fifty years ago, to the dispersion of which his labours have so largely contributed. Geology was then beset by wild and untenable speculations on the one hand, whilst on the other even its most calm and rational theories were received by many with distrust or with ridicule—and by others with aversion, as likely to interfere with those convictions on which the best hopes of man repose.

Under such circumstances Geology needed the support and open advocacy of men who, by their intellect and acquirements, and by the respect attached to their individual characters, their profession, or social position, might be able on the one hand to repress wild fancies, and on the other to rebut the unfounded assertions of those who opposed the discussion of scientific truth. Such a man was Professor Sedgwick, and such was the influence he exerted. It may be well to make this allusion on an occasion like the present, because it often happens, not unnaturally, that those who are most occupied with the questions of the day, in an advancing science, retain but an imperfect recollection of the obligations due to those who laid the first foundation of our subsequent knowledge.

More than forty years have passed since Professor Sedgwick began those researches among the older rocks of England which it became the main purpose of his life to complete. In 1822 was begun that full and accurate survey of the Magnesian Limestone of the North of England which to this day holds its high place in the estimation of geologists as the foundation of our knowledge of this important class of deposits, whether we regard their origin, form of deposition, peculiarities of structure, or organic contents.

Contemporaneously with this excellent work, he examined the Whin Sill of Upper Teesdale, showed its claims to be treated as a rock of fusion, and discussed the perplexed question of its origin.

Advancing to one of the great problems which occupied his thoughts for many years, he combined in 1831 the observations of the older rocks of the Lake Mountains which he had commenced in 1822, and added a special memoir on the great dislocations by which they are sharply defined and separated from the Pennine chain of Yorkshire. Memoirs followed in quick succession on the New Red Sandstone of the Vale of Eden; on the stratified and unstratified rocks of the Cumbrian Mountains, and on the Limestone and Granite Veins near Shap. Thus, thirty years since, before the names of Cambrian and Silurian were ever heard, under which we now thankfully class the strata of the English lakes, those rocks had been vigorously assailed and brought into a lucid order and system which is to this day unchanged, though by the same hands which laid the

foundations many important additions have been made, one of signal value in 1851—the lower palæozoic rocks at the base of the carboniferous chain between Ravenstonedale and Ribblesdale. Perhaps no district in the world affords an example of one man's researches begun so early, continued so long, and ending so successfully. By these persevering efforts, the Geology of the Lake district came out into the light; and there is no doubt, and can be no hesitation in ascribing to them the undivided honour of the first unrolling of the long series of deposits which constitute the oldest groups of British Fossiliferous Rocks.

Still more complete, however, was the success of that work which was undertaken immediately afterwards on the coeval rocks of Wales; by which Professor Sedgwick and Sir Roderick Murchison, toiling in separate districts, unravelled the intricate relations of those ancient rocks, and determined the main features of the successive groups of ancient life which they enclose. These labours began in 1831–32, and in 1835 the two great explorers had advanced so far in their research as to present a united memoir to the British Association in Dublin, showing the progress each had made in the establishment of the Cambrian and Silurian systems, as they were then called; Professor Sedgwick taking the former, and Sir Roderick Murchison the latter for his special field of study.

In 1843 Professor Sedgwick produced two memoirs on the structure of what he then termed the Protozoic rocks of North Wales. Many excellent sections were given in detail in these memoirs; those exhibiting the structure of the western part of the district about Carnarvonshire being principally taken from his observations in 1831–32, while the more detailed sections of the eastern part were from those of 1842–43. These two papers gave the complete outline or framework, as it were, of the geological structure of this intricate region. In several subsequent years he continued to fill up this outline with further details, observed almost entirely by himself, giving numerous general and local sections, by which he determined the dip and strike of the beds, normal and abnormal, and all the great anticlinal and synclinal lines on which the fundamental framework depends.

Further and still minuter details were subsequently given, as was to be expected, by the Government Surveyors; but the general arrangement, finally recognized on the map of the Survey, is essentially the same as that previously worked out by his unaided labours.

It was a principle always advocated by Professor Sedgwick, that the geological structure of a complicated district could never be accurately determined by fossils alone without a detailed examination of its stratification. He always proceeded on this principle; nor (from the paucity of organic remains) would it have been possible on any other principle to have determined the real geological character of those older districts which he investigated so successfully. His arrangement and nomenclature of the Cambrian rocks in North Wales (the Lower Silurians of Sir Roderick

Murchison) are given in his "Synopsis of the Classification of the British Palæozoic Rocks," 1855. It possesses the weight which must always be recognized as appertaining to the authority of the geologist who, by his own labours, first solved the great problem of the physical structure of the district.

There are other important memoirs of Professor Sedgwick's of which time forbids more than a very passing notice. The memoir "On the Structure of large Mineral Masses," published in 1831, was the first, and remains to this day the best descriptive paper which has yet appeared on joints, planes of cleavage, nodular concretions, &c.

Always attentive to the purpose of preparing a complete and general classification of the Palæozoic Strata, Professor Sedgwick at an early period in his career printed a memoir "On the Physical Structure of the Older Strata of Devon and Cornwall;" and another "On the Physical Structure of the Serpentine District of the Lizard." Of later date are several papers written by him, conjointly with Sir Roderick Murchison, respecting the Devonian System. The principal of these, published in 1840, comprised the work of several previous years, and made known the true nature of the *Culm Beds* of North Devon, as belonging to the Carboniferous series, and their position in a trough of the subjacent rocks, which rocks, on account of their position and their organic contents, were concluded to belong to the Devonian, or Old Red Sandstone period, a conclusion which was at first controverted, but was ultimately admitted. In another memoir by the same authors in 1828, they conclude that the coarse old red conglomerate along the north-western coast of Scotland and in Caithness is of about the same age as the Old Red Sandstone of South Wales and Herefordshire, and therefore of the Devonian period. They also published in 1840 an account of their general observations on the Palæozoic Formations of Belgium and the Banks of the Rhine, the results of which were considered to harmonize with those derived from other localities. Finally, we may notice another joint memoir by these authors in 1830, "On the Structure of the Eastern Alps," which, however, had no immediate relation to the researches on the Palæozoic formations.

It will be observed that the memoirs which have been noticed are for the most part pervaded by a certain unity of purpose. The investigations were not on points of merely local interest, but were essential for the elucidation of the geological history of our planet during those early periods of which the records are most difficult to unfold. Few persons perhaps can have an adequate idea of the difficulties he had to contend with when he first entered North Wales as a geologist. Geologically speaking, it was a *terra incognita* of which he undertook to read the geological history before any one had deciphered the characters in which it is written. Moreover, besides the indistinctness and complexity of the stratification, and the obscurity which then prevailed as to the distinction between planes of stratification and planes of cleavage, there was also the

difficulty of what may be called "mountain geometry"—that geometry by which we unite in imagination lines and surfaces observed in one part of a complicated mountain or district with those in another, so as to form a distinct geometrical conception of the arrangement of the intervening masses. This is not an ordinary power; but Mr. Sedgwick's early mathematical education was favourable to the cultivation of it. We think it extremely doubtful whether any other British geologist forty years ago could have undertaken, with a fair chance of success, the great and difficult work which he accomplished.

Such are the direct and legitimate claims of Professor Sedgwick to the honour conferred upon him by the award of the Copley Medal. But there are also other claims, less direct, but which it would be wrong to pass altogether unnoticed. It is not only by written documents that knowledge and a taste for its acquirement are disseminated; and those who have had the good fortune to attend Professor Sedgwick's lectures, or may have enjoyed social intercourse with him, will testify to the charm and interest he frequently gives to geology by the happy mixture of playful elucidation of the subject with the graver and eloquent exposition of its higher principles and objects.

PROFESSOR SEDGWICK,

Accept this Medal, the highest honour which it is in the power of the Royal Society to confer, in testimony of our appreciation of the importance of the researches which have occupied so large a portion of your life, and which have placed you in the foremost rank of those eminent men by whose genius and labours Geology has attained its present high position in our country.

The Council has awarded a Royal Medal to the Reverend Miles Joseph Berkeley for his researches in Cryptogamic Botany, especially in Mycology.

Mr. Berkeley's labours as a cryptogamic botanist for upwards of thirty-five years, during which they have been more especially devoted to that extensive and most difficult order of plants the Fungi, have rendered him, in the opinion of the botanical members of the Council, by far the most eminent living author in that department. These labours have consisted in large measure of the most arduous and delicate microscopic investigation. Besides papers in various journals on Fungi from all parts of the globe, and in particular an early and admirable memoir on British Fungi, the volume entitled 'Introduction to Cryptogamic Botany,' published in 1857, is one which especially deserves to be noticed here. It is a work which he alone was qualified to write. It is full of sagacious remarks and reasoning; and particular praise is due to the special and conscientious care bestowed on the verification of every part, however minute and difficult, upon which its broad generalizations are founded. Mr. Berkeley's merits are not confined to description or classification; there

are facts of the highest significance, which he has been the first to indicate, and which in many cases he has also proved by observation and by experiments. We refer to his observations on the development of the reproductive bodies of the three orders of Thallogens (Algæ, Lichens, and Fungi), and on the conversion under peculiar conditions of certain forms of their fruit into others;—to the exact determination of the relations, and sometimes of the absolute specific identity of various forms of Fungi previously referred to different tribes; and to the recognition, in many species and genera, of a diversity of methods of reproduction in giving origin to parallel series of forms. As intimately connected with the life-history of Fungi, the intricate subject of vegetable pathology has been greatly elucidated by him; and he is indeed the one British authority in this department. His intimate acquaintance with vegetable tissues, and with the effects of external agents, such as climate, soil, exposure, &c., has enabled him to refer many maladies to their source; and to propose methods, which in some cases have proved successful, of averting, checking, and even curing diseases in some of our most valuable crops. In this line of research he has also demonstrated, on the one hand, that many so-called epiphytal and parasitic Fungi are nothing but morbid conditions of the tissues of the plant; on the other hand, that microscopic Fungi lurk and produce the most disastrous results where their presence had been least suspected.

Mr. Berkeley,

I present you with this Medal, in testimony of the high opinion which the Botanical Members of the Council of the Royal Society entertain of your researches in Cryptogamic Botany, especially Mycology; in which latter department your writings entitle you, in their judgment, to be considered as the most eminent living author.

The Council has awarded a Royal Medal to John Peter Gassiot, Esq., for his researches on the Voltaic Battery and Current, and on the Discharge of Electricity through Attenuated Media.

These contributions, most of which are recorded in our Transactions, are of high value, and in some respects peculiar. Their experimental part has been conducted on a scale of magnitude and power unmatched since the days of Davy and of Children, with apparatus of the highest perfection, and with consummate dexterity and skill; and the discussion and interpretation of the facts observed are characterized by sound theory and sober judgment.

It would trespass too much on your time were I to give a detailed account of them, and I shall only select a few which are examples of what Bacon has called "Instantiæ Crucis," such as, when the mind is undecided between several paths, point out the true one.

1. The first decides a question which was long debated with great vehemence, whether the energy of the Voltaic Battery arises from the contact of its metals, or from chemical action. The first of these opinions was mainly supported by the fact that, when two dissimilar metals are made to

touch, they show signs of opposite electricities when separated. Mr. Gassiot showed, in 1844, that the same occurs when the metals are separated by a thin stratum of air without having been in previous contact.

2. The identity of voltaic with frictional electricity was denied by many, because it gave no spark through an interval of air. Davy had indeed asserted the contrary in his 'Elements of Chemical Philosophy,' but his statement seems to have been doubted or unheeded. Mr. Gassiot, in the Transactions for 1844, has put the fact beyond dispute; he showed that by increasing the number of cells and carefully insulating them, sparks can be obtained even with the feeblest elements. With 3520 cells, zinc and copper excited with rain-water, he obtained sparks in rapid succession through $\frac{1}{50}$th of an inch of air; and a little later added to this a fact of still higher significance, that by exalting the chemical action in the cells, the same or even greater effect could be produced by a much smaller series. The battery of 500 Grove's cells which was constructed for these experiments is probably in some respects the most powerful that was ever made.

3. The currents produced by electric or magnetic induction are of the highest interest, and the employment of them as a source of electric power is almost daily enriching physical science with precious results. In this new field Mr. Gassiot has been one of the most successful explorers. So early as 1839 he showed that the induction current gives a real spark, and he found that in the flame of a spirit-lamp it could strike at a distance of $\frac{3}{4}$ths of an inch.

4. The splendid phenomena produced by the discharge of the induction current through rarefied gases or vapours are well known; in particular the stratification of the light. The cause of this is not yet fully understood, but Mr. Gassiot has made some very important additions to our knowledge of it in the Bakerian Lecture for 1858 and his subsequent communications to the Society. Among these may be named his explanation of the occasionally reversed curvature of the strata, and his discovery of the Reciprocating discharge, which, seeming single, is composed of two, opposite in direction, but detected by the different action of a magnet on each of them—a beautiful test, which is of wide application in such researches. Again, the Torricellian vacuum which he used at first, even when absolutely free from air, contains mercurial vapour: by applying to his tubes a potent freezing mixture, he found that as this vapour condensed, the strata vanished, the light and transmission of electricity decreased, till at a very low temperature both ceased entirely. It follows from this that a perfect vacuum does not conduct—a fact of cosmical importance, which had been surmised before, but not proved; and the desire of verifying this discovery led him to a means of far higher rarefaction. A tube containing a piece of fused hydrate of potassa is filled with dry carbonic acid, exhausted to the limit of the air-pump's power, and sealed; then by heating the potassa, the residual carbonic acid is mostly, or even totally absorbed. Vessels so exhausted, though still containing vapour of potassa, and perhaps of water, have a better vacuum than had been previously obtained, and often cease

to conduct till a little of the alkali is vaporized by heating them, and the gradual progress of the exhaustion gives a wide range of observation.

5. The current of an induction machine is necessarily intermittent, and it has been supposed that the strata are in some way caused by the intermittence, and are possibly connected with the mode of action of the contact-breaker. Mr. Gassiot has, however, shown that they are perfectly developed in the discharge of an extended voltaic battery through exhausted tubes. The large water-battery already mentioned shows them in great beauty; the discharge, however, is still intermittent.

6. The same appearance is exhibited by a Grove's battery of 400 well-insulated cells; but in this case a new and remarkable phenomenon presents itself. At first the discharge resembles that obtained from the water-battery, and is like it intermittent; but *suddenly* it changes its character from intermittent to continuous (so far at least as can be decided by a revolving mirror), and everything indicates that we have now the true voltaic arc. The discharge is now of dazzling brilliancy, *and is stratified as before*, whence it appears that strata are capable of being produced by the true arc discharge.

7. This change is accompanied by a remarkable alteration in the heating of the two electrodes. Mr. Gassiot had previously shown that, in the ordinary voltaic arc, formed in air of the usual pressure, the *positive* electrode is that which is the more heated, whilst in the discharge of an induction machine, whether sent through air at the ordinary pressure between electrodes of thin wire, or through an exhausted tube, it is the *negative*. The discharge through the large Grove's battery, so long as it was intermittent, agreed with the induction discharge in this character as in others, that the *negative* electrode was that which became heated; but when the discharge suddenly and spontaneously passed from the intermittent to continuous, the previously heated negative electrode became cool, and the positive was intensely heated.

These brief references will suffice to show what a high place Mr. Gassiot holds amongst those who are investigating this new track, which promises such great advance in our knowledge of those molecular forces in the study of which all physical science must ultimately centre. I may be permitted to add, that in his whole career he has sought not his own fame, but the advancement of science; he has rejoiced as much in the discoveries of others as in his own, and aided them by every appliance in his power. I cannot refrain from mentioning a recent instance in which this liberal and unselfish spirit has been strikingly exhibited. He has had executed a grand spectroscope, furnished with no less than nine faultless prisms, a design in which he has been ably seconded by the skill of the optician Mr. Browning, to whom the construction was entrusted. This magnificent instrument he has placed at the disposal of any Fellow of the Society who may happen to be engaged in researches requiring the use of such powerful apparatus. The instrument is at present at the Kew Observatory, where it is in contemplation to undertake the construction of a highly elaborate map of the spectrum.

Mr. Gassiot is still pursuing his electrical researches, and we may be assured that he will feel this acknowledgment of his labours by the Royal Society not merely as a recompense for that he has accomplished, but as an obligation to continued exertion and new discoveries.

MR. GASSIOT,

You will receive this Medal as a mark of the deep interest which the Royal Society takes in the investigations in which you are engaged, and of the high value which it attaches to the results with which you have already enriched our Transactions.

These are the grounds on which the Medal has been awarded to you by the Council. But it may be permitted to me to express the hope that you will also associate with it—as it is impossible that we should not do—the Society's recognition of the generous and kindly spirit which has manifested itself, as elsewhere, so also in all your pursuit of Science; and of which one memorial amongst others will remain in future times connected with the Society, in the establishment of the Scientific Relief Fund.

On the motion of Professor Owen, seconded by Mr. Gwyn Jeffreys, it was resolved—"That the thanks of the Society be returned to the President for his Address, and that he be requested to allow it to be printed."

The Statutes for the election of Council and Officers having been read, and Dr. W. Farr and Mr. Evans having been, with the consent of the Society, nominated Scrutators, the votes of the Fellows present were collected, and the following were declared duly elected as Council and Officers for the ensuing year:—

President.—Major-General Edward Sabine, R.A., D.C.L., LL.D.

Treasurer.—William Allen Miller, M.D., LL.D.

Secretaries.—{ William Sharpey, M.D., LL.D.
George Gabriel Stokes, Esq., M.A., D.C.L.

Foreign Secretary.—Prof. William Hallows Miller, M.A.

Other Members of the Council.—James Alderson, M.D.; George Busk, Esq., Sec. L.S.; Col. Sir George Everest, C.B.; Hugh Falconer, M.A., M.D.; John Hall Gladstone, Esq., Ph.D.; Joseph Dalton Hooker, M.D.; Henry Bence Jones, M.A., M.D.; Prof. James Clerk Maxwell, M.A.; Prof. William Pole, C.E.; Archibald Smith, Esq., M.A.; Prof. Henry J. Stephen Smith, M.A.; The Earl Stanhope, P.S.A., D.C.L.; Prof. James Joseph Sylvester, M.A.; Thomas Watson, M.D., D.C.L.; Prof. Charles Wheatstone, D.C.L.; Rev. Prof. Robert Willis, M.A.

On the motion of Mr. Brayley, seconded by Mr. Balfour Stewart, the thanks of the Society were voted to the Scrutators. The Society then adjourned.

Receipts and Payments of the Royal Society between December 1, 1862, and November 30, 1863.

	£	s.	d.
Balance at Bank, and on hand	635	7	3
Annual Subscriptions and Compositions	1740	8	0
Rents	354	2	8
Dividends on Stock	922	5	10
Ditto, Ditto, Trust Funds	281	7	0
Ditto, Ditto, Stevenson Bequest	466	7	0
Sale of Transactions, Proceedings, &c.	440	19	7
Chemical Society, Proceedings, 1861–63 (2 years)	100	0	0
Tea Expenses and Gas, repaid	77	17	2
Parcel Charges recovered	14	15	9
Sale of £107 13*s.* 5*d.* Consols (Rumford Fund)	100	0	0

Estates and Property of the Royal Society, including Trust Funds.

Estate at Mablethorpe, Lincolnshire (55 A. 2 R. 2 P.), £117 6*s.* 4*d.* per annum.

Estate at Acton, Middlesex (34 A. 3 R. 11 P.), £110 0*s.* 0*d.* per annum.

Fee farm rent in Sussex, £19 4*s.* per annum.

One-fifth of the clear rent of an estate at Lambeth Hill, from the College of Physicians, £3 per annum.

£14,000 Reduced 3 per Cent. Annuities.

£26,969 15*s.* 7*d.* Consolidated Bank Annuities.

£513 9*s.* 8*d.* New 2½ per Cent. Stock.

£5133 10 3

	£	s.	d.
Salaries, Wages, and Pension	996	9	0
The Scientific Catalogue	194	10	9
Books for the Library and Binding	198	15	10
Printing Transactions and Proceedings, Paper, Binding, Engraving, and Lithography	2292	11	3
Upholstery, Painting, and Miscellaneous House Expenses	66	16	5
Coal and Lighting	97	5	2
Tea Expenses	53	3	6
Fire Insurance	42	1	6
Shipping Expenses	23	12	9
Fittings and Gas for Illumination	33	2	6
Taxes	16	6	3
Law Expenses	11	4	4
Stationery	6	8	9
Postage, Parcels, and Petty Charges	27	10	0
Subscription, Mablethorpe Schools	2	2	0
Donation Fund	126	19	5
Rumford Fund	237	10	4
Wintringham Fund	34	13	0
Copley Medal Fund	4	15	1
Mr. Sorby, Bakerian Lecture	3	17	0
Prof. Lister, Croonian Lecture	2	17	6
Rev. T. S. Evans, Fairchild Lecture	2	17	10
Balance at Bank	641	14	0
„ Catalogue Account	10	3	9
„ Petty Cash „	6	2	4
	£5133	10	3

WILLIAM ALLEN MILLER,

Treasurer.

WILLIAM ALLEN MILLER,
Treasurer.

Statement of Income and Expenditure (apart from Trust Funds) during the Year ending November 30, 1863.

	£	s.	d.
Annual Subscriptions	1144	8	0
Admission Fees	190	0	0
Compositions	406	0	0
Rents	354	2	8
Dividends on Stock (exclusive of Trust Funds)	922	5	10
„ on Stevenson Bequest	466	7	0
Sale of Transactions, Proceedings, &c.	440	19	7
Chemical Society, for Proceedings, 1862–63	50	0	0
Chemical Society, Tea Expenses — 14 6 4			
Geological Society, Tea Expenses and Gas — £15 5 10			
Linnean Society, Tea Expenses — 14 6 4			
Chemical, Geographical, and Linnean Societies, and University, part cost of Illumination, £6 each — 24 0 0			
Geographical Society, Gas at Evening Meetings — 9 0 0			
Social Science Association, and St. George's Rifles, ditto — 0 18 8			
	77	17	2
Parcel Charges recovered	14	15	9
	4066	16	0
Receivable on this Year's account.			
Miss Burdett Coutts Half cost of Plates to Dr. Heer's and Mr. Pengelly's Papers	90	0	0
Income available for the Year ending Nov. 30, 1863	4156	16	0
Expenditure in the Year ending Nov. 30, 1863	4062	0	0
Excess of Income over Expenditure in the Year ending Nov. 30, 1863	£94	16	0

	£	s.	d.	£	s.	d.
Salaries, Wages, and Pension				996	9	0
The Scientific Catalogue				194	10	9
Books for the Library	£109	18	11			
Binding ditto	88	16	11	198	15	10
Printing Transactions, Parts I. & II. 1862, and Part I. 1863	835	13	11			
Ditto Proceedings, Nos. 52–57	209	15	0			
Ditto Miscellaneous	110	11	8	2292	11	3
Paper for Transactions and Proceedings	328	7	4			
Binding and Stitching ditto	95	17	10			
Engraving and Lithography	712	5	6			
Painting and Cleaning				20	6	4
Upholstery, Repairs, and Miscellaneous House Expenses				46	10	1
Coal and Lighting				97	5	2
Tea Expenses				53	3	6
Fire Insurance				42	1	6
Fittings and Gas for Illumination				33	2	6
Shipping Expenses				23	12	9
Taxes				16	6	3
Law Expenses				11	4	4
Stationery				6	8	9
Postage, Parcels, and Petty Charges				27	10	0
Subscription to Mablethorpe Schools				2	2	0
				£4062	0	0

WILLIAM ALLEN MILLER,
Treasurer.

Scientific Relief Fund.

Investments up to July 1863, New 3 per Cent. Annuities	£5300 0 0
	£5300 0 0

Dr.	£	s.	d.	*Cr.*	£	s.	d.
To Subscriptions and Dividends.....	377	17	0	By Grants	75	0	0
				Purchase of Stock.....	74	9	0
				Balance	228	8	0
	£377	17	0		£377	17	0

The following Table shows the progress and present state of the Society with respect to the number of Fellows:—

	Patron and Honorary.	Foreign.	Having compounded.	Paying £2 12*s.* annually.	Paying £4 annually.	Total.
December 1, 1862..	5	49	327	4	275	660
Since compounded..	.....	.....	+2	.....	−2	
Since elected	+1	+3	+7	.....	+11	+22
Since admitted	.....	.....	.....	.....	+1	+1
Since readmitted ..	.....	.....	.....	.....	+1	+1
Since withdrawn ..	.....	.....	.....	.....	−2	−2
Since deceased	.....	−3	−12	.....	−10	−25
November 30, 1863.	6	49	324	4	274	657

December 10, 1863.

Major-General SABINE, President, in the Chair.

The President announced that he had appointed the following Members of the Council to be Vice-Presidents :—

The Treasurer.
Dr. Falconer.
Dr. Hooker.
Professor Wheatstone.
The Rev. Professor Willis.

The Very Reverend Dr. Stanley, Dean of Westminster, was admitted into the Society.

The following communications were read :—

I. "On the Spectra of some of the Chemical Elements." By William Huggins, Esq., F.R.A.S. Communicated by Dr. W. A. Miller, V.P. and Treas. R.S. Received November 5, 1863.

(Abstract.)

The author has been engaged for some time in association with Prof. W. A. Miller in observing the spectra of the fixed stars. For the purpose of comparing the spectra of these with the spectra of the terrestrial elements, no maps of the latter were found that were conveniently available. Kirchhoff's maps and tables, besides their partial incompleteness, were not suited for night work when the sun could not be simultaneously observed.

The author adopts the lines of the spectrum of common air as the fiducial points of a standard scale to which the spectra of the elements are referred. The air-spectrum has the advantage of being always visible with the spectra of the metals without increased complication of apparatus.

The observations were made with a spectroscope of six prisms of heavy glass of large size. The total deviation of the light with this train of prisms is for the *D* ray about 198°. The telescope and the collimator have both an aperture of 1·7 inch. The focal length of the telescope is 16·5 inches. The measures were partly taken from the readings of a finely divided arc of brass, which the arm carrying the telescope traverses, and partly from the readings of a wire micrometer attached to the eye-end of the telescope. The scale of measurement adopted gives five divisions for the interval between the components of the double line D. The excellent performance of this instrument is shown by the great distinctness of the finer lines of the solar spectrum. All those mapped by Kirchhoff are seen, and many others in addition to these.

The spark of an induction coil was employed, into the secondary circuit

of which a battery of nine Leyden jars was introduced. The Leyden jars are arranged in three batteries of three jars each, and the batteries connected in series.

The relative intensities and distinctive characters of the lines are represented by figures and letters, placed against the numbers in the Tables.

The spectrum, which extends from *a* to *H*, is divided, and forms two maps. The air-spectrum and the principal solar lines are placed at the top of each map, and below these the spectra of the following metals:—Sodium, potassium, calcium, barium, strontium, manganese, thallium, silver, tellurium, tin, iron, cadmium, antimony, gold, bismuth, mercury, cobalt, arsenic, lead, zinc, chromium, osmium, palladium, and platinum.

The lines of the air-spectrum are referred to the components of air to which they severally belong. An unexpected result was observed; two strong lines of the air-spectrum, one of them a double line, were seen to be common to the spectra of oxygen and nitrogen. These gases were obtained from different sources with identical results. The strong red line of the air-spectrum is shown to be due to the presence of aqueous vapour, and to coincide with the line of hydrogen. The carbonic acid in the air is not revealed by spectrum analysis.

Three pairs of lines and one band of haze are given in the sodium spectrum in addition to the double D line. As these might be due to impurities of the commercial sodium employed, the observation was confirmed by an amalgam of sodium prepared by the voltaic method from pure chloride of sodium. Two of these pairs of lines have been recognized in the spectrum of a saturated solution of pure nitrate of soda.

The two stronger pairs appear to agree in position with solar lines having the following numbers in Kirchhoff's scale:—864·4 and 867·1, and 1150·2 and 1154·2.

The spectrum from electrodes of potassium contains many new lines. For the spectra of calcium, lithium, and strontium, metallic calcium, lithium, and strontium were employed.

Barium was mapped from an amalgam of barium prepared by electricity from chloride of barium.

The following metals were employed in the form of electro-deposits upon platinum:—manganese, silver, tin, iron, cadmium, antimony, bismuth, cobalt, lead, zinc, and chromium. Care was taken that the other metals should be reliable for purity.

II. "On the Acids derivable from the Cyanides of the Oxy-radicals of the Di- and Tri-atomic Alcohols." By Maxwell Simpson, A.B., M.B., F.R.S. Received November 7, 1863.

From every glycol it is possible to obtain two radicals—one monatomic, the other diatomic. From every glycerine it is possible to obtain three

radicals, which are respectively mono-, di- and tri-atomic. The compounds which these radicals form with the metalloids have been long since prepared and thoroughly studied. Our knowledge of the compounds which they form with cyanogen, whose behaviour so much resembles the metalloids, is not in so forward a state. At present we are only acquainted with a few of the cyanides of those of them which are destitute of oxygen, and the acids they form when submitted to the action of potash. The object of the present investigation is to extend our knowledge in this direction. With this view I propose to myself the following questions:—Is it possible to prepare also the cyanides of the oxy-radicals of glycol or glycerine? And if it be possible, is the action of potash on these cyanides analogous to its action on the ordinary cyanides? If the foregoing questions be answered in the affirmative, we shall then be able to obtain in this way, from every glycol, two, and from every glycerine three acids. A glance at the following Table will make this intelligible:—

Diatomic Alcohol (*Glycol*).

		Cyanide.	Acid.	
Chlorhydrine of Glycol	$C_4H_5O_2Cl$	$C_4H_5O_2Cy$	$C_6H_6O_6$	Lactic?
Chloride of Ethylene	$C_4H_4Cl_2$	$C_4H_4Cy_2$	$C_8H_8O_8$	Succinic* (bibasic).

Triatomic Alcohol (*Glycerine*).

		Cyanide.	Acid.	
Monochlorhydrine	$C_6H_7O_4Cl$	$C_6H_7O_4Cy$	$C_8H_8O_8$	
Dichlorhydrine	$C_6H_6O_2Cl_2$	$C_6H_6O_2Cy_2$	$C_{10}H_8O_{10}$	(Bibasic)
Trichlorhydrine	$C_6H_5Cl_3$	$C_6H_5Cy_3$	$C_{12}H_8O_{12}$	(Tribasic)†

In the present paper I propose to take up the study of the acid $C_{10}H_8O_{10}$ in the glycerine series, which I succeeded in preparing in the following manner:—

A mixture of one equivalent of dichlorhydrine and two equivalents of pure cyanide of potassium, together with a quantity of alcohol, was maintained at the temperature of 100° Cent. for twenty-four hours in well-closed soda-water bottles. At the expiration of this time it was found that all the cyanide of potassium had been converted into chloride. The contents of the bottles were then filtered, and to the filtered liquor, which no doubt contained the body $C_6H_6O_2Cy_2$ in solution, solid potash was added. To this, heat was applied in such a manner as to prevent the escape of the alcohol by evaporation; and its application continued till ammonia ceased to be evolved. As soon as this was observed, the alcohol was distilled off, and the residue treated with nitric acid, which was afterwards removed by evaporation at a low temperature. The nitric acid accomplishes two objects: it destroys in a great measure the tarry matter which is present in large quantity, and at the same time sets free the

* Philosophical Transactions for 1861, p. 61.

† Proceedings of the Royal Society, vol. xii. p. 236.

organic acid combined with the potash. The free acid was then separated from the nitrate of potash by means of alcohol.

On evaporating the alcohol a dark-coloured residue was obtained, which was dissolved in hot water and treated with chlorine. Finally a silver-salt of the acid was prepared by the following kind of fractional precipitation:—About one-third of the neutralized acid was first precipitated by the cautious addition of a solution of nitrate of silver. The liquor was then filtered, and the remainder of the acid was converted into the silver-salt. By these means I obtained, instead of a brown, a perfectly white precipitate, which yielded an acid in colourless crystals when decomposed by sulphuretted hydrogen. Dried at 100° Cent. these crystals gave on analysis numbers which agree tolerably well with the formula $C_{10}H_8O_{10}$, as will be seen from the following Table:—

	Theory.	Experiment.	
		I.	II.
C_{10}	40·54	41·62	41·61
H_8	5·40	5·17	5·16
O_{10}	54·06	——	——
	100·00		

These analyses were performed on specimens prepared at different times. This acid is soluble in water, alcohol, and ether. It has a pure acid taste. It melts at about 135° Cent., and at a higher temperature suffers decomposition. The free acid gives an abundant white precipitate with acetate of lead, soluble in strong acetic acid. It is not precipitated by lime-water. The neutralized acid yields a bulky white precipitate with corrosive sublimate, and a pale brown with perchloride of iron. Copper salts give a bluish-white precipitate. Chloride of barium is not affected. The formation of this acid may be explained by the following equation:—

$$C_8H_8O_2Cy_2+2\left(\begin{matrix}K\\H\end{matrix}\right\}O_2\Big)+4HO=C_{10}\left.\begin{matrix}H_6\\K_2\end{matrix}\right\}O_{10}+2NH_3.$$

I have also analyzed the silver-salt of this acid. As it suffers decomposition at the temperature of boiling water, I was obliged to effect its desiccation by placing it *in vacuo* over sulphuric acid. It is slightly soluble in water. The numbers it yielded on analysis agree very well with the formula $C_{10}\left.\begin{matrix}H_6\\Ag_2\end{matrix}\right\}O_{10}$:—

	Theory.	Experiment.	
		I.	II.
C_{10}	16·57	16·61	——
H_6	1·65	1·39	——
O_{10}	22·17	——	——
Ag_2	59·61	——	60·67
	100·00		

The ether of this acid is readily prepared by passing hydrochloric acid gas through its solution in absolute alcohol. On evaporating the alcohol an oily residue was obtained, which was washed with a solution of carbonate of soda and distilled. The greater portion passed over between 295° and 300° Cent. The analysis of this portion gave numbers which indicate the formula $C_{10}\,\overset{H_6}{(C_4H_5)_2}\,O_{10}$:—

	Theory.	Experiment. I.	Experiment. II.
C_{18}	52·94	54·61	54·32
H_{16}	7·84	8·09	6·91
O_{10}	39·22	——	——
	100·00		

This ether suffers partial decomposition during distillation; hence the discrepancy between the theoretical and experimental numbers in the first analysis. The specimen which served for the second was not distilled at all, but simply purified by solution in ether. It is a colourless neutral oil with a very acrid taste. It is somewhat soluble in water. Heated with solid potash it yields alcohol, and the acid is regenerated. I regret to say I have not succeeded in obtaining the cyanide ($C_6H_6O_2Cy_2$), which generates this acid, in a state of purity.

The compositions of the ether and silver-salt of this acid prove it to be bibasic. It is highly probable that the basicity of an acid produced in this way depends on the atomicity of the radical in the cyanide which generates it. If this be so, the cyanides of the mono-, di- and tri-atomic radicals of the glycols and glycerines should then yield by decomposition with potash respectively mono-, bi- and tri-basic acids. If it would be possible to prepare the acid $C_8H_8O_8$ from the cyanide $C_6H_7O_4Cy$, it would be interesting to examine its bearing on this point. Would it prove monobasic or bibasic?

This acid bears the same relation to pyrotartaric that malic bears to succinic acid:—

Succinic acid	$C_8H_6O_8$	Pyrotartaric acid	$C_{10}H_8O_8$
Malic acid	$C_8H_6O_{10}$	New acid	$C_{10}H_8O_{10}$

It has the composition of the homologue of malic acid. Whether it is actually the homologue of that acid or not I cannot yet say. I propose to call it oxy-pyrotartaric acid. Formulated according to the carbonic acid type it is thus written:—

$$2HO, C_6H_6O_2'' \left\{\begin{matrix} C_2O_2 \\ C_2O_2 \end{matrix}\right\} O_2.$$

We may now, I think, safely answer in the affirmative the questions put at the commencement of this Paper. The cyanides of the oxy-radicals of the di- and tri-atomic alcohols *can* be formed, and the action of the potash on these cyanides *is* analogous to its action on the ordinary cyanides.

The foregoing research was finished many months ago, but I delayed publishing it in the hope of being able to announce at the same time the formation of lactic acid by a similar process. I find, however, from the 'Annalen der Chemie und Pharmacie' of last month that I have been anticipated by Wislicenus, who has succeeded in forming lactic acid in the manner I have just described.

December 17, 1863.

Major-General SABINE, President, in the Chair.

The following communications were read:—

I. "First Analysis of 177 Magnetic Storms, registered by the Magnetic Instruments in the Royal Observatory, Greenwich, from 1841 to 1857." By GEORGE BIDDELL AIRY, Astronomer Royal. Received November 28, 1863.

(Abstract.)

The author first refers to his paper in the Philosophical Transactions, 1863, "On the Diurnal Inequalities of Terrestrial Magnetism as deduced from Observations made at the Royal Observatory, Greenwich, from 1841 to 1857." These results were obtained by excluding the observations of certain days of great magnetic disturbance; it is the object of the present paper to investigate the results which can be deduced from these omitted days.

The author states his reasons for departing from methods of reduction which have been extensively used, insisting particularly on the necessity of treating every magnetic storm as a coherent whole. And he thinks that our attention ought to be given, in the first instance, to the devising of methods by which the complicated registers of each storm, separately considered, can be rendered manageable; and in the next place, to the discussion of the laws of disturbance which they may aid to reveal to us, and to the ascertaining of their effects on the general means in which they ought to be included.

The author then describes the numerical process (of very simple character) by which, when the photographic ordinates have been converted into numbers, any storm can be separated into two parts, one consisting of waves of long period, and the other consisting of irregularities of much more rapid recurrence. He uses the term "Fluctuation" in a technical sense, to denote the area of a wave-curve between the limits at which the wave-ordinate vanishes. The Waves, Fluctuations, and Irregularities, as inferred from separate treatment of each storm, constitute the materials from which the further results of the paper are derived.

Table I. exhibits the Algebraic Sum of Fluctuations for each storm, with the Algebraic Mean of Disturbances, and Tables II. and III. exhibit the

Aggregate or Mean for each year, and the Aggregate for the seventeen years. The Aggregate for the Northerly Force is negative in every year. That for the Westerly Force is on the whole negative; the combination of the two indicates that the mean force is directed about 10° to the east of south. That for the Nadir Force appears negative, but its existence is not certain.

Some peculiarities of the numbers of waves with different signs are then pointed out. For Westerly Force and also for Nadir Force, the numbers of +waves and −waves are not very unequal; but for Northerly Force there are 177+waves and 277−waves. In Nadir Force it is almost an even chance whether a storm begins with a +wave or with a −wave; and the same with regard to its ending; in Westerly Force the chances at beginning and ending are somewhat in favour of a +wave; but in Northerly Force two storms out of three begin with a −wave, and ten storms out of eleven end with a −wave.

The beginnings and ends of the storms are also arranged by numeration of the combination of waves of different character in the different elements (as, for instance, Westerly Force + with Northerly Force −, Northerly force + with Nadir Force +, &c.); but no certain result appears to follow, except what might be expected from the special preponderances mentioned above, leaving the relative numbers of the combinations a matter of chance in other respects.

Tables IV., V., VI. exhibit the Absolute Aggregates of Fluctuations and Absolute Means of Disturbances without regard to sign. In interpreting these it is remarked that the large − mean force in the northerly direction necessarily increases the Aggregate and diminishes the Number of Waves. With probable fair allowance for this, it appears that the Numbers of Waves are sensibly equal, that the Sums of Fluctuations are sensibly equal, and that the Means of Disturbances are sensibly equal for Westerly Force and for Northerly Force. But the Number of Waves for Nadir Force is less than half that for the other forces; while the Sum of Fluctuations is almost three times as great as that for the others, and the Mean of Disturbances almost three times as great.

Attempts are made to compare the epochs of the waves in the different directions, but no certain result is obtained.

Tables VII., VIII., IX. exhibit for each storm, and for each year, and for the whole period the Number of Irregularities, the Absolute Sum of Irregularities, and the Mean Irregularity. It appears that the value of Mean Irregularity is almost exactly the same in the three directions, that the number of irregularities is almost exactly the same in Westerly Force and in Northerly Force, but that the number in Nadir Force is almost exactly half of the others.

It is certain that the times of Irregularities in the Westerly and Northerly directions do not coincide. There appears some reason to think that Nadir Irregularities frequently occur between Westerly Irregularities.

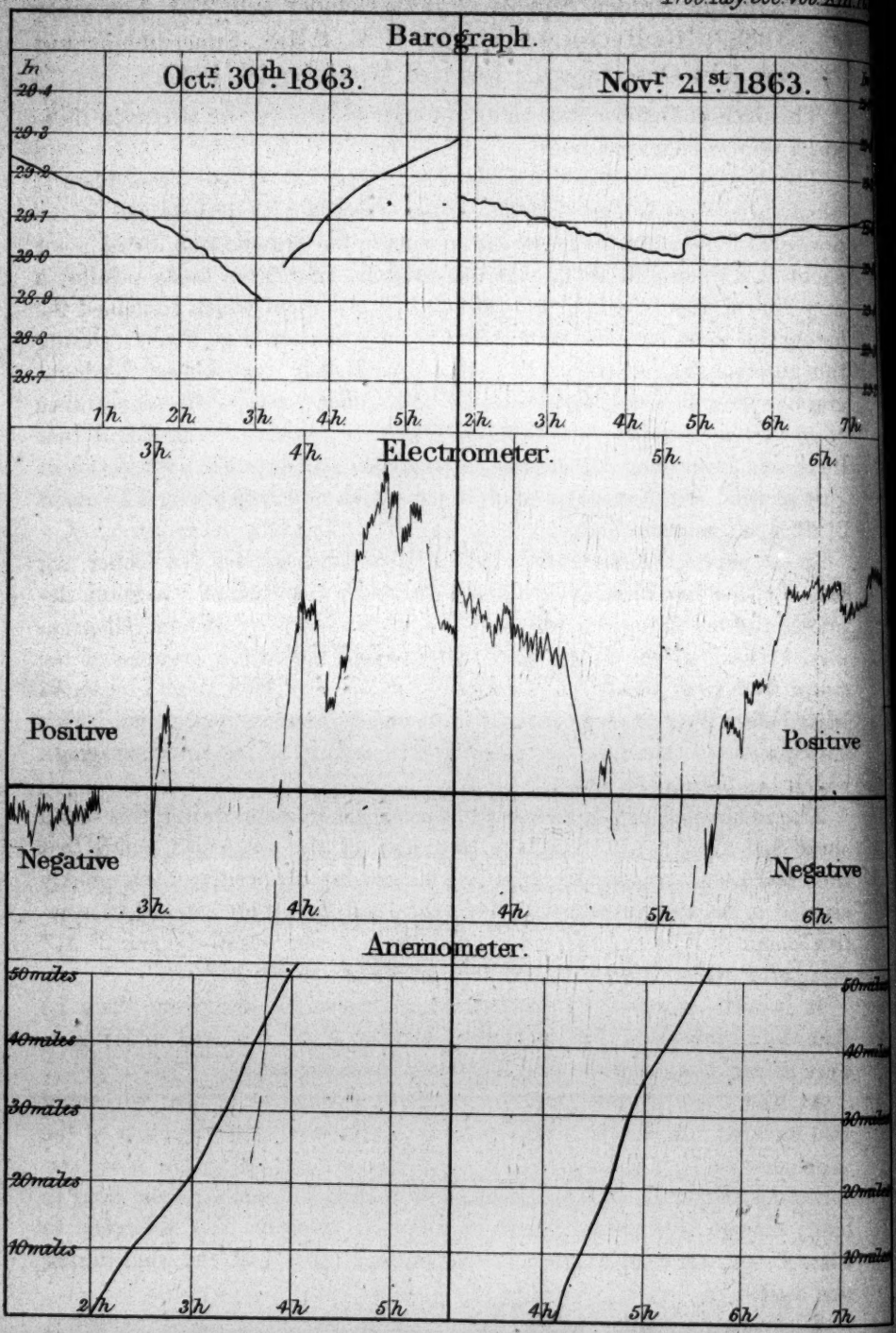
In Table X. the Aggregates of Fluctuations and Irregularities are arranged

by months, but no certain conclusions follow. In Table XI. the Wave-disturbances and the Irregularities are arranged by hours; for the Wave-disturbances results are obtained which may be compared with those of previous investigators; in Table XII. it is shown that these may be represented by a general tendency of wave-disturbances, different at different hours, which general tendency is itself subject to considerable variations. For the Irregularities it is found that the coefficient is largest in the hours at which storms are most frequent. It does not appear that any sensible correction is required to the Diurnal Inequalities of the former paper on account of these disturbed days.

The author then treats of the physical inference from these numerical conclusions. And in the first place he states his strong opinion that it is impossible to explain the disturbances by the supposition of definite galvanic currents or definite magnets suddenly produced in any locality whatever. The absolute want of simultaneity (especially in the Irregularities), and the great difference of numbers between the Waves and Irregularities for the Nadir Force (in which the Irregularities are just as strongly marked as in the Westerly and Northerly, and the Wave-disturbances are much more strongly marked), and those for the other Forces, appear fatal to this.

It is then suggested that the relations of the forces found from the investigations above, bear a very close resemblance to what might be expected if we conceived a fluid (to which for facility of language the name "Magnetic Ether" is given) in proximity to the earth, to be subject to occasional currents produced by some action or cessation of action of the sun, which currents are liable to interruptions or perversions of the same kind as those in air and water. He shows that in air and in water the general type of irregular disturbance is travelling circular forms, sometimes with radial currents, but more frequently with tangential currents, sometimes with increase of vertical pressure in the centre, but more frequently with decrease of vertical pressure; and in considering the phenomena which such travelling forms would present to a being over whom they travelled, he thinks that the magnetic phenomena would be in great measure imitated.

The author then remarks that observations at five or six observatories, spread over a space less than the continent of Europe, would probably suffice to decide on these points. He would prefer self-registering apparatus, provided that its zeros be duly checked by eye-observations, and that the adjustments of light give sufficient strength to the traces to make them visible in the most violent motions of the magnet. For primary reduction he suggests the use of the method adopted in this paper, with such small modifications as experience may suggest.



J. Basire lith.

II. "On the Sudden Squalls of 30th October and 21st November 1863." By Balfour Stewart, M.A., F.R.S., Superintendent of the Kew Observatory. Received December 10, 1863.

The 30th of October was windy throughout, and in the afternoon there was a very violent squall.

The barograph at the Kew Observatory, as will be seen from Plate I. which accompanies this communication, records a very rapid fall in the pressure of the atmosphere, which appears to have reached its lowest point about $3^h\ 9^m$ P.M., G. M. T. At this moment, from some cause, possibly a very violent gust of wind, the gas-lights in the room which contained the barograph went out, and were again lit in a quarter of an hour. During this interval the barometer had risen considerably; and indeed the barograph curve, although unfortunately incomplete, presents the appearance of an extremely rapid rise. It may therefore perhaps be supposed that there was a very sudden increase of pressure accompanied with a violent gust of wind at the moment when the gas went out, which would be about $3^h\ 9^m$ P.M., as above stated.

In a paper communicated to the Royal Society on November 23, Mr. Glaisher has remarked that at Greenwich the time of maximum depression of the barometer was $3^h\ 30^m$ P.M., while at the Radcliffe Observatory, Oxford, it was $2^h\ 30^m$ P.M. This would indicate a progress of the storm from west to east, in accordance with which Kew should be somewhat before Greenwich as regards the time of maximum depression. This anticipation is therefore confirmed by the record of the Kew barograph which has been given above.

The indications of the Kew self-recording electrometer during this squall show that about $2^h\ 39^m$ P.M. the electricity of the air, which before that time had been very slightly negative, became rapidly positive, then quickly crossed to negative, became positive again, and once more crossed to negative about $3^h\ 3^m$ P.M., recrossing again from strong negative about $3^h\ 51^m$ P.M., after which it settled down into somewhat strong positive.

It is well, however, to state (what may also be seen from Plate I.) that the variations of this instrument between $3^h\ 3^m$ P.M. and $3^h\ 51^m$ P.M. were so rapid as not to be well impressed upon the paper.

At Kew there is often occasion to move the dome, so that we cannot well have an instrument which records continuously the direction of the wind; but we have a Robinson's anemometer, which records the space traversed by the wind, and thus enables us to find its velocity from hour to hour, though not perhaps from moment to moment. A reference to Plate I. will show an increase in the average velocity of the wind during this squall.

A somewhat similar squall took place in the afternoon of Saturday, November 21st, about 4 o'clock.

In this case the Kew barograph presents a *rapid* (and, in the curve,

ragged) fall of the atmospheric pressure, which reached its minimum about $4^h\ 45^m$ P.M. There was then a very abrupt and nearly perpendicular rise of about five hundredths of an inch of pressure, or rather less, after which the rise still went on, but only more gradually.

Through the kindness of the Rev. R. Main, of the Radcliffe Observatory, I have been favoured with a copy of the trace afforded by the Oxford barograph during this squall, in which there appears a very sudden rise of nearly the same extent as that at Kew, but which took place about four o'clock, and therefore, as on the previous occasion, somewhat sooner than at Kew. This change of pressure at Oxford was accompanied by a very rapid fall of temperature of about 8° Fahr.

The minimum atmospheric pressure at Kew was 29·52 inches, while at Oxford it was 29·28 inches.

It will be seen from the Plate that at Kew the electricity of the air fell rapidly from positive to negative about $4^h\ 30^m$ P.M., and afterwards fluctuated a good deal, remaining, however, generally negative until $5^h\ 22^m$ P.M., when it rose rapidly to positive.

We see also from the Plate that there was an increase in the average velocity of the wind at Kew during the continuance of this squall. To conclude, it would appear that in these two squalls there was in both cases an exceedingly rapid rise of the barometer from its minimum both at Oxford and at Kew, this taking place somewhat sooner at the former place than at the latter; and that in both cases the air at Kew remained negatively electrified during the continuance of the squall, while the average velocity of the wind was also somewhat increased.

The Society then adjourned over the Christmas recess to Thursday January 7, 1864.

"On the Equations of Rotation of a Solid Body about a Fixed Point." By William Spottiswoode, M.A., F.R.S., &c. Received March 21, 1863.*

In treating the equations of rotation of a solid body about a fixed point, it is usual to employ the principal axes of the body as the moving system of coordinates. Cases, however, occur in which it is advisable to employ other systems; and the object of the present paper is to develope the fundamental formulæ of transformation and integration for any system. Adopting the usual notation in all respects, excepting a change of sign in the quantities F, G, H, which will facilitate transformations hereafter to be made, let

$$A=\Sigma m(y^2+z^2), \quad B=\Sigma m(z^2+x^2), \quad C=\Sigma m(x^2+y^2),$$
$$-F=\Sigma myz, \quad -G=\Sigma mzx, \quad -H=\Sigma mxy;$$

* Read April 16, 1863: see abstract, vol. xii. p. 523.

and if p, q, r represent the components of the angular velocity resolved about the axes fixed in the body, then, as is well known, the equations of motion take the form

$$\left.\begin{aligned} A\frac{dp}{dt}+H\frac{dq}{dt}+G\frac{dr}{dt} &= -F(q^2-r^2)+(B-C)qr+Hrp-Gpq,\\ +H\frac{dp}{dt}+B\frac{dq}{dt}+F\frac{dr}{dt} &= -G(r^2-p^2)-Hqr+(C-A)rp+Fpq,\\ +G\frac{dp}{dt}+F\frac{dq}{dt}+C\frac{dr}{dt} &= -H(p^2-q^2)+Gqr-Frp+(A-B)pq. \end{aligned}\right\} \quad (1)$$

To obtain the two general integrals of this system: multiplying the equations (1) by p, q, r, respectively adding and integrating, we have for the first integral

$$Ap^2+Bq^2+Cr^2+2(Fqr+Grp+Hpq)=h, \quad . \quad . \quad . \quad . \quad (2)$$

where h is an arbitrary constant. Again, multiplying (1) by

$$\begin{aligned} &Ap+Hq+Gr,\\ &Hp+Bq+Fr,\\ &Gp+Fq+Cr, \end{aligned}$$

respectively adding and integrating, we have for the second integral

$$(Ap+Hq+Gr)^2+(Hp+Bq+Fr)^2+(Gp+Fq+Cr)^2=k^2, \quad . \quad (3)$$

where k^2 is another arbitrary constant. This equation may, however, be transformed into a more convenient form as follows: writing, as usual,

$$\left.\begin{aligned} &\mathfrak{A}=BC-F^2, \quad \mathfrak{B}=CA-G^2, \quad \mathfrak{C}=AB-H^2, \quad \nabla=\begin{vmatrix} A & H & G\\ H & B & F\\ G & F & C \end{vmatrix}.\\ &\mathfrak{F}=GH-AF, \quad \mathfrak{G}=HF-BG, \quad \mathfrak{H}=FG-CH,\\ &\qquad\qquad A+B+C=S, \end{aligned}\right\} \quad (4)$$

and bearing in mind the inverse system, viz

$$\left.\begin{aligned} &\nabla A=\mathfrak{B}\mathfrak{C}-\mathfrak{F}^2, \quad \nabla B=\mathfrak{C}\mathfrak{A}-\mathfrak{G}^2, \quad \nabla C=\mathfrak{A}\mathfrak{B}-\mathfrak{H}^2,\\ &\nabla F=\mathfrak{G}\mathfrak{H}-\mathfrak{A}\mathfrak{F}, \quad \nabla G=\mathfrak{H}\mathfrak{F}-\mathfrak{B}\mathfrak{G}, \quad \nabla H=\mathfrak{F}\mathfrak{G}-\mathfrak{C}\mathfrak{H},\\ &\qquad\qquad \mathfrak{A}+\mathfrak{B}+\mathfrak{C}=\mathfrak{S}, \end{aligned}\right\} \quad . \quad (5)$$

we may transform (3) into the following form:—

$$\left.\begin{aligned} &(AS-\mathfrak{B}-\mathfrak{C})p^2+2(FS+\mathfrak{F})qr\\ &+(BS-\mathfrak{C}-\mathfrak{A})q^2+2(GS+\mathfrak{G})rp\\ &+(CS-\mathfrak{A}-\mathfrak{B})r^2+2(HS+\mathfrak{H})pq=k^2, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (6)$$

which in virtue of (2) becomes

$$(\mathfrak{A}-\mathfrak{S})p^2+(\mathfrak{B}-\mathfrak{S})q^2+(\mathfrak{C}-\mathfrak{S})r^2+2(\mathfrak{F}qr+\mathfrak{G}rp+\mathfrak{H}pq)=k^2-Sh. \quad (7)$$

This form of the integral is very closely allied with the inverse or reciprocal form of the first integral (2), and is the one used below.

In order to find the third integral, we must find two of the variables in terms of the third by means of (2) and (7), and substitute in the corre-

sponding equation of motion. The most elegant method of effecting this is to transform (2) and (7) simultaneously into their canonical forms. If

$$\begin{matrix} \alpha & \beta & \gamma \\ \alpha_1 & \beta_1 & \gamma_1 \\ \alpha_2 & \beta_2 & \gamma_2 \end{matrix} .$$

be the coefficients of transformation, and if $\square$ be the determinant formed by them, the terms involving the products of the variables will be destroyed by the conditions

$$\left.\begin{aligned} &(\mathrm{A} \quad \ldots \quad \mathrm{F} \ldots)(\beta\beta_1\beta_2)(\gamma\gamma_1\gamma_2) = 0, \\ &(\mathrm{A} \quad \ldots \quad \mathrm{F} \ldots)(\gamma\gamma_1\gamma_2)(\alpha\alpha_1\alpha_2) = 0, \\ &(\mathrm{A} \quad \ldots \quad \mathrm{F} \ldots)(\alpha\alpha_1\alpha_2)(\beta\beta_1\beta_2) = 0, \\ &(\mathfrak{A}-\mathfrak{S} \ldots \mathfrak{F} \ldots)(\beta\beta_1\beta_2)(\gamma\gamma_1\gamma_2) = 0, \\ &(\mathfrak{A}-\mathfrak{S} \ldots \mathfrak{F} \ldots)(\gamma\gamma_1\gamma_2)(\alpha\alpha_1\alpha_2) = 0, \\ &(\mathfrak{A}-\mathfrak{S} \ldots \mathfrak{F} \ldots)(\alpha\alpha_1\alpha_2)(\beta\beta_1\beta_2) = 0, \end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

from the last two of which we have

$$\left.\begin{aligned} &\beta_1\gamma_2-\beta_2\gamma : \beta_2\gamma-\beta\gamma_2 : \beta\gamma_1+\beta_1\gamma \\ &= \mathrm{A}\alpha+\mathrm{H}\alpha_1+\mathrm{G}\alpha_2 = (\mathfrak{A}-\mathfrak{S})\alpha+\mathfrak{H}\alpha_1+\mathfrak{G}\alpha_2 \\ &: \mathrm{H}\alpha+\mathrm{B}\alpha_1+\mathrm{F}\alpha_2 : \mathfrak{H}\alpha+(\mathfrak{B}-\mathfrak{S})\alpha_1+\mathfrak{F}\alpha_2 \\ &: \mathrm{G}\alpha+\mathrm{F}\alpha_1+\mathrm{C}\alpha_2 : \mathfrak{G}\alpha+\mathfrak{F}\alpha_1+(\mathfrak{C}-\mathfrak{S})\alpha_2 ; \end{aligned}\right\} \quad . \quad . \quad (9]$$

whence, θ being a quantity to be determined,

$$\left.\begin{vmatrix} \mathfrak{A}-\mathfrak{S}-\mathrm{A}\theta, & \mathfrak{H} & -\mathrm{H}\theta, & \mathfrak{G} & -\mathrm{G}\theta \\ \mathfrak{H} & -\mathrm{H}\theta, & \mathfrak{B}-\mathfrak{S}-\mathrm{B}\theta, & \mathfrak{F} & -\mathrm{F}\theta \\ \mathfrak{G} & -\mathrm{G}\theta, & \mathfrak{F} & -\mathrm{F}\theta, & \mathfrak{C}-\mathfrak{S}-\mathrm{C}\theta \end{vmatrix} = 0. \right\} \quad . \quad (10)$$

Proceeding to develope this expression, we have the term independent of θ

$$\begin{aligned} &= \nabla^2-(\mathfrak{B}\mathfrak{C}+\mathfrak{C}\mathfrak{A}+\mathfrak{A}\mathfrak{B})\mathfrak{S}+\mathfrak{S}^3-\mathfrak{S}^2 \\ &\quad -(\mathfrak{F}^2+\mathfrak{G}^2+\mathfrak{H}^2)\mathfrak{S} \\ &= \nabla^2-\mathrm{S}\mathfrak{S}\nabla. \end{aligned}$$

The coefficient of $-\theta$

$$\begin{aligned} &= \mathrm{A}\{\nabla\mathrm{A}-(\mathfrak{B}+\mathfrak{C})\mathfrak{S}+\mathfrak{S}^2\}+\mathrm{H}(\nabla\mathrm{H}+\mathfrak{H}\mathfrak{S})+\mathrm{G}(\nabla\mathrm{G}+\mathfrak{G}\mathfrak{S}) \\ &\quad + \ldots\ldots \\ &= \nabla(\mathrm{A}^2+\mathrm{H}^2+\mathrm{G}^2)+\nabla\mathfrak{S} \\ &+ \nabla(\mathrm{H}^2+\mathrm{B}^2+\mathrm{C}^2)+\nabla\mathfrak{S} \\ &+ \nabla(\mathrm{G}^2+\mathrm{F}^2+\mathrm{C}^2)+\nabla\mathfrak{S} \\ &= \nabla\{\mathrm{A}^2+\mathrm{B}^2+\mathrm{C}^2+3(\mathrm{BC}+\mathrm{CA}+\mathrm{AB})-\mathrm{F}^2-\mathrm{G}^2-\mathrm{H}^2\} \\ &= \nabla(\mathrm{S}^2+\mathfrak{S}). \end{aligned}$$

The coefficient of $-\theta^3$

$$=\nabla.$$

Hence (dividing throughout by ∇) (10) becomes

$$\theta^3+2.S\theta^2+(S^2+\mathfrak{S})\theta+S\mathfrak{S}-\nabla=0;$$

or, what is the same thing,

$$(\theta+S)^3-S(\theta+S)^2+\mathfrak{S}(\theta+S)-\nabla=0; \quad . \quad . \quad (11)$$

or, as it may also be written,

$$\begin{vmatrix} A-(\theta+S), & H, & G \\ H, & B-(\theta+S), & F \\ G, & F, & C-(\theta+S) \end{vmatrix}=0.$$

It will be seen by reference to (9) that the values of θ determined by this equation are equal to the ratios of the coefficients of the squares of the new variables respectively in the equivalents of (2) and (7). The coefficients of transformation are nine in number; if therefore to the six equations of condition (8) we add three more, the system will be determinate.

Let three new conditions be

$$\left.\begin{aligned} (A\ldots F\ldots)(\alpha\alpha_1\alpha_2)^2&=1,\\ (A\ldots F\ldots)(\beta\beta_1\beta_2)^2&=1,\\ (A\ldots F\ldots)(\gamma\gamma_1\gamma_2)^2&=1, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

then the variable terms of (2) will take the form of the sum of three squares, and the roots of (11) will be the coefficients of the transformed expression for (7). Or, if θ, θ_1, θ_2 be the roots of (11), (2) and (7) take the forms

$$\left.\begin{aligned} p_1^2+q_1^2+r_1^2&=h,\\ \theta p_1^2+\theta_1 q_1^2+\theta_2 r_1^2&=k^2-Sh. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In order to determine the values of the coefficients of transformation α, α_1, α_2, we have from (9),

$$\left.\begin{aligned} (\mathfrak{A}-\mathfrak{S}-A\theta)\alpha+(\mathfrak{H}-H\theta)\alpha+(\mathfrak{G}-G\theta)\alpha_2&=0,\\ (\mathfrak{H}-H\theta)\alpha+(\mathfrak{B}-\mathfrak{S}-B\theta)\alpha+(\mathfrak{F}-F\theta)\alpha_2&=0,\\ (\mathfrak{G}-G\theta)\alpha+(\mathfrak{F}-F\theta)\alpha+(\mathfrak{C}-\mathfrak{S}-C\theta)\alpha_2&=0; \end{aligned}\right\} \quad . \quad . \quad . \quad (14)$$

from the last two of which

$$\begin{aligned} \alpha:\ &\mathfrak{B}\mathfrak{C}-(\mathfrak{B}+\mathfrak{C})\mathfrak{S}+\mathfrak{S}^2-(\mathfrak{B}C+\mathfrak{C}B-\overline{B+C}\mathfrak{S})+BC\theta^2\\ &-\mathfrak{F}^2 \qquad\qquad -2\mathfrak{F}F\theta \qquad\qquad -F^2\theta^2\\ =\alpha:\ &\nabla A+\mathfrak{A}\mathfrak{S}+(\overline{B+C}\mathfrak{A}+B\mathfrak{B}+C\mathfrak{C}+2F\mathfrak{F})\theta+\mathfrak{A}\theta^2\\ =\alpha:\ &\nabla A+\mathfrak{A}\mathfrak{S}+(2\nabla-H\mathfrak{H}-G\mathfrak{G}-A\mathfrak{A}+S\mathfrak{A})\theta+\mathfrak{A}\theta^2\\ =\alpha:\ &\nabla A+\mathfrak{A}\mathfrak{S}+(\nabla+S\mathfrak{A})+\mathfrak{A}\theta^2\\ =\alpha:\ &\nabla(A+\theta)+\mathfrak{A}(\mathfrak{S}+S\theta+\theta^2); \end{aligned}$$

or, writing for brevity

$$\mathfrak{S}+S\theta+\theta^2=T,$$

the expression becomes

$$\begin{aligned}
&\alpha : \nabla(A+\theta)+T\mathfrak{A}\\
&=\alpha : \mathfrak{F}\mathfrak{E}-(F\mathfrak{E}+G\mathfrak{F})\theta+FG\theta\\
&\quad -\mathfrak{C}\mathfrak{H}\mathfrak{F}+\mathfrak{H}\mathfrak{S}+(C\mathfrak{H}+H\mathfrak{S})\theta-CH\theta^2\\
&=\alpha : \nabla H+\mathfrak{H}\mathfrak{S}+S\mathfrak{H}\theta+\mathfrak{H}\theta^2\\
&=\alpha_1 : \nabla H+T\mathfrak{H}\\
&=\alpha_2 : \nabla G+T\mathfrak{E};
\end{aligned}$$

whence the system

$$\left.\begin{array}{llllll}
\alpha : \alpha_1 : \alpha_2 & & & & & \\
=\nabla(A+\theta)+T\mathfrak{A} = \nabla H & +T\mathfrak{H} = \nabla G & +T\mathfrak{E} & \\
: \nabla H \quad +T\mathfrak{H} : \nabla(B+\theta)+T\mathfrak{B} & : \nabla F & +T\mathfrak{F} & \\
: \nabla G \quad +T\mathfrak{E} : \nabla F & +T\mathfrak{F} : \nabla(C+\theta)+T\mathfrak{C}, &
\end{array}\right\} \quad . \quad (15)$$

with similar expressions for β, β_1, β_2; γ, γ_1, γ_2, obtained by writing θ_1, T_1; θ_2, T_2 respectively for θ, T,

Returning to the equations of motion (1), and transforming by the formulæ

$$\left.\begin{aligned}
p&=\alpha\, p_1+\beta\, q_1+\gamma\, r_1,\\
q&=\alpha_1 p_1+\beta_1 q_1+\gamma_1 r_1,\\
r&=\alpha_2 p_1+\beta_2 q_1+\gamma_2 r_1,
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

we have

$$\left.\begin{aligned}
&(A\alpha+H\alpha_1+G\alpha_2)p'_1=[-F(\alpha_1^2-\alpha_2^2)+(B-C)\alpha_1\alpha_2+H\alpha_2\alpha-G\alpha\alpha_1]p_1^2\\
&+(A\beta+H\beta_1+G\beta_2)q'_1+[-F(\beta_1^2-\beta_2^2)+(B-C)\beta_1\beta_2+H\beta_2\beta-G\beta\beta_1]q_1^2\\
&+(A\gamma+A\gamma_1+G\gamma_2)r'_1+[-F(\gamma_1^2-\gamma_2^2)+(B-C)\gamma_1\gamma_2+H\gamma_2\gamma-G\gamma\gamma_1]r_1^2\\
&\quad +[-2F(\beta_1\gamma_1-\beta_2\gamma_2)+(B-C)(\beta_1\gamma_2+\beta_2\gamma_1)\\
&\qquad +H(\beta_2\gamma+\beta\gamma_2)-G(\beta\gamma_1+\beta_1\gamma)]q_1r_1\\
&\quad +[-2F(\gamma_1\alpha_1-\gamma_2\alpha_2)+(B-C)(\gamma_1\alpha_2+\gamma_2\alpha_1)\\
&\qquad +H(\gamma_2\alpha+\gamma\alpha_2)-G(\gamma\alpha_1+\gamma_1\alpha)]r_1p_1\\
&\quad +[-2F(\alpha_1\beta_1-\alpha_2\beta_2)+(B-C)(\alpha_1\beta_2+\alpha_2\beta_1)\\
&\qquad +H(\alpha_2\beta+\alpha\beta_2)-G(\alpha\beta_1+\alpha_1\beta)]p_1q_1\\
&\quad =[\alpha_2(H\alpha+B\alpha_1+F\alpha_2)-\alpha_1(G\alpha+F\alpha_1+C\alpha_2)]p_1^2\\
&\quad +[\beta_2(H\beta+B\beta_1+F\beta_2)-\beta_1(G\beta+F\beta_1+C\beta_2)]q_1^2\\
&\quad +[\gamma_2(H\gamma+B\gamma+F\gamma_2)-\gamma_1(G\gamma+F\gamma_1+C\gamma_2)]r_1^2\\
&\quad +[\beta_2(H\gamma+B\gamma_1+F\gamma_2)-\beta_1(G\gamma+F\gamma+C\gamma_2)\\
&\qquad +\gamma_2(H\beta+B\beta_1+F\beta_2)-\gamma_1(G\beta+F\beta+C\beta_2)]q_1r_1\\
&\quad +[\gamma_2(H\alpha+B\alpha_1+F\alpha_2)-\gamma_1(G\alpha+F\alpha_1+C\alpha_2)\\
&\qquad +\alpha_2(H\gamma+B\gamma_1+F\gamma_2)-\alpha_1(G\gamma+F\gamma_1+C\gamma_2)]r_1p_1\\
&\quad +[\alpha_2(H\beta+B\beta_1+F\beta_2)-\alpha_1(G\beta+F\beta_1+C\beta_2)\\
&\qquad +\beta_2(H\alpha+B\alpha_1+F\alpha_2)-\beta_1(G\alpha+F\alpha_1+C\alpha_2)]p_1q_1,
\end{aligned}\right\} \quad (17)$$

with similar expressions for the two other equations. Multiplying the system so formed by $\gamma, \gamma_1, \gamma_2$ respectively and adding, the coefficients of p'_1, q'_1 will vanish, and that of r'_1 will $=1$ in virtue of (12); and as regards the right-hand side of the equation, the coefficient of p_1^2

$$= \begin{vmatrix} A\alpha+H\alpha_1+G\alpha_2, & \alpha, & \gamma \\ H\alpha+B\alpha_1+F\alpha_2, & \alpha_1, & \gamma_1 \\ G\alpha+F\alpha_2+C\alpha_3, & \alpha_2, & \gamma_2, \end{vmatrix}$$

which, omitting common factors,

$$\begin{vmatrix} (S+\theta)A+\mathfrak{A}+(S+\theta)\theta, & \nabla A+T\mathfrak{A}+\nabla\theta, & \nabla A+T_2\mathfrak{A}+\nabla\theta_2 \\ (S+\theta)H+\mathfrak{H}, & \nabla H+T\mathfrak{H}, & \nabla H+T_2\mathfrak{H} \\ (S+\theta)G+\mathfrak{G}, & \nabla G+T\mathfrak{G} & \nabla G+T_2\mathfrak{G} \end{vmatrix}$$

$$= \{(S+\theta)\theta \begin{vmatrix} \nabla H+T\mathfrak{H} & \nabla H+T_2\mathfrak{H} \\ \nabla G+T\mathfrak{G} & \nabla G+T_2\mathfrak{G} \end{vmatrix} + \nabla\theta \begin{vmatrix} \nabla H+T_2\mathfrak{H} & (S+\theta)H+\mathfrak{H} \\ \nabla G+T_2\mathfrak{G} & (S+\theta)G+\mathfrak{G} \end{vmatrix}$$
$$+ \nabla\theta_2 \begin{vmatrix} (S+\theta)H+\mathfrak{H} & \nabla H+T\mathfrak{H} \\ (S+\theta)G+\mathfrak{G} & \nabla G+T\mathfrak{G} \end{vmatrix}$$

$$= \{(S+\theta)\theta\nabla(T_2-T)+\nabla\theta\big(\nabla-T_2(S+\theta)\big)+\nabla\theta_2\big(T(S+\theta)-\nabla\big)\}(H\mathfrak{G}-\mathfrak{H}G)$$
$$= \nabla(\theta_2-\theta)\{T(S+)\theta-\nabla\}(H\mathfrak{G}-\mathfrak{H}G).$$

But

$$T(S+\theta)-\nabla = (S+\theta)(\theta^2+S\theta+\mathfrak{S})-\nabla = (S+\theta)\{(S+\theta)^2-(S+\theta)+\mathfrak{S}\}-\nabla$$
$$= (S+\theta)^3-S(S+\theta)^2+\mathfrak{S}(S+\theta)-\nabla$$
$$= 0.$$

Hence, finally, the coefficient of p_1^2 vanishes.

So likewise the coefficient of q_1^2

$$= \begin{vmatrix} A\beta+H\beta_1+G\beta_2, & \beta & \gamma \\ H\beta+B\beta_1+F\beta_2, & \beta_1 & \gamma_1 \\ G\beta+F\beta_1+C\beta_2, & \beta_2 & \gamma \end{vmatrix} = 0.$$

And that of r_1^2,

$$\begin{vmatrix} A\gamma+H\gamma_1+G\gamma_2, & \gamma & \gamma \\ H\gamma+B\gamma_1+F\gamma_2, & \gamma_1 & \gamma_1 \\ G\gamma+F\gamma_1+C\gamma_2, & \gamma_2 & \gamma_2 \end{vmatrix} = 0.$$

Similarly the coefficients of $q_1 r_1$, and $r_1 p_1$ will be found to vanish; and lastly, the coefficient of $p_1 q_1$

$$= \alpha\{A(\beta_1\gamma_2-\beta_2\gamma_1)+H(\beta_2\gamma-\beta\gamma_2)+G(\beta\gamma_1-\beta_1\gamma)\}$$
$$+\alpha_1\{H(\beta_1\gamma_2-\beta_2\gamma_1)+B(\beta_2\gamma-\beta\gamma_2)+F(\beta\gamma_1-\beta_1\gamma)\}$$
$$+\alpha_2\{G(\beta_1\gamma_2-\beta_2\gamma_1)+F(\beta_2\gamma-\beta\gamma_2)+C(\beta\gamma_1-\beta_1\gamma)\}$$
$$-\beta\{A(\gamma_1\alpha_2-\gamma_2\alpha_1)+H(\gamma_2\alpha-\gamma\alpha_2)+G(\gamma\alpha_1-\gamma_1\alpha)\}$$
$$-\beta_1\{H(\gamma_1\alpha_2-\gamma_2\alpha_1)+B(\gamma_2\alpha-\gamma\alpha_2)+F(\gamma\alpha_1-\gamma_1\alpha)\}$$
$$-\beta_2\{G(\gamma_1\alpha_2-\gamma_2\alpha_1)+F(\gamma_2\alpha-\gamma\alpha_2)+C(\gamma\alpha_1-\gamma_1\alpha)\},$$

which, by reference to (9), may be transformed into

$$\square\ \{(A\alpha+H\alpha_1+G\alpha_2)^2+(H\alpha+B\alpha_1+F\alpha_2)^2+(G\alpha+F\alpha_1+C\alpha_2)^2$$
$$-(A\beta+H\beta_1+G\beta_2)^2+(H\beta+B\beta_1+F\beta_2)^2+(G\beta+F\beta_1+C\beta_2)^2\}$$

$$=\square\ \{(A\alpha^2+B\alpha_1{}^2+C\alpha_2{}^2+2F\alpha_1\alpha_2+2G\alpha_2\alpha+2H\alpha\alpha_1)S$$
$$-(A\beta^2+B\beta_1{}^2+C\beta_2{}^2+2F\beta_1\beta_2+2G\beta_2\beta+2H\beta\beta_1)S$$
$$+(\mathfrak{A}-\mathfrak{S})(\alpha^2-\beta^2)+(\mathfrak{B}-\mathfrak{S})(\alpha_1{}^2-\beta_1{}^2)+(\mathfrak{C}-\mathfrak{S})(\alpha_2{}^2-\beta_2{}^2)$$
$$+2\mathfrak{F}(\alpha_1\alpha_2-\beta_1\beta_2)+2\mathfrak{G}(\alpha_2\alpha-\beta_2\beta)+2\mathfrak{H}(\alpha\alpha_1-\beta\beta_1)\};$$

in which the coefficient of S vanishes in virtue of (12); so that the coefficient of p_1, q_1

$$=\square\{(\mathfrak{A}-\mathfrak{S},\mathfrak{B}-\mathfrak{S},\mathfrak{C}-\mathfrak{S},\mathfrak{F},\mathfrak{G},\mathfrak{H}\rangle\!\langle\alpha,\alpha_1,\alpha_2)^2$$
$$-(\mathfrak{A}-\mathfrak{S},\mathfrak{B}-\mathfrak{S},\mathfrak{C}-\mathfrak{S},\mathfrak{F},\mathfrak{G},\mathfrak{H}\rangle\!\langle\beta,\beta_1,\beta_2)^2\};$$

but, by (12),

$$(\mathfrak{A}-\mathfrak{S},\mathfrak{B}-\mathfrak{S},\mathfrak{C}-\mathfrak{S},\mathfrak{F},\mathfrak{G},\mathfrak{H}\rangle\!\langle\alpha\alpha_1\alpha_2)^2=\theta,$$
$$(\mathfrak{A}-\mathfrak{S},\mathfrak{B}-\mathfrak{S},\mathfrak{C}-\mathfrak{S},\mathfrak{F},\mathfrak{G},\mathfrak{H}\rangle\!\langle\beta\beta_1\beta_2)^2=\theta_1.$$

Hence the coefficient in question

$$=\square(\theta-\theta_1), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and the equations of motion become

$$\left.\begin{aligned} p_1' &= \square(\theta_1-\theta_2)q_1r_1, \\ q_1' &= \square(\theta_2-\theta)r_1p_1, \\ r_1' &= \square(\theta-\theta_1)p_1q_1. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

To find the value of $\square$ in terms of A, B, C, F, G, H, we have from (12)

$$A\alpha+H\alpha_1+G\alpha_2=\square^{-1}(\beta_1\gamma_2-\beta_2\gamma_1),$$
$$A\beta+H\beta_1+G\beta_2=\square^{-1}(\gamma_1\alpha_2-\gamma_2\alpha_1),$$
$$A\gamma+H\gamma_1+G\gamma_2=\square^{-1}(\alpha_1\beta_2-\alpha_2\beta_1),$$
$$G\alpha+B\alpha_1+F\alpha_2=\square^{-1}(\beta_2\gamma-\beta\gamma_2),$$
$$H\beta+B\beta_1+F\beta_2=\square^{-1}(\gamma_2\alpha-\gamma\alpha_2),$$
$$H\gamma+B\gamma_1+F\gamma_2=\square^{-1}(\alpha_2\beta-\alpha\beta_2),$$
$$H\alpha+F\alpha_1+C\alpha_2=\square^{-1}(\beta\gamma_1-\beta_1\gamma),$$
$$G\beta+F\beta_1+C\beta_2=\square^{-1}(\gamma\alpha_1-\gamma_1\alpha),$$
$$G\gamma+F\gamma_1+C\gamma_2=\square^{-1}(\alpha\beta_1-\alpha_1\beta).$$

And forming the determinant of each side of this system, there results

$$\nabla\square=\square^{-3}\square^2,$$

or

$$\nabla\ =\square^{-2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

whence the equations of motion (19) become

$$\left.\begin{aligned} p'_1 &= \nabla^{-\frac{1}{2}}(\theta_1-\theta_2)q_1r_1, \\ q'_1 &= \nabla^{-\frac{1}{2}}(\theta_2-\theta)\,r_1p_1, \\ r'_1 &= \nabla^{-\frac{1}{2}}(\theta-\theta_1)\,p_1q_1. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

In order to compare these results with the ordinary known form, we must make

$$F=0, \quad G=0, \quad H=0,$$

$$p_1=A^{\frac{1}{2}}p, \quad q_1=B^{\frac{1}{2}}q, \quad r_1=C^{\frac{1}{2}}r;$$

which values reduce (13) to the following:

$$(A^{\frac{1}{2}}p)^2+(B^{\frac{1}{2}}q)^2+(C^{\frac{1}{2}}r)^2=h,$$
$$-(B+C)Ap^2-(C+A)Bq^2-(A+B)Cr^2=k^2-Sh;$$

which last is equivalent to

$$(A-S)(A^{\frac{1}{2}}p)+(B-S(B^{\frac{1}{2}}q)^2+(C-S)(C^{\frac{1}{2}}r)^2=k-Sh,$$

or

$$A(A^{\frac{1}{2}}p)^2+B(B^{\frac{1}{2}}q)^2+C(C^{\frac{1}{2}}r)^2=k^2.$$

Also, on the same supposition,

$$\nabla=ABC, \quad \theta=-(B+C), \quad \theta_1=-(C+A), \quad \theta_2=-(A+B),$$

which, when substituted in the above, give

$$Ap'^{\frac{1}{2}}=(ABC)^{-\frac{1}{2}}(B-C)B^{\frac{1}{2}}C^{\frac{1}{2}}qr, \quad B^{\frac{1}{2}}q'=\ldots, \quad C^{\frac{1}{2}}r'=\ldots,$$

or

$$Ap'=(B-C)qr, \quad Bq'=(C-A)rp, \quad Cr'=(A-B)pq,$$

as usual.

It remains only to determine the absolute values of the coefficients of transformation, the ratios of which are given in (15). For this purpose let

$$\left.\begin{aligned} \nabla(A+\theta_0)+T_0\mathfrak{A}&=\mathfrak{A}_0, & \nabla F+T_0\mathfrak{F}&=\mathfrak{F}_0, \\ \nabla(B+\theta_0)+T_0\mathfrak{B}&=\mathfrak{B}_0, & \nabla G+T_0\mathfrak{G}&=\mathfrak{G}_0, \\ \nabla(C+\theta_0)+T_0\mathfrak{C}&=\mathfrak{C}_0, & \nabla H+T_0\mathfrak{H}&=\mathfrak{H}_0. \end{aligned}\right\} \quad . \quad . \quad . \quad (22)$$

Then, from (15),

$$\alpha=\frac{\mathfrak{A}_0}{(A\ldots H\ldots)(\mathfrak{A}_0\mathfrak{H}_0\mathfrak{G}_0)^2}=\frac{\mathfrak{H}_0}{(A\ldots)(\mathfrak{H}_0\mathfrak{B}_0\mathfrak{F}_0)^2}=\frac{\mathfrak{G}_0}{(A\ldots)(\mathfrak{G}_0\mathfrak{F}_0\mathfrak{C}_0)^2},$$

$$\alpha_1=\frac{\mathfrak{H}_0}{(A\ldots H\ldots)(\mathfrak{A}_0\mathfrak{H}_0\mathfrak{G}_0)^2}=\frac{\mathfrak{B}_0}{(A\ldots)(\mathfrak{H}_0\mathfrak{B}_0\mathfrak{F}_0)^2}=\frac{\mathfrak{F}_0}{A\ldots)(\mathfrak{G}_0\mathfrak{F}_0\mathfrak{C}_0)^2},$$

$$\alpha_2=\frac{\mathfrak{G}_0}{(A\ldots H\ldots)(\mathfrak{A}_0\mathfrak{H}_0\mathfrak{G}_0)^2}=\frac{\mathfrak{F}_0}{(A\ldots)(\mathfrak{H}_0\mathfrak{B}_0\mathfrak{F}_0)^2}=\frac{\mathfrak{C}_0}{(A\ldots)(\mathfrak{G}_0\mathfrak{H}_0\mathfrak{C}_0)^2}.$$

From these relations it follows that

$$\left.\begin{aligned} \mathfrak{B}_0\mathfrak{C}_0-\mathfrak{F}_0^2=0, \quad & \mathfrak{G}_0\mathfrak{H}_0-\mathfrak{A}_0\mathfrak{F}_0=0, \\ \mathfrak{C}_0\mathfrak{A}_0-\mathfrak{G}_0^2=0, \quad & \mathfrak{H}_0\mathfrak{F}_0-\mathfrak{B}_0\mathfrak{G}_0=0, \\ \mathfrak{A}_0\mathfrak{B}_0-\mathfrak{H}_0^2=0, \quad & \mathfrak{F}_0\mathfrak{G}_0-\mathfrak{C}_0\mathfrak{H}_0=0, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (23)$$

which relations may be also verified as follows:—

$$\begin{aligned} \mathfrak{G}_0\mathfrak{H}_0-\mathfrak{A}_0\mathfrak{F}_0 &= (\nabla G+T_0\mathfrak{G})(\nabla H+T_0\mathfrak{H})-(\nabla A+T_0\mathfrak{A}+\nabla\theta_0)(\nabla F+T_0\mathfrak{F}) \\ &= \nabla^2\mathfrak{F}+\nabla T_0(G\mathfrak{H}+H\mathfrak{G}-A\mathfrak{F}-F\mathfrak{A})+T_0^2\nabla F-\nabla\theta_0(\nabla F+T_0\mathfrak{F}) \\ &\quad \nabla\{\nabla\mathfrak{F}-T_0(S\mathfrak{F}+\mathfrak{S}F)+T_0^2F-\nabla\theta_0F-\nabla\mathfrak{F}+ST_0\mathfrak{F}\mathfrak{G}\}; \end{aligned}$$

Since

$$G\mathfrak{H}+F\mathfrak{B}+C\mathfrak{F}=0,$$

$$H\mathfrak{G}+B\mathfrak{F}+F\mathfrak{C}=0,$$

and

$$(\theta+S)T-\nabla=0,$$

or

$$\theta T=\nabla-ST.$$

Hence

$$\begin{aligned} \mathfrak{G}_0\mathfrak{H}_0-\mathfrak{A}_0\mathfrak{F}_0 &= \nabla F\{T_0^2-T_0\mathfrak{S}-\nabla\theta_0\} \\ &= \nabla F\{T\theta_0(S+\theta_0)-\nabla\theta_0\} \\ &= 0. \end{aligned}$$

From these relations it follows that the first denominator, viz.

$$\begin{aligned} &(A, B, C, F, G, H,)(\mathfrak{A}_0\mathfrak{H}_0\mathfrak{G}_0)^2 \\ &=A\mathfrak{A}_0^2+B\mathfrak{H}_0^2+C\mathfrak{G}_0^2+r(F\mathfrak{H}_0\mathfrak{G}_0+G\mathfrak{G}_0\mathfrak{A}_0+H\mathfrak{A}_0\mathfrak{H}_0) \\ &=\mathfrak{A}_0\{A\mathfrak{A}_0+B\mathfrak{B}_0+C\mathfrak{C}_0+2(F\mathfrak{F}_0+G\mathfrak{G}_0+H\mathfrak{H}_0\} \\ &=\mathfrak{A}_0\nabla\{A^2+B^2+C^2+2(F^2+G^2+H^2)+3T_0+S\theta_0\} \\ &=\mathfrak{A}_0\nabla\{S^2-2\mathfrak{S}+3T_0+S\theta_0\} \\ &=\mathfrak{A}_0\nabla\ 3\theta_0^2+4S\theta_0+\mathfrak{S}+S^2\} \\ &=\mathfrak{A}_0\nabla\{(S+\theta_0)(S+3\theta_0)+\mathfrak{S}\}. \end{aligned}$$

Hence, writing $(S+\theta_0)(S+3\theta_0)+S=\mathfrak{T}_0$, we have, finally,

$$\alpha=\frac{1}{\mathfrak{T}_0},\ \alpha_1=\frac{\mathfrak{H}_0}{\mathfrak{A}_0\mathfrak{T}_0},\ \alpha_2=\frac{\mathfrak{G}_0}{\mathfrak{A}_0\mathfrak{T}_0}.$$

From this we may obtain the following system:

$$\left.\begin{aligned} \alpha=\frac{1}{\mathfrak{T}_0},\ \alpha_1=\frac{\mathfrak{H}_0}{\mathfrak{A}_0\mathfrak{T}_0},\ \alpha_2=\frac{\mathfrak{G}_0}{\mathfrak{A}_0\mathfrak{T}_0} \\ =\frac{\mathfrak{B}_0}{\mathfrak{H}_0\mathfrak{T}_0} \quad =\frac{\mathfrak{F}_0}{\mathfrak{H}_0\mathfrak{T}_0} \\ =\frac{\mathfrak{F}_0}{\mathfrak{G}_0\mathfrak{T}_0} \quad =\frac{\mathfrak{C}_0}{\mathfrak{G}_0\mathfrak{C}_0}, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (24)$$

with similar expressions for β, β_1, β_2; $\gamma, \gamma_1, \gamma_2$, obtained by writing the suffixes 1 and 2 respectively for 0. By means of these we may write the equations connecting the variables as follow:—

$$\left.\begin{aligned} p &= \frac{1}{\mathfrak{T}_0}\, p_1 + \frac{1}{\mathfrak{T}_1}\, q_1 + \frac{1}{\mathfrak{T}_2} r_1, \\ q &= \frac{\mathfrak{H}_0}{\mathfrak{A}_0 \mathfrak{T}_0} p_1 + \frac{\mathfrak{B}_1}{\mathfrak{H}_1 \mathfrak{T}_1} q_1 + \frac{\mathfrak{F}_2}{\mathfrak{G}_2 \mathfrak{T}_2} r_1, \\ r &= \frac{\mathfrak{G}_0}{\mathfrak{A}_0 \mathfrak{T}_0} p_1 + \frac{\mathfrak{F}_1}{\mathfrak{H}_0 \mathfrak{T}_1} q_1 + \frac{\mathfrak{C}_2}{\mathfrak{G}_2 \mathfrak{T}_2} r_1. \end{aligned}\right\} \quad . \quad . \quad . \quad (25)$$

Lastly, to complete the transformations, the values of p_1, q_1, r_1 should be determined in terms of p, q, r. Now

$$\begin{aligned} \mathfrak{A}_0\mathfrak{H}_1 + \mathfrak{H}_0\mathfrak{B}_1 + \mathfrak{G}_0\mathfrak{F}_1 &= (\nabla A + T_0\mathfrak{A} + \nabla\theta_0)(\nabla H + T_1\mathfrak{H}) \\ &\quad + (\nabla H + T_0\mathfrak{H})(\nabla B + T_1 B + T\theta_1) \\ &\quad + (\nabla G + T_0\mathfrak{G})(\nabla F + T_1\mathfrak{H}) \\ &= \nabla^2\{(A+B)H + FG\} + T_0T_1\{(\mathfrak{A}+\mathfrak{B})\mathfrak{H} + \mathfrak{F}\mathfrak{G}\} + \nabla^2 H(\theta_0+\theta_1) \\ &\quad + \nabla\mathfrak{H}(\theta_0 T_1 + \theta_1 T_0) \\ &= \nabla^2(SH + \mathfrak{H}) + T_0T_1(\mathfrak{S}\mathfrak{H} + \nabla H) + \nabla^2 H(\theta_0+\theta_1) + \nabla\mathfrak{H}(\theta_0 T_1 + \theta_1 T_0) \\ &= \nabla\{\nabla(S+\theta_0+\theta_1) + T_0T_1\}H + (\nabla\theta_0 T_1 + \nabla\theta_1 T_0 + \mathfrak{S}T_0T_1 + \nabla^2) \\ &= T_0T_1\{[-(S+\theta_0)(S+\theta_1)(S+\theta_2) + \nabla]H + [\theta_0(S+\theta_0) + \theta_1(S+\theta_1) \\ &\quad + S + (S+\theta_0)(S+\theta_1)]\mathfrak{H}\}, \end{aligned}$$

since

$$\nabla = T_0(S+\theta_0) = T_1(S+\theta_1) = T_2(S+\theta_2).$$

Moreover by (11) we have

$$(S+\theta_0)(S+\theta_1)(S+\theta_2) = \nabla,$$

and consequently the coefficient of H vanishes. And it may be noticed, as a useful formula for verification, that, from the relations last above written, we may at once deduce the following:

$$T_0T_1T_2 = \nabla^2.$$

Again, the coefficient of $\mathfrak{H}$ may be thus written:

$$\begin{aligned} &(S+\theta_0+\theta_2)(S+\theta_0) + (S+\theta_0+\theta_1)(S+\theta_1) + S + (S+\theta_0)(S+\theta_1) \\ &- \quad (S+\theta_2)(S+\theta_0) - \quad (S+\theta_0)(S+\theta_1) \\ &= - \ (S+\theta_1)(S+\theta_0) - (S+\theta_2)(S+\theta_1) - (S+\theta_0)(S+\theta_2) + S \\ &= 0, \end{aligned}$$

in virtue of (11). Hence the whole expression vanishes, or

$$\mathfrak{A}_0\mathfrak{H}_1+\mathfrak{H}_0\mathfrak{B}_1+\mathfrak{G}_0\mathfrak{F}_1=0; \quad \ldots\ldots (26)$$

and similarly

$$\mathfrak{A}_0\mathfrak{G}_2+\mathfrak{H}_0\mathfrak{F}_2+\mathfrak{G}_0\mathfrak{C}_2=0.$$

Moreover, in virtue of (23), we have

$$\mathfrak{A}_0^2+\mathfrak{H}_0^2+\mathfrak{G}_0^2=\mathfrak{A}_0\mathfrak{S}_0.$$

Hence multiplying (25) first by $\mathfrak{A}_0$, $\mathfrak{H}_0$, $\mathfrak{G}_0$ respectively and adding,

secondly by $\mathfrak{H}_1$, $\mathfrak{B}_1$, $\mathfrak{F}_1$,

thirdly by $\mathfrak{G}_2$, $\mathfrak{F}_2$, $\mathfrak{C}_2$,

we shall obtain the inverse system

$$\left.\begin{aligned}\frac{\mathfrak{S}_0}{\mathfrak{T}_0}p_1&=\mathfrak{A}_0p+\mathfrak{H}_0q+\mathfrak{G}_0r,\\ \frac{\mathfrak{S}_1}{\mathfrak{T}_1}q_1&=\mathfrak{H}_1p+\mathfrak{B}_1q+\mathfrak{F}_1r,\\ \frac{\mathfrak{S}_2}{\mathfrak{T}_2}r_1&=\mathfrak{G}_2p+\mathfrak{F}_2q+\mathfrak{C}_2r.\end{aligned}\right\} \quad \ldots\ldots (27)$$

Returning to the integrals (13), we derive

$$\begin{aligned}(\theta_1-\theta)q_1^2+(\theta_2-\theta)r_1^2&=k^2-(\mathrm{S}+\theta)h,\\ (\theta_2-\theta_1)r_1^2+(\theta-\theta_1)p_1^2&=k^2-(\mathrm{S}+\theta_1)h,\\ (\theta-\theta_2)p_1^2+(\theta_1-\theta_2)q_1^2&=k^2-(\mathrm{S}+\theta_2)h.\end{aligned}$$

Let

$$p_1=\sqrt{\frac{k^2-(\mathrm{S}+\theta_2)h}{\theta-\theta_2}}\cos\chi;$$

then

$$q_1=\sqrt{\frac{k^2-(\mathrm{S}+\theta_2)h}{\theta_1-\theta_2}}\sin\chi;$$

and

$$\begin{aligned}r_1&=\sqrt{\frac{k^2-(\mathrm{S}+\theta)h}{\theta_2-\theta}}\sqrt{1-\frac{\theta_1-\theta}{k^2-(\mathrm{S}+\theta)h}q_1^2}\\ &=\sqrt{\frac{k^2-(\mathrm{S}+\theta)h}{\theta_2-\theta}}\sqrt{1-\frac{\theta_1-\theta}{\theta_1-\theta_2}\frac{k^2-(\mathrm{S}+\theta_2)h}{k^2-(\mathrm{S}+\theta)h}\sin^2\chi}.\end{aligned}$$

Substituting in the equations of motion (21) (*e. g.* the first of them) and dividing throughout by $\sin\chi\sqrt{k^2-(\mathrm{S}+\theta_2)h}$, we have

$$\frac{1}{\sqrt{\theta-\theta_2}}\frac{d\chi}{dt}+\frac{1}{\sqrt{\theta_1-\theta_2}}\sqrt{\frac{k^2-(S+\theta)h}{\theta_2-\theta}}\sqrt{1-\frac{\theta_1-\theta}{\theta_1-\theta_2}\frac{k^2-(S+\theta)h}{k^2-(S+\theta)h}\sin^2\chi},$$

or

$$\frac{d\chi}{dt}+\nabla^{-\frac{1}{2}}(\theta_1-\theta_2)\sqrt{\frac{k^2-(S+\theta)h}{\theta_2-\theta_1}}\sqrt{1-\frac{\theta-\theta_1}{\theta_2-\theta_1}\frac{k^2-(S+\theta_2)h}{k^2-(S+\theta)h}\sin^2\chi},$$

or

$$\frac{d\chi}{\sqrt{1-\frac{\theta_1-\theta}{\theta_1-\theta_2}\frac{k^2-(S+\theta_2)h}{k^2-(S+\theta)h}\sin^2\chi}}=\sqrt{\frac{\theta_2-\theta}{\nabla}}_1\sqrt{k^2-(S+\theta)h}dt;$$

then

$$\chi=am\left(\sqrt{\frac{\theta_2-\theta}{\nabla}}_1\sqrt{k^2-(S+\theta)h}t+f\right),$$

and

$$p_1=\sqrt{\frac{k^2-(S+\theta_2)h}{\theta-\theta_2}}\cos am\left(\sqrt{\frac{\theta_2-\theta_1}{\nabla}}\sqrt{k^2-(S+\theta)h}t+f\right),$$

$$q_1=\sqrt{\frac{k^2-(S+\theta_2)h}{\theta_1-\theta_2}}\sin am\left(\sqrt{\frac{\theta_2-\theta_1}{\nabla}}\sqrt{k^2-(S+\theta)h}t+f\right),$$

$$r_1=\sqrt{\frac{k^2-(S+\theta)h}{\theta_2-\theta}}\Delta am\left(\sqrt{\frac{\theta_2-\theta_1}{\nabla}}\sqrt{k^2-(S+\theta)h}t+f\right).$$

These, then, are the integrals of the equations of motion when no external forces are acting. The next step is to determine the variations of the arbitrary constants, due to the action of disturbing forces, when, as in the case of nature, those forces are small. With a view to this, it will be convenient to change the arbitrary constants into the following,

$$\sqrt{k^2-(S+\theta_2)h}=m \quad \sqrt{k^2-(S+\theta)h}=n,$$

whence

$$(\theta-\theta_2)h=m^2-n^2,$$

$$(\theta-\theta_2)k^2=(S+\theta)m^2-(S+\theta_2)n^2;$$

also, for brevity, let

$$\sqrt{\frac{\theta_2-\theta_1}{\nabla}}=l, \quad am(lnt+f)=\chi, \quad \frac{\theta_1-\theta}{\theta_1-\theta_2}\frac{m}{n^2}=k_1^2.$$

Then the equations of motion become

$$p_1=\frac{m}{\sqrt{\theta-\theta_2}}\cos am(lnt+f),$$

$$q_1=\frac{m}{\sqrt{\theta_1-\theta_2}}\sin am(lnt+f),$$

$$r_1=\frac{n}{\sqrt{\theta_2-\theta}}\Delta am(lnt+f).$$

Now it is known by the theory of elliptic functions that

$$\frac{d\cos am x}{dx} = -\sin am\, x \Delta\, am\, x,$$

$$\frac{d\sin am\, x}{dx} = \cos am\, x \Delta\, am\, x,$$

$$\frac{d\Delta\, am\, x}{dx} = -k_1^2 \sin am\, x \cos am\, x.$$

Whence P_1, Q_1, R_1 being the moments of the disturbing forces about the present axes,

$$P_1 = \frac{1}{\sqrt{\theta - \theta_2}}\left\{\cos\chi\frac{dm}{dt} - m\sin\chi\Delta\chi\left(lt\frac{dn}{dt} + \frac{df}{dt}\right)\right\},$$

$$Q_1 = \frac{1}{\sqrt{\theta_1 - \theta_2}}\left\{\sin\chi\frac{dm}{dt} + m\cos\chi\Delta\chi\left(lt\frac{dn}{dt} + \frac{df}{dt}\right)\right\},$$

$$R_1 = \frac{1}{\sqrt{\theta_2 - \theta}}\left\{\Delta\chi\frac{dn}{dt}nk_1^2\sin\chi\cos\chi\left(lt\frac{dn}{dt} + \frac{df}{dt}\right)\right\}.$$

From these we derive

$$\frac{dm}{dt} = \sqrt{\theta - \theta_2}P_1\cos\chi + \sqrt{\theta_1 - \theta_2}\,Q_1\sin\chi,$$

$$m\Delta\chi\left(lt\frac{dn}{dt} + \frac{df}{dt}\right) = -\sqrt{\theta - \theta_2}P_1\sin\chi + \sqrt{\theta_1 - \theta_2}\,Q_1\cos\chi,$$

$$\Delta\chi\frac{dn}{dt} = \sqrt{\theta_2 - \theta}R_1 + \frac{n}{m}k_1^2\frac{\sin\chi\cos\chi}{\Delta\chi}\{-\sqrt{\theta - \theta_2}P_1\sin\chi + \sqrt{\theta_1 - \theta_2}Q_1\cos\chi\},$$

or

$$\frac{dn}{dt} = \sqrt{\theta_2 - \theta}\frac{R_1}{\Delta\chi} + \frac{\theta_1 - \theta}{\theta_1 - \theta_2}\frac{m}{n}\frac{\sin\chi\cos\chi}{(\Delta\chi)^2}\{-\sqrt{\theta - \theta_2}P_1\sin\chi + \sqrt{\theta_1 - \theta_2}\,Q_1\cos\chi\}$$

$$= \sqrt{\theta_2 - \theta}\frac{R_1}{\chi} + \frac{\theta_1 - \theta}{\theta_1 - \theta_2}\frac{1}{n}\frac{\sin\chi\cos\chi}{(\Delta\chi)^2}\{-\sqrt{\theta - \theta_2}P_1\sin\chi + \sqrt{\theta_1 - \theta_2}\,Q_1\cos\chi\}$$
$$\int\{\sqrt{\theta - \theta_2}P_1\cos\chi + \sqrt{\theta_1 - \theta_2}\,Q_1\sin\chi\}dt.$$

And lastly,

$$\frac{df}{dt} = -lt\frac{dn}{dt} - \frac{1}{\Delta\chi}\frac{-\sqrt{\theta - \theta_2}P_1\sin\chi + \sqrt{\theta_1 - \theta_2}\,Q_1\cos\chi}{\int\{\sqrt{\theta - \theta_2}P_1\cos\chi + \sqrt{\theta_1 - \theta_2}\,Q_1\sin\chi\}dt}.$$

"Experiments, made at Watford, on the Vibrations occasioned by Railway Trains passing through a Tunnel." By Sir James South, LL.D., F.R.S., &c., one of the Visitors of the Royal Observatory of Greenwich. Received June 17, 1863*.

In the year 1846 an attempt was made to obtain the consent of the Lords of the Admiralty to run a railway through Greenwich Park, distant only 860 feet from the Royal Observatory, which would in the opinion of many competent judges have been most injurious to that Establishment. Such consent their Lordships refused; but as I was assured on high authority that this attempt was to be repeated, and that too with the fullest confidence of success on the part of its projectors and supporters, I determined to make experiments which might bear more decisively on the question of railway tremors, as affecting that Observatory, than those previously made by myself and others.

For this purpose it seemed indispensable that the station selected for making them should geologically resemble that of Greenwich, and that the astronomical means employed to detect the existence and determine the intensity of the tremors should be, optically, at least equal to the telescope of the Greenwich Mural Circle.

As much importance was attributed by the advocates of this railway to the supposed power of a tunnel to render the vibrations imperceptible, it was also desirable that *it* should be one of the conditions of these trials.

Having but little more than a popular knowledge of geology, I relied on my old and valued friend the late Mr. Warburton, who had recently been President of the Geological Society, to guide me in the choice of a station; and it was on his authority that I fixed on the Watford Tunnel and its immediate vicinity.

There, under a light gravelly soil of 18 or 20 inches deep, lies a bed of gravel of considerable but variable thickness, sometimes compact, at other times loose, and immediately under it chalk with occasional flints.

The tunnel, of which the bearing is 41° 19′ to N.W. of the meridian, and by my measurement is 1812 yards long, passes principally through chalk; its arch is about 24 feet in diameter, the crown of it being about 21·5 feet above the rails. The thickness of the brickwork is about 18 inches; the mean thickness of the chalk above the crown of the arch about 50 feet, whilst that of the gravel, though subject to great irregularity, may perhaps be regarded as 14 feet. If so, we have outside the tunnel above the horizontal plane of the rails 87 feet of chalk, flint, gravel and soil, constituting an assemblage of which the power of transmitting tremors must be comparatively feeble.

There are five shafts in the tunnel, four of which are circular, 8·5 feet diameter, and one quadrangular, about 26 feet by 34.

* Read June 18, 1863: see Abstract, vol. xii. p. 630.

The tunnel runs under the park of the Earl of Essex; and though I had not the honour of a personal acquaintance with the Noble Earl, nor any introduction to him, yet on learning my objects he transmitted to me by return of post, from Carlsbad, a carte-blanche to erect my observatory wherever I pleased, though it were in the very heart of his choicest game preserves. To him therefore is mainly due whatever benefit may accrue to science or to the Royal Observatory from the experiments recorded in this communication.

The point I selected was 302 yards distant from the centre of the line; and the perpendicular from it on the axis of the tunnel meets that at a point 567 yards from the southern or London end of the tunnel, 1245 yards from the Tring or north end, and 594·5 from the fourth shaft. This is the centre of the Observatory which I erected there: it is of wood, as small as is consistent with the necessary accommodation, both for portability and that it might be less agitated by the wind.

It is quadrangular, 12 feet by 10, and its length is in the meridian; the eaves are 8·5 feet, and the ridge of the roof 10 feet above the floor, this last being 4 inches above the ground, which is nearly level with that over the tunnel. The roof is covered with tarpaulins very well secured, so as not to be torn by a gale of wind. In the south and west sides are four windows, which can be opened or shut at pleasure, to light the Observatory by day, or to see powder or other signals at night. In the roof is no opening; but in its northern side there is one which can be shut as required: it is little larger than what is absolutely necessary to allow the reflected rays from the Pole-star to pass uninterruptedly to the observer's eye through its whole revolution.

At its centre, parallel with its sides and resting on the undisturbed gravel 4 feet below the surface, is a mass of brickwork laid in excellent Roman cement, 8 by 3·5 feet at bottom, 7 by 3·5 at top, its length running east and west. On this stand two piers of similar brickwork, 18 inches by 14, and 46 inches higher than the floor: they are capped by two Portland stones of similar horizontal section 8 inches thick. In the interior faces of these stones are firmly fixed the Y-plates, which carry the Ys on which the instrument's pivots rest.

Eighteen inches north of the brick massive, but in the same plane with its base, is the centre of the base of another pier, brought up also in Roman cement, 24 inches from N. to S., 18 from E. to W.; and it rises 12 inches above the floor. The upper surface is perfectly horizontal, and serves to support a vessel which contains mercury. Both this pier and the massives are insulated from the floor, and touch the ground only at their bases. The mercury-vessel was 18 inches by $4\frac{3}{4}$, with its length in the meridian.

The transit-instrument of the Campden Hill Observatory is far too precious to be exposed to the risks of such an expedition; I therefore had one constructed which might be considered an excellent substitute. The object-glass (which under favourable circumstances will bear a power of 1000) is 87 inches focus and 4·75 aperture. The transverse axis is 31 inches; and

the Y has sufficient azimuthal motion to enable me to follow the Pole-star in its whole course, so that at any hour (if clear) I could have the reflected image of the star in the mercurial vessel ready to testify against the tremors caused by a train.

Supported by timber passing into the ground, but unconnected with the floor and convenient to a writing-desk which occupies the S.E. angle of the building, stands a journeyman clock. It is set by my excellent gold pocket-chronometer, Molyneux No. 963, and rarely deviates from that more than one- or two-tenths of a second in three or four hours. The clock of the Watford Station was compared with the chronometer, going and generally returning, for the purpose of identifying particular trains.

These details will, I hope, suffice to prove that every precaution was taken to obtain accurate results, and that those which I did obtain may be fairly considered as identical with what would have been found in a first-class observatory under the same circumstances of locality and traffic.

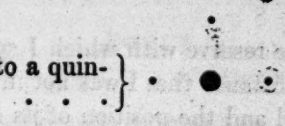
I was at my post to commence observations on December 22nd, 1846; but that and the three following nights were starless. The 26th was fine, but, owing to the irregularity of the trains, and the want of well-organized signals, I could only satisfy myself that all was in good working order, and that the trains caused great disturbance. For thirteen following nights I was at my post, but in vain; all was dark, with the thermometer from 22° to 31°.

On January 11th, 1847, it cleared, and I observed seven trains with decisive results, being able to *announce* their presence before it was known to my assistants, who were on the watch outside the observatory.

The Pole-star's image as reflected from the mercurial surface, when no train was near, appeared

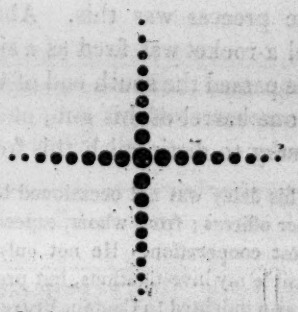
As a very small, perfectly steady disk, thus— . . . . . ● (1.)

which as the train approached broke up into a quintuple, thus— . . . . . (2.)



As the disturbance increased, the form became linear at right angles to the length of the mercury-vessel, thus— . . . (3.)

When the train was considerably advanced in the tunnel, a cross formed, thus— . (4.)



And when near the perpendicular from the observatory, three parallel lines of disks appeared, thus— . . . . . (5.)

still parallel to No. 3. As the tremors became more distant, these transformations of the image take place in a reverse order, until the star resumes its original disk-like form.

These results were strongly conspicuous even in a fully illuminated field, and equally so whether the magnifying power was 60, 200, or 750. The phenomena are very striking, from the contrast between the smaller images, which are blue, while the larger ones are reddish, and from the sudden way in which they break out.

The nights of the 13th and 14th were fine, and so thoroughly confirmed my previous observations that I felt it my duty to lose no time in informing the late Lord Auckland, then First Lord of the Admiralty, of the preceding details and of my conclusions from them, that a tunnel did not prevent great tremors from being propagated from it when a train was traversing it, certainly to the distance of 643 yards, and probably much further.

The impression which these facts made on his Lordship he expressed in the following letter.

Copy of a Letter from the Earl of Auckland to Sir James South.

"Admiralty, January 26th, 1847.

"Sir,—I have to return you many thanks for the very interesting report which you sent to me of your experiments upon the distance to which the vibration caused by steam-carriages within a tunnel extend; and I cannot but admire the enterprise and ability with which these experiments were conducted. They would be quite conclusive if the question of carrying a tunnel through Greenwich Park were again to be agitated.

"I am, very faithfully yours,

"*To Sir James South, &c. &c.*" "Auckland."

The reserve with which I spoke of that further distance arose from the circumstance that I was not in possession of the exact measurements of the tunnel and the position of its shafts. I had twice applied for them in vain to the railway authorities, and was obliged at last to execute the measures myself*. This consumed some time, and the observations were not completely resumed till February 24, 1847.

The process was this. About 600 yards before the entrance of the tunnel a rocket was fired as a signal for attention. At the instant that the engine passed the south end of the tunnel, one of Lord Essex's game-keepers fired one barrel of his gun, and the other about a second after, which was necessary to distinguish this from the shots of poachers, who were often at

* This delay was not occasioned by any want of courtesy on the part of the Directors or other officers; from whom, especially from Mr. Creed, their Secretary, I received the heartiest cooperation. He not only directed all the officers along the line to aid as far as possible my investigations, but pressed on me free passes for myself and my assistants. I was also indebted to Captain Bruyeres for the character of the trains, and to Mr. Stubbs, the Superintendent of the Watford Station, for the zeal with which he followed out the Secretary's instructions at much personal inconvenience.

work around me. Similar shots were fired when the engine was at the centre of the 4th shaft (which could be seen from above). The times of these signals were taken by an assistant. During this time I was at the telescope, and noticed the second when any peculiar phase of disturbance appeared.

The computation of the distance of the engine from the eye at a given time is very simple. From the known distance of the south end of the tunnel and the 4th shaft from the eye, we know the times taken by the sound of the gun to reach the observatory. The temperature was during the whole series so near 32° that the velocity of sound for that temperature, 363·13 yards, may be used without sensible error. The effect of wind must also have been insensible. Hence the signal from the south entrance was $1^{s}\cdot77$ too late, that from the shaft $1^{s}\cdot84$.

Correcting the times and dividing by their difference the distance of the shaft from the entrance, 1162 yards, we have the velocity of the train (which, however, I have given in miles per hour, as affording a more familiar measure of the disturbing power). Then the difference of the time of phase and corrected time of entrance gives the place of the engine on the line, and the perpendicular is given.

In the following record of the observations, the first column contains the number, the second the times, the third the facts observed, and the fourth gives the distance, then follow occasional remarks. In the disturbances, I specially recorded as most definite the cross (4), and the arrangement of bars of parallel stars (5). The slighter disturbances which precede or follow the former were seldom entered, though quite sensible.

1847, February 24.—I.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
1	7 18 43	Cross very distinct.....	845	Velocity 11·00, miles an hour; weight of train 77·5 tons; twelve carriages.
	7 19 21	Shaft gun.		
	7 22 57	South gun.		
2	7 23 8	Lost sight of cross.....	704	

II.

3	7 34 0	Cross	699	Velocity 16·6 miles; train 69·5 tons, 231 feet long; ten carriages. Thermometer 24°.
	34 8	Shaft gun.		
	36 31	South gun.		
4	36 48	Lost sight of cross.....	780	

III.

5	7 44 40	Cross; star very faint...	680	Velocity 13·8 miles. Star invisible to the naked eye. Train 58·5 tons; engine 14·5 tons; length 185 feet.
	44 44	Shaft gun.		
	47 38	South gun.		
6	7 47 42	Lost cross	678	

1847, February 24.—IV.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
	7 59 6	Shaft gun.		Velocity 11·4 miles; train 89·5 tons; engine 18 ditto; length 308 feet. Wind E. Therm. 24°.
	8 2 30	Star became visible.		
	8 2 34	South gun; star bright.		
7	8 3 10	Cross disappeared	834·5	

1847, February 27.—I.

	7 28 0	Shaft gun.		Velocity 15·4 miles; train 54 tons; engine 14·5 tons; length of train 172 feet.
	29 7	Cross first seen, but star very faint.		
	30 34	South gun.		
8	7 30 44	Cross lost; star very faint.....	722	

II.

9	7 44 43	Cross seen	736	Velocity 25·6 miles; train 49·5 tons; engine 14·5 tons; length of train 150 feet.
	7 44 51	Shaft gun.		
10	45 8	Cross very strong	470	
11	45 27	Line very strong	326	
	46 24	South gun.		
12	46 46	Cross lost	915	

III.

13	7 56 21	Cross seen	706	Velocity 17·6 miles; train 270·5 tons; two engines 29·5 tons; length of train 663 feet; 37 carriages.
	56 31	South gun.		
14	57 45	Cross very strong	314	
	58 46	Shaft gun.		
15	58 53	Cross lost	736	

IV.

16	8 3 36	Cross very strong	736	Velocity 31·7 miles; train 112 tons; engine 21 tons; length of train 394 feet; carriages 17. Wind N.E. Thermometer 26°.
	3 44	Shaft gun.		
17	4 6	Cross very fine	377	
18	4 14	Triple line, upper and lower stars blue	319	
	4 59	South gun.		
19	5 28	Cross lost	1086	

V.

20	8 10 56	Cross seen	727	Velocity 18·7 miles; train 51·5 tons; engine 12·5 tons; length of train 187 feet. A train of empty cattle-waggons.
	11 8	South gun.		
21	11 56	Triple line strong	322	
	13 15	Shaft gun.		
		Cross lost from cloud...	lost	

1847, March 11.—I.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
22	7 18 44	Cross very distinct.....	802	Velocity 17·7; train 147·5 tons; engine 12·5 tons; length of train 355 feet.
	19 6	South gun.		
	21 20	Shaft gun, cloud.		

II.

	8 25 3	Shaft gun, cloud.		Velocity 33·0 miles; train 122 tons; engine 21 tons; length of train 416 feet. Cross so strong, but for the cloud it might have been seen 15 or even more seconds longer.
	26 15	South gun, cloud cleared		
23	26 32	Cross lost by cloud.....	921	

1847, March 12.—I.

24	6 56 22	Cross very distinct.....	822	Velocity 28·33 miles; train 68 tons; engine 15 tons; length of train 231 feet; many carriages but mostly empty, many wheels and axles; agitation excessive. Seemed to keep time with the jolts of the train.
	56 38	South gun.		
25	56 52	Cross very strong	461	
26	57 16	Star tossed about 3 or 4 of its diameters	302·4	
	58 2	Shaft gun.		
27	58 8	Cross lost	766	

II.

28	7 13 15	Cross plain	811	Velocity 35·5 miles; train 59·5 tons; engine 15 tons; length of train 192 feet. Train does not stop at Watford.
	13 26	Shaft gun.		
29	13 44	Cross very strong	392	
30	13 56	Triple line very strong .	305	
31	14 20	Cross very strong	480	
	14 33	South gun.		
32	14 58	Cross lost	1074	

III.

33	7 57 16	Image much agitated...	1077	Velocity 30·9 miles; train's weight 124 tons; two engines 21 tons and 14 tons; length of train 375 feet. Wind N., very weak. Thermometer 31°·5.
34	57 30	Cross	877	
	57 47	Shaft gun.		
35	58 0	Cross very strong	478	
36	58 10	Parallel lines (5) very strong	374	
37	58 16	(5) still stronger.....	329	
38	58 25	(5) ten lines, quite cover field of telescope.....	302	
39	58 45	Cross very strong	431	
	59 4	South gun.		
40	59 14	Cross strong	803	
41	59 31	Cross lost	1045	

1847, March 12.—IV.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
42	8 40 11	Cross	855	Velocity 37·7 miles; train 50 tons; engine 14·5 tons; train's length 152 feet.
	40 24	Shaft gun.		
43	40 38	Cross very strong	428	
44	40 54	Parallel lines; image trembles	302	The image trembled very much during the whole time of passage through the field.
45	41 10	Strong lines	416	
	41 27	South gun.		
46	41 48	Cross lost	1031	

1847, March 15.—I.

47	7 21 14	Cross	1176	Velocity 20·5 miles; train 125·5 tons; engine 22 tons; length of train 409 feet; 18 carriages.
48	22 6	Cross strong	686	
	22 10	Shaft gun.		
49	22 37	Line brilliant; changed suddenly to	430	
50	22 50	Parallel lines (5)	349	
51	23 5	(5) very strong	303	
52	23 15	(5) still strong	311	
53	23 22	Cross very strong	335	
54	24 6	South gun; cross very strong.		
55	24 51	Cross still seen	1078	

II.

56	7 25 55	Cross	775	Velocity 22·6 miles; train 209·5 tons; length of train 172 feet; slow goods train.
	26 10	South gun.		
57	27 1	Parallel lines very strong	303	
58	27 10	Do. do. very beautiful...	324	
	27 55	Shaft gun.		
59	28 18	Cross lost	922	

III.

60	7 31 19	Cross	1032	Velocity 21·6 miles; train 91·5 tons; engine 14·5 tons; train's length 319 feet.
61	31 52	Cross very strong	706	
	31 58	Shaft gun.		
62	32 30	Single line very strong .	384	
63	32 32	Changed to (5) parallel lines.....	371·5	
64	32 48	(5) very strong	308	
	33 48	South gun.		
65	34 22	Cross lost	992	

IV.

66	7 43 37	Cross	786	Velocity 14·8 miles; train 49·5 tons; engine 14·5 tons; train's length 150 feet; six carriages.
67	43 44	Cross strong	740	
	43 57	Shaft gun.		
68	44 22	Line strong.....	502	
69	45 0	Cross very strong	328	
70	45 18	Trace of (5) parallel lines	302	
71	45 37	Line very strong.....	336	
72	45 52	Cross strong	391	
	46 38	South gun.		[leaving the tunnel.
73	46 42	Cross lost	679	I never saw it cease so soon after

1847, March 15.—V.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
74	8 10 16	Cross	1029	Velocity 33·0 miles; train 106 tons; engine 21 tons; train's length 364 feet.
75	10 35	Cross very strong	741	
	10 42	Shaft gun.		
76	10 52	Line very strong.....	504	
77	11 16	(5) brilliant.....	303	
	11 54	South gun, cross very strong.		
78	12 16	Cross lost	997	

VI.

79	8 25 57	Cross	854	Velocity 15·9 miles. This train could not be identified.
	26 25	Shaft gun.		
80	28 12	Cross very strong	394	
	28 54	South gun.		
81	29 12	Cross lost	782	

VII.

82	8 41 29	Cross	926	Velocity 23·7 miles. Newcastle Express.
	41 55	Shaft gun.		
83	42 21	(5) parallel bars	406	
	43 35	South gun.		
84	44 2	Cross lost	950	

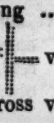
1847, March 16.—I.

85	6 44 49	Cross	1157	Velocity 34 miles; train 75 tons; engine 15 tons; length of train 282 feet.
	45 21	Shaft gun.		
86	46 8	Cross very strong	393	
	46 28	South gun; cross very strong.		
87	46 58	Cross last seen	1157	

II.

88	6 54 14	Cross	935	Velocity 24·8 miles; train 67 tons; engine 14 tons; length of train 231 feet.
	54 42	South gun.		
89	55 34	(5) parallel lines.....	314	
	56 18	Shaft gun.		
90	56 42	Lost cross, but cart within hearing	959	

III.

91	6 58 18	Cross	915	Velocity 11·4 miles; train 322 tons; engine 14 tons; train's length 857 feet. A heavy goods train.
	59 9	Shaft gun.		
92	7 0 11	Line very strong.....,	382	
93	0 20	(5) very strong	352	
94	0 30	(5) magnificent	328	
95	0 40	Cross very strong	308	
96	0 55	Cross double  very beautiful.	309	
	2 36	South gun; cross very strong.		
97	4 4	Cross lost	1110	

1847, March 16.—IV.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
98	7 8 5	Cross	870	Velocity 21·9 miles.
	8 30	South gun.		
	10 18	Shaft gun.		
99	10 38	Lost cross	878	

V.

100	7 18 32	Cross	1038	Velocity 25·7 miles; weight of train 69·5 tons; engine 14·5 tons; train's length 194 feet.
	19 5	Shaft gun.		
	20 37	South gun.		
101	21 5	Lost cross	988	

VI.

102	7 42 2	Cross	824	Velocity 35·8 miles; train 53·5 tons; engine 14·5 tons; train's length 168 feet.
	42 14	Shaft gun.		
103	42 43	(5)	305	
	43 20	South gun.		
104	43 45	Lost cross	1079	

VII.

105	8 31 42	Cross	846	Velocity 36·9 miles; train 98·5 tons; engine 21 tons; length of train 322 feet. Wind S.E.; fresh.
	31 55	Shaft gun.		
106	32 9	(5)	428	
	32 59	South gun.		
107	33 22	Lost cross	1058	

VIII.

108	8 43 48	Cross very faint.....	668	Velocity 20·1 miles; train 55·75 tons; engine 23·75 tons; train's length 146 feet. Tremors unusually small.
	43 50	Shaft gun.		
	45 48	South gun.		
109	46 8	Lost the cross.....	821	

1847, March 17.—I.

110	6 42 45	Cross	1055	Velocity 33·9 miles; train 104 tons; engine 19 tons; length of train 362 feet.
	43 12	Shaft gun.		
111	43 40	(5) beautiful	318	
112	43 55	(5) ditto	337	
	44 22	South gun; cross very strong.		
113	44 54	Cross lost	1166	

II.

114	6 55 26	Cross	1076	Velocity 28·7 miles; train 70 tons; engine 12 tons; train's length 247; feet. Wind S.E.; scarcely sensible; image very unsteady.
	56 1	South gun.		
115	56 37	Line of stars very beautiful.	304	
116	56 46	(5)	315	
	57 24	Shaft gun.		
117	57 35	Cross still very strong...	832	
118	57 47	Cross lost	1004	

1847, March 17.—III.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
119	7 10 23	Cross	828	Velocity 22·9 miles; train 74·5 tons; engine 12·5; train's length 171 feet.
	10 43	South gun.		
120	11 32	Cross strong	302	
121	11 45	Line strong	336	
	12 27	Shaft gun.		
122	12 47	Cross lost.....	892	

1847, March 18.—I.

123	6 18 12	Cross well seen	961	Velocity 27·3 miles; train's weight 87 tons; engine 15 tons; length of train 345 feet.
	18 38	South gun		
124	19 25	(5) strong.	314	
125	20 3	Cross strong	666	
	20 5	Shaft gun.		
126	20 20	Cross still strong.....	875	
127	20 26	Cross lost	950	The image oscillating in every direction.

II.

128	6 38 32	Cross strong	902	Velocity 30·4 miles; train 78 tons; engine 19 tons; length of train 246 feet.
	38 51	Shaft gun.		
129	39 20	(5) very strong	331	
	40 9	South gun.		
130	40 11	Strong cross	692	
131	40 24	Cross lost	870	

III.

132	6 55 11	Cross	992	Velocity 25·6 miles; train 63 tons; engine 14 tons; train's length 212 feet.
133	55 32	Line strong.....	746	
	55 43	South gun.		
134	56 22	Cross very strong	307	
135	56 33	(5) beautiful	313	
	57 16	Shaft gun.		
136	57 43	Cross lost	1003	

IV.

137	7 14 12	Cross strong	824	Velocity 31·6 miles; train 72·5 tons; engine 14·5 tons; train's length 254 feet.
	14 25	Shaft gun.		
138	15 0	(5) beautiful	303	
139	15 12	(5) still fine.....	343	
140	15 26	Cross strong	484	
	15 40	South gun.		
141	16 7	Cross lost	1057	

V.

142	7 21 59	Cross	705	Velocity 22·9 miles; a light goods train.
	22 7	South gun.		
143	22 45	Cross strong	326	
	23 51	Shaft gun.		
144	24 5	Cross lost	830	

1847, March 18.—VI.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
145	7 36 54	Cross	888	Velocity 18·2 miles; train's weight 84 tons; engine 16 tons; train's length 187 feet. All the images inosculated during the train's passage through the tunnel; yet when it was gone the star was perfectly steady. Wolverton goods train.
	37 26	South gun.		
146	38 20	Cross strong	311	
147	38 30	All confusion	303	
148	38 40	Line very strong.....	320	
	39 37	Shaft gun.		
149	40 3	Cross lost.....	895	

VII.

150	7 41 0	Cross	896	Velocity 29·7 miles; train's weight 62·5 tons; engine 14·5 tons; train's length 204 feet. As in last the images inosculated; even the lines of (5) ran into each other. Peterborough light train.
151	41 13	Cross strong	721	
	41 19	Shaft gun.		
152	41 43	(5)	373	
153	42 0	(5) strong	303	
	42 39	South gun.		
154	43 14	Cross lost.....	1141	

VIII.

155	7 59 5	Cross	1111	Velocity 40·2 miles; train 128 tons; engine 19 tons.; length of train 458 feet.
156	59 25	Cross very strong	740	
	59 31	Shaft gun.		
157	8 0 10	(5)	367	
	0 30	South gun.		
158	0 54	Cross lost	1115	

IX.

159	8 38 36	Cross	818	Velocity 41·62 miles; train 61·25 tons; engine 23·75 tons; length of train 144 feet.
	38 46	Shaft gun.		
160	39 0	Cross very strong	406	
161	39 10	(5)	310	
162	39 18	Cross	481	
	39 43	South gun.		
163	40 3	Cross lost	1076	

1847, March 19.—I.

164	6 39 55	Cross seen	858	Velocity 37·9 miles; train 92·75 tons.; engine 23·75 tons; train's length 284 feet.
	40 8	Shaft gun.		
165	40 10	Line very distinct	606	
166	40 25	Cross extremely bright..	390	
	41 0	South gun.		
167	41 37	Cross lost	1128	Observed by the Marquis of Blandford

1847, March 19.—II.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
168	6 56 52	Cross	919	Velocity 32·8 miles; train 67 tons; engine 14 tons; train's length 231 feet.
	57 12·5	South gun.		
169	57 30	Cross very bright	397	
170	57 49	(5) very distinct	306	
	58 25	Shaft gun.		
171	58 44	Cross lost	978	Observed by the Marquis of Blandford.

III.

172	7 15 20	Cross	697	Velocity 32·1 miles; train 68·5 tons; engine 14 tons; train's length 234 feet.
	15 24	Shaft gun.		
173	15 45	Cross still distinct	383	
174	15 55	(5) slightly	313	
	16 38	South gun.		
175	16 46	Cross lost	781	Observed by Lord Alfred Churchill.

IV.

176	7 24 40	Cross	1042	Velocity 24·0 miles; train 98·5 tons; engine 12·5 tons; train's length 156 feet. This night very unfavourable. Many observations lost from clouds, and the stars when seen often faint.
177	24 57	Cross very strong	848·5	
	25 18	South gun.		
178	26 5	Cross extremely strong..	302	
179	26 10	(5)	309	
	26 57	Shaft gun.		
180	27 26	Cross lost	1005	

1847, March 22.—I.

181	7 22 12	Cross	759	Velocity 23·6 miles; train 86·5 tons; engine 12·5 tons; train's length 203 feet.
	22 24	South gun.		
182	23 22	Cross strong	323	
	24 5	Shaft gun.		
183	24 33	Cross lost	964	

II.

184	7 38 32	Cross	1025	Velocity 20·7 miles; train 68 tons; engine 14·5 tons; train's length 233 feet.
185	39 10	Cross strong	668	
	39 12	Shaft gun.		
186	40 12	(5)	304	
	41 7	South gun.		
187	41 36	Cross lost	950	

III.

188	7 58 20	Cross	827	Velocity 31·4 miles; train 88 tons; engine 21 tons; length of train 288 feet.
	58 33	Shaft gun.		
189	59 4	(5) beautiful	312	
	59 47	South gun.		
190	60 8	Cross lost	972	

1847, March 29.—I.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
191	6 54 58	Cross	837	Velocity 33·1 miles; train 108 tons; two engines, 15 tons and 13 tons; train's length 334 feet.
	55 13	South gun; cross very strong.		
192	55 42	Cross extremely strong..	310	
193	56 0	All lost in a flare	374	
	56 25	Shaft gun.		
194	56 49	Cross lost	1057	

II.

195	7 36 0	Cross	849	Velocity 33·4 miles; train 67·5 tons; engine 14·5 tons; train's length 226 feet. At $36^m\ 45^s$ the stars lost shape and were inosculated with each other.
	36 14	Shaft gun.		
196	36 40	Cross very strong	333	
	37 25	South gun.		
197	37 43	Cross lost	940	

III.

	7 47 43	Shaft gun.		Velocity 20·8 miles; train 48 tons; engine 13 tons; train's length 150 feet. It was stopped by the police at the entrance of the tunnel, and went slowly through it—"crawling," in the words of the signal-man.
198	48 10	Cross first seen	426	
199	48 41	Cross strong	302	
	49 37	South gun.		
200	49 39	Cross lost	677	

IV.

201	8 4 6	Cross	698	Velocity 33·4 miles; train 92·5 tons; engine 15 tons; train's length 328 feet.
	4 10	Shaft gun.		
202	4 45	Cross strong	302	
	5 21	South gun.		
203	5 44	Cross lost	1018	

V.

204	8 33 12	Cross	774	Velocity 39·5 miles; train 55·5 tons; engine 21 tons; train's length 144 feet. The night unfavourable from clouds.
	33 20	Shaft gun.		
	34 20	South gun.		
205	34 37	Cross lost	978	

1847, March 30.—I.

206	6 49 26	Cross	771	Velocity 29·0 miles; train 122 tons; engine 18 tons; train's length 404 feet.
	49 36	Shaft gun.		
207	50 10	(5) two faint parallel lines	314	
208	50 15	Stars entirely confused..	302	Observed by the late Professor James, M^{c}Cullagh, of Trinity College, Dublin.
	50 58	South gun.		
209	51 16	Cross lost	899	

1847, March 30.—II.

No.	Time.	Observations.	Yards.	Remarks.
	h m s			
210	6 57 57	Cross	940	Velocity 38·4 miles; train 89·5 tons;
211	58 10	Cross strong	714	engine 19 tons; length of train 293 feet.
	58 16	South gun.		
212	58 40	(5) but confused.....	313	
	59 18	Shaft gun.		
213	59 36	Cross lost	1014	Observed by Prof. James McCullagh.

III.

214	7 2 2	Cross	798	Velocity 29·0 miles.; a pilot engine.
	2 14	Shaft gun.		
215	3 36	South gun.		
		Star which had been faint was now totally clouded.		Observed by Prof. James McCullagh.

IV.

216	7 17 25	Cross	883	Velocity 43·1 miles; train 49·5 tons;
	17 38	Shaft gun.		engine 14·5 tons; length 150 feet.
217	17 58	(5)	330	
	18 33	South gun.		
218	18 48	Cross lost	969	

V.

219	7 44 17	Cross	924	Velocity 28·3; train 53·5 tons; engine
	44 39	Shaft gun, cross very strong.		14·5 tons; length of train 167 feet.
220	45 8	(5) brilliant. All the stars blue except the centre.	346	
221	45 27	(5) changes to cross ...	316	
	46 3	South gun.		
222	46 26	Cross lost	958	

VI.

223	8 1 9	Cross	965	Velocity 34·4 miles; train 114·5 tons;
	1 30	Shaft gun.		engine 21 tons; length of train 408
224	1 44	(5)	446	feet.
225	2 12	(5) very strong	335	
	2 39	South gun.		
226	3 2	Cross lost	1029	

VII.

227	8 39 20	Cross	916	Velocity 45·6 miles; train 50·5 tons;
	39 34	Shaft gun.		engine 15 tons; train's length 152
228	39 55	(5)	313	feet. Newcastle Express.
	40 26	South gun.		
229	40 47	Cross lost	1116	

Date.	Cross begins.	(5) begins.	(5) ends.	Cross ends.	Exit.	Train's velocity.	Train's weight.	
1847.	yards.	yards.	yards.	yards.		miles.	tons.	
Feb. 24	845	...	...	704	S.	11·0	77·5	
	699	...	...	780	S.	16·6	69·5	
	680	...	...	678	S.	13·8	58·5	
	...	...	...	834·5	S.	11·4	89·5	
Feb. 27	...	...	...	722	S.	15·4	54·0	
	736	...	...	915	S.	25·6	49·5	
	706	...	...	736	N.	17·6	270·5	
	736	319	...	1086	S.	31·7	112	
	727	322	...	cloud		18·7	51·5	
Mar. 11	802	...	...	cloud		17·7	147·5	
	cloud	...	...	921+	S.	33·0	122	
Mar. 12	822	303	...	766	N.	28·3	68	
	811	305	...	1074	S.	35·5	59·5	
	877	374	302+	1045	S.	30·9	124	Image much agitated at 1077 yards.
	855	302	416	1031	S.	37·7	50	
Mar. 15	1176	349	311+	1078+	S.	20·5	125·5	Train very long.
	775	303	324+	922	N.	22·6	209·5	
	1032	371	308+	992	S.	21·6	91·5	
	786	302	...	679	S.	14·8	49·5	
	1029	303	...	997	S.	33·0	106	
	854	...	...	782	S.	15·9		
	926	406	...	950	S.	23·7		
Mar. 16	1157	...	...	1157	S.	35·4	75	
	935	314	...	959	N.	24·8	67	
	915	352	308+	1110	S.	11·4	322	Train very long.
	870	...	...	878	N.	21·9		
	1038	...	...	988	S.	25·7	69·5	
	824	305	...	1079	S.	35·8	53·5	
	846	428	...	1058	S.	36·9	98·5	Long train.
	668	...	...	821	S.	20·1	55·75	
Mar. 17	1055	318	337+	1166	S.	33·9	104	Rather long.
	1076	315	...	1004	N.	28·7	70	
	828	...	...	892	N.	22·9	74·5	
Mar. 18	961	314	...	950	N.	27·3	87	Image oscillating.
	902	331	...	870	S.	30·4	78	
	992	313	...	1003	N.	25·6	63	
	824	303	343	1057	S.	31·6	72·5	
	705	...	...	830	N.	22·9	...	Light goods train.
	808	303	320	895	N.	18·2	84	Images confused.
	896	373	303+	1141	S.	29·7	62·3	Ditto ditto.
	1111	...	...	1115	S.	40·2	128	Long train.
	818	310	...	1076	S.	41·6	61·25	
Mar. 19	858	...	...	1128	S.	37·9	92·75	} Observed by the Marquis of Blandford.
	919	306	...	978	N.	32·8	67	
	697	313	...	781	S.	32·1	68·5	Observed by Lord Alfred Churchill.
	1042	309	...	1005	N.	24·0	98·5	
Mar. 22	759	...	...	964	N.	23·6	86·5	
	1025	304	...	950	S.	20·7	68	
	827	312	...	972	S.	31·4	88	
Mar. 29	837	374	...	1057	N.	33·1	108	Long train.
	849	...	...	940	S.	33·4	67·5	Images confused.
	426	...	...	677	S.	20·8	48	Stopped at entrance.
	698	...	...	1018	S.	33·4	92·5	
	774	...	...	978	S.	39·5	55·5	Cloudy.
Mar. 30	771	314	...	899	S.	29	122	} Observed by Professor James McCullagh.
	940	313	...	1014	N.	38·4	89·5	
	798	...	...	...	S.	29	pilot	
	883	330	...	969	S.	43·1	49·5	
	924	346	316	958	S.	28·3	53·5	
	965	446	335	1029	S.	34·4	114·5	Long and heavy engine.
	916	313	...	1116	S.	45·6	50·5	

That these results may be more easily appreciated, I have condensed the most important of them into the preceding Table, which gives in one view the distance at which that amount of disturbance begins and ends which produces the cross, that at which the far greater one occurs causing the appearance (5) (a system of three or more parallel rows) wherever it does appear, and the velocities and weights of the trains when known.

It is evident from this Table that the tremor which is sufficient to produce that disturbance of the mercury which shows a cross of stars is propagated to considerable distances—in one case to 1176 yards; and 24 per cent. of the entire are above 1000. Such distances do not pass the northern end of the tunnel, but go far beyond the southern. In the latter case the vibrations are excited while the train is in an open cutting; and those who suppose that the tunnel has much power in deadening them would of course expect that they would be sensible at a greater distance than at the other end. This does not seem, however, to be the case: and the Table shows that in this respect there is very little difference, if we take into account another cause of inequality, namely, that the tremor is manifested further at the exit than at the entrance of the train. The column headed "Exit" shows by S. that the exit was at the South end, and the entrance at the North.

Now, when the observations are examined where both were noted, we find that the limit of the cross is greater at the exit than at the entrance in 29 out of 39, or 74 per cent. of S., and 12 out of 16, or 75 per cent. of N. The reason of this, I suppose, is that the long-continued action of the train on the rails tends to produce a greater and more prolonged undulation in the mercury.

But the equal percentage shows that there is really no protecting power in the tunnel against the lateral propagation of tremors, whatever may be the case immediately above the crown.

In general one might expect trains to produce disturbance in proportion to their speed and their weight. To a certain degree this is true; but the exceptions are sufficient to show that other influences must be taken into consideration. Examples of high speed with comparatively small effect are afforded by the observations on March 18, II.; 19, III.; and 29, V.

Others of the reverse conditions are given by February 24, I., II.; March 18, VI.; and specially March 16, III., in which with a velocity of only 11·4 miles the cross was shown at 1110 yards. This it may be remarked is a decisive proof that any plan of protecting an observatory by slackening the speed of trains passing near it is entirely useless, even if it could be enforced.

It is probable that one cause of this high disturbing-power in slow trains is that already referred to, the long-continued accumulation of vibration, the quick ones passing beyond distance before the mercury has got into full vibration, the others having full time to do their work though with less intrinsic force. On this account also long trains are more disturbing than short.

The engine is not so paramount a disturber as might be expected,—the heaviest, and even a pair of them, not causing more tremor than occurs with the smaller ones.

In taking the cross of stars as the test of disturbance, I must observe that I do so, not because it is the earliest which appears, but because it marks distinctly an agitation greater than what is likely to occur at an observatory subject to ordinary perturbations. These produce in such a mercury-vessel as I used a single line of stars perpendicular to the length of the vessel. It should seem that then only one set of undulations fit to produce these images is excited in the mercury, the direction of which is regulated by the sides of the vessel *. The existence of the cross shows that a second set of waves perpendicular to the first has been developed : this always happens if the sides of the vessel are equal ; and its occurring when they are so unequal as in the present case seems to indicate a corresponding excess of the power which causes them. If the agitation be still greater, it seems as if each of the images which form the cross became the origin of a row of secondary images, the result of which is the form (5), a series of parallel rows of stars varying from two to ten, or even filling the whole field. This token of ultra disturbance is confined between lines making angles of 45° with the perpendicular to the rails—in other words, to distances under 427 yards, and when the train is nearly in the centre of the tunnel. It is (except in two instances) only seen when the cross is visible beyond 1000 yards : when the agitation is still further increased the images vibrate in every direction, and with yet more of it the whole becomes a mass of nebulous light ; of both which some examples may be found in these observations.

The opinion maintained by the late Mr. Robert Stephenson, that much of these railway tremors were due to the sound of the train, although not probable, induced me to try some experiments by firing cannon, maroons, and rockets at various distances.

One of these cannons (for I had two, each $\frac{3}{4}$ of a pound calibre) heavily loaded, at 300 yards produced (5), cross, and line simultaneously with my hearing the reports ; but all disturbance was over in about 1·5 second. At 2020 yards there was the cross synchronous with the report, and of the same momentary character ; and even at 3000 yards the cross could be traced. This seems to have been due to the momentary impulse of the sound-wave, for the continuous roar of two-pound rockets fired at 82 feet from the mercury, though very loud, disturbed it very little ; while the explosions of eight ounces of powder in their heads about 800 yards above the ground produced all, the (5), cross, and line. A still more interesting experiment was, firing the cannon in the tunnel at the point where the perpendicular from the observatory met it. In this case two disturbances were seen—one propagated through the ground, the other through the air with about a second of time interval. The sound probably made its way chiefly

* For details on this I may refer to my Report to the Admiralty, published by order of the House of Commons, July 6, 1846.

through the shafts; but even had they been closed, it seems unquestionable that the report, and of course the sound of a train, would travel through the earth *.

I should have prosecuted these researches further, especially in reference to the velocity with which these tremors are propagated through the ground, but that Lord Auckland's letter to me led me to hope that all danger to the Royal Observatory was past, never to return. I therefore contented myself with reducing the observations I had made. As, however, the Railway Moloch seems never likely to be satiated with victims, and as the observatories of Oxford, Armagh, and again that of Greenwich have been marked for sacrifice, it seems to me a duty to place before the public the facts which had been collected at a great expense of labour, and some pecuniary outlay.

They were made without any bias, or any motive but a desire to ascertain the actual truth; and in addition to their bearing on practical astronomy, I hope that they may not be without use in reference to some other departments of science.

January 7, 1864.

Dr. W. A. MILLER, Treasurer and Vice-President, in the Chair.

The following communications were read:—

I. "Extract of a letter to General Sabine from Dr. Otto Torell, dated from Copenhagen, Dec. 12, 1863." Received December 18, 1863.

The Swedish Diet has given the necessary money to complete the survey for the measurement of an Arc of the Meridian at Spitzbergen.

When the proposal was submitted to the Diet by our Government, at the instance of the Academy of Sciences at Stockholm, it was passed without opposition in the three first houses of the Diet (viz. the Nobles, the Clergy, and the Burghers). In the fourth house (the Peasants), only one Member opposed the proposal, on the ground of the high amount of the Budget. He was replied to by seven or eight other Members, advising that the house should not oppose a grant which had for its object to advance science. In

* An interesting fact was observed with the maroons. They were fired vertically from a mortar twenty feet from the observatory, and had fuses which gave them flight for six seconds. The mercury showed the usual intense disturbance when the mortar was fired, and also at the explosion of the maroons in the air. But there was also an intermediate disturbance which I cannot explain but by supposing it to be as it were an echo of the earth-wave caused by the discharge of the mortar and reflected from *the masonry of the tunnel*. I showed it to the Marquis of Blandford, to Lord Alfred Churchill, and to Professor James M^c^Cullagh; unfortunately the nights Dr. Robinson and Mr. Warburton accompanied me to Watford, not a single star was visible. On repeating the experiments at Campden Hill, nothing of the sort occurred.

consequence the money was also voted by the House of Peasants. There is every reason to expect that the question of the practicability of the undertaking will be settled in the next summer, and I hope that the result may be satisfactory.

The Diet has with the same liberality given the necessary money for the Swedish share in the proposed large Middle-European Triangulation from Palermo to Trondjem, and has also made a grant of the money which will be required to erect a new Astronomical Observatory at the University of Lund. I expect therefore that the excellent astronomer at the University, Mr. Möller, will read with intense interest the correspondence regarding the Melbourne Telescope, which even to me has been of great interest.

II. "Results of hourly Observations of the Magnetic Declination made by Sir Francis Leopold M'Clintock, R.N., and the Officers of the Yacht 'Fox,' at Port Kennedy, in the Arctic Sea, in the Winter of 1858–59; and a Comparison of these Results with those obtained by Captain Maguire, R.N., and the Officers of H.M.S. 'Plover,' in 1852, 1853, and 1854, at Point Barrow." By Major-General Sabine, R.A., President. Received December 21, 1863.

(Abstract.)

When about to undertake a voyage to the Arctic Sea in 1857, in the yacht 'Fox,' in search of the ships of Sir John Franklin's expedition, Captain M'Clintock requested that the Royal Society would supply him with such information and instructions as might enable him to make the best use of the opportunity which the voyage was likely to afford for the prosecution of magnetical and meteorological observations.

As the present communication is limited to a discussion of the hourly observations of the declination made by Captain M'Clintock and his officers from December 1, 1858 to March 31, 1859 inclusive, the portion of the instructions with which Captain M'Clintock was supplied which relates to such observations forms an appropriate introduction. It is followed by a full statement from Captain M'Clintock himself of the circumstances under which an observatory was established on the ice at a distance of 220 yards from the ship, and hourly observations maintained during five months of the arctic winter, being only discontinued when, on the return of a more genial season, the services of both officers and sailors were required in prosecuting the more immediate objects of the expedition.

On the return of the 'Fox' to England, the observations were sent, through the Royal Society, to the Woolwich establishment for the reduction and publication of magnetic observations. The results of the observations treated of in this paper are discussed in comparison with those obtained from similar observations made by Captain Maguire and the officers of H.M.S. 'Plover' at Point Barrow, on the shore of the Arctic

Sea, 1200 miles distant from Port Kennedy (Captain M'Clintock's station), in the winters of 1852–53, and 1853–54, published in the Phil. Trans. for 1857, Art. xxiv.

The first point established conclusively by this comparison is, that, after due allowance has been made for the difference in the antagonistic force of the horizontal portion of the earth's magnetism by which any disturbing action on the declination-magnet is opposed at the two stations, the intensity of the disturbing force is considerably less at Port Kennedy than at Point Barrow—that is to say, less at the station which is nearest to the points of 90° of dip, and of the maximum of the total terrestrial magnetic force, than at the station which is more distant from those points. The indication thus derived from the magnetic record at the two stations accords with the fact of the far greater frequency of the aurora at Point Barrow, where in the two winters its appearance is recorded on six days out of every seven, whilst the proportion at Port Kennedy is not more than one day in four.

For the purpose of examining the periodical laws of the disturbances at Port Kennedy, those which exhibited the largest differences from their respective normals of the same month and hour, amounting to between one-fourth and one-fifth of the whole body of the hourly observations, were separated from the others, and were subjected to analysis in the customary manner. It is thus shown that both at Port Kennedy and at Point Barrow the disturbances so treated form themselves into distinct categories of easterly and westerly deflection,—the curve representing the easterly deflections having the same general form and single maximum as that of the easterly deflections at Kew, exhibited in Pl. XIII. fig. 1 of the Phil. Trans. for 1863, Art. XII.; and the westerly curve having the same general form and double maximum as is seen in fig. 2 of the same Plate, representing the westerly deflections at Kew.

A remarkable correspondence is pointed out in regard to the hours at which the maxima of easterly and westerly deflection take place at Port Kennedy and Point Barrow. The maximum of easterly deflection occurs at the same hour of *absolute* time at the two stations; and the maximum of westerly deflection at the same hour of *local* time at the two stations.

The author concludes by taking a general review of the phenomena of the solar-diurnal variation, particularly in the vicinity of the dip of 90°, where the geographical and magnetical directions of the magnetic needle are often strongly contrasted. At Port Kennedy the normal direction of the magnet is 35° to the east of south, and at Point Barrow 41° to the west of north: the contrast at the two stations in this respect is therefore nearly as great as can exist in any part of the globe, wanting only 6° of 180°, or of being diametrically opposite. The solar-diurnal variation at these stations furnishes an apt illustration of the author's exposition.

He further takes the occasion of the phenomena of the disturbance diurnal variation at Port Kennedy, and at Nertschinsk in Siberia, to show

the caution which is necessary in endeavouring to derive the epochs of the decennial period from the magnitude of the diurnal range of the declination-magnet, and the preference due to the variation in the amount of the disturbances in different years.

January 14, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read:—

I. "Examination of *Rubia munjista*, the East-Indian Madder, or Munjeet of Commerce." By JOHN STENHOUSE, LL.D., F.R.S. Received December 21, 1863.

(Abstract.)

As a portion of this paper has already appeared in the 'Proceedings,' vol. xii. p. 633, I shall confine myself in this abstract to briefly noticing the additional observations which I have subsequently made.

By numerous analyses of munjistine crystallized out of spirits and dried at 100° C., and likewise of sublimed munjistine, I find that its formula is $C_{16}H_6O_6$. This result has been confirmed by analyses of the lead-salt, the formula of which I find to be $5(C_{16}H_6O_6)$, $6PbO$, exactly corresponding to the purpurine compound described by Wolff and Strecker*.

A comparison of the subjoined formula of alizarine, purpurine, and munjistine,

Alizarine	$C_{20}H_6O_6$,
Purpurine	$C_{18}H_6O_6$,
Munjistine.....	$C_{16}H_6O_6$,

indicates the very close relationship between these three substances, the only true colouring principles of the different species of madder with which we are acquainted.

Tinctorial Power of Munjeet.

From a numerous series of experiments I have just completed, I find that the garancine from munjeet has about half the tinctorial power of the garancine made from the best madder, viz. Naples roots. These, however, yield only about 30 to 33 per cent. of garancine, while munjeet, according to my friend Mr. Higgin of Manchester, yields from 52 to 55 per cent. The actual amount of colouring matter in munjeet and the best madder are very nearly the same; but the inferiority of munjeet as a dye-stuff results mainly from its containing only the comparatively feeble colouring matters purpurine and munjistine. The latter in large quantity is positively injurious; so much is this the case, that when the greater part of the munjistine is removed from munjeet-garancine by boiling water, it yields much richer shades with alumina mordants.

* Annalen der Chemie, vol. lxxv. p. 24.

Purpureine.

When purpurine is dissolved in dilute ammonia, and exposed to the air for about a month in a warm place, ammonia and water being added from time to time as they evaporate, the purpurine disappears, whilst a new colouring matter is formed, which dyes unmordanted silk and wool of a fine rose-colour, but is incapable of dyeing vegetable fabrics mordanted with alumina.

This new substance, which, from its mode of formation and physical properties, is so analogous to orceine, I have called *purpureine*. When pure, it forms fine long needles of a deep crimson colour, insoluble in dilute acids, slightly soluble in pure water, and very soluble in alcohol and in water rendered slightly alkaline. Professor Stokes has examined purpureine optically, and finds the spectrum the same in character as that of purpurine, but different in position, the bands of absorption being severally nearer to the red end.

From the analyses, purpureine seems to yield the formula $C_{66}H_{24}N_2O_{20}$.

Nitropurpurine.

When purpurine is dissolved in a small quantity of nitric acid, spec. grav. about ·135, and heated to 100° C., it gives off red fumes, and on being allowed to cool, a substance separates in fine scarlet prisms, somewhat like chromate of silver, only of a brighter colour. It is quite insoluble in water, but slightly soluble in spirit; it is, however, soluble in strong nitric acid. When heated, it deflagrates. From this circumstance, and considering its mode of formation, it is evidently a nitro-substitution compound. I have therefore called it nitropurpurine.

When alizarine and munjistine are subjected, in the manner above described for purpurine, to the joint action of ammonia and oxygen, substantive colours are produced, neither of which are crystalline.

Action of Bromine on Alizarine.

When alcoholic solutions of alizarine are mixed with water, and aqueous solution of bromine added, a yellow precipitate is produced; the solution filtered from this, after expelling the spirit by heat, deposits a deep orange-coloured crystalline compound, which, from the analyses of six specimens prepared at different times, I find has the composition

$$C_{60}H_{16}Br_2O_{18}=C_{20}H_6O_6,\ 2(C_{20}H_5BrO_6).$$

Purpurine, when treated with bromine in a similar manner, does not yield a corresponding compound.

II. "On the Magnetic Variations observed at Greenwich." By Professor Wolf of Zurich. Communicated by G. B. Airy, F.R.S., Astronomer Royal. Received December 21, 1863.

(Translation.)

In April 1863 Professor Airy kindly communicated to me the Mean

daily variations of the Declination as given by the Greenwich Observations 1841–1857, and as entered in the subjoined Table under v_1.

Year.	Ratios r_1.	Mean absolute variat. v_1.	Calculated by (I). v_1.	Diff.	Calculated by (II). v_1.	Diff.	Hourly mean of the Declination. Maximum.	At	Minimum.	At	Max. − Min. v_2.	Calculated by (III). v_3.	Diff.	Calculated by (IV). v_3.	Diff.
		′	′	′	′	′	° ′	h	° ′	h	′	′	′	′	′
1841	29·7	10·1	11·61	−1·51	11·59	−1·49	23 22·1	2	23 13·4	12	8·7	7·83	+0·87	9·01	−0·31
1842	19·5	13·3	11·04	+2·26	10·97	+2·33	23 20·2	2	23 11·9	12	8·3	7·41	+0·89	8·40	−0·10
1843	8·6	11·6	10·43	+1·17	10·30	+1·30	23 17·6	2	23 9·4	20	8·2	7·01	+1·19	7·78	+0·42
1844	13·0	11·6	10·68	+0·92	10·57	+1·03	23 20·8	2	23 13·0	12	7·8	7·18	+0·62	7·83	−0·03
1845	33·0	12·1	11·80	+0·30	11·79	+0·31	23 2·6	2	22 54·1	20	8·5	7·96	+0·54	8·54	−0·04
1846	47·0	13·6	12·58	+1·02	12·65	+0·95	22 55·8	2	22 47·2	20	8·6	8·50	+0·10	8·96	−0·36
1847	79·4	17·8	14·40	+3·40	14·62	+3·18	22 58·2	2	22 48·4	18	9·8	9·77	+0·03	10·08	−0·28
1848	100·4	15·3	15·57	−0·27	15·90	−0·60	22 59·8	2	22 48·5	20	11·3	10·59	+0·71	10·70	+0·60
1849	95·6	14·3	15·30	−1·00	15·61	−1·31	22 44·8	2	22 34·5	20	10·3	10·40	−0·10	10·29	+0·01
1850	64·5	12·9	13·56	−0·66	13·71	−0·81	22 30·5	2	22 20·5	20	10·0	9·19	+0·81	8·93	+1·07
1851	61·9	11·6	13·42	−1·82	13·56	−1·96	22 23·7	2	22 15·8	21	7·9	9·08	−1·18	8·64	−0·74
1852	52·2	13·0	12·87	+0·13	12·96	+0·04	22 23·4	2	22 15·4	9	8·0	8·71	−0·71	8·12	−0·12
1853	37·7	11·3	12·06	−0·76	12·08	−0·78	22 15·0	2	22 8·0	11	7·0	8·14	−1·14	7·45	−0·45
1854	19·2	11·3	11·03	+0·27	10·95	+0·35	22 5·7	2	21 58·5	11	7·2	7·42	−0·22	6·68	+0·52
1855	6·9	10·6	10·43	+0·26	10·20	+0·40	21 53·3	2	21 46·4	11	6·9	6·94	−0·04	6·14	+0·76
1856	4·2	8·7	10·19	−1·49	10·04	−1·34	21 47·6	2	21 41·8	20	5·8	6·83	−1·03	5·93	−0·13
1857	21·6	9·0	11·16	−2·16	11·10	−2·10	21 39·4	1	21 33·8	10	5·6	7·51	−1·91	6·32	−0·72
		Sum of squares 34·7350			..... 35·1531						Sum of squares 12·7773			..... 4·1458	
1858	50·9		12·80		12·88							8·66		7·03	
1859	96·4		15·35		15·66							10·43		8·13	
1860	98·6		15·47		15·79							10·52		7·97	
1861	77·4		14·28		14·50							9·69		7·19	
1862	59·4		13·28		13·40							8·99		6·55	

I naturally lost no time in proceeding to try whether I could not repre-

sent them from my Sun-spot-ratios r in the same way that I had succeeded in doing with those obtained at numerous other stations (see different Nos. of my "Mittheilungen über die Sonnenflecken"), and I obtained the formula

$$v_1 = 9'{\cdot}95 + 0{\cdot}056.r. \quad \ldots\ldots\ldots \quad (\mathrm{I})$$

The comparison contained in the Table between the observed values and the values computed by formula (I) gave me, however, a strikingly less good accord than I had obtained for Munich, Prague, Christiania, &c.; and this induced me to try how it would be if I formed groups of the years "rich," "medium," and "poor" in sun-spots, and compared for each group the mean variation with the mean ratio. Thus

Years.	Sun-spots.		Mean.	Calculated by (II).	
	Number.	Mean r.	v_1.	v_1.	Diff.
			′	′	′
1841–1842	Medium	24·60	11·70	11·28	+0·42
1843–1844	Poor ...	10·80	11·60	10·44	+1·16
1845–1846	Medium	40·00	12·85	12·22	+0·63
1847–1849	Rich ...	91·80	15·80	15·38	+0·42
1850–1852	Medium	59·53	12·50	13·41	−0·91
1853–1854	Medium	28·45	11·30	11·52	−0·22
1855–1857	Poor ...	10·90	9·43	10·44	−1·01
				Sum of squares	3·9919

And I then obtained the formula

$$v_1 = 9'{\cdot}78 + 0{\cdot}061.r, \quad \ldots\ldots\ldots \quad (\mathrm{II})$$

the comparison of which with the values derived from the observations shows a much better accord, without the individual years being much worse represented than by (I). It follows that the Greenwich observations also give on the whole a march corresponding to that of the sun-spots, but at the same time with materially greater deviations than appear in the continental stations which I have previously treated.

When I communicated to Professor Airy the unexpected result of my calculations, he called my attention to the circumstance that his variations were *absolute* ones, *i. e.* the means of the differences between the daily extremes, while the variations at other stations which I had employed were probably obtained from observations at definite hours; and on my informing him that such was really the case, he gave me in addition the Means of the Declinations which corresponded at Greenwich to the prescribed hours of Göttingen time. It is from these subsequently communicated values that I have derived the maxima and minima and their differences entered in the Table under v_2. The calculation of the quantities in v_2 then led me to make the formula

$$v_1 = 6'{\cdot}67 + 0{\cdot}039.r; \quad \ldots\ldots\ldots \quad (\mathrm{III})$$

and the comparison in the Table of the values computed by this formula with those derived from the observations does in fact show a much greater accord. I was, however, further led to infer that the constants in the formula, which I had perceived to vary slowly with the lapse of time at other stations, must at Greenwich also have changed materially in the 17 years (1841–1857), and thus I was finally led to construct the formula

$$v_2 = 6{\cdot}66 - 0{\cdot}123(t-1849) + [0{\cdot}038 - 0{\cdot}001(t-1849)].r, \quad \text{(IV)}$$

which, as the Table shows, suits very well with the observations. For application to longer periods it will still require some further modification, and, in particular, to be augmented by corrections from the term $(t-1849)^2$. In conclusion, I also computed the variations for the years 1858–1862 by all the four formulæ, and have entered them in the Table for future comparison.

January 21, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read :—

I. "A Description of the Pneumogastric and Great Sympathetic Nerves in an Acephalous Fœtus." By Robert James Lee, Esq., B.A. Cantab. Communicated by Robert Lee, M.D. Received November 20, 1863. With Supplement, received January 20, 1864.

(Abstract.)

The author observes that hitherto no account has been given of the origin and distribution of the par vagum or pneumogastric nerve in any instance of a fœtus born with brain entirely or partially wanting. This reason has been thought sufficient for communicating to the Royal Society the description of a dissection of the pneumogastric and sympathetic nerves in a fœtus born at the full period, in which the cerebellum and medulla oblongata were absent. At the time of birth it cried, moved, and for the space of one hour might be said to live. All the thoracic and abdominal viscera were found properly formed, and the upper and lower extremities properly developed. The eyes, nose, and mouth were present. The head, when regarded as a whole, seemed as though the posterior and superior parts had been entirely removed, thus leaving the spinal cord and base of the skull exposed. Some tough cerebral matter, covered only by a dense membrane, was seen in two small masses exposed in the cranium, not continuous with the spinal cord (which terminated abruptly at the base of the cranium and was entirely exposed at this point), but separated from it by a bony prominence arising from the floor of the cranial cavity.

After the removal of the extremities, the abdomen was opened and the viscera of the abdominal cavity removed. The anterior halves of the ribs

were cut away, and the thorax with its contents washed and immersed in alcohol. The dissection was conducted in that liquid, with the assistance of an ordinary lens magnifying six diameters.

The pneumogastric nerve having been traced down the neck and thorax, was found to be distributed in the usual way. Its several ganglia, its communications with the sympathetic, and its branches to the larynx, trachea, bronchi, and œsophagus, appeared in no respect different from what is their usual condition in a perfectly formed fœtus of the same age.

Certain ganglionic enlargements formed on the superior laryngeal and recurrent nerves were likewise seen as they have been described by Dr. Robert Lee. Respecting these two principal branches of the pneumogastric in the neck, the author observes that, if they be separately examined, each will be found to be composed of two portions—one descending, the other ascending—which unite and form a single cord to be again divided into many filaments for the supply of various parts. From this he concludes that the pneumogastric is not derived from the brain; for otherwise we should expect to find branches from it composed only of descending fibres, whereas we find its two chief branches equally made up of fibres from above and from below.

The hypoglossal, glossopharyngeal and communicating branch of the accessory of the eighth pair were disposed as usually.

The sympathetic nerve was also dissected in the neck and thorax, and found to present its usual arrangement; but, besides its commonly recognized ganglia, the author discovered certain other bodies connected with it in the thorax, which he considers to be nervous ganglia, and which he thus describes:—"Just beneath the costal pleura some small stellate bodies are seen lying internal to the ganglia of the sympathetic, and at variable distances from them. Their size is that of a small pea, colour pink, and structure apparently nervous. From the circumference of one of them, fine vessels or nervous tubes are seen to radiate and join in some cases the ganglia of the sympathetic. In the angles of the rays are some pigmentary particles of brown colour, not connected, however, with the central mass. Many of these bodies are found in different parts of the thorax; and there can be no doubt of their nature, from their intimate connexion with the sympathetic."

The dissection of the nerves supplying the stomach, liver, and alimentary canal was completed in those viscera removed, with the heart and lungs, from a fœtus of six months in which neither brain nor spinal cord were present.

The stomach had numerous filaments ramifying on its surface, which could be traced down to the lining membrane. Similarly the liver was found to be pervaded with numerous fibres which followed generally the course of the blood-vessels.

Portions of the intestinal canal of the fœtus first described were examined; so that there is every reason to believe that this fœtus was supplied with nerves in the neck, thorax, and abdomen in the same manner as

one endowed with those parts thought most essential to life, the cerebellum and medulla oblongata.

Since the foregoing paper was communicated, the author has had the opportunity of examining two anencephalous fœtuses which had been preserved for many years in the Museum of St. George's Hospital. In the first specimen examined the bones forming the roof and sides of the cranium were wanting, as well as the greater part of the basis behind the foramen magnum; the brain was absent, and the bony cavity was not formed.

In this instance nerves were seen passing through foramina in the basis of the cranium. There was no spinal canal, though a membrane from which nerves proceeded occupied the position of the spinal cord. At the commencement of this there was a round body of the size of a small bean, of nervous substance, which was exposed most clearly before the preparation was removed from the bottle in which it had been preserved. The connexion was easily seen between this body and the spinal membrane, and nerves proceeded from its under surface in different directions.

There were small pieces of cartilage, apparently portions of vertebræ, found here and there in the spinal region, but there was no sacrum, and the rectum was exposed.

Such were the general appearances presented by this fœtus. In other respects it was well developed. The large venous and arterial vessels were seen to supply appropriate parts.

The pneumogastric nerves were traced by following their branches in the neck upward and downward. That on the left side was seen perforating the floor of the cranium, with a ganglionic enlargement formed on it soon after it issued from the canal, and thence passing downwards to supply the usual organs. The nerve on the right side was not clearly traced through the floor of the cranium; but as there was an opening corresponding to that on the left side, there is no reason to doubt the similarity between the two nerves.

A question which naturally presented itself on perceiving the small round body described above, was whether this might not correspond to the medulla oblongata. That this is not probable appeared from the fact that the pneumogastric nerves left the cranium at a distance of nearly two inches from it, and no connexion could be seen to exist between them and that body. No history has been preserved of this fœtus; so that whether it showed any signs of independent existence cannot be ascertained.

The distribution of the nerves in the thorax presented nothing very abnormal, the various parts being supplied with their proper branches. The sympathetic nerve was of small size on both sides, and its extent greatly diminished.

The second of these dissections bore some resemblance to the fœtus described in the paper, with the exception that there was no trace of cerebral matter whatever. A membrane from which nerves proceeded was all

that was seen. There was a proper canal for the spinal cord, but it had no osseous covering. The deep groove bifurcated at the cranial extremity into grooves of half its size, which took a direction at right angles to that of the former. The left pneumogastric nerve was seen passing through the base of the cranium to the surface, where it appeared to have come from the membrane from which other nerves proceeded. After descending to the cervical region and giving off the recurrent, the principal branch was not continued to the lungs and œsophagus, but directly to the ganglion of the sympathetic in the upper part of the thorax, so that the sympathetic chain of ganglia in the thorax appeared to be simply a continuation of the pneumogastric. To compensate for this absence of nervous supply on the left side, the nervous plexuses on the roots of the lungs were found to be enormously increased on the opposite side. A large branch ascended from the solar plexus and united with the divisions of the right pneumogastric. The splanchnic on this side was large, and was composed of filaments from the upper thoracic ganglia, not merely from those below the sixth. The action of the heart and the functions of the liver, kidneys, and other organs must have continued during the uterine existence of the fœtus.

The author expects to be afforded further means of prosecuting his dissections of the nerves of acephalous monsters, in which case he will communicate the results of his examinations to the Royal Society.

II. "On the Conditions, Extent, and Realization of a Perfect Musical Scale on Instruments with Fixed Tones." By Alexander J. Ellis, B.A., F.C.P.S. Communicated by C. Wheatstone, Esq., F.R.S. Received January 7, 1864.

Euler*, perceiving that the relative pitches of all musical notes might be represented by $2^m . 3^n . 5^p$, formed different "genera musica" by allowing n and p to vary from 0 to fixed limits. His "genus diatonicum hodiernum" (*op. cit.* p. 135) limits n to 3 and p to 2, and consists of 12 tones. These tones and 12 others are contained in his "genus cujus exponens est $2^m . 3^7 . 5^2$," that is, which limits n to 7 and p to 2. He has further (*ib.* p. 161) given a scheme in which each manual of an instrument should represent two sounds, the primary belonging to the first 12 tones, and the secondary to the additional 12. He says (*ib.* p. 162), "Soni secundarii summo rigore ab iisdem clavibus edi nequeunt, quia vero tam parum a primariis discrepant, ad eos exprimendos hæ claves *sine sensibili harmoniæ jactura* tuto adhiberi possunt. Nam *etiamsi ab acutioribus auribus comma seu diaschisma*, quibus intervallis soni secundarii a primariis differunt, *distingui queat*, tamen quia soni secundarii cum primariis *neque*

* Tentamen Novæ Theoriæ Musicæ ex certissimis Harmoniæ principiis dilucide expositæ auctore Leonhardo *Eulero*. Petropoli, 1739.

in eadem consonantia neque in duarum consonantiarum successione misceri possunt, error etiam ab acutissimo auditu percipi non poterit." * It will appear in the sequel that these assertions, when tested by experiments on instruments with fixed tones, are all incorrect.

The musical scale has formed the subject of many recent investigations*; but I have been unable to find a complete account of the necessary conditions to be fulfilled by a perfect scale, the least number of fixed tones required, and the practical means of producing them uncurtailed without inconvenience to the performer, although instruments which produce a *limited* number of just tones have been practically used by Gen. Perronet Thompson, Mr. Poole, Prof. Helmholtz, Prof. Wheatstone, myself, and others. This is therefore the subject of the present paper.

The following notation is employed. I have introduced it for the purpose of supplying a want which has been greatly felt by all writers on the theory of music. It is founded on the principle of retaining the whole of the usual notation unaltered, but restricting its signification so as to prevent ambiguity, and introducing the smallest possible number of additional signs to express the required shades of sound with mathematical accuracy, selecting such signs as are convenient for the printer, and harmonize with the ordinary notation of accidentals on the staff.

A letter, as *C*, called a *note*, will represent both a certain *tone* and its *pitch*, defined to be the number of *double vibrations* in one second, to which the tone is due. The letters *D*, *E*, *F*, *G*, *A*, *B* represent other tones and pitches, so that

$$8D = 9C,\ 4E = 5C,\ 3F = 4C,\ 2G = 3C,\ 3A = 5C,\ 8B = 15C.$$

Small and numbered letters will be so used that

$$c = \frac{1}{2}c^2 = \frac{1}{4}c^4 = \frac{1}{8}c^8 = \ldots,\ 2C = 4C^4 = 8C^8 = \ldots,$$

and similarly for other letters. The pitch of *c* is that of the "tenor or middle *c*," usually written on the leger line between the treble and bass staves; and the other letters are noted on the staff as usual in the scale of *C* major.

* *Gen. T. Perronet Thompson*, F.R.S., Instructions to my daughter for playing on the Enharmonic Guitar, 1829; Just Intonation, 6th ed. 1862. *H. W. Poole*, On a perfect musical Intonation, Silliman's American Journal of Science, 2nd ser. vol. ix. pp. 68 and 199. *W. S. B. Woolhouse*, Essay on Musical Intervals, 1835. *Prof. A. De Morgan*, Cambridge Philosophical Transactions, vol. x. p. 129. *M. Hauptmann*, Die Natur der Harmonik, Leipzig, 1853. *M. W. Drobisch*, Abhandlungen der Fürstlich Jablonowskischen Gesellschaft, 1846; Poggendorff's Annalen, vol. xc. *C. E. Naumann*, Ueber die verschiedenen Bestimmungen der Tonverhältnisse, Leipzig, 1858. *Prof. H. Helmholtz*, Lehre von den Tonempfindungen, Braunschweig, 1863. To this last writer we owe the first satisfactory theory of consonance and dissonance.

The following symbols always represent the fractions, and are called by the names written against them :

1. *Following a Note.*

$\sharp = \frac{135}{128}$ = sharp, or greater limma.

$\flat = \frac{128}{135}$ = flat, or hypolimma.

$\times = \sharp.\sharp$ = double sharp; $\flat\flat = \flat.\flat$ = double flat.

2. *Preceding a Note.*

$\dagger = \frac{81}{80}$ = acute, or comma.

$\ddagger = \frac{80}{81}$ = grave, or hypocomma.

$\wr = \frac{63}{64}$ = septime (an inverted 2).

$\P = \frac{32805}{32768} = 1{\cdot}001129150390625$ = schisma.

$\natural = \frac{32768}{32805} = 0{\cdot}99887212315$ = hyposchisma.

The name and pitch of the tones represented by any such notes as

$\ddagger c\sharp$ = grave c sharp $= \frac{80}{81}.c.\frac{135}{128} = \frac{25}{24}c$,

$\dagger e\flat$ = acute e flat $= \frac{81}{80}.e.\frac{128}{135} = \frac{24}{25}e$,

and the ratio of their pitches to the corresponding notes in the scale of *C* major is therefore precisely indicated. In ordinary musical notation on the staff, it is only necessary to prefix the signs †, ‡, ≀, ¶, ♮, to those already in use. These symbols suffice for writing any tone whose index is the product $2^m.3^n.5^p.7^q$ (see Tables I. and III.). For equally tempered tones, when it is necessary to distinguish them, the sign ‖ is prefixed to the usual names, and read "equal." Since

$$\|g : c = \sqrt[12]{2^7} : 1 = 0{\cdot}998871384584 \times \tfrac{3}{2},$$

and

$$\natural g : c \qquad = 0{\cdot}99887212315 \times \tfrac{3}{2},$$

we may without sensible error consider $\|g = \natural g$, and hence represent the equally tempered scale

c, $\|d\flat$, $\|d$, $\|e\flat$, $\|e$, $\|f$, $\|f\sharp$, $\|g$, $\|a\flat$, $\|a$, $\|b\flat$, $\|b$ by

c, $\P^5\ddagger d\flat$, $\natural^2 d$, $\P^3 e\flat$, $\natural^4\dagger e$, $\P f$, $\natural^6\dagger f\sharp$, $\natural g$, $\P^4 a\flat$, $\natural^3\dagger a$, $\P^2 b\flat$, $\natural^5\dagger b$.

In calculating relative pitches or intervals, and in all questions of tem-

perament, it is most convenient to use *ordinary* logarithms to five places, because the actual pitches, and the length of the monochord (which is the reciprocal of the relative pitch), can be thus most easily found. In Table I. the principal intervals are given as fractions, logarithms, and *degrees*. If we call 0·00568 one *degree*, then 53 degrees=0·30104=log 2−0·00001, and 31 degrees=0·17608=log $\frac{3}{2}$−0·00001. If we moreover represent the addition and subtraction of 0·00035 (or one-sixteenth of a degree) by an acute or grave accent respectively, then 17′ degrees=0·09691=log $\frac{5}{4}$, and 1` degree=0·00533=log $\frac{81}{80}$−0·00007. Two numbers of degrees which differ by a single accent of the same kind, as 17′, 17″ represent notes whose real interval is a schisma (thus *e* has 17′ degrees; and $d\times$, $=\P e$, has 17″ degrees), having a difference of logarithm=0·00049 or 0′ degrees +0·00014. By observing this, degrees may be very conveniently used for all calculation of intervals between tones of pitches represented by $2^m \cdot 3^n \cdot 5^p$. Table IV. contains a list of tones which differ from each other by a schisma, and will be useful hereafter.

The conditions of a perfect musical scale are *not* discovered by taking all the tones which can be expressed by one of Euler's "exponents," nor by forming all the tones which are consonant with a certain tone, and then all the tones consonant with these, as Drobisch has done. Such processes produce many useless, and omit many necessary tones. Since modern music depends on the relations of harmonies, and not on scales, it is necessary to find what consonant chords of three tones are most closely connected*.

Three tones whose pitches are as 4 : 5 : 6, or 10 : 12 : 15 form a major or minor consonant chord respectively. The same names are used when any one or more of the pitches is multiplied or divided by a power of 2, notwithstanding the dissonant effect in some cases. Thus, $C : E : G = 4 : 5 : 6$ is a major, and $e : \dagger e\flat : g = 10 : 12 : 15$ is a minor chord, and the same names are applied to $e : g^2 : c^4 = 5 : 2\times 6 : 2^2\times 4$, and $G : \dagger e\flat : c^2 = 15 : 2\times 12 : 2^2\times 10$, although these chords are really dissonant (Helmholtz, *ib.* p. 333–4). I shall consequently use a group of capitals, as *C E G*, to represent a major chord, and a group of small letters, as $c\,\dagger e\flat\, g$, to represent a minor chord, irrespective of the octaves. The three notes in this order, being the first, third and fifth of the major or minor scale commencing with the first, are called the first, third and fifth of the chords respectively. Both chords contain a fifth, a major and a minor third. If the interval of the *fifth* is contained by the same tones in a major and minor chord, as

* There are consonant chords of four tones, such as $g\,b\,d^2\,\dagger f^2$, and these are insisted on by Poole (*loc. cit.*); but, though they are quite consonant and agreeable, and much pleasanter than the dissonant chords by which they are replaced, such as $g\,b\,d^2\,f^2$, they do not form a part of modern music, for reasons clearly laid down by Helmholtz (*op. cit.* p. 295). Dissonant chords must always arise from the union of tones belonging to two consonant chords, or from the inversions of consonant chords; and therefore their tones are determined with those of the others.

C E G, *c* †*e*♭ *g*, or *A* ‡*C*♯ *E*, *a c e*, the chords are here termed *synonymous*. If the interval of the *major third* is contained by the same tones, as *C E G*, *a c e*; or †*E*♭ *G* †*B*♭, *c* †*e*♭ *g*, they are termed *relative*. If two chords, major or minor, have the fifth tone of the one the same as the first tone of the other, as *F A C*, *C E G*; *f* †*a*♭ *c*, *c* †*e*♭ *g*; *f* †*a*♭ *c*, *C E G*; *F A C*, *c* †*e*♭ *g*, they are here termed *dominative*. If a chain of three such dominative chords be formed (as *F A C*, *C E G*, *G B D*, or *f* †*a*♭ *c*, *c* †*e*♭ *g*, *g* †*b*♭ *d*, the minor and major chords being interchanged at pleasure), the first is called the *subdominant*, the second the *tonic*, and the third the *dominant*. Three such chords contain seven tones, and if such octaves of these tones are taken that all seven tones may lie within the compass of one octave they form a scale, of which 24 varieties can be formed by varying the major and minor chords, and beginning with the first of any one of the three chords. These scales include all the old ecclesiastical modes and several others. If all three chords are major and the scale begins on the first of the tonic chord, the result is the *major scale*, *C, D, E, F, G, A, B, c*. If all three chords are minor and the scale begins on the first of the tonic chord, the result is the *minor descending scale*, c^2, †*b*♭, †*a*♭, *g, f*, †*e*♭, *d, c*. If the first and second are minor, and the third major, or if the first and third are major and the second minor, we have the two usual *ascending minor scales*, *c, d*, †*e*♭, *f, g*, †*a*♭, *b*, c^2, or *c, d*, †*e*♭, *f, g, a, b*, c^2. Three major chords may therefore be considered to represent a major scale, but both major and minor chords are necessary for the various minor scales. If to each of three dominative major chords we form the relative and synonymous minor chords, the synonymous and relative majors of these, and the relative minor of this synonymous major, we shall have a group of 9 major and 9 minor chords, which I shall call the *key of the first of the tonic chord*. Thus the following is the

Key of C.

Relative Ma. (of Syn. Mi.).	Synonymous Minor.	Primary MAJOR.	Relative Minor.	Synon. Ma. (of Rel. Mi.).	(Sub-) Relative Minor.
†A♭ C †E♭	f †a♭ c	F A C	‡d f a	‡D ‡F♯ A	‡b ‡d ‡f♯
†E♭ G †B♭	c †e♭ g	C E G	a c e	A ‡C♯ E	‡f♯ a ‡c♯
†B♭ D †F	g †b♭ d	G B D	e g b	E ‡G♯ B	‡c♯ e ‡g♯

These chords contain 16 tones, which, when reduced to the compass of the same octave, form the *complex scale* *c*, ‡*c*♯, ‡*d*, *d*, †*e*♭, *e*, *f*, (†*f*), ‡*f*♯, *g*, ‡*g*♯, †*a*♭, *a*, †*b*♭, (‡*b*), c^2, of which the acute fourth (†*f*), and the grave seventh (‡*b*), have been enclosed in parentheses, as being of rare occurrence. From this complex scale 54 scales of 7 tones each may be formed, similar to the 24 scales already named. A selection of 12 tones, such as *c*, ‡*c*♯, *d*, †*e*♭, *e*, *f*, ‡*f*♯, *g*, †*a*♭, *a*, †*b*♭, *b*, c^2 forms the so-called *chromatic scale*, which, however, has no proper existence except in equal temperament.

Now proceed to form a series of seven dominative major chords, as

E♭ ‡*G B*♭, *B*♭ ‡*D F*, *F A C*, *C E G*, *G B D*, *D F*♯ †*A*, †*A C*♯ †*E*, and form the five related chords of each as before. The result will be *five* keys, as those of *B*♭, *F*, *C*, *G*, *D*, such that the primary major scales of each will have either two major chords, or one major chord in common with the original primary major scale. I call these five keys the *postdominant*, *subdominant*, *tonic*, *dominant*, and *superdominant* keys, and the whole group of 21 major and 21 minor chords, with the 30 tones which they contain, I term the *system* of the first tone of the tonic chord of the original primary major scale, which tone may be called the *tonic of the system*.

A piece of music is written in a certain system, determined by the compass or quality of tone of the instruments or voices which have to perform it, and rarely exceeds that system*. It is only in the system that the true relation of the tones of a piece of music, the course and intention of the modulation, and the return to the original key or scale can be appreciated. I have not yet found these relations fully expressed in any theoretical work on music; but their full expression was necessary to the solution of the problem here proposed.

It will be found practically that only 11 systems are used in music. These are, in dominative order, the systems of ‡*D*♭, *A*♭, *E*♭, *B*♭, *F*, *C*, *G*, *D*, †*A*, †*E*, †*B*, which contain the 11 keys of the same name, together with the 4 keys of ‡*C*♭, ‡*G*♭, and †*F*♯, †*C*♯. In Table V., columns III. to VIII., the whole of the major and minor chords of these 15 keys are exhibited in dominative order§. This Table, therefore, furnishes the tones which must be contained in a perfect musical scale of fixed tones, or the *conditions* of the problem.

On examination it will be found that these six columns contain 72 different notes. Hence the *extent* of a perfect scale is fixed at 72 tones to the octave. It is therefore six times as extensive as the equally tempered scale. Some means of reducing this unwieldy extent is required. The most obvious is that proposed by Euler, in the passage already quoted, namely, the use of certain tones for others which differ from them by a comma or diaschisma. Such substitution *within the same chord* creates intolerable dissonance. But in melody and in successions of chords it might seem feasible. I have had a concertina tuned, so that the three chords of

* The use of the equally tempered scale has much diminished the feeling for the relations of the system, by confounding tones originally distinct, and has thus led to the confusion of the corresponding notes. Thus such a note as ‖*g*♯ will have to be read as ‡*g*♯, *g*♯, †*g*♯; ‡*a*♭, *a*♭ or †*a*♭, according to the requirements of the system, for all six tones are represented by one on the equally tempered scale.

§ The Table of Key-relationships (*Tonartenverwandtschaften*) in Gottfried Weber's *Theorie der Tonsetzkunst* (3rd ed. 1830, vol. ii. p. 86), may be formed from Table V., by suppressing the signs †, ‡, supposing all the notes to represent tempered tones, contracting the names of the chords to their first notes, and extending the Table indefinitely in all directions.

the major scale of *D* are played as *G B D*, *D F♯ †A*, and *A ‡C♯ E*, instead of *†A C♯ †E*. The dominant chord is therefore too flat by a comma, and in passing from the chord of *A* to that of *D*, as in the ordinary cadence, the note *A* has to be changed into *†A*. If *A* is the highest or lowest note in the chords, the effect is decidedly bad. The flatness of the "leading note" *‡C♯*, in place of *C♯*, although only a comma in extent, is felt as annoying in the succession *‡c♯, d*. The result is such that it would not be worth while to invent new instruments with such a defect in common scales. On the same instrument I have the three chords of the major scale of *A* tuned as *D F♯ †A*, *A ‡C♯ E*, *E ‡G♯ B*, in which the subdominant chord is now a comma too sharp. As the subdominant is a much less important chord than the dominant, the effect is better, but trouble arises from having occasionally to alter the tonic note *A* itself. Even the dissonance of the dominant seventh, when played as *E ‡G♯ B d* is perceptibly harsher than the correct *E ‡G♯ B ‡d* (both forms lie on the instrument), although the added seventh *d* now forms a true minor third with the fifth *B*, whereas the correct note *‡d* forms a dissonant Pythagorean minor third with the same note *B*. When, however, the first *E* is omitted, the chord of the diminished fifth *‡G♯ B ‡d* is not so pleasant as *‡G♯ B d*. Again, on the same instrument, instead of having *‡D ‡F♯ A*, as the synonymous major of *‡d f a* in the scale of *a* minor, I have only *D F♯ †A*, which is a comma too sharp. The rarity of the chord, however, renders the bad effect of less importance. Again, I am obliged to modulate from *D* major to *‡d* minor instead of *d* minor. Even here the error of a comma is perceptible. The general result, therefore, is that *commatic substitution, even within the same melody or succession of chords, is inadmissible in just intonation.*

Professor Helmholtz (*op. cit.* pp. 433 & 484) has suggested what may be termed *schismatic substitution*, or the use of one note for another which only differs from it by a schisma, the eleventh part of a comma. Having one concertina tuned to equal temperament, and another to just intervals, the equation *‖g = ♭g* has enabled me to test this suggestion by practice. I find that in slow chords, the altered fifth *c ♭g*, the altered major third *♭g b*, and the altered minor third *e ♭g* are all decidedly, though only slightly, dissonant. In rapid chords the effect would be necessarily much less perceptible. Such chords as *C E ♭G*, *e ♭g b* are far superior either to the Pythagorean *C †E G*, *†e g †b* (of which I can produce the counterparts *F †A C*, *d f †a*), or the still worse tempered chords *C ‖E ‖G*, *‖e ‖g ‖b*. If we modified Professor Helmholtz's suggestion, and, where practicable, used only *entire* chords which are too flat or too sharp by a schisma, so that the schismatic errors would only occur in harmonies where a note was prolonged from a chord to which it belonged into another for which it was too sharp or too flat by a schisma, then there could be no objection whatever to schismatic substitution, which would be quite inappreciable in melody.

Now schismatic substitution will materially reduce the number of different tones required. By referring to Table IV. it will be seen that all the

tones in Table V., lines 1 to 8, throughout all the columns are exactly *one schisma flatter* than the corresponding tones in lines 10 to 17. Hence we only require the tones in lines 5 to 13 in order to reproduce the whole Table, with the help of schismatic substitution. It is, however, more convenient to use columns III., IV., lines 14 to 17, in place of columns I. and II., lines 5 to 8; and columns VII., VIII., lines 1 to 4, in place of columns IX. and X., lines 10 to 13. In this case only 48 tones will be required. If the schismatic substitution of ‡*f*, *a*♭, *c*♭ for *e*♯, *g*♯, †*b* were allowed, which would introduce three schismatic errors of no great importance, the number of tones would be reduced to 45, which is the lowest possible number of tones by which a complete scale can be played. All these tones are enumerated in Table III.

There are several ways of realizing such a scale in whole or in part*. The following appears to be the most feasible, as it would render the mere mechanism of playing a perfect scale on an organ or harmonium easier than that of playing the tempered scale on the same instruments.

On a board of manuals similar to that now in use for the organ, introduce two additional *red* manuals (of the same shape as the black, but with a serrated front edge to be recognizable by blind and colour-blind performers, as in some cases on General Perronet Thompson's organ) in the two gaps between *B* and *C*, and between *E* and *F*, so as to make 14 manuals in all. Let there be 16 stops worked as pedals with the foot, as in Mr. Poole's Euharmonic Organ (*loc. cit.* p. 209). Let one of these stops give the equally tempered tones to the manuals, so that any piece could be played in the tempered scale, and thus compared with the same piece when played with just intervals. Let the 15 other stops give the tones required for the 15 keys ‡*C*♭ to †*C*♯, as shown in Table II., and be numbered 7♭, 6♭ ... 1♭, natural, 1♯ .... 7♯. When any pedal is put down, let the seven white manuals give the seven tones of the primary major scale of the corresponding key, and the seven coloured manuals give seven out of the nine other tones required to complete the key, omitting the acute fourth (which would be found in the key of the dominant) and the grave seventh (which would be found in the key of the subdominant). To the right of each white manual let there be its conjugate coloured manual, of such a value that, if the seven tones of the major scale be indicated by the numbers 1 to 7, the tones corresponding to the manuals in any key may be

Coloured..	‡1♯	‡2	†3♭	‡4♯	‡5♯	†6♭	†7♭
White....	1	2	3	4	5	6	7.

Table II. shows the tones associated with the manuals in each stop; capital letters indicate white manuals, small letters black, and small

* Singers and performers on bowed instruments and trombones can produce any scale whatever. Other instruments are more limited in range and would require special treatment, similar to the "crooks" of the horns and the various clarinets.

capitals red*. By this arrangement the fingering of every key would be the same. The performer would disregard the signature except as naming the pedal, and play as if the signature were natural. Table V. would inform him whether the accidentals belonged to the key, its dominant, or any other key; and if they indicated another key, he would change the pedal. It would be convenient to mark where a new pedal had to be used; but no change would be required in the established notation §.

Mr. Poole's organ, which suggested the above arrangement, has 11 stops, from 5♭ to 5♯, and only 12 manuals, which appear to be associated with the following tones on each stop:

Black..		♮2		(♮2♯)		(4♯)		♮5♯		♮7♭
White..	1		2		3	4	5		6		7

The two manuals whose notes are put in parentheses are inadequately described. Mr. Poole's scale does not include the synonymous minor chords, which he plays by commatic substitution.

Another method of realizing such a scale is by additional manuals and additional boards of manuals. Thus three boards of manuals, each with 23 manuals, containing the tones in Table V. cols. III. to VIII., lines 4

* On examining Table II. it will be found that 10 different tones lie on each pair of manuals, so that there are only 70 different tones. The two missing tones are, necessarily, ♮♮*f*♯ (the acute fourth of the key of ♮*C*♯), and ♮♮*b*♭ (the grave seventh of the key of ♮*C*♭); and to this extent the scheme is defective. It would probably be more convenient to the instrument-maker to use all the 70 tones in this arrangement than to take the inferior number 45 due to schismatic substitution. A full-sized harmonium at present employs from 48 to 60 vibrators to the octave, so that the mechanical difficulties to be overcome in introducing 70 are comparatively slight. By omitting the two very unusual keys of ♮*C*♭ and ♮*C*♯, the 8 tones denoted by ♮♮*d*♭, ♮*F*♭, ♮♮*f*, *d*♭♭ and ♮*D*♯, *g*×, ♮*B*♯, ♮♮*b* in Table II. would be saved, and the number of vibrators required would be reduced to 62, nearly the same as that actually in use. As each new key introduces 4 additional tones, and the key of *C* has 14 tones, the number of vibrators required for any extent of scale is readily calculated. Thus for the 11 keys from 5 flats to 5 sharps, or ♮*D*♭, *A*♭, *E*♭, *B*♭, *F*, *C*, *G*, *D*, *A*, *E*, *B*, which is Mr. Poole's range, and is sufficiently extensive for almost all purposes, only 4×10+14=54 vibrators to the octave would be required, distributed over 11 stops (exclusive of the tempered notes); and such a number of vibrators and stops is in common use.

§ If in Table V. we reject the marks ♮, ♮, consider 16 *A*=27 *C*, 64 *E*=81 *C*, 128 *B*=243 *C*, ♯=$\frac{2187}{2048}$, ♭=$\frac{2048}{2187}$, leaving the value of the other letters unchanged, the Table will represent the Pythagorean relations expressed by the usual notation (which is quite unsuited to the equally tempered scale). The chords thus formed were too dissonant for the Greek or Arabic ear to endure, although Drobisch and Naumann (*loc. cit.* ad finem) desire this system to be acknowledged as "the sole, really sufficient acoustical foundation for the theory of music" (*als einzige, wahrhaft genügende akustische Grundlage der theoretisch-mukalischen Lehre*).

to 8, 7 to 11, and 10 to 14 respectively would be nearly complete. The manuals might be similar to those on General T. Perronet Thompson's Enharmonic Organ, which has 3 boards, with 20, 23 and 22 manuals respectively, and contains the chords in Table V. cols. III., lines 6 to 11; IV. 6 to 12; V., VI., VII., 5 to 12; VIII. and IX., 6 to 12 (four chords belonging to col. IX., lines 6 to 9, are not in the Table, but can be readily supplied, as well as the additional lines 0, —1, named below).

Euler's "genus cujus exponens est $2^m . 3^7 . 5^2$," as developed in his *Tentamen*, p. 161, must be considered as adapted for an instrument with two boards of ordinary manuals, such as some harmoniums are now constructed. His "soni primarii" would occupy the lower, and his "soni secundarii" the upper board. If to these we add their schismatic equivalents, inclosed in brackets, and distinguish white and black manuals by capital and small letters as in Table II., Euler's scheme will appear as follows, where the notation interprets his arithmetical expressions of pitch ("soni"), and not his notes ("signa sonora"), which are too vague.

Euler's Double Scheme.

Upper Board.

Schism. Equival... [‡*C*, ‡*d*♭, ‡*D*, *e*♭, *F*♭, ‡*F*, *g*♭, ‡*G*, *a*♭, *B*♭♭, *b*♭, *C*♭

"Soni Secundarii" *B*♯, *c*♯, *C*×, *d*♯, †*E*, *E*♯, †*f*♯, *F*×, *g*♯, †*A*, †*a*♯, †*B*.

Lower Board.

"Soni Primarii" . *C*, ‡*c*♯, *D*, ‡*d*♯, *E*, *F*, *f*♯, *G*, ‡*g*♯, *A*, *a*♯, *B*

Schism. Equival... [‡*D*♭♭, ‡‡*d*♭, *E*♭♭, ‡*e*♭, ‡*F*♭, ‡*G*♭♭, ‡*g*♭, *A*♭♭, ‡*a*♭, ‡*B*♭♭, ‡*b*♭, ‡*C*♭

Although it is evident from his notation that Euler regarded schismatic equivalents as identities, he has not especially alluded to them. The above scheme would contain Table V. col. V., lines 0 to 14, and the major third †*F*♯ †*A*♯ in 15 (with the schismatic error of ¶*B*♭ ¶‡*D F* for *B*♭ ‡*D F*), col. VI. 1 to 15; VII. 9 to 24; VIII. 10 to 24; IX. 18 to 24; III. —1 to 5; IV. 0 to 6. It would be therefore nearly complete in major scales, but would have only ‡*d*, *a*, *e*, *b*, *f*♯, *c*♯, *g*♯ minor, and their comparatively useless schismatic equivalents. It would have no single complete key, and would therefore require many commatic substitutions in modulation, and the use of the Pythagorean major third in the major chords of the comparatively common minor scales of ‡*f*, ‡*c*, ‡*g*. If only the "soni primarii" of the lower board are used the substitutions become very harsh, as for example *A*♯ *D F*, *D F*♯ *A* for *B*♭ ‡*D F*, *D F*♯ †*A*.

Euler's "soni primarii" may be compared with Rameau's scale *, which was as follows,

C, ‡*c*♯, ‡*D*, †*e*♭, *E*, *F*, ‡*f*♯, *G*, ‡*g*♯, *A*, †*b*♭, *B*,

* Traité de l'Harmonie, 1721. The values of the tones are determined from his arithmetical expression of the intervals.

and therefore only contained the following perfect harmonies, and *two* perfect scales, *A* major and *a* minor:—

		—♯*D F*		
		F A C	♯*d f a*	♯*D* ♯*F*♯ *A*
†*E*♭ *G B*♭,	*c* †*e*♭ *g*	*C E G*	*a c e*	*A* ♯*C*♯ *E*
	g †♭—	*G B* —	*e g b*	*E* ♯*G*♯ *B*.

Prof. Helmholtz has tuned an harmonium with two boards of manuals, somewhat in Euler's manner, as follows:—

Helmholtz's Double Scheme.

Upper Board.

Schism. equiv. [♯*C*, *d*♭, ♯*D*, *e*♭, *F*♭, *F*, *g*♭, ♯*G*, *a*♭, *A*, *b*♭, *C*♭]
Tones tuned.. *B*♯, †*c*♯, *C*×, *d*♯, †*E*, †*E*♯, †*f*♯, *F*×, *g*♯, *G*×, †*a*♯, †*B*.

Lower Board.

Tones tuned.. *C*, *c*♯, *D*, ♯*d*♯, *E*, *E*♯, *f*♯, *G*, ♯*g*♯, †*A*, *a*♯, *B*
Schism. equiv. [♯*D*♭, ♯*d*, *E*♭, ♯*e*♭, ♯*F*♭, ♯*F*, ♯*g*♭, *A*♭, ♯*a*♭, *b*♭, ♯*b*♭, ♯*C*♭].

This scheme has nearly the same extent and the same defects as Euler's.

The concertina, invented by Prof. Wheatstone, F.R.S., has 14 manuals to the octave, which were originally tuned thus, as an extension of Euler's 12-tone scheme.

C, ♯*c*♯, *D*, ♯*d*♯, *E*, †*e*♭, *F*, *f*♯, *G*, ♯*g*♯, *A*, †*a*♭, *B*, †*b*♭.

It possessed the perfect major and minor scales of *C* and *E*. The harshness of the chords †*B*♭ *D F*, *D F*♯ *A*, for *B*♭ ♯*D F*, *D F*♯ †*A* has, however, led to the abandonment of this scheme, and to the introduction of a tempered scale. I have taken advantage of the 14 manuals to contrive 4 different methods of tuning, so that 4 concertinas would play in all the common major and minor scales. Two of these I have in use, and find them effective and very useful for experimental purposes. The following gives the arrangement of the manuals in each, together with the scales possessed by each instrument, major in capitals, and minor in small letters. Where commatic substitution makes the dominant chord too flat in major scales, parentheses ( ) are used; where it makes the subdominant chord too sharp, brackets [ ] are used. Minor scales in brackets have only the subdominant tone too sharp.

The major chord *G B D* and the tone *C* being common to all four instruments, determine their relative pitch. The method of tuning these and all justly intoned or *teleon** instruments is very simple. *C* being tuned to any standard pitch, the fifths above and below it are tuned perfect. To any convenient tone thus formed, as *C* itself, form the major thirds above,

* A convenient name, formed from τέλεον διάστημα, a perfect interval.

as E, $\ddagger G\sharp$, $\ddagger B\sharp$, &c., and below as $\dagger A\flat$, $\dagger F\flat$, &c., and then the fifths above and below these tones. The names of the tones thus tuned are apparent from Table V. This tuning is much simpler than any system of temperament, and can be successfully conducted by ear only, taking care to avoid all beats in the middle octave c to c^2.

Scheme for Four Concertinas.

1. $\dagger A\flat$ *Concertina.*

Manuals..C $b\flat$, D $d\flat$, E $\dagger e\flat$, F $\dagger f$, G $g\flat$, A $\dagger a\flat$, B $\dagger b\flat$,
Scales....$D\flat$, $\dagger A\flat$, $\dagger E\flat$, $(\dagger B\flat)$, $[F]$, C; $[b\flat]$, f, c.

2. $\dagger B\flat$ *Concertina.*

Manuals..C $\dagger c$, D $c\sharp$, $\dagger E$ $\dagger e\flat$, $\dagger F$ $f\sharp$, G $\dagger g$, $\dagger A$ $\dagger a\flat$, B $\dagger b\flat$,
Scales....$\dagger E\flat$, $\dagger B\flat$, $\dagger F$, $(\dagger C)$, $[G]$, D; $[c]$, g, d.

3. *C Concertina.*

Manuals..C $\ddagger c\sharp$, $\ddagger Dd$, E $\ddagger d\sharp$, F $f\sharp$, G $\ddagger g\sharp$, A $\dagger a$, B $b\flat$,
Scales....F, C, G, (D), $[A]$, E; $[\ddagger d]$, a, e.

4. *D Concertina.*

Manuals..C $c\sharp$, D $\ddagger d\sharp$, E $\dagger e$, $E\sharp$ $f\sharp$, G $g\sharp$, $\dagger A$ $a\sharp$, B $\dagger b$,
Scales....G, D, $\dagger A$, $[\dagger E]$, $[B]$, $F\sharp$; $[e]$, b, $f\sharp$.

In Table III. the first column shows the number of degrees of any tone, two tones whose degrees differ by one-sixteenth being schismatic equivalents. The second column contains the notes of the tones. The third column contains the logarithm of the ratio of their pitch to that of c, whence the ratio itself, the absolute pitch, and the length of the monochord are readily found. In the fourth column E marks Euler's primary, and E^a his secondary tones; H, H^a the tones on Helmholtz's lower and upper board; T, the 40 tones of General T. Perronet Thompson's Enharmonic Organ; P, the 50 tones of Mr. Poole's Euharmonic Organ; t, the 72 tones of Table V., cols. III. to VIII.; s, the 24 tones out of these 72 which may be played as their adjoining schismatic substitutes without injuring the harmony; se, the 3 tones which, if played as their schismatic equivalents, would produce a slight but sensible error; t, not followed by either s or se, the 45 tones which form the minimum number of a justly intoned or teleon scale; et, the 12 tones of the equally tempered scale. The seven tones of the major scale of C are printed in capitals in the second column.

Table I.—Principal Musical Intervals.

Name.	Example.	Ratio.	or	Log.	Deg.
Unison	c : c	1 : 1	1 : 1	·00000	0
¶ Schisma	†B♯ : c	32805 : 32768	$3^8 . 5 : 2^{15}$	·00049	0′
†♭ Diaschisma	c : B♯	2048 : 2025	$2^{11} : 3^4 . 5^2$	·00491	1‴
† Comma	†c : c	81 : 80	$3^4 : 2^4 . 5$	·00540	1‵
†¶ Pythagorean Comma §	††B♯ : c	531441 : 52488	$3^{12} : 2^{19}$	·00589	1
††♭ Diesis	d♭ : ‡c♯	128 : 125	$2^7 : 5^3$	·01030	2‴
‡♯ Minor Semitone	‡c♯ : c	25 : 24	$5^2 : 2^3 . 3$	·01773	3″
‖♯ Limma	c : †B	256 : 243	$2^8 : 3^5$	·02263	4
♯ Sharp, or Greater Limma	c♯ : c	135 : 128	$3^3 . 5 : 2^7$	·02312	4′
†♭7♯ Equal Semitone*	‖c♯ : c	$\sqrt[12]{2} : 1$	$*2^{94} : 3^{49} . 5^7$	·02509	$4\frac{7}{12}$
Greater Semitone	c : B	16 : 15	$2^4 : 3 . 5$	·02803	5‵
Greatest Semitone	†c♯ : c	2187 : 2048	$3^7 : 2^{11}$	·02852	5
Greatest Limma	†d : c♯	27 : 25	$3^3 : 5^2$	·03343	6‴
Minor Tone	e : d	10 : 9	$2 . 5 : 3^2$	·04576	8′
Greater Tone	d : c	9 : 8	$3^2 : 2^3$	·05115	9
Extended Tone	g : 𝄫f	8 : 7	$2^3 : 7$	·05799	$10\frac{1}{4}$
Contracted 3rd	𝄫f : d	7 : 6	$7 : 2 . 3$	·06695	$11\frac{4}{5}$
Pythagorean Minor 3rd	f : d	32 : 27	$2^5 : 3^3$	·07379	13
Minor 3rd	g : e	6 : 5	$2 . 3 : 5$	·07918	14‵
Major 3rd	e : c	5 : 4	$5 : 2^2$	·09691	17′
Pythagorean Major 3rd	†e : c	81 : 64	$3^4 : 2^6$	·10231	18
Fourth, or Perfect 4th	f : c	4 : 3	$2^2 : 3$	·12494	22
False 4th	d : A	27 : 20	$3^3 : 2^2 . 5$	·13033	23‵
Contracted 5th	𝄫f : b	7 : 5	$7 : 5$	·14613	$25\frac{5}{7}$
Diminished 5th	f : b	64 : 45	$2^6 : 3^2 . 5$	·15297	27‵
False 5th	a : d	40 : 27	$2^3 . 5 : 3^3$	·17070	30′
Equal 5th	‖g : c	$\sqrt[12]{2^7} : 1$	$*2^{14} : 3^7 . 5$	·17560	31‵
Fifth, or Perfect 5th	g : c	3 : 2	$3 : 2$	·17609	31
Pythagorean Minor 6th	c : †E	128 : 81	$2^7 : 3^4$	·19872	35
Minor 6th	c : E	8 : 5	$2^3 : 5$	·20461	36‵
Major 6th	a : c	5 : 3	$5 : 3$	·22185	39′
Pythagorean Major 6th	†a : c	54 : 32	$2 . 3^3 : 2^5$	·22724	40
Diminished 7th	f : ‡G♯	128 : 75	$2^7 : 3 . 5^2$	·23215	41‴
Extended 6th	d : 𝄫F	12 : 7	$2^2 . 3 : 7$	·23408	$41\frac{3}{14}$
Perfect 7th	𝄫f : G	7 : 4	$7 : 2^2$	·24304	$42\frac{4}{5}$
Minor 7th	f : G	16 : 9	$2^4 : 3^2$	·24988	44
Acute Minor 7th	†b♭ : c	9 : 5	$3^2 : 5$	·25527	45‵
Major 7th	b : c	15 : 8	$3 . 5 : 2^3$	·27300	48′
Octave	c : C	2 : 1	$2 : 1$	·30103	53

§ Hence the symbol ¶ for Pythagoras, with the † (comma) prefixed.
* Approximately.

Table II.—Stops on a Justly Intoned, or *Teleon* Organ or Harmonium.

[Capitals represent *white*, small capitals *red*, small letters *black* manuals: a prefixed asterisk (*) marks the key-note of each stop.]

Name of Stop.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Tempered	C	c♯	D	d♯	E	D♯	F	f♯	G	g♯	A	a♯	B	A♯
7 flats	*‡C♭	‡‡c	‡D♭	‡‡d♭	‡E♭	E♭♭	‡F♭	‡‡f	‡G♭	‡‡g	‡A♭	a♭♭	‡B♭	B♭♭
6 flats	‡C♭	‡‡c	‡D♭	‡‡d	‡E♭	E♭♭	‡F	f♭	*‡G♭	‡‡g	A♭	‡a♭	‡B♭	B♭♭
5 flats	‡C	c♭	*‡D♭	‡‡d	E♭	‡E♭	‡F	f♭	‡G♭	‡‡g	A♭	‡a	‡B♭	B♭♭
4 flats	‡C	c♭	‡D♭	‡‡d	E♭	‡E	‡F	f♭	‡G	g♭	*A♭	‡a	B♭	‡B♭
3 flats	‡C	c♭	‡D	d♭	*E♭	‡E	F	‡f	‡G	g♭	A♭	‡a	B♭	‡B
2 flats	C	‡c	‡D	d♭	E♭	‡E	F	‡f♯	‡G	g♭	A	‡a♭	*B♭	‡B
1 flat	C	‡c♯	‡D	d♭	E	†E♭	*F	‡f♯	G	‡g	A	†a♭	B♭	‡B
Natural	*C	‡c♯	D	‡d	E	†E♭	F	‡f♯	G	‡g♯	A	†a♭	B	†B♭
1 sharp	C	‡c♯	D	‡d♯	E	†E♭	F♯	†f	*G	‡g♯	†A	a	B	†B♭
2 sharps.....	C♯	†c	*D	‡d♯	†E	E	F♯♯	†f	G	‡g♯	†A	a♯	B	†B♭
3 sharps.....	C♯	†c	D	‡d♯	†E	E♯	F♯♯	†f	G♯	†g	*†A	a♯	†B	B
4 sharps.....	C♯	†c	D♯	†d	*†E	E♯	†F♯♯	f♯	G♯	†g	†A	a♯	†B	B♯
5 sharps.....	†C♯	c♯	D♯	†d	†E	E♯	†F♯♯	f×	G♯	†g	†A♯	††a	*†B	B♯
6 sharps.....	†C♯	c×	D♯	†d	†E♯	††E	*†F♯♯	f×	†G♯	g♯	†A♯	††a	†B	B♯
7 sharps.....	*†C♯	c×	†D♯	d♯	†E♯	††E	†F♯	f×	†G♯	g×	†A♯	††a	†B♯	††B

Table III.

General List of Musical Tones.

Deg.	Note.	Log.	Remarks.	Deg.	Note.	Log.	Remarks.
0	C	·00000	E, H, T, P, t, et.	19‵	†♯e	·10770	t.
0′	†♭♯	·00049	t, s.	20″	‡♯f	·11365	t.
1‵	†c	·00540	T, t.	20‴	‡e♯	·11464	T.
2‴	‡♯c♯	·01233	T.	20⅘	𝄾f	·11810	P.
2⅘	𝄾‡d♭	·01579	P.	21′	‡f	·11954	T, P, t.
3′	‡♯d♭	·01724	t, s.	21″	e♯	·12003	E², H, P, t, se.
3″	‡c♯	·01773	E, T, P, t.	22	F	·12494	E, T, P, t.
4	‡d♭	·02263	P, t, s.	22′	†e♯	·12543	H², t, s.
4′	c♯	·02312	E², H, T, P, t.	22 1/12	‖f	·12543	et=†e♯.
4 5/12	‖c♯	·02509	et.	23‵	†f	·13033	T, t.
5‵	d♭	·02803	T, t.	23⅗	𝄾‡g♭	·14073	P.
5	†c♯	·02852	H², P, t, s.	25″	‡f♯	·14267	T, P, t.
7″	‡♯d	·04036	t.	26	‡g♭	·14757	P, t, s.
7⅗	𝄾d	·04431	P.	26′	f♯	·14806	E, H, T, P, t.
8′	‡d	·04576	T, P, t.	26½	‖f♯	·15051	et.
8″	c×	·04625	E², H², t, s.	27‵	g♭	·15297	T, t.
8 5/6	‖d	·05017	et.	27	†f♯	·15346	E², H², P, t, s.
9‵	e♭♭	·05066	t, s.	28‵	†♯f♯	·15886	t.
9	D	·05115	E, H, T, P, t.	29″	‡♯g	·16530	t.
10‵	†d	·05655	t.	29‴	‡f×	·16579	T.
11‴	‡♯d♯	·06349	T.	29⅘	𝄾g	·16925	P.
11⅗	𝄾e♭	·06695	P.	30′	‡g	·17070	T, P, t.
12′	‡e♭	·06839	t, s.	30″	f×	·17119	E², H², P, t, s.
12″	‡d♯	·06888	E, H, T, P, t.	31‵	a♭♭	·17560	t, s.
13	e♭	·07379	T, P, t.	30 11/12	‖g	·17560	et=a♭♭.
13′	d♯	·07428	E², H², P, t, s.	31	G	·17609	E, H, T, P, t.
13¼	‖d♯	·07526	et.	32‵	†g	·18149	t.
14‵	†e♭	·07918	T, t.	33‴	‡♯g♯	·18843	T.
14	†d♯	·07967	t, s.	33⅗	𝄾a♭	·19189	P.
16″	‡e	·09151	T, P, t.	34′	‡a♭	·19333	t, s.
16⅘	𝄾†e	·09547	P.	34″	‡g♯	·19382	E, H, T, P, t.
17	‡f♭	·09642	t, s.	35	a♭	·19873	T, P, t.
17′	E	·09691	E, H, T, P, t.	35′	g♯	·19922	E², H², P, t, se.
17⅔	‖e	·10034	et.	35⅓	‖g♯	·20068	et.
1‵	f♭	·10181	t, s.	36‵	†a♭	·20412	T, t.
18	†e	·10231	E², H², T, P, t.	36	†g♯	·20461	t, s.

TABLE III. (*continued*).

Deg.	Note.	Log.	Remarks.	Deg.	Note.	Log.	Remarks.
38″	‡a	·21645	T, P, t.	45′	†b♭	·25527	T, t.
38⅘	ƨ†a	·22040	P.	45⅘	ƨ‡c♭	·26567	P.
39′	A	·22185	E, T, P, t.	47″	‡b	·26761	T, P, t.
39″	g×	·22234	H², t, ṡ.	48	‡c♭	·27251	t, s.
				48′	B	·27300	E, H, T, P, t.
39¾	‖a	·22577	et.				
				48 7/12	‖b	·27594	et.
40′	b♭♭	·22675	t, s.				
40	†a	·22724	E², H, T, P, t.	49′	c♭	·27791	t.
41′	††a	·23264	t.	49	†b	·27840	E², H², P, t, se.
42″	‡‡b♭	·23908	t.	50′	††b	·28380	t.
42‴	‡a♯	·23958	T.	51″	‡‡c	·29024	t.
42⅘	ƨb♭	·24304	P.	51‴	‡b♯	·29073	T.
43′	‡b♭	·24448	P, t, s.	51⅘	ƨc	·29419	P.
43″	a♯	·24497	E, H, T, P, t.	52′	‡c	·29563	T, P, t.
44	b♭	·24988	T, P, t.	52″	b♯	·29612	E², H², P, t, s.
44′	†a♯	·25037	E², H², P, t, s.				
44⅙	‖a♯	·25086	et.				

January 28, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read:—

I. "On the Osteology of the genus *Glyptodon*." By THOMAS HENRY HUXLEY, F.R.S. Received December 30, 1863.

In 1862 the author communicated to the Royal Society an account of the more remarkable features of the skeleton of a specimen of the extinct genus *Glyptodon* which had been recently added to the Museum of the Royal College of Surgeons; and he then promised to give a full description of the skeleton, illustrated by appropriate figures, in a memoir to be presented in due time to the Royal Society. The present communication consists of Part I., and Sections 1 and 2 of Part II., of the promised memoir. Part I. contains the history of the discovery and determination of the remains of the Hoplophoridæ, or animals allied to, or identical with *Glyptodon clavipes*. Part II. is destined to comprehend the description of the skeleton of *Glyptodon clavipes* (Owen)—*Hyplophorus Selloi*? (Lund); and the Sections 1 and 2 now given contain descriptions of the skull and the vertebral column.

The preliminary notice already published in the Proceedings (Dec. 18, 1862, vol. xii. p. 316) will serve as an abstract.

TABLE IV.—Se

¶c = †B♯	W
¶d = †c ×	¶ †c
¶e = d ×	¶¶ ††c
¶f = †e♯	¶¶¶ †c
¶g = †f ×	¶¶¶¶
¶a = g ×	¶¶¶¶¶ †e
¶b = †a ×	¶¶¶¶¶¶ ††e
	¶¶¶¶¶¶¶ †g
♭♭c = ‡d♭♭	¶¶¶¶¶¶¶¶
♭♭d = e♭♭	
♭♭e = ‡f♭	
♭♭f = ‡g♭♭	
♭♭g = a♭♭	
♭♭a = ‡b♭♭	
♭♭b = ‡c♭	

TABLE V

	MAJOR. I.	*Minor.* II.	MAJOR. III.	*Minor.* IV.	MAJO V
1	C♭♭ E♭♭ G♭♭	a♭♭ c♭♭ e♭♭	A♭♭ ‡C♭ E♭♭	‡f♭ a♭♭ ‡c♭	‡F♭ ‡A
2	G♭♭ B♭♭ D♭♭	e♭♭ g♭♭ b♭♭	E♭♭ ‡G♭ B♭♭	‡c♭ e♭♭ ‡g♭	‡C♭ ‡E
3	D♭♭ F♭ †A♭♭	b♭♭ d♭♭ f♭	B♭♭ ‡D♭ F♭	‡g♭ b♭♭ ‡d♭	‡G♭ ‡B
4	†A♭♭ C♭ †E♭♭	f♭ †a♭♭ e♭	F♭ A♭ C♭	‡d♭ f♭ a♭	‡D♭ ‡F
5	†E♭♭ G♭ †B♭♭	c♭ †e♭♭ g♭	C♭ E♭ G♭	a♭ c♭ e♭	A♭ ‡C
6	†B♭♭ D♭ †F♭	g♭ †b♭♭ d♭	G♭ B♭ D♭	e♭ g♭ b♭	E♭ ‡G
7	†F♭ †A♭ †C♭	d♭ †f♭ †a♭	D♭ F †A♭	b♭ d♭ f	B♭ ‡D
8	†C♭ †E♭ †G♭	†a♭ †c♭ †e♭	†A♭ C †E♭	f †a♭ c	F A
9			†E♭ G †B♭	c †e♭ g	C E
10			†B♭ D †F	g †b♭ d	G B
11			†F †A †C	d †f †a	D F
12			†C †E †G	†a †c †e	†A C
13			†G †B †D	†e †g †b	†E G
14			†D †F♯ ††A	†b †d †f♯	†B D
15			††A †C♯ ††E	†f♯ ††a †c♯	†F♯ A
16			††E †G♯ ††B	†c♯ ††e †g♯	†C♯ E
17			††B †D♯ ††F♯	†g♯ ††b †d♯	†G♯ B
	I.	II.	III.	IV.	

TABLE IV.—Sc

¶c = †B♯	W
¶d = †c×	¶ ‡c
¶e = d×	¶‡‡d
¶f = †e♯	¶ ‡d
¶g = †f×	¶ e
¶a = g×	¶ ‡e
¶b = †a×	¶ ‡f
	¶ f
♭c = ‡d♭♭	¶ ‡g
♭d = e♭♭	¶ a
♭e = ‡f♭	¶ ‡a
♭f = ‡g♭♭	¶ ‡b
♭g = a♭♭	¶ b
♭a = ‡b♭♭	
♭b = ‡c♭	

TABLE V.—

	MAJOR. I.	*Minor.* II.	MAJOR. III.	*Minor.* IV.	MAJOR. V.
1	C♭♭ E♭♭ G♭♭	a♭♭ c♭♭ e♭♭	A♭♭ ‡C♭ E♭♭	‡f♭ a♭♭ ‡c♭	‡F♭ ‡A♭ ‡C
2	G♭♭ B♭♭ D♭♭	e♭♭ g♭♭ b♭♭	E♭♭ ‡G♭ B♭♭	‡c♭ e♭♭ ‡g♭	‡C♭ ‡E♭ ‡G
3	D♭♭ F♭ †A♭♭	b♭♭ d♭♭ f♭	B♭♭ ‡D♭ F♭	‡g♭ b♭♭ ‡d♭	‡G♭ ‡B♭ ‡D
4	†A♭♭ C♭ †E♭♭	f♭ †a♭♭ e♭	F♭ A♭ C♭	‡d♭ f♭ a♭	‡D♭ ‡F A
5	†E♭♭ G♭ †B♭♭	c♭ †e♭♭ g♭	C♭ E♭ G♭	a♭ c♭ e♭	A♭ ‡C E
6	†B♭♭ D♭ †F♭	g♭ †b♭♭ d♭	G♭ B♭ D♭	e♭ g♭ b♭	E♭ ‡G B
7	†F♭ †A♭ †C♭	d♭ †f♭ †a♭	D♭ F †A♭	b♭ d♭ f	B♭ ‡D F
8	†C♭ †E♭ †G♭	†a♭ †c♭ †e♭	†A♭ C †E♭	f †a♭ c	F A C
9			†E♭ G †B♭	c †e♭ g	C E G
10			†B♭ D †F	g †b♭ d	G B D
11			†F †A †C	d †f †a	D F♯ †A
12			†C †E †G	†a †c †e	†A C♯ †E
13			†G †B †D	†e †g †b	†E G♯ †B
14			†D †F♯ ††A	†b †d †f♯	†B D♯ †F♯
15			††A †C♯ ††E	†f♯ ††a †c♯	†F♯ †A♯ †C♯
16			††E †G♯ ††B	†c♯ ††e †g♯	†C♯ †E♯ †G♯
17			††B †D♯ ††F♯	†g♯ ††b †d♯	†G♯ †B♯ †D♯
	I.	II.	III.	IV.	V.

[*To face* p. 108.

LE IV.—Schismatic Equivalents.

Whence	and
¶ ♯c♭ = b	𝄫♮c♯ = d♭
¶ ♯♯d♭ = ♯c♯	𝄫 c× = ♯d
¶ ♯d♭ = c♯	𝄫 d♯ = e♭
¶ e♭♭ = d	𝄫♮d♯ = ♮e♭
¶ ♯e♭ = ♯d♯	𝄫 e♯ = ♯f
¶ ♯f♭ = e	𝄫♮e♯ = f
¶ f♭ = ♮e	𝄫♮f♯ = g♭
¶ ♯g♭ = f♯	𝄫 f× = ♯g
¶ a♭♭ = g	𝄫 g♯ = a♭
¶ ♯a♭ = ♯g♭	𝄫♮g♯ = ♮a♭
¶ ♯b♭ = a♯	𝄫 g× = a
¶ b♭♭ = ♮a	𝄫♮a♯ = b♭
	𝄫♮b = c♭
	𝄫 b♯ = ♯c
	𝄫♮b♯ = c

ABLE V.—Related Systems.

MAJOR. V.			*Minor.* VI.			MAJOR. VII.			*Minor.* VIII.			MAJOR. IX.			*Minor.* X.			
♯A♭	♯C♭		♯♯d♭	♯f♭	♯a♭	♯♯D♭	♯♯F	♯A♭	♯♯b♭	♯♯d♭	♯♯f							1
♯E♭	♯G♭		♯a♭	♯c♭	♯e♭	♯A♭	♯♯C	♯E♭	♯♯f	♯a♭	♯♯c							2
♯B♭	♯D♭		♯e♭	♯g♭	♯b♭	♯E♭	♯♯G	♯B♭	♯♯c	♯e♭	♯♯g							3
♯F	A♭		♯b♭	♯d♭	♯f	♯B♭	♯♯D	♯F	♯♯g	♯b♭	♯♯d							4
♯C	E♭		♯f	a♭	♯c	♯F	♯A	♯C	♯♯d	♯f	♯a							5
♯G	B♭		♯c	e♭	♯g	♯C	♯E	♯G	♯a	♯c	♯e							6
♯D	F		♯g	b♭	♯d	♯G	♯B	♯D	♯e	♯g	♯b							7
A	C		♯d	f	a	♯D	♯F♯	A	♯b	♯d	♯f♯							8
E	G		a	c	e	A	♯C♯	E	♯f♯	a	♯c♯							9
B	D		e	g	b	E	♯G♯	B	♯c♯	e	♯g♯	♯C♯	♯E♯	♯G♯	♯a♯	♯c♯	♯e♯	10
F♯	♯A		b	d	f♯	B	♯D♯	F♯	♯g♯	b	♯d♯	♯G♯	♯B♯	♯D♯	♯e♯	♯g♯	♯b♯	11
C♯	♯E		f♯	♯a	c♯	F♯	A♯	C♯	♯d♯	f♯	a♯	♯D♯	♯F×	A♯	♯b♯	♯d♯	♯f×	12
G♯	♯B		c♯	♯e	g♯	C♯	E♯	G♯	a♯	c♯	e♯	A♯	♯C×	E♯	♯f×	a♯	♯c×	13
D♯	♯F♯		g♯	♯b	d♯	G♯	B♯	D♯	e♯	g♯	b♯	E♯	♯G×	B♯	♯c×	e♯	♯g×	14
♯A♯	♯C♯		d♯	♯f♯	♯a♯	D♯	F×	♯A♯	b♯	d♯	f×	B♯	♯D×	F×	♯g×	b♯	♯d×	15
♯E♯	♯G♯		♯a♯	♯c♯	♯e♯	♯A♯	C×	♯E♯	f×	♯a♯	c×	F×	A×	C×	♯d×	f×	a×	16
♯B♯	♯D♯		♯e♯	♯g♯	♯b♯	♯E♯	G×	♯B♯	c×	♯e♯	g×	C×	E×	G×	a×	c×	e×	17
V.			VI.			VII.			VIII.			IX.			X.			

II. "On the Great Storm of December 3, 1863, as recorded by the Self-registering Instruments at the Liverpool Observatory." By JOHN HARTNUP, F.R.A.S., Director of the Observatory. Communicated by General SABINE, P.R.S. Received January 21, 1864.

[This Paper is accompanied by a diagram, which is deposited, for reference, in the Archives of the Royal Society, and of which the author gives the following explanation.]

The accompanying diagram exhibits the strength and direction of the wind, the height of the barometer, and the rain-fall for three days preceding, two days following, and during the great storm of December 3, 1863, as recorded by the self-registering instruments at the Liverpool Observatory. The barometer-tracing is a *facsimile* of the original record produced by King's self-registering barometer; the force and direction of the wind and the rain-fall have been taken from the sheets of Osler's anemometer and rain-gauge; the time-scale for the anemometer has been slightly increased to adapt it to that of the barometer, and the scale of wind-pressure for each five pounds has been made uniform, instead of leaving the spaces greater or less according to the strength of the springs as in the original record. The tracings of the recording-pencils for the direction of the wind and the rain-fall are faithfully represented, but it is scarcely possible to copy the delicate shadings and every gust recorded on the original sheets by the pencil which registers the force of the wind; all the heavy pressures are, however, correctly represented, and may be taken from the diagram as accurately as from the original sheets. The figures at the bottom of the diagram show the readings of the dry- and wet-bulb thermometers and the maximum and minimum thermometers as recorded at the Observatory during the six days; the wet- and dry-bulb thermometers were read each day at 8 and 9 A.M. and at 1, 3 and 9 P.M.; the registering dry thermometers were read and readjusted each day at 1 P.M. The time marked on the diagram for all the instruments is Greenwich mean time.

For four days previous to the 30th of November the barometer had been high and steady, the readings ranging from 30·13 in. to 30·33 in., the latter at noon on the 29th being the highest; from this time to midnight the fall was slow and pretty uniform; from midnight November 29 to midnight December 5 the changes of barometric pressure, the strength and direction of the wind, and the rain-fall are shown on the diagram. The fall of the barometer on the day of the great storm was rapid from midnight to 6 A.M.; heavy rain and hail fell from $3^h\,30^m$ to $7^h\,20^m$; and from $5^h\,50^m$ to $6^h\,45^m$ it was nearly calm, during which time the wind shifted from E. through S. to W. Between $6^h\,45^m$ and $8^h\,15^m$ the pressure of the wind increased from 0 to 16 lbs. on the square foot, and at about twenty-five

minutes past eight it increased from 16 to 43 lbs. in the short space of two or three minutes; the barometer, being at its minimum, suddenly rose about three-hundredths of an inch, and during the heaviest part of the storm it continued to rise at the rate of about one-tenth of an inch an hour. The oscillations in the mercurial column, as will be seen by the diagram, were large and frequent during the storm, one of the most remarkable being immediately after 10^h A.M. and nearly coincident with two of the heaviest gusts of wind; the depression in this case amounted to between four and five hundredths of an inch, the rise following the fall so quickly that the clock moved the recording-cylinder only through just sufficient space to cause a double line to be traced by the pencil.

III. "On the Criterion of Resolubility in Integral Numbers of the Indeterminate Equation

$$f=ax^2+a'x'^2+a''x''^2+2bx'x''+2b'xx''+2b''x'x=0."$$

By H. J. Stephen Smith, M.A., F.R.S., Savilian Professor of Geometry in the University of Oxford. Received January 20, 1864.

It is sufficient to consider the case in which f is an indefinite form of a determinant different from zero. We may also suppose that f is primitive, *i. e.* that the six numbers a, a', a'', b, b', b'' do not admit of any common divisor. We represent by Ω the greatest common divisor of the minors of the matrix of f, by $\Delta\Omega^2$ the determinant of f, and by ΩF the contravariant of f, *i. e.* the form

$$(b^2-a'a'')x^2+\ldots;$$

$\Omega\Delta^2$ will then be the determinant of F, and Δf its contravariant. By $\overline{\Omega}$, $\overline{\Delta}$, and $\overline{\Omega\Delta}$ we denote the quotients obtained by dividing Ω, Δ, and $\Omega\Delta$ by the greatest squares contained in them respectively; ω is any uneven prime dividing $\overline{\Omega}$, but not $\overline{\Delta}$; δ is any uneven prime dividing $\overline{\Delta}$, but not $\overline{\Omega}$; and θ is any uneven prime dividing both $\overline{\Omega}$ and $\overline{\Delta}$, and consequently not dividing $\overline{\Omega\Delta}$. We may then enunciate the theorem—

"The equation $f=0$ will or will not be resoluble in integral numbers different from zero according as the equations included in the formulæ

$$\left(\frac{\overline{\Omega}}{\delta}\right)=\left(\frac{F}{\delta}\right),\qquad \left(\frac{\overline{\Delta}}{\omega}\right)=\left(\frac{f}{\omega}\right),\qquad \left(\frac{-\overline{\Omega\Delta}}{\theta}\right)=\left(\frac{f}{\theta}\right)\left(\frac{F}{\theta}\right)$$

are or are not satisfied."

The symbols $\left(\frac{\overline{\Omega}}{\delta}\right)$, $\left(\frac{\overline{\Delta}}{\omega}\right)$, and $\left(\frac{-\overline{\Omega\Delta}}{\theta}\right)$ are the quadratic symbols of Legendre; the symbols $\left(\frac{F}{\delta}\right)$, $\left(\frac{F}{\theta}\right)$, $\left(\frac{f}{\omega}\right)$, $\left(\frac{f}{\theta}\right)$ are generic characters of f (see the Memoir of Eisenstein, "Neue Theoreme der höheren Arithmetik," in his 'Mathematische Abhandlungen,' p. 185, or in Crelle's Journal, vol. xxxv. p. 125).

The theorem includes those of Legendre and Gauss on the resolubility

of equations of the form $ax^2+a'x'^2+a''x''^2=0$ (Legendre, Théorie des Nombres, vol. i. p. 47; Gauss, Disq. Arith. arts. 294, 295, & 298). It is equally applicable whether the coefficients and indeterminates of f are real integers, or complex integers of the type $p+qi$.

It will be observed that if f, f', f'' . . . are forms contained in the same genus, the equations $f=0, f'=0, f''=0$, &c. are either all resoluble or all irresoluble.

IV. "Results of a Comparison of certain traces produced simultaneously by the Self-recording Magnetographs at Kew and at Lisbon; especially of those which record the Magnetic Disturbance of July 15, 1863." By Senhor Capello, of the Lisbon Observatory, and Balfour Stewart, M.A., F.R.S. Received January 14, 1864.

The National Portuguese Observatory established at Lisbon in connexion with the Polytechnic School, and under the direction of Senhor da Silveira, has not been slow to recognize the advantage to magnetical science to be derived from the acquisition of self-recording magnetographs. Accordingly that institution being well supported by the Portuguese Government, despatched Senhor Capello, their principal observer (one of the writers of this communication), with instructions to procure in Great Britain a set of self-recording magnetographs after the pattern of those in use at the Kew Observatory of the British Association.

These instruments were made by Adie of London, and when completed were sent to Kew for inspection and verification, and Senhor Capello resided there for some time in order to become acquainted with the photographic processes. The instruments were then taken to Lisbon, where they arrived about the beginning of last year, and they were forthwith mounted at the Observatory, and were in regular operation by the beginning of July last.

It had been agreed by the writers of this paper that the simultaneous magnetic records of the two observatories at Kew and Lisbon should occasionally be compared together, and the opportunity for such a comparison soon presented itself in an interesting disturbance which commenced on the 15th of July last. The curves were accordingly compared together, and the results are embodied in the present communication.

We shall in the first place compare the Kew curves by themselves, secondly the Lisbon curves in the same manner, and lastly the curves of the two Observatories together.

Comparison of Kew Curves.

The disturbance, as shown by the Kew curves, commenced on July 15th, at $9^h\ 13^m{\cdot}5$ G.M.T., at which moment the horizontal-force curve recorded an abrupt augmentation of force. The vertical component of the earth's magnetic force was simultaneously augmented, but to a smaller extent; while only a very small movement was visible in the declination curve.

The disturbance, which began in this manner, continued until July 25th, if not longer; but during the period of its action there was not for any of the elements a very great departure from the normal value; probably in this respect the declination was more affected than either of the other components.

While frequently there is an amount of similarity between the different elements as regards disturbances of long period, yet there is often also a want of likeness. If, however, we take the small but rapid changes of force, or *peaks* and *hollows*, as has been done by one of the writers of this paper in a previous communication to the Royal Society (Phil. Trans. 1862, page 621), we shall find that a disturbance of this nature which increases or diminishes the westerly declination at the same time increases or diminishes both elements of force. This will be seen more distinctly from the following Table, in which + denotes an increase and − a diminution of westerly declination, horizontal, and vertical force respectively, and the proportions are those of the apparent movements of the elements on the photographic paper.

TABLE I.

Date.	Greenwich Mean Time.	Declination.	Horizontal force.	Vertical-force change = unity in each instance.
1863.				
July 17	2 46·5	−1·0	−1·9	−1·0
17	2 53·5	−1·1	−2·0	−1·0
17	3 21·5	−1·0	−2·0	−1·0
17	7 58·5	not similar.	−2·0	−1·0
17	16 13·0	+3·5	+2·2	+1·0
18	21 23·5	+3·0	+2·0	+1·0*
19	0 15·5	not similar.	+1·8	+1·0
19	2 13·5	+1·1	+1·9	+1·0
19	2 38·0	+1·1	+1·7	+1·0
19	3 22·5	+1·0	+1·9	+1·0
19	17 51·0	+2·8	+2·0	+1·0†
19	18 0·0	+3·6	+2·0	+1·0†
19	20 29·5	+3·2	+2·0	+1·0
20	3 21·0	+1·0	+1·6	+1·0
20	18 52·5	+4·0	+2·2	+1·0
21	0 22·0	−1·6	−2·3	−1·0
21	2 2·0 to 2 15·0	−1·4	−2·1	−1·0
21	5 38·0	+1·4	+2·0	+1·0
22	19 20·5	−4·0	−2·0	−1·0
22	19 32·5	−3·4	−2·0	−1·0
22	21 40·0	+3·5	+2·0	+1·0
23	18 34·5	+3·5	+2·2	+1·0
23	19 26·5	+3·4	+2·0	+1·0†
24	3 31·0	+1·2	+2·0	+1·0
24	16 44·5	+3·1	+2·0	+1·0†

* Doubtful. † Vertical force too small to be accurately measured, but horizontal-force change reckoned = 2·0.

From this Table it will be seen that the signs are always alike for the different elements, and also that the small and rapid movements of the horizontal force are double of those of the vertical force—a result in conformity with that already obtained by one of the writers in a previous communication. On the other hand, the declination peaks and hollows do not bear an invariable proportion to those of the horizontal and vertical force, but present the appearance of a daily range, being great in the early morning hours, and small in those of the afternoon. Indeed this is evident by a mere glance at the curves, which, it so happens, present unusual facilities for a comparison of this nature.

Comparison of Lisbon Curves.

1. *Declination- and vertical-force curves.*—The peaks of the waves, or the elevations in the curve of declination, are always shown in hollows or depressions in the vertical-force curve, and *vice versâ*. We have never seen an instance to the contrary either in the curves under comparison or during the whole time of the operation of these instruments. This curious relation is exhibited in a Plate appended to this communication, from which it will be seen that we have not only a reversal, but also a very nearly constant ratio between the ordinates of the two curves. At Lisbon therefore an increase of westerly declination corresponds to a diminution of vertical force, and *vice versâ*; also an almost constant proportion obtains between the corresponding changes of these two elements.

2. *Bifilar and Declination Curves.*—July 15. A great disturbance, which at $8^h\ 37^m$ Lisbon mean time, or $9^h\ 13^m{\cdot}5$ Greenwich mean time, abruptly and suddenly augmented the horizontal force.

The curve of the declination continues nevertheless nearly undisturbed for about 30 minutes after this, and only at $9^h\ 41^m{\cdot}5$ G.M.T. it commences to descend very slowly.

July 16.—At about $13^h\ 6^m$ G.M.T., a very regularly shaped prominence of some duration occurs in the declination, but is quite invisible in the horizontal force.

July 17.—We see in the bifilar curve half-a-dozen small peaks reproduced in the declination in the same direction, but to a smaller extent.

July 18.—One or two accordant peaks. A large prominence of some duration in the declination at about $17^h\ 56^m$ G.M.T. is reproduced as a slight depression in the horizontal force.

July 19.—A reproduction in the declination of several small peaks of the horizontal force; nevertheless there are others also small which one does not see there, or only reproduced to a small extent. Not much accordance between the great and long-continued elevations and depressions.

July 20.—An accordance between the small peaks.

July 21.—The same.

July 22.—The curve is well marked with small peaks. Coincidence of several small peaks, but a want of agreement between the more remarkable

peaks. The peaks of the horizontal force more developed than those of the declination.

July 23.—The same appearance of the horizontal-force curves. One remarks on 22nd and 23rd that the small peaks of the declination and horizontal-force are more numerous and more developed in the morning hours.

July 24.—Agreement between the small peaks. A strong disturbance about $10\frac{1}{2}^h$ G.M.T., no agreement between the waves. A well-marked prominence of declination ($15\frac{1}{2}^h$) does not alter at all the horizontal-force curve.

We derive the following conclusions from the comparison which we have made between the Lisbon curves:—

1. The waves and the peaks and hollows of declination are always reproduced at the same instant in the vertical force, but in an opposite direction; that is to say, that when the north pole of the declination-needle goes to the east, the same pole of the vertical-force magnet is invariably plunged below the horizon, and *vice versâ*. During five months of operation of these instruments there has not been an example of the contrary.

2. The more prominent disturbances of the horizontal force do not in general agree with those of the declination or vertical force either in duration or time.

It is certain that when one of the two elements (bifilar or declination) is disturbed, the other is also; and sometimes one appears to see even for several periods of one of the curves, an imitation of the general march of the other; but when this is examined a little more minutely, and rigorous measures are attempted, one easily perceives that the phases do not arrive at the same time, but sometimes later and sometimes earlier, without any fixed rule.

In the same curve one generally sees contradictions of this kind. Nevertheless it is certain that the agreement in direction and time is more complete when the elevations or depressions are of shorter duration.

3. The small peaks and hollows are generally simultaneous for the three curves. The direction of these is the same for the horizontal force and declination, while that for the vertical force is opposite.

The ratio in size of the peaks and hollows is generally variable between the horizontal force and the declination, while it is always constant between the latter and the vertical force.

Our next deduction requires a preliminary remark. It has been shown by General Sabine, that if the disturbances of declination at various places be each divided into two categories, easterly and westerly, these obey different laws of daily variation, this difference not being the same for all stations.

This would seem to indicate that for every station there are at least two simultaneous disturbing forces acting independently, and superposed upon one another.

This interesting conclusion, derived by General Sabine, appears to be

[*To face page* 115.

Table II.—Comparison of the time of the principal corresponding point

		h m	h m	h m	h m	h m	h m	h m	h m	h m
July 15 ...	Kew.....	9 10	9 15	9 34	10 17	11 43	13 43	17 51	19 42	20 40
	Lisbon.....	8 37	8 42	9 1	9 43	11 9	13 10	17 15	19 10	20 9
	Differences	0 33	0 33	0 33	0 34	0 34	0 33	0 36	0 32	0 31
16 ...	Kew.....	0 23 (15)	0 32 (15)	1 24	1 40	2 33	2 58	3 50	4 09	4 42
	Lisbon.....	23 44	23 54	0 46	1 2	1 55	2 22	3 13	3 31	4 4
	Differences	0 39	0 38	0 38	0 38	0 38	0 36	0 37	0 38	0 38
17 ...	Kew.....	1 30	1 32	1 37	1 56	2 12	2 25	2 42	2 51	3 20
	Lisbon.....	0 57	1 00	1 3	1 22	1 39	1 51	2 8	2 16	2 47·5
	Differences	0 33	0 32	0 34	0 34	0 33	0 34	0 34	0 35	0 32·5
18 ...	Kew.....	1 23	1 48	2 5	5 52	6 43	12 10	14 56		
	Lisbon.....	0 47	1 11	1 29	5 15	6 7	11 35	14 20		
	Differences	0 36	0 37	0 36	0 37	0 36	0 35	0 36		
19 ...	Kew.....	(18) 23 24	0 14 (18)	2 12	3 21	4 42	5 48	10 35	13 12	19 17
	Lisbon.....	22 50	23 41	1 38	2 47	4 9	5 13	10 1	12 37	18 42
	Differences	0 34	0 33	0 34	0 34	0 33	0 35	0 34	0 35	0 35
20 ...	Kew.....	0 12 (19)	2 42	3 19·5	4 15	4 58	5 13	10 33	11 04	18 51
	Lisbon.....	23 40	2 10	2 46·5	3 42	4 23	4 39	9 58	10 29	18 18·5
	Differences	0 32	0 32	0 33	0 33	0 35	0 34	0 35	0 35	0 32·5
21 ...	Kew.....	1 32·5	2 00·5	5 36·5	7 00	10 45	11 47	15 1		
	Lisbon.....	0 57	1 24·5	5 00	6 22	10 12	11 11	14 26		
	Differences	0 35·5	0 36	0 36·5	0 38	0 33	0 36	0 35		
22 ...	Kew.....	1 24	4 02	5 6	8 26	12 57	15 49	16 27	19 19	19 31
	Lisbon.....	0 51	3 29	4 33	7 53	12 26	15 18	15 56	18 47	18 58
	Differences	0 33	0 33	0 33	0 33	0 31	0 31	0 31	0 32	0 33
23 ...	Kew.....	2 32	3 11	3 35	7 4	7 18	10 12	11 34	12 21	16 10
	Lisbon.....	1 57	2 37	3 1	6 29	6 42	9 40	11 1	11 48	15 37
	Differences	0 35	0 34	0 34	0 35	0 36	0 32	0 33	0 33	0 33
24 ...	Kew.....	3 29·5	4 21	4 50	5 25	5 56	8 40	9 53	11 27	12 54
	Lisbon.....	2 56	3 49	4 19	4 53	5 23	8 6	9 20	10 55	12 23
	Differences	0 33·5	0 32	0 31	0 32	0 33	0 34	0 33	0 32	0 31

ding points of the Horizontal-Force Curves at Kew and at Lisbon.

m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
42	20 40											
10	20 9											
32	0 31											
09	4 42	9 50	18 43	20 1								
31	4 4	9 12	18 6	19 22								
38	0 38	0 38	0 37	0 39								
51	3 20	4 6	5 43	6 38	6 59	7 6	7 22	7 57	11 00	11 12	11 51	
16	2 47·5	3 32	5 09	6 4	6 25	6 32	6 48	7 22·5	10 25	10 38	11 18	
35	0 32·5	0 34	0 33	0 34	0 34	0 34	0 34	0 34·5	0 35	0 34	0 33	
12	19 17	20 28	21 58									
37	18 42	19 54	21 24									
35	0 35	0 34	0 34									
04	18 51											
29	18 18·5											
35	0 32·5											
19	19 31	21 38·5										
47	18 58	21 5										
32	0 33	0 33·5										
21	16 10	18 33	21 52									
48	15 37	18 00	21 19									
33	0 33	0 33	0 33									
27	12 54	16 23	18 41									
55	12 23	15 48	18 8									
32	0 31	0 35	0 33									

[*To face page* 115.

Table II.—Comparison of the time of the

		h m	h m	h m	h m	h
July 15 ...	Kew.....	9 10	9 15	9 34	10 17	11
	Lisbon.....	8 37	8 42	9 1	9 43	11
	Differences	0 33	0 33	0 33	0 34	0
16 ...	Kew.....	0 23	0 32	1 24	1 40	2
		(15)	(15)			
	Lisbon.....	23 44	23 54	0 46	1 2	1
	Differences	0 39	0 38	0 38	0 38	0
17 ...	Kew.....	1 30	1 32	1 37	1 56	2
	Lisbon.....	0 57	1 00	1 3	1 22	1
	Differences	0 33	0 32	0 34	0 34	0
18 ...	Kew.....	1 23	1 48	2 5	5 52	6
	Lisbon.....	0 47	1 11	1 29	5 15	6
	Differences	0 36	0 37	0 36	0 37	0
19 ...		(18)				
	Kew.....	23 24	0 14	2 12	3 21	4
		(18)	(18)			
	Lisbon.....	22 50	23 41	1 38	2 47	4
	Differences	0 34	0 33	0 34	0 34	0
20 ...	Kew.....	0 12	2 42	3 19·5	4 15	4
		(19)				
	Lisbon.....	23 40	2 10	2 46·5	3 42	4
	Differences	0 32	0 32	0 33	0 33	0
21 ...	Kew.....	1 32·5	2 00·5	5 36·5	7 00	10
	Lisbon.....	0 57	1 24·5	5 00	6 22	10
	Differences	0 35·5	0 36	0 36·5	0 38	0
22 ...	Kew.....	1 24	4 02	5 6	8 26	12
	Lisbon.....	0 51	3 29	4 33	7 53	12
	Differences	0 33	0 33	0 33	0 33	0
23 ...	Kew.....	2 32	3 11	3 35	7 4	7
	Lisbon.....	1 57	2 37	3 1	6 29	6
	Differences	0 35	0 34	0 34	0 35	0
24 ...	Kew.....	3 29·5	4 21	4 50	5 25	5
	Lisbon.....	2 56	3 49	4 19	4 53	5
	Differences	0 33·5	0 32	0 31	0 32	0

verified by the behaviour of the Lisbon curves. From the relation, always invariable, between the waves of declination and vertical force, as well as from the almost total absence of agreement between these two curves and the horizontal force, one has a right to conclude—

1. That there is approximately only one independent force which acts at Lisbon, if we consider the vertical plane bearing (magnetic) east and west. Now the ratio of the disturbing forces for the vertical force and declination is, in units of force, between 26 : 48 and 26 : 36. This would give the inclination of the resultant between 29° and 36°.

2. The absence of agreement in time, and the variability in direction, between the waves of the horizontal force and those of the declination and vertical force, appear to lead to the conclusion that there is another disturbing force besides that already mentioned, which acts in the direction of the magnetic meridian and almost horizontally.

Comparison of the Kew and Lisbon Curves (14–24 July).

1. *Horizontal force* (*north and south disturbing force*).—The curves of the horizontal force at Kew and at Lisbon exhibit a very great similitude *, as will be seen at once from the Plate appended to this communication. Almost all the waves and peaks and hollows are reproduced at both places. At the same time one does not see the same resemblance during the great disturbance of 15th July. In the commencement, and for the first four hours, there is a resemblance for all the waves, but from that time until $19\frac{1}{2}^h$ G.M.T. one remarks little agreement between the different elevations and depressions. But from $19\frac{1}{2}^h$ until the end of the disturbance the likeness reappears. There are, however, one or two cases of small resemblance in the other curves, but these are of short duration.

In order to demonstrate the similarity between the two curves, reference is made to Table II., in which the principal points are compared together with respect to time; that employed being the mean time for both stations.

From this Table it will be found that the average difference between the local times of corresponding points is $34^m{\cdot}3$, while that due to difference of longitude is $35^m{\cdot}3$. We attribute this apparent want of simultaneity to various causes :—

(1) Loss of time in the commencement of movements of the registering cylinder.

(2) Difficulty in estimating precisely the commencement of certain curves.

(3) It was only in the month of August that the exact Lisbon time of the astronomical observatory was obtained by a telegraphic connexion.

(4) To these must be added the uncertainty in estimating the exact turning-point of an elevation or depression of a blunt or rounded form.

* We speak of the Kew curves reversed so as to have their base-lines above, the disposition of the registering arrangement at Kew being the opposite of that at Lisbon. This reversal has been made in the Plate which accompanies this paper.

The following Table exhibits approximately the proportion between the disturbance-waves of the horizontal force at Lisbon and at Kew.

TABLE III.

Date.	Proportion between the disturbance-waves of the horizontal force reduced at both places to English units (Lisbon wave = unity).
July 15.	Variable between 1 : 1·3 and 1 : 1·9
16.	,, 1 : 1·8 and 1 : 1·9
17.	,, 1 : 1·6
18.	,, 1 : 1·9 and 1 : 2·5
19.	,, 1 : 1·7
20.	,, 1 : 1·5 and 1 : 2·0
21.	,, 1 : 1·5 and 1 : 2·0
22.	,, 1 : 1·7
23.	,, 1 : 2·0
24.	,, 1 : 2·0
	Mean . . 1 : 1·8

From this Table it will be seen that while this proportion is variable, yet one may generally regard the disturbing force at Kew as greater than that at Lisbon in the proportion of 1·8 to 1.

2. *Declination* (*east and west disturbing force*).—The declination-curves for Kew and Lisbon are very like each other, and the waves as well as the peaks and hollows are for the most part simultaneously produced in the two collections of curves. Since, however, at Kew the waves are greater, one does not always easily perceive the resemblance. Certain peaks or waves very prominent at Kew, are reproduced but slightly at Lisbon; but a careful scrutiny shows that all, or very nearly all, of the Kew waves and peaks occur at Lisbon also.

In Table IV. we have a comparison of the principal points of the declination-curves with respect to time. From this Table it will be found that the average difference between the local times of corresponding points is $34^{m}\cdot0$, that due to difference of longitude being $35^{m}\cdot3$.

The following Table exhibits approximately the proportion between the disturbance-waves of the declination at Lisbon and at Kew.

TABLE V.

Date.	Proportion between the disturbance-waves of the declination reduced at both places to English units (Lisbon wave=unity).
July 15.	Variable from 1 : 1·8 to 1 : 2·1
16.	,, 1 : 1·5
17.	,, 1 : 1·4 to 1 : 1·6
18.	,, 1 : 1·5
19.	,, 1 : 1·5
20.	,, 1 : 1·7
21.	,, 1 : 1·3
22.	,, 1 : 1·4
23.	,, 1 : 1·8
24.	,, 1 : 1·6
	Mean . . 1 : 1·6

TABLE IV.—Comparison of the times of the principal corresponding

		h m	h m	h m	h m	h m	h m	h
July 15 ...	Kew	10 15	13 15	14 36	17 47	18 00	18 53	19
	Lisbon	9 35	12 36	13 56	17 08	17 21	18 16	19
	Differences	0 40	0 39	0 40	0 39	0 39	0 37	0
16 ...	Kew	0 18	2 17	2 30	3 51	12 48	14 11	16
		(15)						
	Lisbon	23 41	1 41	1 54	3 16	12 14	13 37	15
	Differences	0 37	0 36	0 36	0 35	0 34	0 34	0
17 ...		(16)	(16)					
	Kew	22 55	23 49	1 46	2 45	2 52	3 20	
		(16)	(16)					
	Lisbon	22 21	23 16	1 11	2 11	2 20	2 48	
	Differences	0 34	0 33	0 35	0 34	0 32	0 32	0
18 ...		(17)						
	Kew	23 20	0 22	2 7	3 37	7 18	8 5	
		(17)	(17)					
	Lisbon	22 46	23 48	1 33	3 3	6 44	7 32	
	Differences	0 34	0 34	0 34	0 34	0 34	0 33	0
19 ...		(18)						
	Kew	23 20	00 14	2 12	2 36·5	3 21	5 45	
		(18)	(18)					
	Lisbon	22 45	23 42	1 36·5	2 2	2 46	5 12	
	Differences	0 35	0 32	0 35·5	0 34·5	0 35	0 33	
20 ...	Kew	0 54	3 19·5	3 56	4 13	5 52	10 59	
	Lisbon	0 18	2 46·5	3 22	3 41	5 20	10 23	
	Differences	0 36	0 33	0 34	0 32	0 32	0 36	
21 ...	Kew	0 20·5	2 10	2 13	4 19	5 36·5	7 12	
		(20)						
	Lisbon	23 47	1 36	1 40	3 46	5 01·5	6 38	
	Differences	0 33·5	0 34	0 33	0 33	0 35	0 34	
22 ...		(21)						
	Kew	23 20	0 42	3 51	5 6	8 26	12 56	
		(21)						
	Lisbon	22 48	0 8	3 17	4 33	7 51	12 22	
	Differences	0 32	0 34	0 35	0 33	0 35	0 34	
23 ...	Kew	2 24	3 09	3 34	7 26	9 23	10 54	
	Lisbon	1 52	2 34	3 00	6 52	8 48	10 22	
	Differences	0 32	0 35	0 34	0 34	0 35	0 32	
24 ...	Kew	1 14	3 29·5	4 18	6 2	8 36	11 20	
	Lisbon	0 40	2 57	3 46	5 28	8 1·5	10 48	
	Differences	0 34	0 32·5	0 32	0 34	0 34·5	0 32	

rtion between the
Kew.

n is variable, yet
greater than that

The declination-
waves as well as
produced in the
ves are greater,
ertain peaks or
at Lisbon; but
Kew waves and

nts of the decli-
ll be found that
nding points is

on between the

Table IV.—Comparison of the times of the prin

		h m	h m	h m	h m	h
July 15 ...	Kew	10 15	13 15	14 36	17 47	1
	Lisbon	9 35	12 36	13 56	17 08	1
	Differences	0 40	0 39	0 40	0 39	
16 ...	Kew	0 18	2 17	2 30	3 51	1
		(15)				
	Lisbon	23 41	1 41	1 54	3 16	1
	Differences	0 37	0 36	0 36	0 35	
		(16)	(16)			
17 ...	Kew	22 55	23 49	1 46	2 45	
		(16)	(16)			
	Lisbon	22 21	23 16	1 11	2 11	
	Differences	0 34	0 33	0 35	0 34	
		(17)				
18 ...	Kew	23 20	0 22	2 7	3 37	
		(17)	(17)			
	Lisbon	22 46	23 48	1 33	3 3	
	Differences	0 34	0 34	0 34	0 34	
		(18)				
19 ...	Kew	23 20	00 14	2 12	2 36·5	
		(18)	(18)			
	Lisbon	22 45	23 42	1 36·5	2 2	
	Differences	0 35	0 32	0 35·5	0 34·5	
20 ...	Kew	0 54	3 19·5	3 56	4 13	
	Lisbon	0 18	2 46·5	3 22	3 41	
	Differences	0 36	0 33	0 34	0 32	
	Kew	0 20·5	2 10	2 13	4 19	
21 ...		(20)				
	Lisbon	23 47	1 36	1 40	3 46	
	Differences	0 33·5	0 34	0 33	0 33	
		(21)				
22 ...	Kew	23 20	0 42	3 51	5 6	
		(21)				
	Lisbon	22 48	0 8	3 17	4 33	
	Differences	0 32	0 34	0 35	0 33	
23 ...	Kew	2 24	3 09	3 34	7 26	
	Lisbon	1 52	2 34	3 00	6 52	
	Differences	0 32	0 35	0 34	0 34	
24 ...	Kew	1 14	3 29·5	4 18	6 2	
	Lisbon	0 40	2 57	3 46	5 28	
	Differences	0 34	0 32·5	0 32	0 34	

[*To face page* 116.

of the principal corresponding points of the Declination Curves at Kew and Lisbon.

h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
17 47	18 00	18 53	19 49	21 51						
17 08	17 21	18 16	19 12	21 17						
0 39	0 39	0 37	0 37	0 34						
3 51	12 48	14 11	16 22	18 26	20 13	21 16				
3 16	12 14	13 37	15 45	17 52	19 37	20 42				
0 35	0 34	0 34	0 37	0 34	0 36	0 34				
2 45	2 52	3 20	3 59	5 40	5 57	8 33	11 11	16 11·5	18 37	19 51
2 11	2 20	2 48	3 22	5 5	5 22	8 0	10 39	15 39	18 6	19 20
0 34	0 32	0 32	0 37	0 35	0 35	0 33	0 32	0 32·5	0 31	0 31
3 37	7 18	8 5	9 33	12 50	13 58	15 43	16 41	17 54		
3 3	6 44	7 32	8 58	12 16	13 24	15 11	16 10	17 22		
0 34	0 34	0 33	0 35	0 34	0 34	0 32	0 31	0 32		
2 36·5	3 21	5 45	9 14	17 49·5	17 58·5	18 3	20 28			
2 2	2 46	5 12	8 39	17 18	17 25	17 28	19 56			
0 34·5	0 35	0 33	0 35	0 31·5	0 33·5	0 35	0 32			
4 13	5 52	10 59	12 31	15 1	18 51	19 24	20 25	21 4		
3 41	5 20	10 23	11 56	14 26	18 18·5	18 53·5	19 51	20 31		
0 32	0 32	0 36	0 35	0 35	0 32·5	0 30·5	0 34	0 33		
4 19	5 36·5	7 12	11 5	12 7	16 12	17 28	20 15			
3 46	5 01·5	6 38	10 31	11 34	15 39	16 55	19 44			
0 33	0 35	0 34	0 34	0 33	0 33	0 33	0 31			
5 6	8 26	12 56	14 11	16 58	17 39	18 47	19 19	19 31	21 38·5	
4 33	7 51	12 22	13 36	16 25	17 6	18 14	18 47	18 58	21 5	
0 33	0 35	0 34	0 35	0 33	0 33	0 33	0 32	0 33	0 33·5	
7 26	9 23	10 54	11 32	14 36	18 33	19 25	19 50	21 50		
6 52	8 48	10 22	10 59·5	14 3	18 1	18 52	19 18	21 17		
0 34	0 35	0 32	0 32·5	0 33	0 32	0 33	0 32	0 33		
6 2	8 36	11 20	12 30	15 36	16 43	18 8	19 27	20 22	21 39	
5 28	8 1·5	10 48	11 53	15 1	16 6·5	17 35	18 55	19 50	21 6	
0 34	0 34·5	0 32	0 37	0 35	0 36·5	0 33	0 32	0 32	0 33	

It would thus appear that the declination at Kew, judging from the waves, is subject to greater disturbing forces than at Lisbon in the proportion of 1·6 : 1. This ratio is not, however, quite so great as that for the horizontal force.

3. *Vertical disturbing force.*—The curves of vertical force are nearly quite dissimilar. Sometimes the general march of the curves appears to coincide during some time; but in these cases we do not find an appreciable general agreement for the majority of the various points of the wave.

On the other hand, the small peaks and hollows of the Kew curves are generally reproduced in those of Lisbon, but in the opposite direction, that is to say, a sudden augmentation of the vertical force at Kew corresponds to a sudden diminution of the same at Lisbon, and *vice versâ*.

In Table VI. we have a comparison of the principal points of the vertical-force curves with respect to time.

TABLE VI.—Comparison of the time of the principal corresponding points of the Curves of Vertical Force at Kew and Lisbon.

July 15 ...	Kew	h m	h m	h m	h m	h m	h m
	Lisbon ...	No similarity.					
	Differences						
16 ...	Kew.....	2 21	21 17				
	Lisbon.....	1 47	20 45				
	Differences	0 34	0 32				
17 ...	Kew.....	2 45·0	2 52	3 20	7 57	16 11·5	
	Lisbon.....	2 11·5	2 19·5	2 49	7 23	15 39	
	Differences	0 33·5	0 32·5	0 31	0 34	0 32·5	
18 ...	Kew.....	7*00	9*33	21 22			
	Lisbon.....	6 29	9 00	20 50			
	Differences	0 31	0 33	0 32			
19 ...	Kew.....	0*10	2 12	2 36·5	3 21	17 49·5	17 58·5
	Lisbon.....	23 38	1 36·5	2 02	2 46	17 18	17 25
	Differences	0 32	0 35·5	0 34·5	0 35	0 31·5	0 33·5
20 ...	Kew.....	3 19·5	18 51	22 03			
	Lisbon.....	2 46·5	18 18·5	21 31			
	Differences	0 33	0 32·5	0 32			
21 ...	Kew.....	2 10	5 36·5	6 43			
	Lisbon.....	1 36	5 01·5	6 09			
	Differences	0 34	0 35	0 34			
22 ...	Kew.....	5 06	8 24	12 56	19 19	19 31	21 38·5
	Lisbon.....	4 33	7 51	12 22	18 46	18 57	21 4
	Differences	0 33	0 33	0 34	0 33	0 34	0 34·5
23 ...	Kew.....	18 33	18 37	21 54			
	Lisbon.....	18 01	18 05	21 22			
	Differences	0 32	0 32	0 32			
24 ...	Kew.....	3 29·5	4 10	5 59	16 43		
	Lisbon.....	2 56	3 37	5 26	16 8·5		
	Differences	0 33·5	0 33	0 33	0 34·5		

* Only the points marked with this sign are in the same direction, all the others are in the opposite direction; that is to say, an augmentation of force at Kew corresponds to a diminution of the same at Lisbon, and *vice versâ*.

From this Table it will be seen that the average difference between the local times of corresponding points is $33^{m}\cdot1$, while for the horizontal force this was $34^{m}\cdot3$, and for the declination $34^{m}\cdot0$, the mean of the three being $33^{m}\cdot8$. The measurements from which these numbers were obtained were made at Lisbon independently for each element: another set of measurements, made at Kew, but of a less comprehensive description, gave a mean difference in local time of $33^{m}\cdot7$, which is as nearly as possible identical with the Lisbon determination. We have already observed that we attribute the difference between $33^{m}\cdot8$ and $35^{m}\cdot3$, the true longitude-difference of local times, to instrumental errors, and not to want of simultaneity in the corresponding points.

In Table VII. we have a comparison in magnitude and sign of the peaks and hollows at the two stations.

From this Table it will be seen that the magnitude of these is generally greater at Kew than at Lisbon. The curious fact of the reversal in direction of the vertical-force peaks between Kew and Lisbon has been already noticed.

We shall now in a few words recapitulate the results which we have obtained.

1. In comparing the Kew curves together for this disturbance, the peaks and hollows of the horizontal force always bear a definite proportion to those of the vertical force, the proportion being the same as that observed in previous disturbances. On the other hand, the declination peaks and hollows do not bear an invariable proportion to those of the other two elements, but present the appearance of a daily range, being great in the early morning hours, and small in those of the afternoon. The peaks and hollows are in the same direction for all the elements.

2. In comparing the Lisbon curves together, the elevations of the declination-curve always appear as hollows in the vertical-force curve, and *vice versâ*, and there is always a very nearly constant ratio between the ordinates of the two curves. The horizontal-force curve, on the other hand, presents no striking likeness to the other two. We conclude from this that there are at least two independent disturbing forces which jointly influence the needle at Lisbon, but that the declination and vertical-force elements are chiefly influenced by one force.

The peaks and hollows are generally simultaneous for the three curves. The direction of these is the same for the horizontal force and declination, while that for the vertical force is opposite. The ratio in magnitude of the peaks and hollows is generally variable between the horizontal force and the declination, while it is always constant between the latter and the vertical force.

3. When the Kew and Lisbon curves are compared together, there is a very striking likeness between the horizontal-force curves, one perhaps somewhat less striking between the declination-curves, and very little likeness between the vertical-force curves. It is perhaps worthy of note that

TABLE VII.—Magnitude and Sign of the Peaks and Hollows at Kew and Lisbon.

This Table has been constructed with the following values of K, the coefficient for one inch.

KEW.		LISBON.	
	English unit.		English unit.
Horizontal force ...	K=0·041 ...	K (17 and 18) ...	=0·066
		For the other days	K=0·041
Declination	K=0·024 ...	.....	K=0·040
Vertical force	K=0·024 ...	.....	K=0·026

		Declination.			Horizontal force.				Vertical force.			
		Kew.		Lisbon.	Kew.		Lisbon.		Kew.		Lisbon.	
		Inch.	English unit.	Inch.	Inch.	English unit.	Inch.	English unit.	Inch.	English unit.	Inch.	English unit.
July 17		−0·040	0·00096	−0·010	−0·065	0·00267	−0·030	0·00198	−0·035	0·00084	+0·015	0·00039
		−0·040	0·00096	−0·010	−0·075	0·00307	−0·030	0·00198	−0·035	0·00084	+0·015	0·00039
		−0·050	0·00120	−0·010	−0·100	0·00410	−0·040	0·00264	−0·050	0·00120	+0·010	0·00026
		...	...	...	−0·090	0·00369	−0·055	0·00363	−0·045	0·00108		
18		+0·070	0·00168	+0·015	+0·045	0·00184	+0·030	0·00198	+0·020	0·00048	−0·015	0·00039
		+0·045	0·00108	+0·030	+0·030	0·00123	very small	?	?	?	−0·040	0·00104
		...	...	+0·025	+0·100	0·00410	+0·050	0·00205	+0·055	0·00132	−0·025	0·00065
		+0·050	0·00120	?	+0·075	0·00307	+0·055	0·00225	+0·045	0·00108	−0·015	0·00039
		+0·035	0·00084	+0·015	+0·050	0·00205	+0·040	0·00164	+0·030	0·00072	−0·015	0·00039
19		+0·035	0·00084	+0·020	+0·065	0·00267	+0·045	0·00184	+0·035	0·00084	−0·025	0·00065
		+0·035	0·00084	+0·015	+0·025	0·00103	+0·015	0·00061	very small	...	−0·015	0·00039
		+0·065	0·00156	+0·025	+0·035	0·00143	+0·025	0·00102	small	...	−0·025	0·00065
		+0·065	0·00156	+0·035	+0·040	0·00164	+0·010	0·00041	+0·020	0·00048	−0·030	0·00078
		+0·020	0·00048	+0·010	+0·040	0·00164	?	?	+0·025	0·00060	−0·010	0·00026
20		+0·090	0·00216	+0·030	+0·045	0·00184	+0·015	0·00061	+0·020	0·00048	−0·030	0·00078
		−0·025	0·00060	−0·010	−0·035	0·00143	very small	...	−0·015	0·00036	+0·010	0·00026
21		−0·050	0·00120	−0·035	−0·075	0·00307	−0·040	0·00164	−0·035	0·00084	+0·030	0·00078
		+0·035	0·00084	+0·015	+0·050	0·00205	+0·035	0·00143	+0·025	0·00060	−0·010	0·00026
		−0·100	0·00240	−0·035	−0·050	0·00205	−0·030	0·00123	−0·025	0·00060	+0·040	0·00104
22		−0·085	0·00204	−0·035	−0·050	0·00205	−0·025	0·00102	−0·025	0·00060	+0·045	0·00117
		+0·070	0·00168	+0·015	+0·040	0·00164	+0·020	0·00082	+0·020	0·00048	−0·010	0·00026
23		+0·070	0·00168	+0·030	+0·045	0·00184	+0·025	0·00102	+0·020	0·00048	−0·040	0·00104
		+0·060	0·00144	+0·025	+0·035	0·00143	+0·030	0·00123	very small	...	−0·030	0·00078
24		+0·055	0·00132	+0·025	+0·090	0·00369	+0·080	0·00328	+0·045	0·00108	−0·030	0·00078
		+0·050	0·00120	+0·010	+0·030	0·00123	+0·025	0·00102	very small			

the Lisbon horizontal-force curve, in which we may suppose two independent forces to be represented, is probably on the whole the most like the corresponding Kew curve. Corresponding points occur at the same absolute time for both stations.

The disturbance-waves for the horizontal force and declination are greater at Kew than at Lisbon.

The Kew peaks and hollows are simultaneously produced at Lisbon in all the elements, but to a smaller extent than at Kew; also the direction is reversed in the case of the vertical force, so that a sudden small increase of vertical force at Kew corresponds to a diminution of the same at Lisbon.

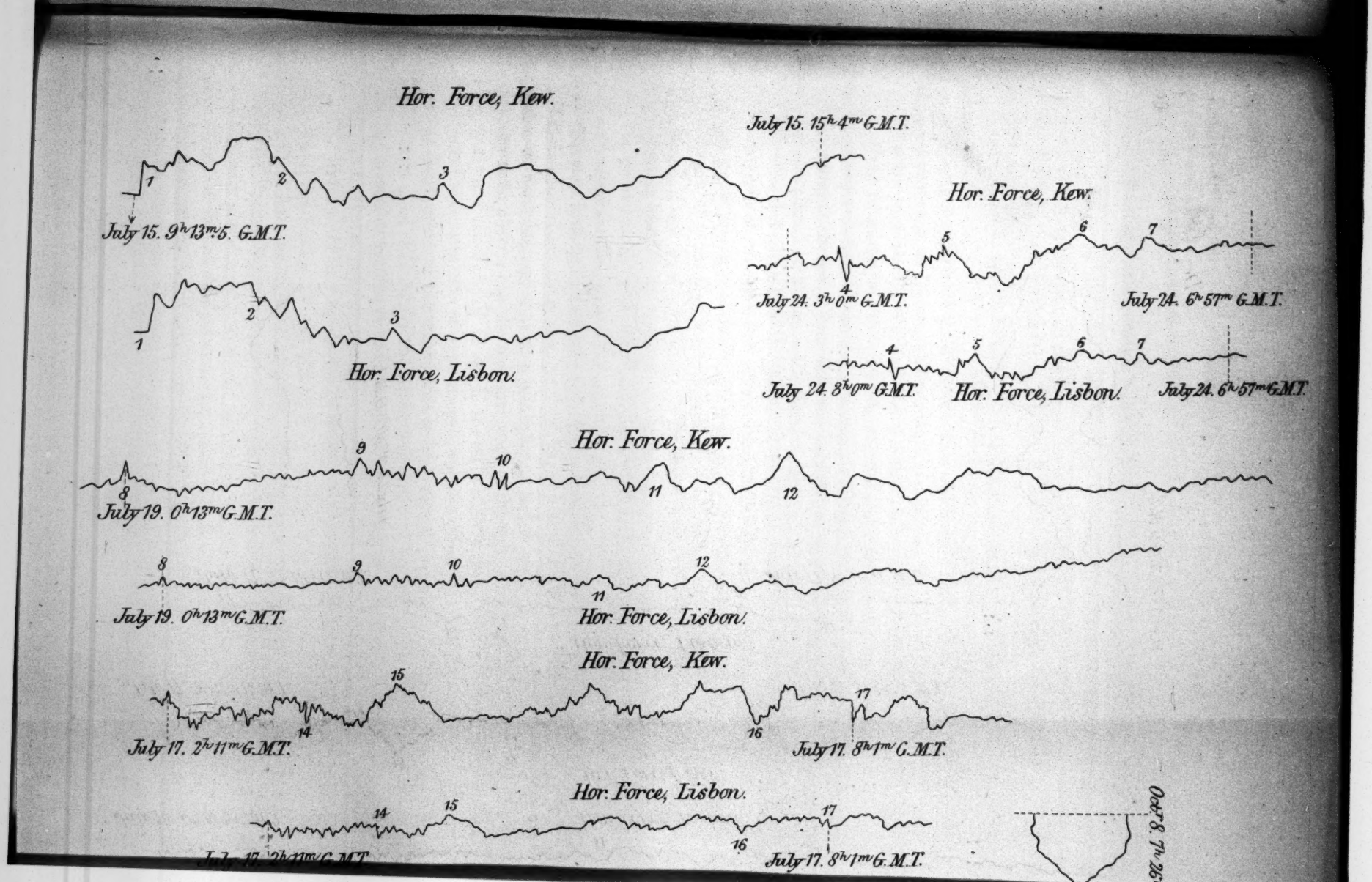
The writers of this paper are well aware that before the various points alluded to in their communication can be considered as established, a more extensive comparison of curves must be made. But as the subject is new and of great interest, they have ventured thus early to make a preliminary communication to the Royal Society. They will afterwards do all in their power to confirm their statements, which in the meantime they submit to this Society as still requiring that proof which only a more prolonged investigation can afford.

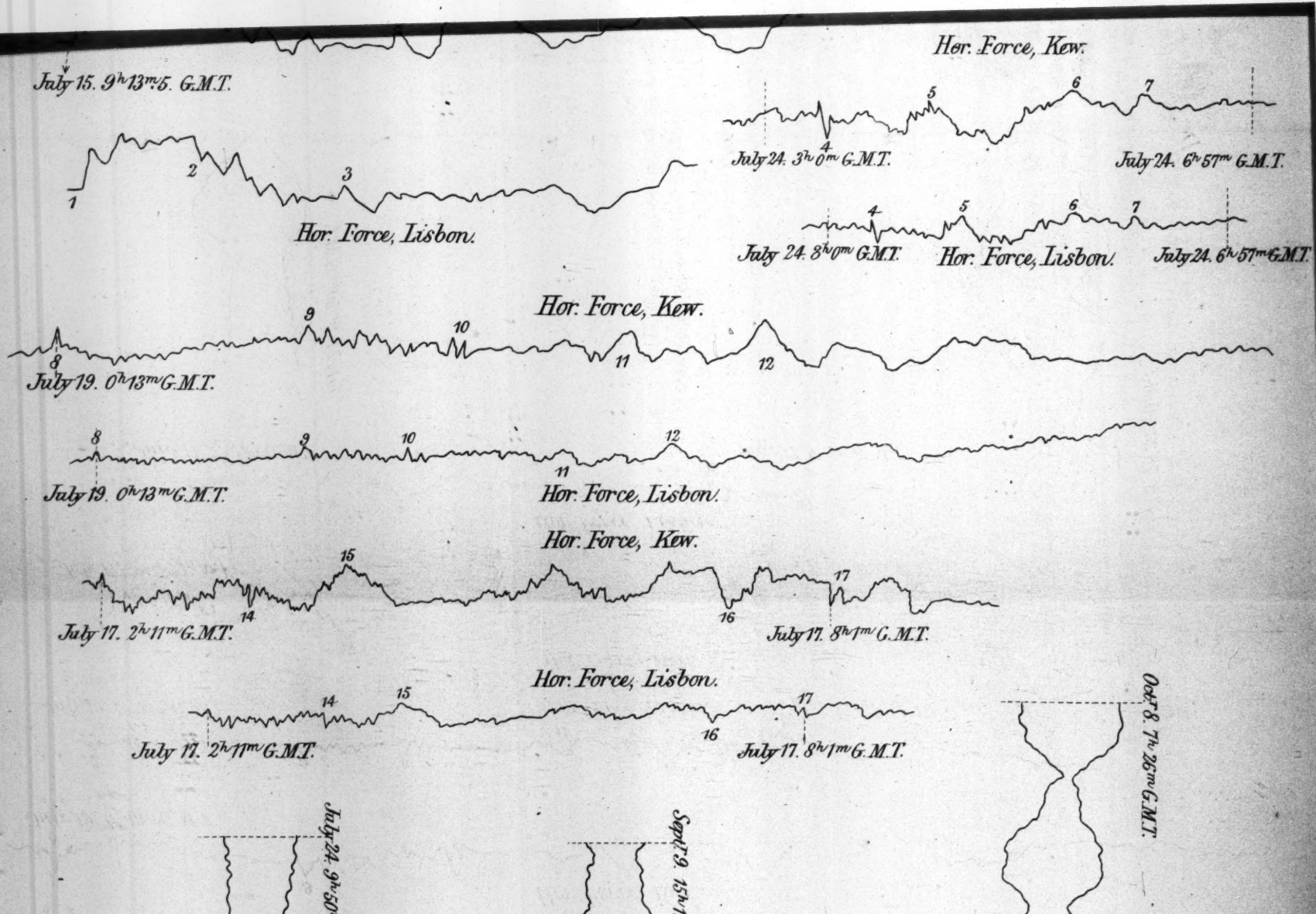
Note regarding the Plates.

Increasing ordinates denote increasing westerly declination, and also increasing horizontal and vertical force.

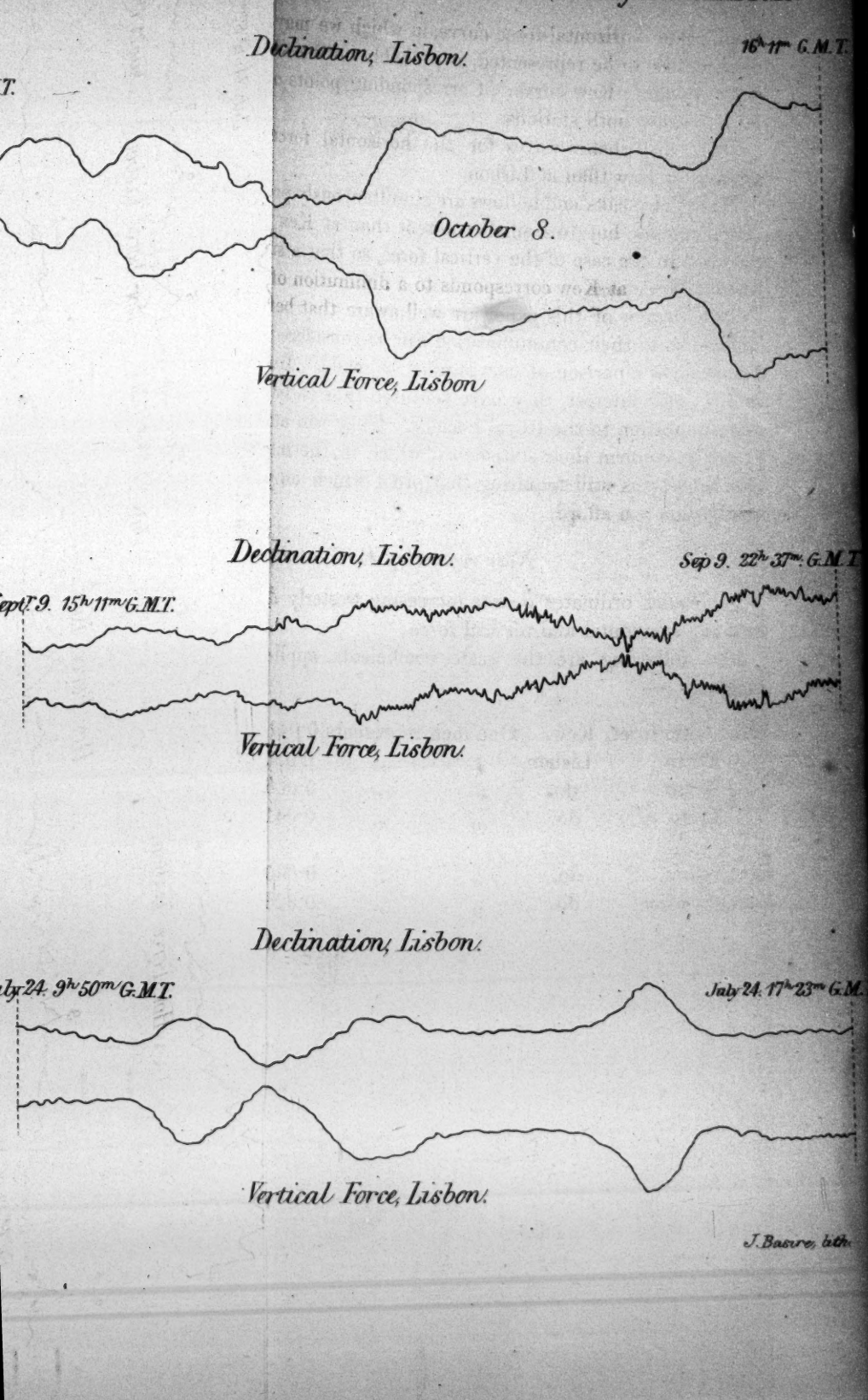
The following are the scale coefficients applicable to the different diagrams:—

Horizontal force,	Kew.	One inch represents		0·041	English unit.	
Ditto	Lisbon.	,,	,,	0·035	,,	for July 15
Ditto	do.	,,	,,	0·066	,,	for July 17.
Ditto	do.	,,	,,	0·041	,,	for the other curves.
Declination	do.	,,	,,	0·040	,,	
Vertical force	do.	,,	,,	0·026	,,	


Hor. Force, Kew.
July 15. 15h 4m G.M.T.
1
2
3
July 15. 9h 13m 5. G.M.T.
Hor. Force, Kew.
5
6
7
4
July 24. 3h 0m G.M.T.
July 24. 6h 57m G.M.T.
1
2
3
Hor. Force, Lisbon.
4
5
6
7
July 24. 8h 0m G.M.T.
Hor. Force, Lisbon.
July 24. 6h 57m G.M.T.
Hor. Force, Kew.
8
9
10
11
12
July 19. 0h 13m G.M.T.
8
9
10
11
12
July 19. 0h 13m G.M.T.
Hor. Force, Lisbon.
Hor. Force, Kew.
14
15
16
17
July 17. 2h 11m G.M.T.
July 17. 8h 1m G.M.T.
Hor. Force, Lisbon.
14
15
16
17
July 17. 2h 11m G.M.T.
July 17. 8h 1m G.M.T.
Oct. 8. 7h 26


July 15. 9h 13m 5. G.M.T.
Hor. Force, Lisbon.
Hor. Force, Kew.
July 24. 3h 0m G.M.T.
July 24. 6h 57m G.M.T.
July 24. 8h 0m G.M.T.
Hor. Force, Lisbon.
July 24. 6h 57m G.M.T.
Hor. Force, Kew.
July 19. 0h 13m G.M.T.
July 19. 0h 13m G.M.T.
Hor. Force, Lisbon.
Hor. Force, Kew.
July 17. 2h 11m G.M.T.
July 17. 8h 1m G.M.T.
Hor. Force, Lisbon.
July 17. 2h 11m G.M.T.
July 17. 8h 1m G.M.T.
July 24. 9h 50
Sept. 9. 15h
Oct. 8. 7h 26m G.M.T.

Proc. Roy. Soc. Vol. XIII. Pl. II.



February 4, 1864.

Major-General SABINE, President, in the Chair.

The following communication was read:—

"Experiments to determine the effects of impact, vibratory action, and a long-continued change of Load on Wrought-iron Girders." By WILLIAM FAIRBAIRN, LL.D., F.R.S. Received January 20, 1864.

(Abstract.)

The author observes that the experiments which were undertaken, nearly twenty years ago, to determine the strength and form of the Tubular Bridges which now span the Conway and Menai Straits, led to the adoption of certain forms of girder, such as the tubular, the plate, and the lattice girder, and other forms founded on the principle developed in the construction of these bridges. It was at first designed that the ultimate strength of these structures should be six times the heaviest load that could ever be laid upon them, after deducting half the weight of the tubes. This was considered a fair margin of strength; but subsequent considerations, such as generally attend a new principle of construction with an untried material, showed the expediency of increasing it; and instead of the ultimate strength being six times, it was in some instances increased to eight times the weight of the greatest load.

The proved stability of these bridges gave increased confidence to the engineer and the public, and for several years the resistance of six times the heaviest load was considered an amply sufficient provision of strength.

But a general demand soon arose for wrought-iron bridges, and many were made without due regard to first principles, or to the law of proportion necessary to be observed in the sectional areas of the top and bottom flanges, so clearly and satisfactorily shown in the early experiments. The result of this was the construction of weak bridges, many of them so ill-proportioned in the distribution of the material as to be almost at the point of rupture with little more than double the permanent load. The evil was enhanced by the erroneous system of contractors tendering by weight, which led to the introduction of bad iron, and in many cases equally bad workmanship.

The deficiencies and break-downs which in this way followed the first successful application of wrought iron to the building of bridges led to doubts and fears as to their security. Ultimately it was decided by the Board of Trade that in wrought-iron bridges the strain with the heaviest load should not exceed 5 tons per square inch; but on what principle this standard was established does not appear.

The requirement of 5 tons per square inch did not appear sufficiently definite to secure in all cases the best form of construction. It is well

known that the powers of resistance to strain in wrought iron are widely different, according as we apply a force of tension or compression; it is even possible so to disproportion the top and bottom areas of a wrought-iron girder calculated to support six times the rolling load, as to cause it to yield with little more than half the ultimate strain or 10 tons on the square inch. For example, in wrought-iron girders with solid tops it requires the sectional area in the top to be nearly double that of the bottom to equalize the two forces of tension and compression; and unless these proportions are strictly adhered to in the construction, the 5-ton strain per square inch is a fallacy which may lead to dangerous errors. Again, it was ascertained from direct experiment that double the quantity of material in the top of a wrought-iron girder was not the most effective form for resisting compression. On the contrary, it was found that little more than half the sectional area of the top, when converted into rectangular cells, was equivalent in its powers of resistance to double the area when formed of a solid top plate. This discovery was of great value in the construction of tubes and girders of wide span, as the weight of the structure itself (which increases as the cubes, and the strength only as the squares) forms an important part of the load to which it is subjected. On this question it is evident that the requirements of a strain not exceeding 5 tons per square inch cannot be applied in both cases, and the rule is therefore ambiguous as regards its application to different forms of structure. In that rule, moreover, there is nothing said about the dead weight of the bridge; and we are not informed whether the breaking-weight is to be so many times the applied weight plus the multiple of the load, or, in other words, whether it includes or is exclusive of the weight of the bridge itself.

These data are wanting in the railway instructions; and until some fixed principle of construction is determined upon, accompanied by a standard measure of strength, it is in vain to look for any satisfactory results in the erection of road and railway bridges composed entirely of wrought iron.

The author was led to inquire into this subject with more than ordinary care, not only on account of the imperfect state of our knowledge, but from the want of definite instructions. In the following experimental researches he has endeavoured to ascertain the extent to which a bridge or girder of wrought iron may be strained without injury to its ultimate powers of resistance, or the exact amount of load to which a bridge may be subjected without endangering its safety—in other words, to determine the fractional strain of its estimated powers of resistance.

To arrive at correct results and to imitate as nearly as possible the strain to which bridges are subjected by the passage of heavy trains, the apparatus specially prepared for the experiments was designed to lower the load quickly upon the beam in the first instance, and next to produce a considerable amount of vibration, as the large lever with its load and shackle was left suspended upon it, and the apparatus was sufficiently elastic for that purpose.

The girder subjected to vibration in these experiments was a wrought-iron plate beam of 20 feet clear span, and of the following dimensions :—

Area of top	4·30 square inches.
Area of bottom	2·40 „
Area of vertical web	1·90 „
Total sectional area	8·60 „
Depth	16 inches.
Weight	7 cwt. 3 qrs.
Breaking-weight (calculated)	12 tons.

The beam having been loaded with 6643 lbs., equivalent to one-fourth of the ultimate breaking-weight, the experiments commenced as follows :—

Experiment I.

Experiment on a wrought-iron beam with a changing load equivalent to one-fourth of the breaking-weight.

Date.	Number of changes of Load.	Deflection produced by Load.	Remarks.
1860.			
March 21	0	0·17	Strap loose on the 24th March.
April 7	202,890	0·17	Strap broken on the 20th April.
May 1	449,280	0·16	
May 14	596,790	0·16	

The beam having undergone about half a million changes of load by working continuously for two months night and day, at the rate of about eight changes per minute, without producing any visible alteration, the load was increased from one-fourth to two-sevenths of the statical breaking-weight, and the experiments were proceeded with till the number of changes of load reached a million.

Experiment II.

Experiment on the same beam with a load equivalent to two-sevenths of the breaking weight, or nearly $3\frac{1}{2}$ tons.

Date.	Number of changes of Load.	Deflection, in inches.	Remarks.
1860.			
May 14	0	0·22	In this experiment the number of changes of load is counted from 0, although the beam had already undergone 596,790 changes, as shown in the preceding Table.
May 22	85,820	0·22	
June 9	236,460	0·21	
June 26	403,210	0·23	The beam had now suffered one million changes of load.

After the beam had thus sustained one million changes of load without apparent alteration, the load was increased to 10,486 lbs., or $\frac{2}{5}$ths of the breaking-weight, and the machinery again put in motion. With this additional weight the deflections were increased, with a permanent set of ·05 inch, from ·23 to ·35 inch, and after sustaining 5175 changes the beam broke by tension at a short distance from the middle. It is satisfactory here to observe that during the whole of the 1,005,175 changes none of the rivets were loosened or broken.

The beam broken in the preceding experiment was repaired by replacing the broken angle-irons on each side, and putting a patch over the broken plate equal in area to the plate itself. A weight of 3 tons was placed on the beam thus repaired, equivalent to one-fourth of the breaking-weight, and the experiments were continued as before.

Experiment III.

Date.	Number of changes of Load.	Deflection, in inches.	Permanent set, in inches.	Remarks.
1860.				
August 9	158	.....	.....	The load during these changes was equivalent to 10,500 lbs., or 4·6875 tons at the centre. With this weight the beam took a large but unmeasured set.
August 11	12,950			During these changes the load in the beam was 8025 lbs., or 3·58 tons.
August 13	25,900	0·22	?	
August 13	25,900	0·18	0	Load reduced to 2·96 tons, or $\frac{1}{4}$th the breaking-weight.
December 1 ...	768,100	0·18	0·01	
1861.				
March 2.....	1,602,000	0·18	0·01	
May 4	2,110,000	0·17	0·01	
September 4 ...	2,727,754	0·17	0·01	
October 16.....	3,150,000	0·17	0·01	

At this point, the beam having sustained upwards of 3,000,000 changes of load without any increase of the permanent set, it was assumed that it might have continued to bear alternate changes to any extent with the same tenacity of resistance as exhibited in the foregoing Table. It was then determined to increase the load from one-fourth to one-third of the breaking-weight; and accordingly 4 tons were laid on, which increased the deflection to ·20.

Experiment IV.

Date.	Number of changes of Load.	Deflection, in inches.	Permanent set, in inches.	Remarks.
1861.				
October 18.....	0	0·20		
November 18...	126,000	0·20	0	
December 18...	237,000	0·20		
1862.				
January 9	313,000	.....	.....	Broke by tension across the bottom web.

Collecting the foregoing series of experiments, we obtain the following summary of results.

Summary of Results.

No. of Expt.	Date.	Weight on middle of the beam, in tons.	Number of changes of Load.	Strain per sq. inch on bottom.	Strain per sq. inch on top.	Deflection, in inches.	Remarks.
1	From March 21 to May 14, 1860...	2	596,790	4·62	2·58	·17	
2	From May 14 to June 26, 1860...	3·50	403,210	5·46	3·05	·23	
3	From July 25 to July 28, 1860...	4·68	5,175	7·31	4·08	·35	Broke by tension a short distance from the centre of the beam.
	Beam repaired.						
4	Aug. 9, 1860	4·68	158	7·31	4·08	...	The apparatus was accidentally set in motion.
5	Aug. 11 & 12	3·58	25,742	3·59	3·12	·22	
6	From Aug. 13, 1860 to Oct. 16, 1861	2·96	3,124,100	4·62	2·58	·18	
7	From Oct. 18, 1861 to Jan. 9, 1862 ...	4·00	313,000	6·25	3·48	·20	Broke by tension as before, close to the plate riveted over the previous fracture.

From these experiments it is evident that wrought-iron girders of ordinary construction are not safe when submitted to violent disturbances equivalent to one-third the weight that would break them. They, however, exhibit wonderful tenacity when subjected to the same treatment with one-fourth the load; and assuming therefore that an iron girder bridge will bear with this load 12,000,000 changes without injury, it is clear that it would require 328 years at the rate of 100 changes per day before its security was affected. It would, however, be dangerous to risk a load of one-third

the breaking-weight upon bridges of this description, as, according to the last experiment, the beam broke with 313,000 changes; or a period of eight years, at the same rate as before, would be sufficient to break it. It is more than prohable that the beam had been injured by the previous 3,000,000 changes to which it had been subjected; and assuming this to be true, it would follow that the beam was undergoing a gradual deterioration which must some time, however remote, have terminated in fracture.

February 11, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read:—

I. "On the Calculus of Symbols.—Fourth Memoir. With Applications to the Theory of Non-linear Differential Equations." By W. H. L. Russell, A.B. Communicated by Professor Cayley. Received July 31, 1863.

(Abstract.)

In the preceding memoirs on the Calculus of Symbols, systems have been constructed for the multiplication and division of non-commutative symbols subject to certain laws of combination; and these systems suffice for linear differential equations. But when we enter upon the consideration of non-linear equations, we see at once that these methods do not apply. It becomes necessary to invent some fresh mode of calculation, and a new notation, in order to bring non-linear functions into a condition which admits of treatment by symbolical algebra. This is the object of the following memoir. Professor Boole has given, in his 'Treatise on Differential Equations,' a method due to M. Sarrus, by which we ascertain whether a given non-linear function is a complete differential. This method, as will be seen by anyone who will refer to Professor Boole's treatise, is equivalent to finding the conditions that a non-linear function may be externally divisible by the symbol of differentiation. In the following paper I have given a notation by which I obtain the actual expressions for those conditions, and for the symbolical remainders arising in the course of the division, and have extended my investigations to ascertaining the results of the symbolical division of non-linear functions by linear functions of the symbol of differentiation.

II. "On Molecular Mechanics." By the Rev. Joseph Bayma, of Stonyhurst College, Lancashire. Communicated by Dr. Sharpey, Sec. R.S. Received January 5, 1864.

The following pages contain a short account of some speculations on molecular mechanics. They will show how far my plan of molecular

mechanics has been as yet developed, and how much more is to be done before it reaches its proper perfection. Of course I can do no more than point out the principles on which, according to my views, this new science ought to be grounded. The *proofs* would require a volume,—and the more so, as existing wide-spread philosophical prejudices will make it my duty to join together both demonstration and refutation. But there will be time hereafter, if necessary, for a complete exposition and vindication of the principles on which I rely; at present it will be enough for me to state them.

The aim of "molecular mechanics" is the solution of a problem which includes all branches of physics, and which may be enunciated, in general terms, as follows:—

"From the knowledge we gain of certain properties of natural substances by observation and experiment, to determine the intrinsic constitution of these substances, and the laws according to which they ought to act and be acted upon in any hypothesis whatever."

In order to clear the way for the solution of this problem, three things are to be done.

First. From the known properties of bodies must be deduced the essential principles and intrinsic constitution of matter.

Secondly. General formulas must be established for the motions of any kind of molecular system, which we conceive may exist *in rerum natura*.

Thirdly. We must determine as far as possible the kinds of molecular systems which are suited to the different primitive bodies; and be prepared to make other applications suitable for the explanation of phenomena.

Of these three things, the first, which is the very foundation of molecular mechanics, can, I think, be done at once. The second also, though it requires a larger treatment, will not present any great difficulty. The third, however, in this first attempt, can be but very imperfectly accomplished; for sciences also have their infancy, nor am I so bold as to expect to be able to do what requires the labour of many: I shall only say so much as may suffice to establish for this science a definite existence and a proper form.

In order to give an idea of my plan, I will now say a few words on each of these three points.

I. Principles of Molecular Mechanics.

First, then, (to say nothing of the name of "*molecular mechanics*," which will be justified later,) in all bodies we find these three things, *extension*, *inertia*, and *active powers*, to one or other of which every property of bodies may be referred. In order therefore to arrive at a clear idea of the constitution of natural substances, these three must be diligently investigated.

Extension.—I have come to the following conclusions on this head, which,

I think, can be established by evident arguments drawn from various considerations.

1. All bodies consist of simple and unextended elements, the sum of which constitute the *absolute mass* of the given body. The extension itself, or *volume*, of the body is nothing but the extension of the space included within the bounding surfaces of the body; and the *extension of space* is nothing but its capability of being passed through (*percurribilitas*) in any direction by means of motion extending from any one point to any other.

2. There is no such thing possible as matter materially and mathematically continuous—that is to say, such that its parts touch each other with true and perfect contact. There must be admitted indeed a continuity of forces ready to act; but this continuity is only virtual, not actual nor formal.

3. Simple elements cannot be at once attractive at greater, and repulsive at less distances. To this extent at least Boscovich's theory must be corrected. If an element is attractive at any distance, it will be so at all distances; and if it be repulsive at any distance, it will be repulsive at all distances. This is proved from the very nature of matter, and perfectly corresponds with the action of molecules and with universal attraction.

4. Simple *elements* must not be confounded with the *atoms* of the chemist, nor with the molecules of which bodies are composed. Molecules are, according to their name, small extended masses, *i. e.* they imply volume; elements are indivisible points without extension. Again, molecules of whatever kind, even those of primitive bodies, are so many systems resulting from elements acting on each other; consequently elements differ from molecules as parts differ from the whole; so that much may be said about separate *elements*, which cannot be said of separate molecules or chemical atoms, and *vice versâ*. *Element, molecule, body* have the same relation to each other in the physical order, that *individual, family, state* bear to each other in the social order; for a body results from molecules, and molecules from elements holding together mechanically, in a similar way to that in which a state results from families, and families from individuals bound together by social ties.

So much regarding *extension*; for I do not now intend to proceed to the demonstration of these statements, but simply to put down what it is I am prepared to prove.

Inertia.—There would scarcely be any need of saying anything on this head, were there not some, even learned men, who entertain false ideas about it, and from not rightly understanding what is said of inertia by physical philosophers, throw out ill-founded doubts, which do more harm than good to science. I say, then,

1. Inertia implies two things: (*a*) that each element of matter is perfectly indifferent to receiving motion in any direction and of any intensity from some external agent; (*b*) that no element of matter can move itself by any action of its own.

2. It follows as a sort of corollary from this, that to be inert does not signify to be *without active power*; and that the very same element, which on account of its inertia cannot act upon itself, may, notwithstanding this inertia, have an active power, by which it may act upon any other element whatever.

3. Inertia is an essential property of matter, and is not greater in one element than in another, but is always the same in all elements, whether they are attractive or repulsive, whether their active power is great or small.

4. That which is called by natural philosophers the *vis inertiæ* is not a special mechanical force added on to the active forces of elements, but is the readiness of a body to react by means of its elementary forces, against any action tending to change the actual condition of that body.

These four propositions will remove many false notions, which give rise to confusion of ideas and impede the solution of many important questions.

Active power.—The questions relating to the active power of matter are of the greatest importance, since on them depends nearly the whole science of nature. On this point I am convinced, and think I can prove, that

1. No other forces exist in the elements of matter except *locomotive* or mechanical forces; for these alone are required, and these alone are sufficient, to account for all natural phenomena. So that we need have no anxiety about the *vires occultæ* of the ancients, nor need we make search after any other kind of *primitive* forces, besides such as are mechanical or locomotive. Hence chemical, electric, magnetic, calorific and other such actions will be all reduced to mechanical actions, complex indeed, but all following certain definite laws, and capable of being expressed by mathematical formulæ as in general mechanics. Hence in treating of *molecular mechanics* we do not make any gratuitous assumption or probable hypothesis, but are engaged on a branch of science founded on demonstrable truths, free from all hypothesis or arbitrary assumption.

2. There are not only attractive, but also repulsive elements; and this is the reason why molecules of bodies (as being made up of both sorts) may at certain distances attract, and at others repel each other.

3. Simple elements, in the whole sphere of their active power, and consequently *also at molecular distances*, act (whether by attracting or repelling) according to the inverse ratio of the squares of the distances. This proposition may seem to contradict certain known laws, as far as regards molecular distances; but the contradiction is only apparent, and this appearance will vanish when we consider that the action of *elements* (of which we are now speaking) is not the same as the action of *molecules*. From the fact that cohesion, *e. g.*, does not follow the inverse ratio of the square of the distance, it will certainly result that *molecules* do not act according to this law, and this is what physical science teaches: but it does not follow that *elements* do not act according to the law. This truth is, as all must see, of the utmost importance, since it is the foundation of molecular

mechanics, of which it would be impossible to treat at all, unless the law of elementary action at infinitesimal distances were known. This truth universalizes Newton's law of celestial attraction by extending it to all elementary action, whether attractive or repulsive, and makes it applicable not only to telescopic, but also to microscopic distances. It is clear therefore that I am bound to prove this law most irrefragably, lest I construct my molecular mechanics on an insecure foundation.

4. The sphere of the activity of matter is indefinite, in this sense, that no finite distance can be assigned at which the action of matter will be null. It by no means, however, follows from this that the force of matter has an infinite intensity.

5. The natural activity of each element of matter is exerted *immediately* on every other existing element at any distance, either by attracting or repelling, according to the agent's nature. Thus, *e. g.*, the action which the earth exerts on each falling drop of rain is exerted *immediately* by each element of the earth on each element of the water (notwithstanding the distance between them); it is not exerted through the material medium of the air, or of ether, or any other substance. The same must be said of the action of the sun on the planets. This proposition, however, it is evident, holds only for the simple action of the elements, *i. e.*, attractive or repulsive. For it is clear that complex actions causing vibratory motions, such as light or sound, are only transmitted through some vibrating medium. This conclusion is also of immense importance, because it solves a question much discussed by the ancients about the nature of action exerted on a distant body, and removes all scruples of philosophers on this head.

6. Bodies do not and cannot act by mathematical contact, however much our prejudices incline us to think the contrary; but *every* material action is *always* exerted on something at a distance from the agent.

7. There is another prejudice which I wish to remove, *i. e.* that one motion is the *efficient cause* of another motion. It is easily shown that this mode of speaking, though sometimes employed by scientific men, is incorrect, and ought to be abandoned, because it tends to the destruction of all natural science. Motion never causes motion, but is only a condition affecting the agent in its manner of acting. For all motion is caused by some agent giving velocity and direction; but the agent gives velocity and direction by means of its own active power, which it exerts differently according as it is found in different local conditions. Now these local conditions of the agent may be differently modified by the movement of the agent itself. The impact of bodies, the change of motion to heat, the communication of velocity from one body to another (always a difficult question), and other points of a like nature can only be satisfactorily explained by this principle.

These are the principal points that have to be discussed, defined, and demonstrated in order that molecular mechanics may be established on solid principles.

II. Mathematical Evolution of these Principles.

After establishing principles, we must proceed to investigate the formulas of motion and of equilibrium, first between the elements themselves, then between the several systems of elements. The difficulties to be overcome in establishing the principles were chiefly philosophical: the difficulties which occur in the present part are mathematical, and can only be overcome by labour and patience.

As long as we confine ourselves to two elements, the mathematical formula expressing their motion is easily found. Thus, if there are two attractive elements of equal intensity, and if v be the action of one for a unit of distance in a unit of time, $2a$ the distance between them at the beginning of motion, x the space passed through by one in the time t, the equation of motion will be

$$t=\sqrt{\frac{2a}{v}}\left(\sqrt{x(a-x)}+a\,.\,\text{arc tang}=\sqrt{\frac{x}{a-x}}\right).$$

And since it is clear, from other considerations, that these two elements must vibrate together indefinitely in vibrations of equal times and constant extent, the time of one oscillation will easily be found from the above formula.

If the two attractive elements have unequal forces, or if one be attractive and the other repulsive, or both repulsive, the equation of motion may easily be obtained.

But when we have to do with a more complex system of elements, after obtaining the differential equations corresponding to the nature of the system, it is scarcely possible to obtain their integration, as will appear from the examples which I shall give below. Consequently, if we wish to deduce anything from such equations, we must proceed indirectly, and a long labour must be undertaken, sometimes with but slender results. This material difficulty will be diminished, or perhaps disappear, either by some new method of integration (which I can scarcely dare to hope for, though it is a great desideratum) or by certain tables exhibiting series of numerical values belonging to different systems.

But there occurs another difficulty in these systems. For since the agglomerations of simple elements can be arranged in an infinite variety, and it would be neither reasonable nor possible to treat of all such agglomerations, we must limit the number of them according to the scope we have in view, *i. e.* according to the use they may be of in explaining natural phenomena. Even this is a very difficult matter. How I have endeavoured to overcome this difficulty I will briefly explain.

First. I considered that the molecules of primitive bodies, such as oxygen, hydrogen, nitrogen, &c., cannot reasonably be supposed to be *irregular*—a conclusion which, though I cannot rigorously demonstrate, yet I can render probable by good reasons. Consequently, while treating of primitive systems I may confine myself to the examen of forms that are *regular*.

Secondly. I divided these regular systems into different classes according to their geometrical figure. Of these I have investigated the tetrahedric, octahedric, hexahedric, octohexahedric, pentagonal-dodecahedric, and icosahedric.

I then divided these classes into different species, viz. *pure centratæ*, *centro-nucleatæ*, *centro-binucleatæ*, *centro-trinucleatæ*, &c., also into *acentratæ* (without centre), *truncatæ*, &c. To enumerate the whole would take too long; indeed I only mention these to show how in such a multiplicity of systems I endeavoured to introduce the order necessary for me to be able to speak distinctly about them.

Lastly, besides classes and species, it was requisite also to consider certain distinct varieties under the same species. And in this way I seemed to myself to have embraced all the regular systems of elements possibly conceivable.

Thirdly. The several parts of which any system of elements can consist are reduced by me to a *centre*, *nuclei* to any number, and an external *envelope*. And thus I obtained not only a method of nomenclature for the different systems (a most important point), but also a method of exhibiting each system under brief and intelligible symbols. Thus, *e. g.*, the tetrahedric system *pure centratum* (*i. e.* without any nucleus), in which the centre is an attractive element, and the four elements of the envelope repulsive, will be represented thus,

$$m=\mathrm{A}+4\mathrm{R},$$

in which expression m signifies the absolute mass of the system (in this case $m=5$), A represents the attractive centre, and 4R the four repulsive elements of the envelope. The letters A and R are not *quantities*, but only *indices* denoting the nature of the action.

In a similar way, the following expression

$$m=\mathrm{R}+6\mathrm{A}+8\mathrm{R}'$$

denotes a system whose centre R is repulsive, whose single nucleus 6A is octahedric and attractive, and whose envelope 8R′ is hexahedric and repulsive: m, which, as before, indicates the absolute mass of the system, here $=15$.

This will suffice to show how the different species and varieties of the afore-mentioned systems may be named and expressed.

Then I had to find mechanical formulas for the motion or equilibrium of the several systems; for it is only from such formulas that we can determine what systems are generally possible in the molecules of bodies. Speaking generally, no system *pure centratum*, of whatever figure it be, can be admitted in the molecules of natural bodies, whether gaseous, liquid, or solid.

Let v represent the action of the centre, and w that of one of the elements of the envelope for a unit of distance; and let r be the radius of the system, *i. e.* the distance o any one of the elements of the envelope from the centre;

the general formula of motion for any system *pure centratum* (expressed as above by $m=A+nR$) will be

$$\frac{d^2r}{dt^2}=-\frac{1}{r^2}(v-Mw),$$

where M signifies a constant, and the actions which tend to increase r are taken as positive.

If the system is	tetrahedric,	$M=0{\cdot}91856$
,,	octahedric,	$M=1{\cdot}66430$
,,	hexahedric,	$M=2{\cdot}46759$
,,	octohexahedric,	$M=4{\cdot}11170$
,,	icosahedric,	$M=4{\cdot}19000$
	pentagonal dodecahedric,	$M=7{\cdot}82419$.

Now none of these varieties satisfies the conditions either of solid, liquid, or gaseous bodies; because they either will not resist compression, or they form masses which are repulsive at all great distances; or if they could constitute gaseous bodies, they do not allow the law of compression to be verified, which we know to hold for all gases.

Passing on to the systems *centro-nucleata*, the formulas will differ according to the several figures of the nuclei and envelope. Taking, *e. g.*, the system

$$m=R+6A+8R',$$

which is hexahedric with an octahedric nucleus, and taking v, v', w to represent respectively the actions of the centre, one element of the nucleus, and one element of the envelope; taking also r and ρ for the radii of the nucleus and envelope, the equations of motion for such a system will be

$$\frac{d^2r}{dt^2}=\frac{v-M'v'}{r^2}+4w\left(\frac{r+\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{3}}\right)^3}}+\frac{r-\rho\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{3}}\right)^3}}\right),$$

$$\frac{d^2\rho}{dt^2}=\frac{v+Mw}{\rho^2}-3v'\left(\frac{\rho+r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2+\frac{2\rho r}{\sqrt{3}}\right)^3}}+\frac{\rho-r\sqrt{\frac{1}{3}}}{\sqrt{\left(\rho^2+r^2-\frac{2\rho r}{\sqrt{3}}\right)^3}}\right),$$

where $M=2{\cdot}46759$, and $M'=1{\cdot}66430$. The conditions of equilibrium will be obtained by making the two first members equal to zero.

What systems of this class (*centro-nucleata*) can satisfy the conditions of solid, liquid, or gaseous bodies, is exceedingly difficult to determine, for reasons which I have above touched on, viz. that the formulæ of these systems are not integrable, and we have consequently to proceed indirectly with great expenditure of time and trouble. It seems to me, however, as far as I can judge, that some of these systems may be found *in rerum natura*.

Passing to another class of systems (*centro-binucleata*), we shall have three equations to express its laws of motion. Taking, *e. g.*, the system

$$m=A+4R+4A'+4R',$$

which is tetrahedric with two tetrahedric nuclei; taking v, v', v'', w for the respective actions of the elements acting from the centre, first and second nuclei, and envelope; taking r, r'', ρ for the radii of the two nuclei and the envelope, the equations of motion will be as follows:

$$\frac{d^2r'}{dt^2} = \frac{Mv'-v}{r'^2} - v''\left(\frac{1}{(r'+r'')^2} + \frac{3r'-r''}{\sqrt{\left(r'^2+r''^2-\frac{2r'r''}{3}\right)^3}}\right)$$
$$-w\left(\frac{1}{(\rho-r')^2} - \frac{3r'+\rho}{\sqrt{\left(\rho^2+r'^2+\frac{2\rho r'}{3}\right)^3}}\right);$$

$$\frac{d^2r''}{dt^2} = \frac{Mv''-v}{r''^2} + v'\left(\frac{1}{(r'+r'')^2} + \frac{3r''-r'}{\sqrt{\left(r'^2+r''^2-\frac{2r'r''}{3}\right)^3}}\right)$$
$$+w\left(\frac{1}{(\rho+r'')^2} + \frac{3r''-\rho}{\sqrt{\left(\rho^2+r''^2-\frac{2\rho r''}{3}\right)^3}}\right);$$

$$\frac{d^2\rho}{dt^2} = \frac{Mw-v}{\rho^2} + v'\left(\frac{1}{(\rho-r')^2} + \frac{3\rho+r'}{\sqrt{\left(\rho^2+r'^2+\frac{2\rho r'}{3}\right)^3}}\right)$$
$$-v''\left(\frac{1}{(\rho+r'')^2} + \frac{3\rho-r''}{\sqrt{\left(\rho^2+r''^2-\frac{2\rho r''}{3}\right)^3}}\right);$$

in which equations $M=0{\cdot}91856$.

The discussion of these equations and similar ones will afford a useful occupation to mathematicians and natural philosophers. Whatever conclusions may be drawn from them cannot fail to throw great light on the question of the nature of bodies.

It is evident that we might go further and pass on to *trinucleate, quadrinucleate,* &c. systems; but the number of equations will increase in proportion, together with the difficulty of dealing with them.

It is not enough to consider the laws of motion and equilibrium in each system separately, but it is also necessary to know what action one system exercises on another, whether like or unlike, placed at a given distance. For since many of the properties of bodies depend on the relation which the different molecules bear to one another, *e. g.*, liquidity, elasticity, hardness, &c., it is not enough to know what is the state of a system of elements (*i. e.* a molecule) in itself, but we must investigate also how several such systems (or molecules) affect each other. Now in this ulterior investigation it is clear that the difficulty increases exceedingly, since the equations become exceedingly complex. Here also then may natural philosophers

find matter for industry and patience. I have done a little in this subject, but not enough to deserve any special mention. In order, however, to diminish the difficulties, the investigation may be provisionally restricted to the mutual actions of the *envelopes*, neglecting for the time that of the *nuclei*, which may be considered as a disturbing cause, for which some correction may afterwards have to be made.

So much then for the mathematical and theoretic development of molecular mechanics. There remains the third part, which, though the most laborious of all, will yet give the greatest pleasure to scientific men; since it is less dry, and opens a way for attaining the end aimed at in the natural sciences. Of this third part I will add a few words.

III. Application of the Principles of Molecular Mechanics.

[Under this head the author points out the various properties of bodies which would have to be explained, and of which he conceives an explanation might be afforded could the mathematical calculations be effected which are required for the elaboration of his theory, and enunciates the following conclusions as deduced from his explanation of the impact of bodies.]

1. If a body does not contain any repulsive elements, it cannot cause any retardation in the movement of any impinging body.

2. Again, if the medium through which a body moves contain no repulsive elements, no retardation of its motion can take place.

3. If a medium does contain repulsive elements, retardation must necessarily take place.

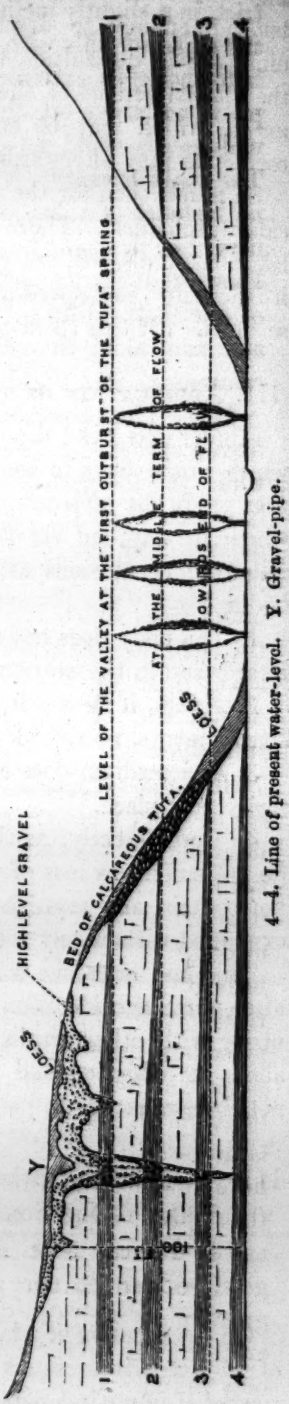
4. Consequently, as the planets in their movements through the æther do not suffer any loss of velocity, it must be concluded that the æther does not contain any repulsive elements at all, and that its elasticity must be explained without any recourse to repulsive forces.

This last inference is somewhat wonderful, and decidedly curious; but after much consideration it appeared to me so natural, and so well harmonizing with other truths and scientific theories, that I ceased to hesitate about its adoption and gave it a most decided assent; whether wisely or not, I leave others to judge.

III. "On some further Evidence bearing on the Excavation of the Valley of the Somme by River-action, as exhibited in a Section at Drucat near Abbeville." By Joseph Prestwich, F.R.S. Received January 29, 1864.

On the occasion of a late visit to Abbeville, I noticed a fact which appears of sufficient interest, as bearing upon and confirming one of the points treated of in my last paper, to induce me to submit a short notice of it to the Royal Society. It occurs in a tributary valley to that of the Somme, but necessarily forms part of the general phenomena affecting the whole basin.

The small stream (the Escardon) which joins the Somme at Abbeville flows through a narrow chalk valley extending a few miles north of Abbeville. Three miles up this valley is the village of Drucat; and on the hill above the village, and about 100 feet above the stream, is a small outlier of high-level gravel which I have before described, and which is remarkable for the number and size of its sand- and gravel-pipes penetrating the underlying chalk. One of these which I measured was 22 feet across at the top and 18 feet at a depth of 30 feet, and I estimated its depth at not less than 100 feet from the surface. It was filled in the usual way with sand and gravel in vertical cylindrical layers. M. Boucher de Perthes has two flint implements which are reported to have come from the pit; but I never myself found any there, or any mammalian remains. The sand and gravel is clean and light-coloured, and very similar in character to some of the beds at Menchecourt, and in so far has the appearance of a fluviatile gravel, and, like it, is overlain by a variable bed of loess. This bed was supposed to form an isolated outlier; but on my last visit I found another bed, though of coarser materials, on a hill of the same height on the opposite side of the valley, above l'Heure. The valley at the foot of the hill on which the Drucat gravel is worked is about a quarter of a mile wide. A lane leads direct down the slope of the hill from a point near the gravel to the valley; and a roadside cutting exposes a section of calcareous tufa or travertin several feet thick, and containing in places numerous land shells, of recent species, and traces of plants. Half a mile beyond, the bed is of sufficient importance to be worked for building-purposes. This bed is overlain by the valley loess, and is in places intercalated with it; it commences a few feet below the level of the gravel at about 70 feet above the valley, and continues to near the foot of the hill.



Now it is well proved that in all purely chalk districts the line of water-level proceeds from the level of the streams and rivers traversing the dis-

trict, in a slightly inclined and continuous plane rising on either side under the adjacent hills with a slope varying from 10 to 40 feet in the mile, the latter being an extreme case. If we take a mean of 20 feet, as the gravel-pit is not above one-third of a mile from the valley, the rise in the water underneath would not probably exceed 10 feet above the level of the stream. The chalk formation is so generally fissured and permeable that I know of no instance of a line of water-level or of springs occurring above the general line dependent upon the level of the adjacent rivers. It is also well known that strong springs are common at the foot of the hills along many of our chalk valleys, as, for instance, that at Amwell, those at Carshalton, and many along the valley of the Thames. These springs are more or less calcareous, often highly so.

It is evident that the travertin at Drucat has been formed by a deposit from a spring of considerable volume; and it further appears that it flowed while the loess was in the course of formation. For the tufa could only have been formed at or near the level of the spring; so that its continued deposit down the slope of the hill shows the spring to have been gradually lowered as the valley became deeper, and while subject to the continued inundations which deposited the loess. The line of present water-level in the chalk here is about 90 feet below the summit of the hill, as proved by a well in an adjacent farmhouse, and at the gravel-pit they have gone down 60 feet without reaching water. But the level of the upper part of the tufa shows the line of water-level or of springs to have been at one time 70 feet above the valley, which could only have happened when the bottom of the valley was on a level 60 to 70 feet higher than it now is. The gradual deepening of the valley is indicated by the gradual lowering of the spring until it reached to within from 20 to 30 feet of the present valley-level, when it became extinct. Further, we have in the adjacent bed of high-level gravel evidence of the origin of this important spring; for the sands and gravel-beds are not only very thick, but they are also perfectly free from calcareous matter and very permeable, and they show in their numerous gravel-pipes how great must have been the volume and solvent power of the rain-water which at one time percolated through them. The water, after passing through the gravel and acting upon the underlying chalk to form these large vertical cavities, would, upon reaching the original line of water-level, have flowed off horizontally and escaped in a strong spring at the base of the adjacent slope. It there parted with its excess of the carbonate of lime, and so formed the calcareous tufa. This case furnishes therefore new and good evidence on two points:—first, on the connexion of the sand- and gravel-pipes with the percolation of fresh water through calcareous rocks; and secondly, on the condition of the former land surface and of the springs, only possible on the hypothesis of former higher levels of the bottom of the valley and of its gradual excavation.

February 18, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read:—

I. "A Contribution to the Minute Anatomy of the Retina of Amphibia and Reptiles." By J. W. HULKE, Esq., F.R.C.S., Assistant-Surgeon to the Middlesex and the Royal London Ophthalmic Hospitals. Communicated by W. BOWMAN, Esq. Received February 4, 1864.

(Abstract.)

The animals of which the retina was examined were the frog, the black and yellow salamander, the edible turtle, the water- and the land-tortoise, the Spanish Gecko, the blindworm, and the common snake. The method adopted was to examine the retina (where possible) immediately after decapitation of the animal, alone and with chemical agents; and to make sections of the retina hardened in alcohol or in an aqueous solution of chromic acid, staining them with iodine or carmine, and adding glycerine, pure and diluted, to make them transparent. The following is a summary of the results of the examination.

1. The rods and cones consist of two segments, the union of which is marked by a bright transverse line.

2. Each segment consists of a membranous sheath and contents.

3. The outer segment, or *shaft*, is a long narrow rectangle (by inference, a prism or cylinder). It refracts more highly than the inner segment. Its contents are structureless, and of an albuminous nature. It is that part which is commonly known as "*the rod*." It is smaller in the cones than in the rods, and in the cones narrows slightly outwards.

4. The outer ends of the shafts rest upon the inner surface of the choroid, and their sides are separated by pigmented processes, prolonged from the inner surface of the choroid between them to the line that marks the union of the shaft with the inner segment. The effect of this is that the shafts are completely insulated, and rays entering one shaft are prevented passing out of it into neighbouring shafts.

5. The inner segment of the rods and cones, or body (the appendage of some microscopists), has a generally flask-shaped form, longer and more tapering in the rods, shorter and stouter in the cones. It is much paler and less conspicuous than the shaft. It fits in an aperture in the membrana limitans externa.

Its inner end always encloses, or is connected by an intermediate band with an outer granule which lies in or below the level of the membrana limitans externa. Its outer end, in cones only, contains a spherical bead nearly colourless in the frog and blindworm, brilliantly coloured in the turtle and water- and land-tortoises, and absent from the common snake and Spanish Gecko. In addition to this bead, where present, and the outer gra-

nule, the body contains an albuminous substance which in chromic acid preparations retires as an opaque granular mass towards the outer end of the body. The inner end of the body is prolonged inwards, in the form of a pale, delicate fibre, which was sometimes followed through the layer of inner granules into the granular layer. It does not appear to be structurally connected with the inner granules. It is essentially distinct from Müller's radial fibres, and bears a considerable resemblance to the axis-cylinder of nerve. That it *ever* proceeds from the outer granule associated with the rod- or cone-body is doubtful, from the consideration (α) that where the body is large, and the granule lies within at some distance from its contour, the fibre is seen to leave the inner end of the body distinct from the granule, and (β) that the fibre appears to proceed from the outer granule only where the body is small, as in the frog, and where the granule does not lie within the body but is joined to this by a band. Ritter's axial fibres are artificial products.

6. The "outer granules" are large, circular, nucleated cells. Each cell is so intimately associated with a rod- or cone-body that it forms an integral part of it.

7. The intergranular layer is a web of connective fibre. It contains nuclei.

8. The inner granules are roundish, in chromic acid preparations polygonal cells. They differ from the outer granules by their higher refraction, by the absence of a nucleus, and by receiving a deeper stain from carmine. They lie in areolæ of connective tissue derived from Müller's radial fibres, and from the intergranular and granular layer. They are more numerous than the outer granules, and consequently than the rods and cones.

9. The granular layer is a very close fibrous web derived in part from Müller's radial fibres, and from other fibres proceeding from the connective frame of the layer of inner granules. It transmits (α) the radial fibres, (β) fibres proceeding radially outwards from the ganglion-cells and bundles of optic nerve-fibres, and (γ) fibres passing inwards from the rod- and cone-bodies.

10. The ganglion-cells communicate by axis-cylinder-like fibres with the bundles of optic nerve-fibres, and send similar fibres outwards, which have been traced some distance in the granular layer.

11. In the frog and Spanish Gecko the author has a few times traced fibres proceeding from the bundles of optic nerve-fibres for some distance in a radial direction in the granular layer.

12. Müller's radial fibres arise by expanded roots at the outer surface of the membrana limitans interna, pass radially through the layers, contributing in their course to the granular layer, to the areolar frame of the layer of inner granules, and end in the intergranular layer and at the inner surface of the membrana limitans externa. They are a connective and not a nervous tissue, and do not communicate between the basilary element and ganglion-cells.

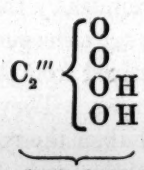
13. The orderly arrangement of the several layers and their elementary parts is maintained by a frame of connective tissue which consists of—1, an unbroken homogeneous membrane bounding the inner surface of the retina, the membrana limitans interna; 2, a fenestrated membrane which holds the rods and cone-bodies, the membrana limitans externa, first correctly described by Schultze; 3, an intermediate system of tie-fibres—Müller's radial fibres—connected with which in the layer of inner granules are certain oblong and fusiform bodies of uncertain nature; 4, the intergranular layer; 5, an areolated tissue, open in the layers of outer and inner granules, and very closely woven in the granular layer.

14. No blood-vessels occur in the reptilian retina.

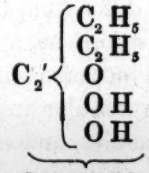
II. "Notes of Researches on the Acids of the Lactic Series.—No. I. Action of Zinc upon a mixture of the Iodide and Oxalate of Methyl." By E. Frankland, F.R.S., Professor of Chemistry, Royal Institution, and B. F. Duppa, Esq. Received February 10, 1864.

In a former communication by one of us*, a process was described by which leucic acid was obtained synthetically by the substitution of one atom of oxygen in oxalic acid by two atoms of ethyl.

The relations of these acids to each other will be seen from the following formulæ†:—



Oxalic acid.



Leucic acid.

This substitution of ethyl for oxygen was effected by acting upon oxalic ether with zincethyl. On distilling the product with water, leucic ether came over, which on treatment with an alkali yielded a salt of leucic acid.

We have since found that this process may be much simplified by generating the zincethyl during the reaction, which is effected by heating a mixture of amalgamated zinc, iodide of ethyl, and oxalic ether in equivalent proportions to the necessary temperature.

The operation may be considered complete when the mixture has solidified to a resinous-looking mass. This, treated with water as in the former reaction and distilled, produces quantities of leucic ether considerably greater than can be obtained from the same materials by the first mode of operating. Thus the necessity for the production of zincethyl is entirely obviated, the whole operation proceeds at the ordinary atmospheric pressure, and a larger product is obtained.

We find that this process is also applicable to the homologous reactions with the oxalates and iodides of methyl and amyl. By it we have obtained

* Proceedings of the Royal Society, vol. xii. p. 396.

† The atomic weights used in this paper are the following:—C=12, O=16 and Zn=65.

numerous other acids belonging to the lactic series, which we have already more or less perfectly studied, and the history of which we propose to lay before the Royal Society as our researches proceed, reserving for a later communication our views regarding the constitution of this series of acids, and the theoretical conclusions arrived at in the course of the inquiry. In the present communication we will describe the application of this reaction to a mixture of iodide of methyl and oxalate of methyl.

Two equivalents of iodide of methyl were mixed with one of oxalate of methyl, and placed in contact with an excess of amalgamated and granulated zinc in a flask, to which an inverted Liebig's condenser, provided with a mercurial safety tube, was attached. The flask was immersed during about twenty-four hours in water maintained at a temperature gradually rising from 70° C. to 100° C. as the reaction progressed towards completion. At the end of that time the mixture had solidified to a yellowish gummy mass, which, on distillation with water, yielded methylic alcohol possessing an etherial odour, but from which we could extract no ether. The residual magma in the flask, consisting of iodide of zinc, oxalate of zinc, and the zinc-salt of a new acid, was separated from the metallic zinc by washing with water. It was then treated with an excess of hydrate of baryta and boiled for a considerable time; carbonic acid was afterwards passed through the liquid until, on again boiling, the excess of baryta was completely removed. To the filtered solution recently precipitated oxide of silver was added until all iodine was removed. The solution separated from the iodide of silver was again submitted to a current of carbonic acid, boiled, and filtered. The resulting liquid, on being evaporated in the water-bath, yielded a salt crystallizing in brilliant needles possessing the peculiar odour of fresh butter. This salt is very soluble in water and in alcohol, but nearly insoluble in ether, and perfectly neutral to test-papers. On being submitted to analysis, it gave numbers closely corresponding with the formula

$$C_2''' \begin{cases} CH_3 \\ CH_3 \\ O \\ OH \\ OBa \end{cases}$$

The acid of this salt, for which we provisionally propose the name *dimethoxalic acid*, is obtained by adding dilute sulphuric acid to the concentrated solution of the baryta-salt and agitating with ether. On allowing the ether to evaporate spontaneously, prismatic crystals of considerable size make their appearance. These yielded, on combustion with oxide of copper, results nearly identical with those required by the formula

$$C_2''' \begin{cases} CH_3 \\ CH_3 \\ O \\ OH \\ OH \end{cases}$$

Dimethoxalic acid is a white solid, readily crystallizing in beautiful prisms resembling oxalic acid. It fuses at 75°·7 C., volatilizes slowly even at common temperatures, and readily sublimes at 50° C., being deposited upon a cool surface in magnificent prisms. It boils at about 212° C., and distils unchanged. Dimethoxalic acid reacts strongly acid, and unites with bases, forming a numerous class of salts, several of which are crystalline. In addition to the baryta-salt above mentioned, we have examined the silver-salt, which is best formed by adding oxide of silver to the free acid, heating to boiling, and filtering, when the salt is deposited in star-like masses of nacreous scales as the solution cools. On analysis, this salt gave numbers closely corresponding with those calculated from the formula

$$C_2''' \begin{cases} C\,H_3 \\ C\,H_3 \\ O \\ O\,H \\ O\,Ag \end{cases}$$

Attempts to produce an ether by digesting the free acid with absolute alcohol at a temperature gradually raised to 160° C. proved abortive, traces only of the ether being apparently formed.

Thus the final result of the action of zinc upon a mixture of iodide and oxalate of methyl is perfectly homologous with that obtained by the action of zincethyl upon oxalic ether. In the methylic reaction, however, no compound corresponding to leucic ether was obtained. This cannot create surprise when it is remembered that dimethoxalic ether approaches closely in composition to lactic ether, which is well known to be instantly decomposed by water. We have sought in vain to obviate this decomposition of dimethoxalic ether by adding absolute alcohol in place of water to the product of the reaction.

February 25, 1864.

Major-General SABINE, President, in the Chair.

I. "On the Joint Systems of Ireland and Cornwall, and their Mechanical Origin." By the Rev. SAMUEL HAUGHTON, M.D., F.R.S., Fellow of Trinity College, Dublin. Received February 8, 1864.

(Abstract.)

This paper is a continuation of a former paper "On the Joints of the Old Red Sandstone of the Co. Waterford," published in the 'Philosophical Transactions' for 1858, and contains the results of the author's observations for some years, in Donegal, the Mourne and Newry Mountains, Cornwall, and Fermanagh, with deductions from theory.

The author establishes the existence in Waterford of a Primary Conjugate System of Joints, and of two Secondary Conjugate Systems, lying at each side of the Primary at angles of 27° 5′ and 37° 11′.

In Donegal there exists a Primary Conjugate System, and a Secondary System, making with the Primary an angle of 32° 24′. In the Mourne and Newry Mountains there is a Primary Conjugate System, and two Secondary Systems at each side of the Primary, making angles of 31° 46′ and 30° 56′. In Cornwall there is a Primary and also a Secondary Conjugate System, making an angle of 27° 28′. And in Fermanagh there are Primary and Secondary Systems, forming an angle of 31° 1′.

Having given, in detail, the observations on which the preceding results are founded, the author says:—"Collecting together into one Table the results of the preceding observations, we find the following Table of Primary and Secondary Joints (True Bearings):—

Name.	Waterford.	Donegal.	Mourne.	Cornwall.	Fermanagh.
Primary System (A)	N. of E. 32° 26′	N. of E. 26° 16′	N. of E. 39° 40′	N. of E. 32° 34′	N. of E. 21° 30′
Primary Conjugate (C).....	W. of N. 31° 37′	W. of N. 29° 35′	W. of N. 38° 31′	W. of N. 32° 55′	W. of N. 25° 48′
First Secondary (A′).....	N. of E. 58° 11′	N. of E. 58° 40′	N. of E. 70° 40′	—	N. of E. 54° 0′
Conjugate to First Secondary (C′)	W. of N. 60° 3′	—	W. of N. 70° 40′	—	W. of N. 55° 20′
Second Secondary (A″).....	S. of E. 5° 50′	—	—	N. of E. 4° 0′	—
Conjugate to Second Secondary (C″).	E. of N. 4° 30′	—	W. of N. 7° 35′	W. of N. 6° 30′	—

The only remarkable agreement as to direction of joints disclosed by the preceding Table is that between Waterford and Cornwall. If we compare together the Primary and Secondary Joints in each locality, we find the following Table of Angles between Primary and Secondary Joints:—

	Waterford.	Donegal.	Mourne.	Cornwall.	Fermanagh.
Between Primary (A, C) and First Secondary (A′, C′)	+27° 5′	+32° 24′	+31° 46′	—	+31° 1′
Between Primary (A, C) and Second Secondary (A″, C″).....	−37° 11′	—	−30° 56′	−27° 28′	—

This Table discloses a very interesting and unexpected result; viz. that in Waterford, Donegal, Mourne, and Fermanagh, the angle between the Primary and first Secondary Joint-Systems ranges between the narrow limits of 27° 5′ and 32° 24′, and that in Waterford, Mourne, and Cornwall, the angle between the Primary and second Secondary Joint-Systems ranges from 27° 28′ to 37° 11′.

The paper concludes with a brief deduction of the observed laws of Conjugate and Secondary Joints from known mechanical principles.

II. "On the supposed Identity of Biliverdin with Chlorophyll, with remarks on the Constitution of Chlorophyll." By G. G. Stokes, M.A., Sec.R.S. Received February 25, 1864.

I have lately been enabled to examine a specimen, prepared by Professor Harley, of the green substance obtained from the bile, which has been named biliverdin, and which was supposed by Berzelius to be identical with chlorophyll. The latter substance yields with alcohol, ether, chloroform, &c., solutions which are characterized by a peculiar and highly distinctive system of bands of absorption, and by a strong fluorescence of a blood-red colour. In solutions of biliverdin these characters are *wholly wanting*. There is, indeed, a vague minimum of transparency in the red; but it is totally unlike the intensely sharp absorption-band of chlorophyll, nor are the other bands of chlorophyll seen in biliverdin. In fact, no one who is in the habit of using a prism could suppose for a moment that the two were identical; for an observation which can be made in a few seconds, which requires no apparatus beyond a small prism, to be used with the naked eye, and which as a matter of course *would* be made by any chemist working at the subject, had the use of the prism made its way into the chemical world, is sufficient to show that chlorophyll and biliverdin are quite distinct.

I may take this opportunity of mentioning that I have been for a good while engaged at intervals with an optico-chemical examination of chlorophyll. I find the chlorophyll of land-plants to be a mixture of four substances, two green and two yellow, all possessing highly distinctive optical properties. The green substances yield solutions exhibiting a strong red fluorescence; the yellow substances do not. The four substances are soluble in the same solvents, and three of them are extremely easily decomposed by acids or even acid salts, such as binoxalate of potash; but by proper treatment each may be obtained in a state of very approximate isolation, so far at least as coloured substances are concerned. The *phyllocyanine* of Fremy* is mainly the product of decomposition by acids of one of the green bodies, and is naturally a substance of a nearly neutral tint, showing however extremely sharp bands of absorption in its neutral solutions, but dissolves in certain acids and acid solutions with a green or blue colour. Fremy's *phylloxanthine* differs according to the mode of preparation. When prepared by removing the green bodies by hydrate of alumina and a little water, it is mainly one of the yellow bodies; but when prepared by hydrochloric acid and ether, it is mainly a mixture of the same yellow body (partly, it may be, decomposed) with the product of decomposition by acids of the second green body. As the mode of preparation of *phylloxantheine*

* Comptes Rendus, tom. l. p. 405.

is rather hinted at than described, I can only conjecture what the substance is; but I suppose it to be a mixture of the second yellow substance with the products of decomposition of the other three bodies. Green sea-weeds (*Chlorospermeæ*) agree with land-plants, except as to the relative proportion of the substances present; but in olive-coloured sea-weeds (*Melanospermeæ*) the second green substance is replaced by a third green substance, and the first yellow substance by a third yellow substance, to the presence of which the dull colour of those plants is due. The red colouring-matter of the red sea-weeds (*Rhodospermeæ*), which the plants contain in addition to chlorophyll, is altogether different in its nature from chlorophyll, as is already known, and would appear to be an albuminous substance. I hope, before long, to present to the Royal Society the details of these researches.

"Continuation of an Examination of *Rubia munjista*, the East-Indian Madder, or Munjeet of Commerce." By JOHN STENHOUSE, LL.D., F.R.S. Received December 21, 1863*.

In the former, preliminary notice of the examination of the *Rubia munjista*†, the mode of extracting munjistine from munjeet, and a number of its properties, have been already described. I now proceed to detail some results which have been subsequently obtained.

When munjistine is extracted from munjeet by boiling solutions of sulphate of alumina, as the whole of the colouring matter is not extracted by a single treatment with the sulphate of alumina, the operation must be repeated five or six times instead of two or three as was formerly stated. During the boiling of the munjeet with sulphate of alumina, a large quantity of furfurol is given off. I may mention, in passing, that the most abundant and economical source of furfurol is found in the preparation of garancine by boiling madder with sulphuric acid. If the wooden boilers in which garancine is usually manufactured were fitted with condensers, furfurol might be obtained in any quantity without expense.

In addition to the properties of munjistine already described, I may mention that acetate of copper produces in solutions of munjistine a brown precipitate but very slightly soluble in acetic acid.

When bromine-water is added to a strong aqueous solution of munjistine, a pale-coloured flocculent precipitate is immediately produced; this when collected on a filter, washed and dissolved in hot alcohol, furnishes minute tufts of crystals, evidently a substitution-product. Unfortunately these crystals are contaminated by a resinous matter, from which I have been unable to free them, and therefore to determine their composition.

When munjistine is strongly heated on platinum-foil, it readily inflames and leaves no residue; when it is carefully heated in a tube, it fuses, and crystallizes again on cooling. If heated very slowly in a Mohr's apparatus,

* Read January 14. See Abstract, page 86. † Proceedings, vol. xii. p. 633.

munjistine sublimes in golden scales and broad flat needles of great beauty; these have all the physical characters and the same composition as the original substance. If the sublimation be continued for a long time at the lowest possible temperature consistent with its volatilization, the whole of it is obtained with scarcely any loss.

The following are the results of the ultimate analysis of different samples of munjistine:—

I. ·314 grm. of munjistine yielded ·732 grm. carbonic acid and ·106 grm. of water.

II. ·228 grm. of munjistine yielded ·535 grm. carbonic acid and ·0765 grm. water.

III. ·332 grm. of munjistine yielded ·7795 grm. of carbonic acid and ·1125 grm. of water.

IV. ·313 grm. of munjistine yielded ·734 grm. of carbonic acid and ·1095 grm. of water.

	Theory.	I.	II.	III.	IV.
$C_{16}=96$	64·00	63·60	64·00	64·04	63·97
$H_6=6$	4·00	3·77	3·73	3·76	3·89
$O_6=48$	32·00	32·63	32·27	32·20	32·14

The carbon in No. I. is rather lower than that of the other three; this is owing to the specimen not being quite free from alumina; moreover it was burnt with oxide of copper, the others with chromate of lead. No. III. is the sublimed munjistine. All the analyses were made on specimens prepared at different times.

Lead Compound.

When aqueous or alcoholic solutions of munjistine and acetate of lead are mixed, a flocculent precipitate of a deep orange-colour falls, which changes to scarlet on the addition of a slight excess of acetate. The best method of preparing it is to dissolve munjistine in hot spirit and add to the filtered solution a quantity of acetate of lead insufficient to precipitate the whole of the munjistine, then to wash thoroughly with spirit, in which the lead compound is but slightly soluble, and dry first *in vacuo*, and then in the water-bath.

I. ·836 grm. lead compound gave ·407 grm. oxide of lead.

II. ·625 grm. lead compound gave ·302 grm. oxide of lead.

III. ·428 grm. lead compound gave ·2075 grm. oxide of lead.

IV. ·523 grm. lead compound gave ·253 grm. oxide of lead.

V. ·2705 grm. lead compound gave ·3445 grm. of carbonic acid and ·0445 grm. water.

VI. ·5350 grm. lead compound gave ·6830 grm. carbonic acid and ·0920 grm. water.

	Theory.	I.	II.	III.	IV.	V.	VI.
$C_{30}=486$	34·93	....	....	....	....	34·73	34·82
$H_{25}=25$	1·82	....	....	....	....	1·83	1·91
$O_{25}=200$	14·55	....	....	....	....	....	....
$6PbO$ 669·6	48·70	48·70	48·32	48·50	48·38	....	....

All the specimens were prepared at different times, except IV. and V., which are analyses of the same specimen. The lead compound therefore seems to approach nearly to the somewhat anomalous formula $5(C_{16}H_8O_5)+6PbO$, being a basic lead-salt; it is, however, perfectly analogous to the lead compound of purpurine, $5(C_{18}H_8O_5)+6PbO$, described by Wolff and Strecker*.

From these analyses of the lead compound and also from the ultimate analyses of munjistine itself, it is pretty evident that its true formula is $C_{16}H_8O_6$.

Neither sublimed munjistine nor that obtained by crystallization from alcohol, when dried at the ordinary temperature *in vacuo*, loses weight at 110° C. It is not improbable, however, that the gelatinous uncrystallizable precipitate, which separates on the cooling of boiling saturated aqueous solutions of munjistine, is a hydrate.

From some experiments made on a considerable scale, I find that ordinary madder does not contain any munjistine. In order to ascertain this fact, a considerable quantity of garancine from Naples Roots, and likewise some which had been subjected to the action of high-pressure steam according to Pincoff and Schunck's process, were treated with boiling bisulphide of carbon, and the product obtained on evaporating the bisulphide repeatedly extracted with large quantities of boiling water; the solution, when acidulated with sulphuric acid, gave an orange-red precipitate from which I was unable to obtain any munjistine. Professor Stokes succeeded, however, in detecting the presence of alizarine, purpurine, and rubiacine in it†.

The production of phthalic acid from alizarine, purpurine, and munjistine, together with a comparison of their subjoined formulæ, indicates the very close relationship between these three substances, the only true colouring principles of the different species of madder with which we are acquainted.

Alizarine	$C_{20}H_8O_6$,
Purpurine.....	$C_{18}H_8O_6$,
Munjistine	$C_{16}H_8O_6$.

Two other very convenient sources of phthalic acid are—first, the dark red resinous matter, combined with alumina, which is left undissolved by the bisulphide of carbon in the preparation of munjistine; secondly, the large quantity of green-coloured resinous matter which remains behind after extracting the alizarine from Professor Kopp's so-called "green alizarine" by means of bisulphide of carbon. I have repeated Marignac's and Schunck's experiments of distilling a mixture of phthalic acid and lime; and, like both of these chemists, I observed a quantity of very aromatic benzol to be produced, which, by the action of strong nitric acid, readily yielded nitrobenzol, and from this, by the action of reducing agents, aniline. The only impurity in the benzol from phthalic acid appears to be a minute

* Annalen der Chemie, lxxv. p. 24.

† He has since informed me that he has succeeded in demonstrating the absence of munjistine.

quantity of an oil, having an aromatic odour, resembling that produced from cinnamic acid by the action of hypochlorite of lime.

Tinctorial power of Munjistine and Munjeet.

Prof. Runge stated, in 1835, that munjeet contains twice as much available colouring matter as the best Avignon madder. This result was so unexpected, that the Prussian Society for the Encouragement of Manufactures, to whom Professor Runge's memoir was originally addressed, referred the matter to three eminent German dyers, Messrs. Dannenberger, Böhm, and Nobiling. These gentlemen reported, as the result of numerous and carefully conducted experiments, that so far from munjeet being richer in colouring-matter than ordinary madder, it contained considerably less. This conclusion has been confirmed by the experience of my friend Mr. John Thom, of Birkacre, near Chorley, one of the most skilful of the Lancashire printers.

From a numerous series of experiments I have just completed, I find that the garancine from munjeet has about half the tinctorial power of the garancine made from the best madder, viz. Naples Roots. These, however, yield only about 30 to 33 per cent. of garancine, while munjeet, according to my friend Mr. Higgin, of Manchester, yields from 52 to 55 per cent. Taking the present prices therefore of madder at 36 shillings per cwt., and munjeet at 30 shillings, it will be found that there will be scarcely any pecuniary advantage in using munjeet for ordinary madder-dyeing. The colours from munjeet are certainly brighter, but not so durable as those from madder, owing to the substitution of purpurine for alizarine. There is, however, great reason to believe that some of the Turkey-red dyers are employing garancine from munjeet to a considerable extent. When this is the case they evidently sacrifice fastness to brilliancy of colour. By treating such a garancine with boiling water, and precipitating by an acid in the way already described, its sophistication with munjeet may very readily be detected. The actual amount of colouring matter in munjeet and the best madder is very nearly the same; but the inferiority of munjeet as a dye-stuff results from its containing only the comparatively feeble colouring matters, purpurine and munjistine, only a small portion of the latter being useful, whilst the presence of munjistine in large quantity appears to be positively injurious. So much is this the case, that when the greater part of the munjistine is removed from munjeet-garancine by boiling water, it yields much richer shades with alumina mordants than before.

PURPUREINE.

Action of Ammonia on Purpurine.

When purpurine is dissolved in dilute ammonia and exposed to the air in a vessel with a wide mouth in a warm place for about a month, ammonia and water being added from time to time as they evaporate, the purpurine almost entirely disappears, whilst a new colouring-matter is formed which dyes unmordanted silk and wool of a fine rose-colour, but is incapable of

dyeing vegetable fabrics mordanted with alumina. If, however, strong ammonia be employed to dissolve the purpurine, considerable heat is produced—a rise of temperature of as much as 20° C. taking place if the bulb of a thermometer be immersed in finely divided purpurine and strong ammonia poured on it.

The purpurine employed in these experiments was prepared by Kopp's process, and I am indebted for it to my friend Professor Crace Calvert.

The solution of the new substance, *purpureine*, is filtered to separate dust, &c., as well as a black substance insoluble in dilute ammonia; it is then added to a considerable quantity of dilute sulphuric acid, boiled for a short time, and allowed to cool. When cold, the impure purpureine is collected on a filter, well washed, and dissolved in a small quantity of hot alcohol. The spirituous solution is again filtered into a quantity of very dilute boiling sulphuric acid, about 1 part acid to from 50 to 100 of water; when cold, the precipitate is collected and again well washed. A crystallization out of boiling very dilute acid now renders it quite pure. This somewhat long and tedious process is necessary to free it from an uncrystallizable black substance, a part of which is separated when the crude purpureine is dissolved in alcohol, and a part is left behind at the last crystallization.

This compound being in its mode of formation and physical properties very analogous to orceine, I have called it purpureine. When crystallized by the spontaneous evaporation of its alcoholic solution, or from boiling dilute sulphuric acid under peculiar conditions of aggregation, it presents a fine iridescent green colour by reflected light; whilst under the microscope it appears as fine long needles of a very deep crimson colour. As obtained by the process above described, it has, however, but little of the iridescent appearance, being of a brownish-red colour with a faint tinge of green. It is almost insoluble in cold dilute acids, and is in great part precipitated from its aqueous solution by common salt, thus greatly resembling orceine. It is almost insoluble in bisulphide of carbon, very slightly so both in ether and in cold water, much more so in hot, and very soluble in spirit both hot and cold and in water rendered slightly alkaline. It is readily soluble in *cold* concentrated sulphuric acid, and is precipitated unaltered by water; on heating, however, it is destroyed.

Its aqueous solution gives a deep-red precipitate with chloride of zinc; with chloride of mercury a purple gelatinous precipitate; and with nitrate of silver a precipitate of a very dark brown colour slightly soluble in ammonia. I have been favoured with the following optical examination by Professor Stokes:—

"Its solutions show bands of absorption just like purpurine in *character*, but in some cases *considerably different in position*. The etherial and acidulated (acetic acid) alcoholic solutions show this strongly. The tint is so different in purpurine and its derivative, that the intimate connexion revealed by the prism would be lost by the eye. A drawing of the spectrum for purpurine would serve for its derivative (purpureine), if the bands were simply pushed a good deal nearer the red end."

I. ·3435 grm. pupureine gave ·8230 grm. carbonic acid and ·1240 grm. of water.

II. ·340 grm. purpureine gave ·813 grm. carbonic acid and ·123 grm. of water.

III. ·336 grm. purpureine gave ·01552 grm. nitrogen.

IV. ·535 grm. purpureine gave ·02453 grm. nitrogen.

		Theory.	I.	II.	III.	IV.
C_{66}	= 396	65·13	65·36	65·22	....	....
H_{24}	= 24	3·95	·4·01	4·02	....	....
N_2	= 28	4·60	....	....	4·62	4·58
O_{20}	= 160	26·32	....	....	....	....
	608	100·00				

The formula therefore appears to be $C_{66}H_{24}N_2O_{20}$?.

Nitropurpureine.

When purpureine is dissolved in a small quantity of moderately strong nitric acid, spec, grav. about 1·35, and heated to 100° C., it gives off red fumes, and on being allowed to cool, a substance separates in magnificent scarlet prisms somewhat like chromate of silver, only of a brighter colour; it is quite insoluble in water, ether, and bisulphide of carbon, and very slightly soluble in spirit, but soluble in hot moderately strong nitric acid, from which it separates on standing for a considerable time. If boiled with strong nitric acid, it is slowly decomposed. When heated, it deflagrates: from this circumstance, and considering its mode of formation, it is evidently a nitro-substitution compound; I have therefore called it nitropurpureine.

Owing to the small quantity which I have hitherto been able to procure, I have not yet determined the composition of this beautiful body, which is finer in appearance than any of the derivatives from madder I have as yet met with.

Action of Ammonia on Alizarine.

The alizarine which was employed for the subjoined experiments was obtained by extracting Professor E. Kopp's so-called green alizarine* with bisulphide of carbon. It yields only about 15 per cent. of orange-red alizarine. This was crystallized three times out of spirit, from which it usually separates as a deep-orange-coloured crystalline powder. Unfortunately this alizarine still contains a quantity of purpurine, from which it is impossible to purify it either by crystallization or sublimation. Accordingly, when treated with ammonia by the method already described for purpurine, while it yields a substance analogous to *purpureine*, the product is impure, being contaminated with purpureine, This mixture has been examined by my friend Professor Stokes, who finds that it contains purpureine, derived from the purpurine present as an impurity in the alizarine employed, and another substance very like alizarine in its optical properties, probably a new substance (*alizareine*), bearing the same relation to alizarine that

* I am also indebted to Professor Calvert for the "green alizarine."

purpureine does to purpurine*. The following is an extract from a letter I received from Professor Stokes:—

"It would be very unlikely *à priori* that such a simple process as that of Kopp should effect a perfect separation of two such similar bodies as alizarine and purpurine; and as I find his purpurine is free from alizarine, it would be almost certain *à priori* that his 'green alizarine' would contain purpurine, and the two would be dissolved by bisulphide of carbon, and might very well afterwards be associated by being deposited in intermingled crystals, if not actually crystallizing together."

Action of Ammonia on Munjistine.

This reaction with munjistine was only tried on a very small scale, but the results were by no means satisfactory. The munjistine was completely destroyed, the greater part being changed into a brown humus-like substance, insoluble in ammonia,—the remainder forming a colouring-substance, analogous to purpureine, but not crystalline. It dyed unmordanted silk a brownish-orange colour.

The combined action of ammonia and oxygen, therefore, on the three colouring-substances alizarine, purpurine, and munjistine, is to change them from adjective to substantive dye-stuffs. I think it not improbable that if this archilizing process were applied to various other colouring matters, they would be found capable of undergoing similar transformations.

Action of Bromine on Alizarine.

A boiling saturated solution of alizarine in alcohol is mixed with about six or eight parts of distilled water, and to this when cold about one or one and a half parts of bromine water are added, when a bright yellow amorphous precipitate is produced. After standing twelve or sixteen hours, the solution is filtered; and if the clear filtrate be now carefully heated so as to expel the spirit, a substance of a deep orange-colour is deposited, consisting of very fine needles, which are contaminated with a small quantity of resin if a great excess of bromine has been employed. These needles are soluble in spirit and ether, insoluble in water, and soluble in bisulphide of carbon, from which they crystallize by spontaneous evaporation, in dark-brown nodules. With soda they give the same purple colour as alizarine. They dye cloth mordanted with alumina a dingy brownish red, very different from the colour produced by ordinary crystallized alizarine. The following optical examination is from a letter of Professor Stokes:—

"*Bromine Derivative of Alizarine.*"

"I can hardly distinguish this substance from alizarine. The solutions

* Since this paper was communicated to the Royal Society, I find by a notice in Kopp and Wills's 'Jahresbericht' for 1862, p. 496, that a similar experiment upon alizarine had been made by Schützenberger and A. Paraf. The product of one preparation which they obtained, and to which they have given the name of *alizarinamid*, yielded a formula $C_{40}H_{15}NO_{12}$, and another preparation gave the formula $C_{80}H_{33}N_3O_{24}$, both being, when dry, nearly *black amorphous* substances. It appears, therefore, from the results of MM. Schützenberger and Paraf's experiments, that these gentlemen were not more successful in obtaining a pure product from the action of ammonia on alizarine than I have been.

in alcohol containing potassa show three bands of absorption just alike in appearance. By measurement it seemed probable that the bromine substance gave the bands a *little* nearer to the red end; but the difference, if real, was very minute. The fluorescent light of the ethereal solution was, I think, a trifle yellower in the bromine substance, that of alizarine being more orange."

The following are the results of the ultimate analysis of the brominated alzarine dried at 100° C.:—

I. ·375 grm. of substance gave ·207 grm. bromide of silver.

II. ·703 grm. of substance gave ·389 grm. bromide of silver.

III. ·401 grm. of substance gave ·221 grm. bromide of silver.

IV. ·543 grm. of substance gave ·300 grm. bromide of silver.

V. ·3575 grm. of substance gave ·695 grm. of carbonic acid and ·0760 grm. of water.

VI. ·454 grm. of substance gave ·8790 grm. of carbonic acid and ·0965 grm. of water.

		Theory.	I.	II.	III.	IV.	V.	VI.
C_{60}	= 360	52·94	....	....	....	....	53·03	52·81
H_{16}	= 16	2·35	....	....	....	....	2·36	2·36
Br_2	= 160	23·53	23·49	23·54	23·45	23·51	....	....
O_{18}	= 144	21·18	....	....	....	....	....	....
	680	100·00						

From this somewhat anomalous formula,

$$C_{60}H_{16}Br_2O_{18} = C_{20}H_6O_6, 2(C_{20}H_5BrO_6),$$

I was for some time inclined to think that it might be a mixture of brominated alizarine with free alizarine; but as all the six samples analyzed were prepared at different times, it is highly improbable that such uniform analytical results could be obtained if they were from a mere admixture of substances. The existence of a brominated compound is also confirmed by its dyeing properties, which differ so remarkably from those of alizarine.

Action of Bromine on Purpurine.

When pure purpurine is dissolved in spirit mixed with a considerable quantity of water, and an aqueous solution of bromine added, as in the case of alizarine, a yellow amorphous precipitate is produced. The solution separated from this by filtration, when heated to expel the spirit, gives *no precipitate whilst hot*; but on cooling, a very small quantity of a brown resinous powder is deposited. From this it is evident that the presence of a small quantity of purpurine in alizarine will not interfere with the production of pure brominated alizarine, if the precaution be taken to collect it from the solution whilst it is still hot.

I think it right to state that the experiments and analyses detailed in the preceding paper have been performed by my assistant, Mr. Charles Edward Groves. I cannot conclude this paper without again acknowledging the essential services I have received from Professor Stokes, who kindly submitted the different products obtained by me to optical examination.

March 3, 1864.

Major-General SABINE, President, in the Chair.

In accordance with the Statutes, the names of the Candidates for election into the Society were read, as follows :—

Alexander Armstrong, M.D.
William Baird, M.D.
Sir Henry Barkly, K.C.B.
Henry Foster Baxter, Esq.
Sir Charles Tilston Bright.
William Brinton, M.D.
John Charles Bucknill, M.D.
Lieut.-Col. John Cameron, R.E.
T. Spencer Cobbold, M.D.
The Hon. James Cockle, M.A.
Henry Dircks, Esq.
Alexander John Ellis, B.A.
John Evans, Esq.
William Henry Flower, Esq.
Sir Charles Fox.
George Gore, Esq.
George Robert Gray, Esq.
Thomas Grubb, Esq.
Henri Gueneau de Mussy, M.D.
William Augustus Guy, M.B.
George Harley, M.D.
Sir John Charles Dalrymple Hay, Bart.
Benjamin Hobson, M.B.
William Charles Hood, M.D.
Fleeming Jenkin, Esq.
William Jenner, M.D.
Edmund C. Johnson, M.D.
Prof. Leone Levi.
Waller Augustus Lewis, M.B.
Sir Charles Locock, Bart., M.D.
Edward Joseph Lowe, Esq.
The Hon. Thomas M'Combie.
Sir Joseph F. Olliffe, M.D.
George Wareing Ormerod, M.A.
Thomas Lambe Phipson, Esq.
John Russell Reynolds, M.D.
William Henry Leighton Russell, B.A.
William Sanders, Esq.
Col. William James Smythe, R.A.
Lieut.-Col. Alexander Strange.
Thomas Tate, Esq.
Charles Tomlinson, Esq.
George Charles Wallich, M.D.
Robert Warington, Esq.
Charles Wye Williams, Esq.
Nicholas Wood, Esq.
Henry Worms, Esq.

The following communication was read :—

"On the Spectra of Ignited Gases and Vapours, with especial regard to the different Spectra of the same elementary gaseous substance." By Dr. JULIUS PLÜCKER, of Bonn, For. Memb. R.S., and Dr. S. W. HITTORF, of Munster. Received February 23, 1864.

(Abstract.)

In order to obtain the spectra of the elementary bodies, we may employ either flame or the electric current. The former is the more easily managed, but its temperature is for the most part too low to volatilize the body to be

examined, or, if it be volatilized or already in the state of gas, to exhibit its characteristic lines. In most cases it is only the electric current that is fitted to produce these lines; and the current furnished by a powerful induction coil was what the authors generally employed.

In the application of the current, different cases may arise. The body to be examined may be either in the state of gas, or capable of being volatilized at a moderate temperature, such as glass will bear without softening, or its volatilization may require a temperature still higher.

In the first two cases the body is enclosed in a blown-glass vessel consisting of two bulbs, with platinum wires for electrodes, connected by a capillary tube. In the case of a gas, the vessel is exhausted by means of Geissler's exhauster, and filled with the gas at a suitable tension. In the case of a solid easily volatilized, a portion is introduced into the vessel, which is then exhausted as highly as possible, and the substance is heated by a lamp at the time of the observation. In the third case the electric current is employed at the same time for volatilizing the body and rendering its vapour luminous. If the body be a conductor, the electrodes are formed of it; but the spectrum observed exhibits not only the lines due to the body to be examined, but also those which depend on the interposed gas. This inconvenience is partly remedied by using hydrogen for the interposed gas, as its spectrum under these circumstances approaches to a continuous one. If the body to be examined be a non-conductor, the metallic electrodes are covered with it. In this case the spectrum observed contains the lines due to the metal of which the electrodes are formed, and to the interposed gas, as well as those due to the substance to be examined.

Among the substances examined, the authors commence with nitrogen, which first revealed to them the existence of two spectra belonging to the same substance. The phenomena presented by nitrogen are described in detail, which permits a shorter description to suffice for the other bodies examined.

On sending through a capillary tube containing nitrogen, at a pressure of from 40 to 80 millimetres, the direct discharge of a powerful Ruhmkorff's coil, a spectrum is obtained consisting, both in its more and in its less refrangible part, of a series of bright shaded bands: the middle part of the spectrum is usually less marked. In each of the two parts referred to, the bands are formed on the same type; but the type in the less refrangible part of the spectrum is quite different from that in the more refrangible. In the latter case the bands have a channeled appearance, an effect which is produced by a shading, the intensity of which decreases from the more to the less refracted part of each band. In a sufficiently pure and magnified spectrum, a small bright line is observed between the neighbouring channels, and the shading is resolved into dark lines, which are nearly equidistant, while their darkness decreases towards the least refracted limit of each band. With a similar power the bands in the less refrangible part of the spectrum are also seen to be traversed by fine dark lines, the arrange-

ment of which, however, while similar for the different bands, is quite different from that observed in the channeled spaces belonging to the more refrangible region.

If, instead of sending the *direct* discharge of the induction coil through the capillary tube containing nitrogen, a Leyden jar be interposed in the secondary circuit in the usual way, the spectrum obtained is totally different. Instead of shaded bands, we have now a spectrum consisting of brilliant lines having no apparent relation whatsoever to the bands before observed. If the nitrogen employed contains a slight admixture of oxygen, the bright lines due to oxygen are seen as well as those due to nitrogen, whereas in the former spectrum a slight admixture of oxygen produced no apparent effect.

The different appearance of the bands in the more and in the less refracted portion of the spectrum first mentioned suggested to the authors that it was really composed of two spectra, which possibly might admit of being separated. This the authors succeeded in effecting by using a somewhat wider tube. Sent through this tube, the direct discharge gave a golden-coloured light, which was resolved by the prism into the shaded bands belonging to the less refrangible part of the spectrum, whereas with a small jar interposed the light was blue, and was resolved by the prism into the channeled spaces belonging to the more refrangible part.

By increasing the density of the gas and at the same time the power of the current, or else, in case the gas be less dense, by interposing in the secondary circuit at the same time a Leyden jar and a stratum of air, the authors obtained lines of dazzling brilliancy which were no longer well defined, but had become of appreciable breadth, while at the same time other lines, previously too faint to be seen, made their appearance. The number of these lines, however, is not unlimited. By the expansion of some of the lines, especially the brighter ones, the spectrum tended to become continuous.

Those spectra which are composed of rather broad bands, which show different appearances according as they are differently shaded by fine dark lines, the authors generally call *spectra of the first order*, while those spectra which show brilliant coloured lines on a more or less dark ground they call *spectra of the second order*.

Incandescent nitrogen accordingly exhibits two spectra of the first, and one of the second order. The temperature produced by the passage of an electric current increases with the quantity of electricity which passes, and for a given quantity with the suddenness of the passage. When the temperature produced by the discharge is comparatively low, incandescent nitrogen emits a golden-coloured light, which is resolved by the prism into shaded bands occupying chiefly the less refrangible part of the spectrum. At a higher temperature the light is blue, and is resolved by the prism into channeled bands filling the more refrangible part of the spectrum. At a still higher temperature the spectrum consists mainly of bright lines,

which at the highest attainable temperature begin to expand, so that the spectrum tends to become continuous.

The authors think it probable that the three different spectra of the emitted light depend upon three allotropic states which nitrogen assumes at different temperatures.

By similar methods the authors obtained two different spectra of sulphur, one of the first and one of the second order. The spectrum of the first order exhibited channeled spaces, like one of the two spectra of that order of nitrogen; but the direction in which the depth of shading increased was the reverse of what was observed with nitrogen, the darker side of each channeled space being in the case of sulphur directed towards the red end of the spectrum.

Selenium, like sulphur, shows two spectra, one of the first and one of the second order.

Incandescent carbon, even in a state of the finest division, gives a continuous spectrum. Among the gases which by their decomposition, whether in flame or in the electric current, give the spectrum of carbon, the authors describe particularly the spectra of cyanogen and olefiant gas when burnt with oxygen or with air, and of carbonic oxide, carbonic acid, marsh-gas, olefiant gas, and methyl rendered incandescent by the electric discharge; they likewise describe the spectrum of the electric discharge between electrodes of carbon in an atmosphere of hydrogen. The spectrum of carbon examined under these various conditions showed great varieties, but all the different types observed were represented, more or less completely, in the spectrum of cyanogen fed with oxygen. The authors think it possible that certain bands, not due to nitrogen, seen in the flame of cyanogen, and not in any other compound of carbon, may have been due to the undecomposed gas.

The spectrum of hydrogen, as obtained by a small Ruhmkorff's coil, exhibited chiefly three bright lines. With the large coil employed by the authors, the lines slightly and unequally expanded. On interposing the Leyden jar, and using gas of a somewhat higher pressure, the spectrum was transformed into a continuous one, with a red line at one extremity, while at a still higher pressure this red line expanded into a band.

The authors also observed a new hydrogen spectrum, corresponding to a lower temperature, but having no resemblance at all to the spectra of the first order of nitrogen, sulphur, &c.

Oxygen gave only a spectrum of the second order, the different lines of which, however, expanded under certain circumstances into narrow bands, but very differently in different parts of the spectrum.

Phosphorus, when treated like sulphur, gave only a spectrum of the second order.

Chlorine, bromine, and iodine, when examined by the electric discharge, gave only spectra of the second order, in which no two of the numerous spectral lines belonging to the three substances were coincident. The

authors were desirous of examining whether iodine would give a spectrum of the first order the reverse of the absorption-spectrum at ordinary temperatures. The vapour of iodine in an oxyhydrogen jet gave, indeed, a spectrum of the first order, but it did not agree with what theory might have led us to expect.

In the electric discharge, arsenic and mercury gave only spectra of the second order. The metals of the alkalies sodium, potassium, lithium, thallium show, even at the lower temperature of Bunsen's lamp, spectra of the second order.

Barium, strontium, calcium in the flame of Bunsen's lamp show bands like spectra of the first order, and in each case a well-defined line-like spectra of the second order. On introducing chloride of barium into an oxyhydrogen jet, the shading of the bands was resolved into fine dark lines, proving that the band-spectrum of barium is in every respect a spectrum of the first order.

Spectra of the first order were observed in the case of only a few of the heavy metals, among which may be particularly mentioned lead, which, when its chloride, bromide, iodide, or oxide was introduced into an oxyhydrogen jet, gave a spectrum with bands which had a channeled appearance in consequence of a shading by fine dark lines.

Chloride, bromide, and iodide of copper gave in a Bunsen's lamp, or the oxyhydrogen jet, spectra with bands, and besides a few bright lines. The bands in the three cases were not quite the same, but differed from one another by additional bands. Manganese showed a curious spectrum of the first order. When an induction discharge passed between electrodes of copper or of manganese, pure spectra of these metals, of the second order, were obtained.

March 10, 1864.

Major-General SABINE, President, in the Chair.

The following communication was read:—

"On the Influence of Physical and Chemical Agents upon Blood; with special reference to the mutual action of the Blood and the Respiratory Gases." By GEORGE HARLEY, M.D., Professor of Medical Jurisprudence in University College, London. Communicated by Dr. SHARPEY, Sec. R.S. Received March 3, 1864.

(Abstract.)

This communication is divided into two parts. The first is devoted to the investigation of the influence of certain physical agencies, viz. simple diffusion, motion, and temperature, and of the conditions of time and the age of the blood itself. The second part includes the consideration of

the influence of chemical agents, especially such as are usually regarded as powerful poisons.

The paper commences with a description of the apparatus employed, and the method followed in conducting the inquiry; and the details of the several experiments are then given. The following is a brief statement of the results.

Part I.

1. The experiments on diffusion showed that venous blood not only yields a much greater amount of carbonic acid than arterial blood, but also absorbs and combines with a larger proportion of oxygen.

2. Motion of the blood was found to increase the chemical changes arising from the mutual action of the blood and the respiratory gases.

3. The results of the experiment on the influence of time led to the conclusion that the blood and air reciprocally act on each other in the same way out of the body as they do within it, and that their action is not instantaneous, but gradual.

4. It was ascertained that a certain degree of heat was absolutely essential to the chemical transformations and decompositions upon which the interchange of the respiratory gases depends. The higher the temperature up to that of 38° C. (the animal heat), the more rapid and more effectual were the respiratory changes; whereas a temperature of 0° C. was found totally to arrest them.

5. The influence of age on the blood was found to be very marked, especially on its relation to oxygen. The older and the more putrid the blood becomes, the greater is the amount of oxygen that disappears from the air; and although at the same time the exhalation of carbonic acid progressively increases with the age of the blood, yet its proportion is exceedingly small when compared with the large amount of oxygen absorbed.

6. The average amount of urea in fresh sheep's blood was ascertained to be 0·559 per cent., and its disappearance from the blood during the putrefactive process was very gradual, there being as much as 0·387 per cent. in blood after it was 304 hours old.

Part II.

The chemical agents employed were animal and vegetable products and mineral substances.

1. The effect of snake-poison was found to be an acceleration of the transformations and decompositions occurring in blood, upon which the absorption of oxygen and the exhalation of carbonic acid depend.

2. The presence of an abnormal amount of uric acid in blood was also found to hasten the chemical changes upon which the absorption of oxygen and exhalation of carbonic acid depend.

3. Animal sugar, contrary to what had been anticipated, retarded the respiratory changes produced in atmospheric air by blood.

4. The influence of hydrocyanic acid was studied both upon ox-blood and human blood, and found to be the same in each case, namely, to arrest respiratory changes.

5. Nicotine was also found to diminish the power of the blood either to take up oxygen or give off carbonic acid gas and thereby become fitted for the purposes of nutrition.

6. The effect of woorara poison, both on the blood in the body and out of it, was ascertained to be in some respects similar to that of snake-poison, namely, to increase the chemical decompositions and transformations upon which the exhalation of carbonic acid depends; but differed in retarding, instead of hastening, the oxidation of the constituents of the blood.

7. Antiar poison and aconite were found to act alike, inasmuch as both of them hastened oxidation and retarded the changes upon which the exhalation of carbonic acid depends; in both respects offering a striking contrast to woorara poison, which, as has just been said, diminishes oxidation and increases the exhalation of carbonic acid.

8. The effect of strychnine on the blood, both in and out of the body, was studied, and found to be in both cases identical, namely, like some of the other substances previously mentioned, to arrest respiratory changes. Moreover, in one experiment in which the air expired from the lungs of an animal dying from the effects of the poison was examined, it was ascertained that the arrest in the interchange of the gases took place before the animal was dead.

9. Brucine acts in a similar manner as strychnine, but in a much less marked degree.

10. Quinine also possesses the power of retarding oxidation of the blood, as well as the elimination of carbonic acid gas.

11. Morphine has a more powerful effect in diminishing the exhalation of carbonic acid gas, as well as the chemical changes upon which the absorption of oxygen by blood depends.

Under this head the effects of anæsthetics upon blood are next detailed; and in the first place, the visible effects of chloroform upon blood are thus described:—If 5 or more per cent. of chloroform be added to blood, and the mixture be agitated with air, it rapidly assumes a brilliant scarlet hue, which is much brighter than the normal arterial tint, and is, besides, much more permanent. When the mixture is left in repose, it gradually solidifies into a red-paint-like mass, which when examined under the microscope is frequently found to contain numerous prismatic crystals of an organic nature. If the blood of an animal poisoned from the inhalation of chloroform be employed in this experiment, the paint-like mass will be found to be composed in greater part of the crystals just spoken of; the crystals in this case being both larger and finer than when healthy blood is employed. Chloroform only partially destroys the blood-corpuscles. Its chemical action is to diminish the power of the constituents of the blood to unite with oxygen and give off carbonic acid.

The action of sulphuric ether upon blood differs in many respects from that of chloroform. In the first place, ether has a powerful effect in destroying the blood-corpuscles, dissolving the cell-walls and setting the contents free. In the second place, ether prevents the blood from assuming an arterial tint when agitated with air. The higher the percentage of the agent, the more marked the effect. In the third place, ether neither diminishes the absorption of oxygen nor the exhalation of carbonic acid by blood; and lastly, it has a much more powerful effect in causing the constituents of the blood to crystallize. For example, if an equal part of ether be added to the blood of a dog poisoned by the inhalation of chloroform, as the ether evaporates groups of large needle-shaped crystals are formed. Under the microscope the crystals are found to be of a red colour and prismatic shape.

Alcohol acts upon blood somewhat like chloroform; it arrests the chemical changes, but in a less marked degree.

Amylene was found to act like ether upon blood, in so far as it did not diminish the absorption of oxygen or retard the elimination of carbonic acid. It differed, however, from ether in not destroying the blood-corpuscles.

In the last place, the action of mineral substances is stated, viz.:—

1. Corrosive sublimate was found to increase the chemical changes which develope carbonic acid, and to have scarcely any effect on those depending upon oxidation; its influence, if any, is rather to diminish them than otherwise.

2. Arsenic seems to retard both the oxidation of the constituents of the blood and the exhalation of carbonic acid.

3. Tartrate of antimony increases the exhalation of carbonic acid gas, while it at the same time diminishes the absorption of oxygen.

4. Sulphate of zinc and sulphate of copper both act like tartrate of antimony, but not nearly so powerfully.

Lastly, phosphoric acid was found to have the effect of increasing the chemical transformations and decompositions upon which the exhalation of carbonic acid depends.

March 17, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read:—

I. "Researches on Radiant Heat.—Fifth Memoir. Contributions to Molecular Physics." By J. Tyndall, F.R.S., &c. Received March 17, 1864.

(Abstract.)

Considered broadly, two substances, or two forms of substance, occupy the universe—the ordinary and tangible matter of that universe, and the

intangible and mysterious ether in which that matter is immersed. The natural philosophy of the future must mainly consist in the examination of the relations of these two substances. The hope of being able to come closer to the origin of the ethereal waves, to get some experimental hold of the molecules whence issue the undulations of light and heat, has stimulated the author in the labours which have occupied him for the last five years, and it is this hope, rather than the desire to multiply the facts already known regarding the action of radiant heat, which prompted his present investigation.

He had already shown the enormous differences which exist between gaseous bodies, as regards both their power of absorbing and emitting radiant heat. When a gas is condensed to a liquid, or a liquid congealed to a solid, the molecules coalesce, and grapple with each other, by forces which were insensible as long as the gaseous state was maintained. But though the molecules are thus drawn together, the ether still surrounds them: hence, if the acts of radiation and absorption depend on the individual molecules, they will assert their power even after their state of aggregation has been changed. If, on the contrary, their mutual entanglement by the force of cohesion be of paramount influence in the interception and emission of radiant heat, then we may expect that liquids will exhibit a deportment towards radiant heat altogether different from that of the vapour from which they are derived.

The first part of the present inquiry is devoted to an exhaustive examination of this question. The author employed twelve different liquids, and operated upon five different layers of each, which varied in thickness from 0·02 of an inch to 0·27 of an inch. The liquids were enclosed, not in glass vessels, which would have materially modified the heat, but between plates of transparent rock-salt, which but slightly affected the radiation. His source of heat throughout these comparative experiments consisted of a spiral of platinum wire, raised to incandescence by an electric current of unvarying strength. The quantities of radiant heat absorbed and transmitted by each of the liquids at the respective thicknesses were first determined; the vapours of these liquids were subsequently examined, the quantities of vapour employed being proportional to the quantities of liquid traversed by the radiant heat. The result of the comparison was that, for heat of the same quality, the order of absorption of liquids and that of their vapours are identical. There was no exception to this law; so that, to determine the position of a vapour as an absorber or radiator, it is only necessary to determine the position of its liquid.

This result proves that the state of aggregation, as far, at all events, as the liquid stage is concerned, is of altogether subordinate moment—a conclusion which will probably prove to be of cardinal moment in molecular physics. On one important and contested point it has a special bearing. If the position of a liquid as an absorber and radiator determine that of its

vapour, the position of water fixes that of aqueous vapour. Water had been compared with other liquids in a multitude of experiments, and it was found that as a radiant and as an absorbent it transcends them all. Thus, for example, a layer of bisulphide of carbon, 0·02 of an inch in thickness, absorbs 6 per cent., and allows 94 per cent. of the radiation from the red-hot platinum spiral to pass through it; benzol absorbs 43, and transmits 57 per cent. of the same radiation; alcohol absorbs 67, and transmits 33 per cent., and it stands at the head of all liquids except one in point of power as an absorber. The exception is *water*. A layer of this substance, of the thickness above given, absorbs 81 per cent., and permits only 19 per cent. of the radiation to pass through it. Had no single experiment ever been made upon the *vapour* of water, we might infer with certainty from the deportment of the *liquid*, that weight for weight this vapour transcends all others in its power of absorbing and emitting radiant heat.

The relation of absorption and radiation to the chemical constitution of the radiant and absorbent substances was next briefly considered.

For the first six substances in the list of those examined, the radiant and absorbent powers augment as the number of atoms in the compound molecule augments. Thus, bisulphide of carbon has 3 atoms, chloroform 5, iodide of ethyl 8, benzol 12, and amylene 15 atoms in their respective molecules; and the order of their powers as radiants and absorbents is that here indicated—bisulphide of carbon being the feeblest, and amylene the strongest of the six. Alcohol, however, excels benzol as an absorber, though it has but 9 atoms in its molecule; but, on the other hand, its molecule is rendered more *complex* than that of benzol by the introduction of a new element. Benzol contains carbon and hydrogen, while alcohol contains carbon, hydrogen, and oxygen. Thus, not only does the idea of *multitude* come into play in absorption and radiation, that of *complexity* must also be taken into account. The author directed the particular attention of chemists to the molecule of water; the deportment of this substance towards radiant heat being perfectly anomalous, if the chemical formula at present ascribed to it be correct.

Sir William Herschel made the important discovery that beyond the limits of the red end of the solar spectrum, rays of high heating power exist which are incompetent to excite vision. The author has examined the deportment of those rays towards certain bodies which are perfectly opaque to light. Dissolving iodine in the bisulphide of carbon, he obtained a solution which entirely intercepted the light of the most brilliant flames, while to the extra-red rays of the spectrum the same iodine was found to be perfectly diathermic. The transparent bisulphide, which is highly pervious to the heat here employed, exercised the same absorption as the opaque solution. A hollow prism filled with the opaque liquid was placed in the path of the beam from an electric lamp; the light-spectrum was completely intercepted, but the heat-spectrum was received upon a

screen, and could be there examined. Falling upon a thermo-electric pile, its presence was shown by the prompt deflection of even a coarse galvanometer.

What, then, is the physical meaning of opacity and transparency, as regards light and radiant heat? The luminous rays of the spectrum differ from the non-luminous ones simply in *period*. The sensation of light is excited by waves of ether shorter and more quickly recurrent than those which fall beyond the extreme red. But why should iodine stop the former, and allow the latter to pass? The answer to this question, no doubt, is, that the intercepted waves are those whose periods of recurrence coincide with the periods of oscillation possible to the atoms of the dissolved iodine. The elastic forces which separate these atoms are such as to compel them to vibrate in definite periods, and when these periods synchronize with those of the ethereal waves the latter are absorbed. Briefly defined, their transparency in liquids, as well as in gases, is synonymous with *discord*, while opacity is synonymous with *accord* between the periods of the waves of ether and those of the molecules of the body on which they impinge. All ordinary transparent and colourless substances owe their transparency to the discord which exists between the oscillating periods of their molecules and those of the waves of the whole visible spectrum. The general discord of the vibrating periods of the molecules of compound bodies with the light-giving waves of the spectrum may be inferred from the prevalence of the property of transparency in compounds, solid, liquid, and gaseous, while their greater harmony with the extra-red periods is to be inferred from their opacity to the extra-red rays.

Water illustrates this transparency and opacity in the most striking manner. It is highly transparent to the luminous rays, which demonstrates the incompetency of its molecules to oscillate in the periods which excite vision. It is as highly opaque to the extra-red undulations, which proves the synchronism of its periods with those of the longer waves.

If, then, to the radiation from any source water shows itself to be eminently or perfectly opaque, it is a proof that the molecules whence the radiation emanates must oscillate in what may be called extra-red periods. Let us apply this test to the radiation from a flame of hydrogen. This flame consists mainly of incandescent aqueous vapour, the temperature of which, as calculated by Bunsen, is 3259° C., so that if transmission augments with temperature, we may expect the radiation from this flame to be copiously transmitted by the water. While, however, a layer of the bisulphide of carbon 0·07 of an inch in thickness transmits 72 per cent. of the incident radiation, and every other liquid examined transmits more or less of the heat, a layer of water of the above thickness is entirely opaque to the radiation from the flame. Thus we establish accord between the periods of the molecules of cold water and those of aqueous vapour at a temperature of 3259° C. But the periods of water have already been proved to be extra-red; hence those of the hydrogen flame

must be extra-red also. The absorption by *dry* air of the heat emitted by a platinum spiral raised to incandescence by electricity was found to be insensible, while that by the ordinary *undried* air was 6 per cent. Substituting for the platinum spiral a hydrogen flame, the absorption by dry air still remained insensible, while that of the undried air *rose to* 20 *per cent. of the entire radiation*. The temperature of the hydrogen flame was as stated, 3259° C., that of the aqueous vapour of the air was 20° C. Suppose, then, the temperature of our aqueous vapour to rise from 20° C. to 3259° C., we must conclude that the augmentation of temperature is applied *to an increase of amplitude*, and not to the introduction of periods of quicker recurrence into the radiation.

The part played by aqueous vapour in the economy of Nature is far more wonderful than hitherto supposed. To nourish the vegetation of the earth, the actinic and luminous rays of the sun must penetrate our atmosphere, and to such rays aqueous vapour is eminently transparent. The violet and the extra-violet rays pass through it with freedom. To protect vegetation from destructive chills, the terrestrial rays must be checked in their transit towards stellar space, and this is accomplished by the aqueous vapour diffused through the air. This substance is the great moderator of the earth's temperature, bringing its extremes into proximity, and obviating contrasts between day and night which would render life insupportable. But we can advance beyond this general statement now that we know the radiation from aqueous vapour is intercepted, in a special degree, by water, and reciprocally, the radiation from water by aqueous vapour; for it follows from this that the very act of nocturnal refrigeration which produces the condensation of aqueous vapour upon the surface of the earth—giving, as it were, a varnish of liquid water to that surface—imparts to terrestrial radiation that particular character which disqualifies it from passing through the earth's atmosphere and losing itself in space.

And here we come to a question in molecular physics which at the present moment occupies the attention of able and distinguished men. By allowing the violet and extra-violet rays of the spectrum to fall upon sulphate of quinine and other substances, Professor Stokes has changed the periods of those rays. Attempts have been made to produce a similar result at the other end of the spectrum—to convert the extra-red periods into periods competent to excite vision—but hitherto without success. Such a change of period the author believed occurs when a platinum wire is heated to whiteness by a hydrogen flame. In this common experiment there is an actual breaking-up of long periods into short ones—a true rendering of invisual periods visual. The change of refrangibility here effected differs from that of Professor Stokes, first, by its being in the opposite direction, that is from lower to higher; and secondly, in the circumstance that the platinum is heated by the collision of the molecules of aqueous vapour, and before their heat has assumed the *radiant form*.

But it cannot be doubted that the same effect would be produced by radiant heat of the same periods, provided the motion of the ether could be rendered sufficiently intense. The effect, in principle, is the same whether we consider the platinum wire to be struck by a particle of aqueous vapour oscillating at a certain rate, or by a particle of ether oscillating at the same rate.

By plunging a platinum wire into a hydrogen flame we cause it to glow, and thus introduce shorter periods into the radiation. These, as already stated, are in discord with water; hence we should infer that the transmission through water will be more copious when the wire is in the flame than when it is absent. Experiment proves this conclusion to be true. Water, from being opaque, opens a passage to 6 per cent. of the radiations from the flame and spiral. A thin plate of colourless glass, moreover, transmitted 58 per cent. of the radiation from the hydrogen flame; but when the flame and spiral were employed 78 per cent. of the heat was transmitted. For an alcohol flame Knoblauch and Melloni found glass to be less transparent than for the same flame with platinum spiral immersed in it; but Melloni afterwards showed that this result was not general, that black glass and black mica were decidedly more diathermic to the radiation from the pure flame. The reason of this is now obvious. Black mica and black glass owe their blackness to the carbon diffused through them. The carbon, as proved by Melloni, is in some measure transparent to the extra-red rays, and the author had in fact succeeded in transmitting between 40 and 50 per cent. of the radiation from a hydrogen flame through a layer of carbon sufficient to intercept the light of the most brilliant flames. The products of combustion of the alcohol flame are carbonic acid and aqueous vapour, the heat of which is almost wholly extra-red. For this radiation the carbon is in a considerable degree transparent, while for the radiation from the platinum spiral it is in a great measure opaque. By the introduction of the platinum wire, therefore, the transparency of the pure glass and the opacity of its carbon were simultaneously augmented; but the augmentation of opacity exceeded that of transparency, and a difference in favour of opacity remained.

No more striking or instructive illustration of the influence of coincidence could be adduced than that furnished by the radiation from a carbonic oxide flame. Here the product of combustion is carbonic acid; and on the radiation from this flame even the ordinary carbonic acid of the atmosphere exerts a powerful effect. A quantity of the gas, only one-thirtieth of an atmosphere in density, contained in a polished brass tube four feet long, intercepted 50 per cent. of the radiation from the carbonic oxide flame. For the heat emitted by solid sources, olefiant gas is an incomparably more powerful absorber than carbonic acid; in fact, for such heat the latter substance, with one exception, is the most feeble absorber to be found among the compound gases. For the radiation from the hydrogen flame, moreover, olefiant gas possesses twice the absorbent power of

carbonic acid; but for the radiation from the carbonic oxide flame at a common tension of one inch of mercury, while carbonic acid absorbs 50 per cent., olefiant gas absorbs only 24. Thus we establish the coincidence of period between carbonic acid at a temperature over 3000° C., the periods of oscillation of both the incandescent and the cold gas belonging to the extra-red portion of the spectrum.

It will be seen from the foregoing remarks and experiments how impossible it is to examine the effect of temperature on the transmission of heat, if different sources of heat be employed. Throughout such an examination the same oscillating atoms ought to be retained. The heating of a platinum spiral by an electric current enables us to do this while varying the temperature between the widest possible limits. Their comparative opacity to the extra-red rays shows the general accord of the oscillating periods of our series of vapours with those of the extra-red undulations; hence, by gradually heating a platinum wire from darkness up to whiteness, we gradually augment the discord between it and our vapours, and must therefore augment the transparency of the latter. Experiment entirely confirms this conclusion. Formic ether, for example, absorbs 45 per cent. of the radiation from a platinum spiral heated to barely visible redness; 32 per cent. of the radiation from the same spiral at a red heat; 26 per cent. of the radiation from a white-hot spiral, and only 21 per cent. when the spiral is brought near its point of fusion. Remarkable cases of inversion as to transparency occurred in these experiments. For barely visible redness formic ether is more opaque than sulphuric; for a bright red heat both are equally transparent, while for a white heat, and still more for a nearly fusing temperature, sulphuric ether is more opaque than formic. This result gives us a clear view of the relationship of the two substances to the luminiferous ether. As we introduce waves of shorter period, the sulphuric augments most rapidly in opacity; that is to say, its accord with the shorter waves is greater than that of the formic. Hence we may infer that the molecules of formic ether oscillate as a whole more slowly than those of sulphuric ether.

When the source of heat was a Leslie's cube filled with boiling water and coated with lampblack, the opacity of formic ether in comparison with sulphuric was very decided; with this source also the position of chloroform, as regards iodide of methyl, was inverted. For a white-hot spiral, the absorption of chloroform vapour being 10 per cent., that of iodide of methyl is 16; with the blackened cube as source, the absorption by chloroform is 22 per cent., while that by the iodide of methyl is only 19. This inversion is not the result of temperature merely; for when a platinum wire heated to the temperature of boiling water was employed as a source, the iodide was the most powerful absorbent. Numberless experiments, indeed, prove that from heated lampblack an emission takes place which synchronizes in an especial manner with chloroform. This may be thus illustrated. For the Leslie's cube coated with lampblack, the absorption by chloroform

is more than three times that by bisulphide of carbon; for the radiation from the most luminous portion of a gas flame the absorption by chloroform is also considerably in excess of that by bisulphide of carbon; while for the flame of a Bunsen's burner, from which the incandescent carbon particles are removed by the free admixture of air, the absorption by bisulphide of carbon is nearly twice that by chloroform; the removal of the incandescent carbon particles more than doubled in this instance the relative transparency of the chloroform. Testing, moreover, the radiation from various parts of the same flame, it was found that for the blue base of the flame the bisulphide was the most opaque, while for all other portions of the flame the chloroform was most opaque. For the radiation from a very small gas flame, consisting of a blue base and a small white top, the bisulphide was also most opaque, and its opacity very decidedly exceeded that of the chloroform when the flame of bisulphide of carbon was employed as a source. Comparing the radiation from a Leslie's cube coated with isinglass with that from a similar cube coated with lampblack, at a common temperature of 100° C., it was found that out of eleven vapours all but one absorbed the radiation from the isinglass most powerfully; the single exception was chloroform. It may be remarked that whenever, through a change of source, the position of a vapour as an absorber of radiant heat was altered, the position of the liquid from which the vapour was derived was changed in the same manner.

It is still a point of difference between eminent investigators as to whether radiant heat up to a temperature of 100° C. is monochromatic or not. Some affirm this, others deny it. A long series of experiments enables the author to state that probably no two substances at a temperature of 100° C. emit heat of the same quality. The heat emitted by isinglass, for example, is different from that emitted by lampblack, and the heat emitted by cloth or paper differs from both. It is also a subject of discussion whether rock-salt is equally diathermic to all kinds of calorific rays,—the differences affirmed to exist by one investigator being ascribed by others to differences of incidence from the various sources employed. MM. De la Provostaye and Desains maintain the former view, Melloni and M. Knoblauch maintain the latter. The question was examined by the author without changing anything but the temperature of the source. Its size, distance, and surroundings remained the same, and the experiments proved that rock-salt shared in some degree the defect of all other substances; it is *not* perfectly diathermic, and it is more opaque to the radiation from a barely visible spiral than to that from a white-hot one.

The author devotes a section of his memoir to the relation of radiation to conduction. Defining radiation, internal as well as external, as the communication of motion from the vibrating molecules to the ether, he arrives by theoretic reasoning at the conclusion that the best radiators ought to prove the worst conductors. A broad consideration of the subject shows at once the general harmony of the conclusion with observed facts. Organic substances are all excellent radiators; they are also extremely bad

conductors. The moment we pass from the metals to their compounds, we pass from a series of good conductors to bad ones, and from bad radiators to good ones. Water, among liquids, is probably the worst conductor; it is the best radiator. Silver, among solids, is the best conductor; it is the worst radiator. In the excellent researches of MM. De la Provostaye and Desains the author finds a striking illustration of what he regards as a natural law—that those molecules which transfer the greatest amount of motion to the ether, or, in other words, radiate most powerfully, are the least competent to communicate motion to each other, or, in other words, to conduct with facility.

II. "Remarks on Sun Spots." By Balfour Stewart, M.A., F.R.S., Superintendent of the Kew Observatory. Received March 8, 1864.

In the volume on Sun Spots which Carrington has recently published, we are furnished with a curve denoting the relative frequency of these phenomena from 1760 to the present time. This curve exhibits a maximum corresponding to 1788·6. Again, in Dalton's 'Meteorology' we have a list of auroræ observed at Kendal and Keswick from May 1786 to May 1793.

The observations at Kendal were made by Dalton himself, and those at Keswick by Crosthwaite. This list gives—

For the year 1787 .... 27 auroræ,	For the year 1790 .... 36 auroræ;
1788 .... 53 „	1791 .... 37 „
1789 .... 45 „	1792 .... 23 „

showing a maximum about the middle, or near the end of 1788. This corresponds very nearly with 1788·6, which we have seen is one of Carrington's dates of maximum sun spots.

The following observation is unconnected with the aurora borealis. In examining the sun pictures taken with the Kew Heliograph under the superintendence of Mr. De la Rue, it appears to be a nearly universal law that the faculæ belonging to a spot appear to the left of that spot, the motion due to the sun's rotation being across the picture from left to right.

These pictures comprise a few taken in 1858, more in 1859, a few in 1861, and many more in 1862 and 1863, and they have been carefully examined by Mr. Beckley, of Kew Observatory, and myself. The following Table expresses the result obtained:—

Year.	No. of cases of facula to left of spot.	No. of cases of facula to right of spot.	No. of cases of facula equally on both sides of spot.	No. of cases of faculæ mostly between two spots.
1858	2	0	0	0
1859	18	0	0	3
1861	9	1	3	0
1862	64	4	7	3
1863	47	0	9	2
1864	18	1	2	1

III. "Description of an Improved Mercurial Barometer." By James Hicks, Esq. Communicated by J. P. Gassiot, F.R.S. Received March 16, 1864.

Having shown this instrument to Mr. Gassiot, he wished me to write a short description of it, which he thought would be of interest to the Royal Society.

Some time since I constructed an open-scale barometer, with a column of mercury placed in a glass tube hermetically sealed at the top, and perfectly open at the bottom. The lower half of the tube is of larger bore than that of the upper.

If a column of mercury, of exactly the length which the atmosphere is able at the time to support, were placed in a tube of glass hermetically sealed at the top, of equal bore from end to end, the mercury would be held in suspension; but immediately the pressure of the atmosphere increased, the mercury would rise towards the top of the tube, and remain there till, on the pressure decreasing, it would fall towards the bottom, and that portion which the atmosphere was unable to support would drop out. But if the lower half of the tube be made a little larger in the bore than the upper, when the column falls, the upper portion passes out of the smaller part of the tube into the larger, and owing to the greater capacity of the latter, the lower end of the column of mercury does not sink to the same extent as the upper end, and the column thus becomes shorter. The fall will continue until the column is reduced to that length which the atmosphere is capable of supporting, and the scale attached thus registers what is ordinarily termed the height of the barometer.

From the above description it will be evident that, by merely varying the proportion in the size of the two parts of the tube, a scale of any length can be obtained. For example, if the tubes are very nearly the same size in bore, the column has to pass through a great distance before the necessary compensation takes place, and we obtain a very long scale, say 10 inches, for every 1-inch rise and fall in the ordinary barometer. But if the lower tube is made much larger than the upper, the mercury passing into it quickly compensates, and we obtain a small scale, say from 2 to 3 inches, for every inch. To ascertain how many inches this would rise and fall for an ordinary inch of the barometer, I attach it, in connexion with a standard barometer, to an air-pump receiver, and by reducing the pressure in the air-pump I cause the standard barometer to fall, say 1 inch, when the other will fall, say 5 inches; and so I ascertain the scale for every inch, from 31 to 27 inches.

It was on this principle that I constructed the open-scale barometer, which has since been extensively used. But having been asked to apply a vernier to one of these barometers graduated in this way, I found this impracticable, as each varied in length in proportion as the bore of the tube varied, so that every inch was of a different length.

I have now remedied this defect, and made what I believe is an absolute standard barometer, by graduating the scale from the centre, and reading it off with two verniers to the $\frac{1}{1000}$th of an inch. The scale is divided from the centre, up and down, into inches, and subdivided into 20ths.

To ascertain the height of the barometer graduated in this way, take a reading of the upper surface of the column of mercury with the vernier, then of the lower surface in the same way, and the two readings added together will give the exact length of the column of mercury supported in the air, which is the height of the barometer at the time.

There is another advantage in this manner of graduating over the former, that if a little of the mercury drops out it will give no error, as the column will immediately rise out of the larger tube into the smaller, and become the same length as before; but by the former scale the barometer would stand too high, until readjusted, which could only be effected by putting the same quantity of mercury in again.

I have introduced Gay-Lussac's pipette into the centre of the tube, to prevent the possibility of any air passing up into the top.

The Society then adjourned over the Easter Recess to Thursday, April 7th.

"On Mauve or Aniline-Purple." By W. H. Perkin, F.C.S. Communicated by Dr. Stenhouse. Received August 19, 1863*.

The discovery of this colouring matter in 1856, and its introduction as a commercial article, has originated that remarkable series of compounds known as coal-tar colours, which have now become so numerous, and in consequence of their adaptibility to the arts and manufactures are of such great and increasing importance. The chemistry of mauve may appear to have been rather neglected, its composition not having been established, although it has formed the subject of several papers by continental chemists. Its chemical nature also has not been generally known; and to this fact many of the discrepancies in the results of the different experimentalists who have worked on this subject are to be attributed.

The first analysis I made of this colouring matter was in 1856, soon after I had become its fortunate discoverer. The product I examined was purified as thoroughly as my knowledge of its properties then enabled me, and the results† obtained agree very closely with those required for the formula I now propose. Since that time I have often commenced the study of this body in a scientific point of view, but other duties have prevented me

* For abstract see vol. xii. p. 713.

† The substance I examined was doubtless the sulphate, of which I made two combustions:—

No. I. gave	71·55	per cent. of carbon and	6·09	per cent. of hydrogen.
No. II. gave	71·60	„ „	5·77	„
Theory requires	71·5	„ „	5·5	„

from completing these investigations; but, although unacquainted with its correct formula, its chemical characters have necessarily been well known to me for a considerable time. When first introduced, commercial mauve appeared as an almost perfectly amorphous body; but now, owing to the great improvements which have been made in its purification, it is sent into the market perfectly pure and crystallized.

On adding a solution of hydrate of potassium to a boiling solution of commercial crystallized mauve, it immediately changes in colour from purple to a blue violet, and after a few moments begins to deposit a crystalline body. After standing a few hours, this crystalline product is collected on a filter, washed with alcohol once or twice, and then thoroughly with water. When dry, it appears as a nearly black glistening substance, not unlike pulverized specular iron ore.

This substance, for which I propose the name Mauveine, is a powerful base. It dissolves in alcohol, forming a blue violet solution, which immediately assumes a purple colour on the addition of acids. It is insoluble, or nearly so, in ether and benzole. It is a very stable body, and decomposes ammoniacal compounds readily. When heated strongly it decomposes, yielding a basic oil, which does not appear to be aniline.

The following analyses were made of specimens dried at 150° C.:—

I. ·301 grm. of substance gave ·8818 of carbonic acid and ·162 of water.

II. ·2815 grm. of substance gave ·8260 of carbonic acid and ·145 of water.

Direct Nitrogen determination.

III. ·3435 grm. of substance gave 41·0 c.c. N at 23° C. and 766 mms. Bar.

$$V' = \frac{41{\cdot}0 \text{ cub. centims. } (766{\cdot}0 \text{ millims.} - 20{\cdot}9)}{824{\cdot}1 \text{ millims.}} = 37{\cdot}7 \text{ cub. centims.}$$

$$37{\cdot}7 \times {\cdot}0012562 \text{ grm.} = {\cdot}04735 \text{ grm. of N.}$$

These numbers correspond to the following percentages:—

	I.	II.	III.
Carbon.....	79·9	80·0	——
Hydrogen.....	5·98	5·72	——
Nitrogen	——	——	13·75

The formula, C_{27}* H_{24} N_4, requires the following values:—

	Theory.		Mean of experiment.
C_{27}	324	80·19	79·95
H_{24}	24	5·94	5·85
N_4.....	56	13·87	13·75
	404	100·	

Hydrochlorate of Mauveine.—This salt is prepared by the direct combination of mauveine and hydrochloric acid. From its boiling alcoholic solution it is deposited in small prisms, sometimes arranged in tufts, possessing

* C=12.

a brilliant green metallic lustre. It is moderately soluble in alcohol, but nearly insoluble in ether. It is also, comparatively speaking, moderately soluble in water.

Different preparations dried at 100° C. gave the following numbers :—

I. ·306 grm. of substance gave ·8255 of carbonic acid and ·162 of water.
II. ·308 grm. of substance gave ·8275 of carbonic acid and ·163 of water.
III. ·310 grm. of substance gave ·8345 of carbonic acid.
IV. ·3165 grm. of substance gave ·851 of carb. acid and ·16525 of water.
V. ·2447 grm. of substance gave ·6603 of carb. acid and ·1356 of water.
VI. ·627 grm. of substance gave ·205 of chloride of silver.
VII. ·560 grm. of substance gave ·195 of chloride of silver.
VIII. ·69 grm. of substance gave ·2266 of chloride of silver.

Direct Nitrogen determination.

IX. ·3497 grm. of substance gave 40 c. c. N at 20° C. and 777·2 mms. Bar.

$$V' = \frac{40 \text{ c. c. } (7772 - 17{\cdot}4)}{815{\cdot}8 \text{ millims.}} = 37{\cdot}2 \text{ c. c. at } 0° \text{ C. and } 760 \text{ millims. Bar.}$$

37·2 cub. centims. × ·0012562 grm. = ·04673 grm. N.

These numbers correspond to the following percentages :—

	I.	II.	III.	IV.
Carbon	73·5	73·27	73·4	73·3
Hydrogen	5·88	5·88	——	5·8
Nitrogen	——	——	——	——
Chlorine	——	——	——	——

	V.	VI.	VII.	VIII.	IX.
Carbon	73·59	——	——	——	——
Hydrogen	6·16	——	——	——	——
Nitrogen	——	——	——	——	13·3
Chlorine	——	8·08	8·06	8·1	——

These numbers agree with the formula $C_{27} H_{24} N_4 H Cl$, as may be seen by the following Table :—

	Theory.		Mean of experiment.
C_{27}	324·	73·55	73·41
H_{25}	25·	5·67	5·93
N_4	56·	12·73	13·30
Cl	35·5	8·05	8·07
	440·5	100·00	

I have endeavoured to obtain a second hydrochlorate containing more acid, but up to the present time have not succeeded.

Platinum-salt.—Mauveine forms a perfectly definite and beautifully crystalline compound with bichloride of platinum. It is obtained by mixing an alcoholic solution of the above hydrochlorate with an excess of an

alcoholic solution of bichloride of platinum; from this mixture the new salt separates as a highly crystalline powder. I have generally preferred to use cold solutions in its preparation; but if moderately hot solutions be employed, the salt will separate as crystals of considerable dimensions.

This platinum-salt possesses the green lustre of the hydrochlorate, but, on being dried, assumes a more golden colour. It is very sparingly soluble in alcohol. The following numbers were obtained from various preparations dried at 100° C.:—

I. ·44125 grm. of substance gave ·072 of platinum.
II. ·4845 grm. of substance gave ·079 „
III. ·511 grm. of substance gave ·083 „
IV. ·510 grm. of substance gave ·083 „
V. ·6345 grm. of substance gave ·1035 „
VI. ·618 grm. of substance gave ·101 „
VII. ·31275 grm. of substance gave ·60525 of carbonic acid and ·118 of water.
VIII. ·30675 grm. of substance gave ·595 of carb. acid and ·110 of water.
IX. ·3795 grm. of substance gave ·27 of chloride of silver.

These results correspond to the percentages in the following Table:—

	I.	II.	III.	IV.	V.	VI.
Carbon	——	——	——	——	——	——
Hydrogen ..	——	——	——	——	——	——
Chlorine	——	——	——	——	——	——
Platinum ..	·16·31	16·3	16·24	16·27	16·3	16·3

	VII.	VIII.	IX.
Carbon.....	52·77	52·86	——
Hydrogen.....	4·19	3·98	——
Chlorine	——	——	17·6
Platinum	——	——	——

The formula, $C_{27}H_{24}N_4HPtCl_3$, requires the following values:—

	Theory.		Mean of experiment.
C_{27}	324·	53·09	52·81
H_{25}	25·	4·09	4·19
N_4	56·	9·2	....
Pt	98·7	16·16	16·28
Cl_3	106·5	17·46	17·6
	610·2	100·00	

Gold-salt.—This compound is prepared in a similar manner to the platinum-salt, only substituting chloride of gold for chloride of platinum. It separates as a crystalline precipitate, which, when moist, presents a much less brilliant aspect than the platinum derivative; it is also more soluble than that salt, and when crystallized appears to lose a small quantity of gold. The following results were obtained from a specimen dried at 100° C.:—

I. ·47175 grm. of substance gave ·1245 of gold.
II. ·35525 grm. of substance gave ·094 of gold.
III. ·309 grm. of substance gave ·495 of carbonic acid and ·101 of water.
Percentage composition :—

	I.	II.	III.
Carbon	——	——	43·68
Hydrogen	——	——	3·6
Gold	26·3	26·46	——

The formula, $C_{27}H_{24}N_4$, $HAuCl_4$, requires the following percentages :—

	Theory.		Mean of experiment.
C_{27}	324	43·53	43·68
H_{25}	25	3·34	3·6
N_4	56	7·44	——
Au	197	26·61	26·38
Cl_4	142	19·08	——
	744	100·00	

Hydrobromate of Mauveine.—This salt is prepared in a similar manner to the hydrochlorate, which it very much resembles, except that it is less soluble in alcohol. Analysis of preparations dried at 100° C. gave the following numbers :—

I. ·3935 grm. of substance gave ·1515 of bromide of silver.
II. ·450 grm. of substance gave ·173 of bromide of silver.
III. ·3265 grm. of substance gave ·79675 of carb. acid and ·158 of water.
IV. ·35125 grm. of substance gave ·86075 of carbonic acid and ·1675 of water.
Percentage composition :—

	I.	II.	III.	IV.
Carbon	——	——	66·55	66·8
Hydrogen	——	——	5·37	5·29
Bromine	16·38	16·37	——	——

These numbers agree with the formula $C_{27}H_{24}N_4HBr$, as shown by the comparisons in the following Table :—

	Theory.		Experiment.
C_{27}	324	66·8	66·67
H_{25}	25	5·15	5·33
N_4	56	11·56	——
Br	80	16·49	16·37
	485	100·00	

Hydriodate of Mauveine.—In preparing this salt from the base, it is necessary to use hydriodic acid which is colourless, otherwise the free iodine will slowly act upon this salt. It crystallizes in prisms having a

green metallic reflexion. It is more insoluble than the hydrobromate. The products used in the subjoined analysis were recrystallized three times, and dried at 100° C.

I. ·5115 grm. of substance gave ·22575 of iodide of silver.

II. ·248 grm. of substance gave ·549 of carb. acid and ·10975 of water.

III. ·2985 grm. of substance gave ·663 of carb. acid and ·1265 of water.

IIV. ·2765 grm. of substance gave ·615 of carb. acid and ·1145 of water.

Percentage composition :—

	I.	II.	III.	IV.
Carbon	——	60·46	60·57	60·65
Hydrogen	——	4·9	4·7	4·7
Iodine	23·8	——	——	——

The formula, $C_{27}H_{24}N_4HI$, requires the following values :—

	Theory.		Experiment.
C_{27}	324·	60·89	60·56
H_{25}	25·	4·69	4·7
N_4	56·	10·54	——
I	127·1	23·88	23·8
	532·1	100·00	

Acetate of Mauveine.—This salt is best obtained by dissolving the base in boiling alcohol and acetic acid. On cooling, it will crystallize out; it should then be recrystallized once or twice. This acetate is a beautiful salt, possessing the green metallic lustre common to most of the salts of mauveine. Two combustions of specimens dried at 100° C. gave the following numbers :—

I. ·28325 grm. of substance gave ·778 of carb. acid and ·153 of water.

II. ·29275 grm. of substance gave ·806 of carb. acid and ·1645 of water.

Percentage composition :—

	I.	II.
Carbon	74·9	75·0
Hydrogen	6·0	6·2

These numbers lead to the formula

$$C_{29}H_{28}N_4O_2 = C_{27}H_{24}N_4, C_2H_4O_2,$$

as shown by the following Table :—

	Theory.		Experiment.
C_{29}	348	75·	74·95
H_{28}	28	6·	6·1
N_4	56	12·06	——
O_2	32	6·94	——
	464	100·00	

Carbonate of Mauveine.—The tendency of solutions of mauveine to combine with carbonic acid is rather remarkable. If a quantity of its solu-

tion be thrown up into a tube containing carbonic acid over mercury, the carbonic acid will quickly be absorbed, the solution in the mean time passing from its normal violet colour to purple. To prepare this carbonate, it is necessary to pass carbonic acid gas through boiling alcohol containing a quantity of mauveine in suspension. It is then filtered quickly, and carbonic acid passed through the filtrate until nearly cold. On standing, this liquid will deposit the carbonate as prisms, having a green metallic reflexion. A solution of this salt, on being boiled, loses part of its carbonic acid and assumes the violet colour of the base. When dry this carbonate rapidly changes, and if heated to 100° C. loses nearly all its carbonic acid and changes in colour to a dull olive; therefore, as it cannot be dried without undergoing a certain amount of change, its composition is difficult to determine. However, I endeavoured to estimate the carbonic acid in this salt by taking a quantity of it freshly prepared and in the moist state, and heating it in an oil-bath until carbonic acid ceased to be evolved. The residual base was then weighed, and also the carbonic acid, which had been collected in a potash bulb, having been previously freed from water by means of sulphuric acid. The following results were obtained:—

I. 1·88 residual base obtained; ·190 carbonic acid evolved.

II. 1·375 residual base; ·1385 carbonic acid evolved. ·190 of CO_2 is equal to ·268 of H_2CO_3; this, added to the residual base, will give the amount of substance experimented with, viz. 2·148. The amount of CO_2 obtained from this quantity, therefore, is 8·8 per cent.

Calculating the second experiment in a similar manner, the amount of carbonate operated upon would be 1·5702 grm.; the percentage of CO_2 obtained is therefore equal to 8·8. A carbonate having the formula $(C_{27}\,H_{24}\,N_4)_2\,H_2\,CO_3$ would contain 5·1 per cent. of CO_2, and an acid carbonate having the formula $O_{27}\,H_{24}\,N_4,\,H_2\,CO_3$ would contain 9·4 per cent. of CO_2.

Considering that this salt when prepared begins to crystallize before it is cold, probably the first portions that deposit are a monocarbonate, while the larger quantity which separates afterwards is an acid carbonate. Hence the deficiency in the amount of CO_2 obtained in the above experiments. I hope to give my attention to this remarkable salt at a future period.

In the analysis of the salts of mauveine great care has to be taken in drying them thoroughly, as most of them are highly hygroscopic.

I am now engaged in the study of the replaceable hydrogen in mauveine, which I hope will throw some light upon its constitution. From its formula I believe it to be a tetramine, although up to the present time I have not obtained any definite salts with more than 1 equiv. of acid.

When mauveine is heated with aniline it produces a blue colouring matter, which will doubtless prove to be a phenyle derivative of that base. A salt of mauveine when heated alone also produces a violet or blue compound. These substances I am now examining, and hope in a short time to have the honour of communicating them to the Society.

April 7, 1864.

Major-General SABINE, President, in the Chair.

The Rev. Dr. Salmon was admitted into the Society.

The following communications were read :—

I. "On the Functions of the Cerebellum." By WILLIAM HOWSHIP DICKINSON, M.D. Cantab., Curator of the Pathological Museum, St. George's Hospital, Assistant Physician to the Hospital for Sick Children. Communicated by Dr. BENCE JONES. Received March 8, 1864.

(Abstract.)

The paper is divided into two Parts; the first gives the results of experiments on animals; the second, of observations upon the human being.

PART I.

Assuming that the great divisions of the brain preserve each the same function through the vertebrate kingdom, it is maintained that experiments which can be performed only on such of the lower animals as are very tenacious of life, will afford deductions of universal application.

The method of proceeding with regard to each species was to remove, first, the whole encephalon, with the exception of the medulla oblongata; then in a similar animal only the cerebrum was taken away. The only difference between the two cases was in the fact that one animal had a cerebellum, and the other had not. A comparison was believed to show, in the powers which one had more than the other, the function of the organ the possession of which constituted the only difference.

Finally it was ascertained in each species what is the effect of taking away the cerebellum alone.

The use of the organ was thus estimated in two ways—by the effect of its addition to the medulla, and of its subtraction from the rest of the nervous system.

The species so treated are arranged in an ascending scale, according to the comparative weight of the cerebellum. The field-snake, frog, salamander, toad, land-tortoise, eel, water-tortoise, pike, perch, tench, dace, carp, gold-fish, rudd, loach, and gudgeon were subjected to these operations; besides which, many experiments of a less systematic character were made upon birds and mammalia.

The results are these :—

In Reptiles, with the exception of the snake, the cord, together with the medulla oblongata, is sufficient to give the power of voluntary or spontaneous motion—limited, but usually enough to allow of feeble locomotion.

With the addition of the cerebellum, all actions dependent on the will appear to be naturally performed.

The removal of the cerebellum shows that the cerebrum by itself is unable to give more than a limited amount of voluntary motion, and that of a kind deficient in balance and adjustment.

It is therefore inferred that the cord, together with the medulla oblongata, is a great source of spontaneous motor power, in which function both the cerebrum and the cerebellum take part, the cerebellum to the greater extent; it also appears that a certain harmony in the use of the muscles depends on the possession of the latter organ.

Regarding *Fishes*, the cord and medulla oblongata seem unequal to the performance of voluntary motion.

When the cerebellum is added, the powers become so far extended that movements are made in obedience to external stimuli. Generally speaking, a determined position is maintained and locomotion accomplished, without the use, however, of the pectoral fins.

If the cerebellum only be taken away, there is a loss of the proper adjustment between the right and left sides; so that oscillation or rotation takes place. All the limbs are used, but apparently with a deficiency of sustained activity.

It is therefore concluded that with Fishes, as with Reptiles, the power of intentional movement is shared by both cerebrum and cerebellum; the former in this case has the larger influence.

Such movements as depend on the cerebrum are destitute of lateral balance, are sudden in being affected by any external cause, and are emotional in their character. Such as depend on the cerebellum are mutually adjusted, of a continuous kind, and less directly under the influence of consciousness.

The same facts were supported by experiments on the higher orders of animals: in these it seemed that the cord and medulla are insufficient to excite voluntary movements. The muscles, as with fishes and reptiles, acknowledge a double rule, from the cerebrum and from the cerebellum. The anterior limbs are most subservient to the cerebrum; the posterior to the cerebellum. The limbs on one side are in connexion chiefly with the lobe of the opposite side. The absence of the cerebellum destroys the power of lateral balance.

From the negative results of the experiments, it is inferred that the cerebellum has nothing to do with common sensation, with the sexual propensity, with the action of the involuntary muscles, with the maintenance of animal heat, or with secretion.

The only function which the experiments assigned to the cerebellum is such as concerns the voluntary muscles, which receive therefrom a regulated supply of motor influence. Each lateral half of the cerebellum affects both sides, but the one opposite to itself most.

The cerebellum has a property distinct from its true voluntary power, which harmonizes the action of the voluntary muscles, and has been described as "coordination."

The voluntary muscles are under a double influence—from the cerebrum and from the cerebellum. The anterior limbs are chiefly under the influence of the cerebrum; the posterior, of the cerebellum. Cerebellar movements are apt to be habitual, while cerebral are impulsive. The cerebellum acts when the cerebrum is removed, though when both organs exist it is under its control.

Part II.

From an analysis of one case of congenital absence of the cerebellum, one of disease of the whole organ, and 46 of disease of a portion of it, the following deductions are stated:—

The only faculty which constantly suffers in consequence of changes in the cerebellum, is the power of voluntary movement.

When the organ is absent or defective congenitally, we have want of action in the muscles of the lower extremities.

When the entire structure is changed by disease, we have loss of voluntary power, either general throughout the trunk, or limited to the lower limbs—which results are about equally frequent.

From the manner in which the paralysis was distributed in cases of disease of a part of the organ, it is inferred that each lobe is in connexion as a source of voluntary movement with all the four limbs, but in the greatest degree with the limbs of the opposite side, and with the lower more than with the upper extremities.

The occasional occurrence of loss of visual power, and alterations of the sexual propensity, is referred to the conveyance of irritation to the corpora quadrigemina in one case, and the spinal cord in the other.

From both sources of knowledge it is concluded that the cerebellum has distinct offices.

It is a source of voluntary motor power to the muscles supplied by the spinal nerves. It influences the lower more than the upper limbs, and produces habitual rather than impulsive movements. Each lobe affects both sides of the body, but most that opposite to itself.

Secondly, the cerebellum has a power which has been described as that of "coordination," which is similarly distributed.

Finally, it is suggested that the outer portion of the organ may be the source of its voluntary motor power, while its inner layer is the means of regulating its distribution.

II. "An Inquiry into Newton's Rule for the Discovery of Imaginary Roots." By J. J. Sylvester, F.R.S., Correspondent of the Institute of France. Received April 6, 1864.

(Abstract.)

In the 'Arithmetica Universalis,' in the chapter "De Resolutione Equationum," Newton has laid down a rule, admirable for its simplicity and generality, for the discovery of imaginary roots in algebraical equations, and

for assigning an inferior limit to their number. He has given no clue towards the ascertainment of the grounds upon which this rule is based, and has stated it in such terms as to leave it quite an open question whether or not he had obtained a demonstration of it. Maclaurin, Campbell, and others have made attempts at supplying a demonstration, but their efforts, so far as regards the more important part of the rule, that namely by which the limit to the number of imaginary roots is fixed, have completely failed in their object. Thus hitherto any opinion as to the truth of the rule rests on the purely empirical ground of its being found to lead to correct results in particular arithmetical instances. Persuaded of the insufficiency of such a mode of verification, the author has applied himself to obtaining a rigorous demonstration of the rule for equations of specified degrees. For the second degree no demonstration is necessary. For cubic equations a proof is found without difficulty. For biquadratic equations the author proceeds as follows. He supposes the equation to be expressed homogeneously in x, y, and then, instituting a series of infinitesimal linear transformations obtained by writing $x+hy$ for x, or $y+hx$ for y, where h is an infinitesimal quantity, shows that the truth of Newton's rule for this case depends on its being capable of being shown that the discriminant of the function $(1, \pm e, e^2, \pm e, 1)(x, y)^4$ is necessarily positive for all values of e greater than unity, which is easily proved. He then proceeds to consider the case of equations of the 5th degree, and, following a similar process, arrives at the conclusion that the truth of the rule depends on its being capable of being shown that the discriminant, say (D) of the function $(1, \epsilon, \epsilon^2, \eta^2, \eta, 1)(x, y)^5$, which for facility of reference may be termed "the (ϵ, η) function," is necessarily positive when $\epsilon^4-\epsilon\eta^2$ and $\eta^4-\eta\epsilon^2$ are both positive. This discriminant is of the 12th degree in ϵ, η. But on writing $x=\epsilon\eta$, $y=\epsilon^5+\eta^5$, it becomes a rational integral function of the 6th degree in x, and of the second degree in y, and such that, on making $D=0$, the equation represents a sextic curve, of which x, y are the abscissa and ordinate, which will consist of a single close. It is then easily demonstrated that all values of ϵ, η which cause the variable point x, y to lie inside this curve, will cause D to be negative (in which case the function ϵ, η has only two imaginary factors), and that such values as cause the variable point to lie outside the curve, will make D positive, in which case the ϵ, η function has four imaginary factors. When the conditions concerning ϵ, η above stated are verified, it is proved that the variable point must be exterior to the curve, and thus the theorem is demonstrated for equations of the 5th degree.

The question here naturally arises as to the significance of the sign of D when such a position is assigned to the variable point as gives rise to *imaginary* values of ϵ, η, which in such case will be conjugate quantities of the form $\lambda+i\mu$, $\lambda-i\mu$ respectively.

The curve D will be divided by another sextic curve into two portions, for one of which the couple ϵ, η corresponding to any point in its interior is

real, and for the other *conjugate*. This brings to view the necessity of there being in general a theory for equations with conjugate coefficients, which for greater brevity may be termed conjugate equations, analogous to that for real equations in respect of the distinction between real and imaginary roots in the latter. A conjugate equation is one in which the coefficients, reckoning from the two ends of the equation, go in pairs of the form $p \pm iq$, with the obvious condition that when there is a middle coefficient this must be real. Such an equation may be supposed to be so prepared that, when thrown into the form $P+iQ$, P and Q shall have no common algebraical factor; and when this is effected, it may easily be shown that the conjugate equation can neither have real roots nor roots paired together of the form $\lambda+i\mu$, $\lambda-i\mu$ respectively. How, then, it may be asked, is the analogy previously referred to possible? On investigation it will be found that the roots divide themselves into two categories, each of exactly the same order of generality,—viz. *solitary* roots of the form $e^{i\theta}$, and *associated* roots which go in pairs, the two roots of each pair being of the form $\rho e^{i\theta}$, $\frac{1}{\rho} e^{i\theta}$ respectively; so that, following the ordinary mode of geometrical representation of imaginary quantities, the roots of a conjugate equation may be denoted by points lying on the circumference of a circle to radius unity (corresponding to solitary roots), and points (corresponding to the associated roots) lying in couples on different radii of the circle at reciprocal distances from the centre, each couple in fact constituting, according to Prof. W. Thomson's definition, electrical images of each other in respect to the circle. If the circle be taken with radius infinity instead of unity (so as to become a straight line), then we have the geometrical *eidolon* of the roots of an ordinary equation, the solitary roots lying on a straight line, and the associated or paired (imaginary) roots on each side of, and at equal distances from the line.

In the inquiry before us, whether the variable point belong to the real or conjugate part of the plane of the D curve, it is shown to remain true that the number of *associated* roots will be two, if it lie inside the curve, and four if it lie outside. The author then suggests a probable extension of Newton's rule to conjugate equations of any degree. In conclusion, he deals with a question in close connexion with, and arising out of the investigation of this rule, relating to equations of the form $\Sigma \pm (ax+b)^m = 0$, to which, for convenience, he gives the provisional name of "superlinear equations" (denoting the function equated to zero as a superlinear form), and establishes a rule for limiting the number of real roots which they can contain, which is, that if such equation be thrown under the form

$$\lambda_1 (x+c_1)^m + \lambda_2 (x+c_2)^m + \ldots + \lambda_n (x+c_n)^m = 0,$$

and $c_1, c_2, \ldots c_n$ be an ascending or descending order of magnitudes, the equation cannot have more real roots than there were variations of sign in the sequence $\lambda_1, \lambda_2, \ldots, \lambda_n, (-)^m \lambda_1$.

This theorem was published by the author, but without proof, in the 'Comptes Rendus' for the month of March in this year.

But the method of demonstration now supplied is deserving of particular attention in itself; for it brings to light a new order of purely tactical considerations, and establishes a previously unsuspected kind of, so to say, algebraical polarity. The proof essentially depends upon the character of every superlinear form being associated with, and capable of definition by means of a pencil of rays, which may be called the type pencil, subject to a species of circulation of a different nature according as the degree of the form is even or odd, which he describes by the terms "per-rotatory" in the one case, and "trans-rotatory" in the other; so that the types themselves may be conveniently distinguished by the names "per-rotatory" and "trans-rotatory." By per-rotatory circulation is to be understood that species in which, commencing with any element of the type, passage is made from it to the next, from that to the one following, from the last but one to the last, from the last to the first, and so on, until the final passage is to the element commenced with from the one immediately preceding. By trans-rotatory circulation, on the other hand, is understood that species in which, commencing with any element and proceeding in the same manner as before to the end element, passage is made from that, not to the end element itself, but to its polar opposite, from that to the polar opposite of the next, and so on, until the final passage is made to the polar opposite of the element commenced with, from the polar opposite of its immediate antecedent. The number of changes of sign in effecting such passages, whether in a per-rotatory or a trans-rotatory type, is independent of the place of the element with which the circulation is made to commence, and may be termed the variation-index of the type, which is always an even number for per-rotatory, and an odd number for trans-rotatory types. A theorem is given whereby a relation is established between the variation-index of a per-rotatory or trans-rotatory and that of a certain trans-rotatory or per-rotatory type capable of being derived from them respectively; and this purely tactical theorem, combined with the algebraical one, that the form $f(x, y)$ cannot have fewer imaginary factors than any linear combination of $\frac{df}{dx}, \frac{df}{dy}$, leads by successive steps of induction to the theorem in question, but under a more general form, which serves to show intuitively that the limit to the number of real roots of a superlinear equation which the theorem furnishes must be independent of any homographic transformation operated upon the form. The author believes that, whilst it is highly desirable that a simple and general method should be discovered for the proof of Newton's rule as applicable to equations of any degree, and that the strenuous efforts of the cultivators of the New Algebra should be directed to the attainment of this object, his labours in establishing a proof applicable as far as equations of the 5th degree inclusive will not

have been unproductive of good, as well on account of the confirmation they afford of the truth of the rule, towards the establishment of which on scientific grounds they constitute the first serious step yet made, as also, and still more, by reason of the accessions to the existing field of algebraical speculation to which they have incidentally led.

III. "Description of a train of Eleven Sulphide-of-Carbon Prisms arranged for Spectrum Analysis." By J. P. Gassiot, F.R.S. Received March 17, 1864.

The principles which should regulate the construction of a battery of prisms have been alluded to in the description of the large spectroscope now at Kew Observatory, which has a train of nine dense glass prisms with refracting angles of 45°*.

While for purposes of exactitude, such as mapping out the solar spectrum, flint glass stands unrivalled; yet when the greatest amount of dispersion is the desideratum, prisms filled with bisulphide of carbon present obvious advantages, on account of the enormous dispersive power of that liquid—the difference of its indices of refraction for extreme rays being, according to Sir David Brewster, as 0·077 against 0·026 for flint glass.

In the fluid prisms of the ordinary construction, the sides are cemented on with a mixture of glue and honey. This cement, on hardening, warps the sides, and confusion of the spectral lines is the consequent result. To obviate this source of error, it has been proposed to attach an additional pair of parallel sides to such prisms, a thin film of castor-oil being interposed between the surfaces. The outer plates are then secured by means of sealing-wax, or some cement, at the corners. In the battery of prisms now about to be described, Mr. Browning has dispensed with this attachment at the corners, which is likely to prove prejudicial, and has secured the second sides in their proper position by extremely light metal frames which clasp the plates only on their edges.

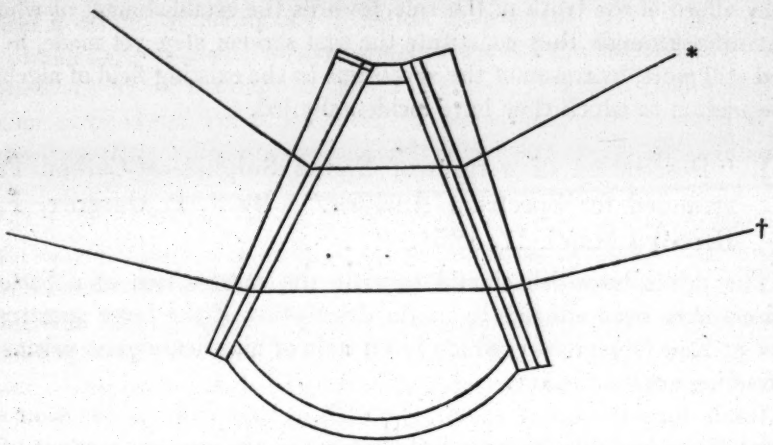
Thus arranged, the frames exert no pressure on the surfaces of the plates, and are quite out of the field of view, and they can be handled without any fear of derangement.

On account of the lower refractive power of bisulphide of carbon, as compared with flint glass, a refractive angle of 50° was given to the fluid prisms. Eight such prisms would cause a ray of light to travel more than a circle, and would be the greatest number that could be employed had the ordinary arrangement been adopted.

In place, however, of giving to the fluid prisms two pairs of parallel sides, Mr. Browning, taking advantage of the difference between the refractive and dispersive properties of crown glass and bisulphide of carbon, has substituted a prism of crown glass having a refracting angle of 6° for one of the outer plates of each prism—the base of this crown-glass prism being brought to correspond with the apex of the fluid prism, thus :—

* Proceedings, vol. xii. p. 536.

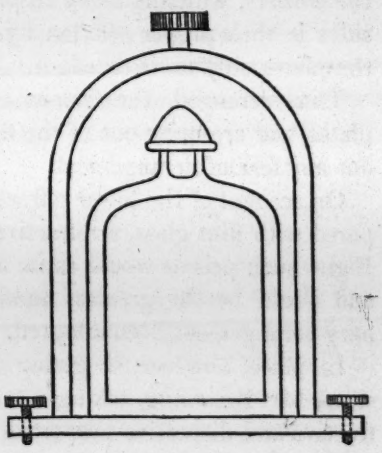
Crown-glass prism.



By this means the angle of minimum deviation of the prisms is so much decreased, that eleven of them thus constructed can be used in a circle instead of eight. An increase of dispersive power, due to refracting angles of 150° of the bisulphide of carbon, is thus gained, minus only the small amount of dispersion counteracted owing to the dispersive power of the crown-glass prisms being employed in the contrary direction.

From the well-known low dispersive power of this medium, however, this loss is inconsiderable, amounting to scarcely more than a fifteenth of the power gained. Owing to the minimum angle of deviation being lowered, the further advantage is also secured of a larger field of view being presented to the telescope by the first and last prism of the train.

Each prism, in addition to the light metal frame referred to, has a separate stand, furnished with screws for adjusting the prisms, and securing them at the angle of minimum deviation for any particular ray. The prism stands within a stirrup furnished with a welled head. By this arrangement the prisms can be removed and replaced without touching their sides —a matter of some importance, as all fluid prisms show different results with every change of temperature.



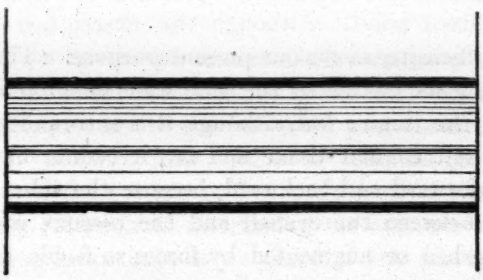
For the sake of simplicity, the metal framing of the prisms, and the various adjusting-screws, have been omitted in the last sketch.

The very unfavourable state of the weather prevented any observations

* Direction of ray as it would pass through two pair of parallel sides.
† Direction of ray as altered by interposing the crown-glass prism.

being made on the solar spectrum with these prisms until Saturday the 12th inst. The results then obtained may probably not be considered devoid of interest. They are as follows:—

The prisms were arranged so as to enable that portion of the spectrum to be observed in which the well-defined D line of Fraunhofer is situated. This line, long since resolved as double, presented an angular separation of 3′ 6″, measured from the centre of one to that of the other principal line, this measurement being made by Mr. Balfour Stewart by means of the micrometer attached to the telescope; the value of the divisions of the micrometer he had previously determined relatively to the divided circle of the spectroscope. A centre line (clearly defined and figured in Kirchhoff and Bunsen's map) was distinctly visible, and nearly equidistant from the centre towards the violet; five clearly defined lines were perceptible, as also two faint lines on each side of the principal lines, between the centre line of Kirchhoff towards the red. Several faint lines were also perceptible.



The lines as represented in the diagram were drawn by Mr. Whipple, one of the assistants in the Observatory, as they were observed by him about 3.45 P.M Some of these may possibly be due to the earth's atmosphere, but the five most refrangible lines were observed at an earlier period of the day by Mr. Stewart, Mr. Browning, and myself.

The great angular separation of the double D line to 3′ 6″ is a proof of the power of this arrangement of the sulphide-of-carbon prisms, and offers the means of mapping out the entire solar spectrum on a scale not hitherto attained.

Received April 6, 1864.

Note.—Since the preceding observations were recorded, an inspection has been made of the region of the spectrum towards the refrangible side of double D; and, from the comparisons made with a map of lines obtained by means of the battery of glass prisms with that given by those of the sulphide-of-carbon prisms, many new lines are produced in addition to those observable by the former, while the battery of glass prisms itself gives a number of additional lines to those that are depicted in Kirchhoff's map.

April 14, 1864.

Major-General SABINE, President, in the Chair.

The CROONIAN LECTURE was delivered by Prof. HERMANN HELMHOLTZ, For. Memb. R.S., "On the Normal Motions of the Human Eye in relation to Binocular Vision."

The Motions of the Human Eye are of considerable interest, as well for the physiology of voluntary muscular motion in general, as for the physiology of vision. Therefore I may be allowed to bring before this Society the results of some investigations relating to them, which I have made myself; and I may venture perhaps to hope that they are such as to interest not only physiologists and medical men, but every scientific man who desires to understand the mechanism of the perceptions of our senses.

The eyeball may be considered as a sphere, which can be turned round its centre as a fixed point. Although this description is not absolutely accurate, it is sufficiently so for our present purpose. The eyeball, indeed, is not fixed during its motion by the solid walls of an articular excavation, like the bone of the thigh; but, although it is surrounded at its posterior surface only by soft cellular tissue and fat, it cannot be moved in a perceptible degree forward and backward, because the volume of the cellular tissue, included between the eyeball and the osseous walls of the orbit, cannot be diminished or augmented by forces so feeble as the muscles of the eye are able to exert.

In the interior of the orbit, around the eyeball six muscles are situated, which can be employed to turn the eye round its centre. Four of them, the so-called *recti* muscles, are fastened at the hindmost point of the orbit, and go forward to fix themselves to the front part of the eyeball, passing over its widest circumference—or its equator, as we may call it, if we consider the foremost and the hindmost points of the eyeball as its poles. These four *recti* muscles are from their position severally named superior, inferior, internal, and external. Besides these, there are two *oblique* muscles, the ends of which come from the anterior margin of the orbit on the side next the nose, and, passing outwards, are attached at that side of the eyeball which is towards the temple—one of them, the superior oblique muscle, being stretched over the upper side of the eyeball, the other, or inferior, going along its under side.

These six muscles can be combined as three pairs of antagonists. The internal and external *recti* turn the eye round a perpendicular axis, so that its visual line is directed either to the right side or to the left. The superior and inferior *recti* turn it round a horizontal axis, directed from the upper end of the nose to the temple; so that the superior *rectus* elevates the visual line, the inferior depresses it. Lastly, the *oblique* muscles turn the eye round an axis which is directed from its centre to the occiput, so

that the superior oblique muscle lowers the visual line, and the inferior raises it; but these last two muscles not only raise and lower the visual line; they produce also a rotation of the eye round the visual line itself, of which we shall have to speak more afterwards.

A solid body, the centre of which is fixed, and which can be turned round three different axes of rotation, can be brought into every possible position consistent with the immobility of its centre. Look, for instance, at the motions of our arm, which are provided for at the shoulder-joint by the gliding of the very accurately spherical upper extremity of the humerus in the corresponding excavation of the scapula. When we stretch out the arm horizontally, we can turn it, first, round a perpendicular axis, moving it forwards and backwards; we can turn it, secondly, round a horizontal axis, raising it and lowering it; and lastly, after having brought it by such motions into any direction we like, we can turn it round its own longitudinal axis, which goes from the shoulder to the hand; so that even when the place of the hand in space is fixed, there are still certain different positions in which the arm can be turned.

Now let us see how far the motions of the eye can be compared to those of our arm. We can raise and lower the visual line, we can turn it to the left and to the right, we can bring it into every possible direction, throughout a certain range—as far, at least, as the connexions of the eyeball permit. So far the motions of the eye are as free as those of the arm. But when we have chosen any determinate direction of the eye, can we turn the eye round the visual line as an axis, as we can turn the arm round its longitudinal axis?

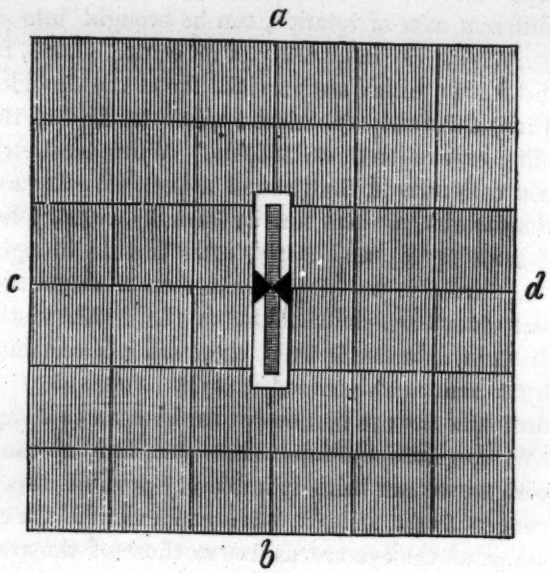
This is a question the answer to which is connected with a curious peculiarity of our voluntary motions. In a purely mechanical sense, we must answer this question in the affirmative. Yes, there exist muscles by the action of which those rotations round the visual line can be performed. But when we ask, "Can we do it by an act of our will?" we must answer, "No." We can voluntarily turn the visual line into every possible direction, but we cannot voluntarily use the muscles of our eye in such a way as to turn it round the visual line. Whenever the direction of the visual line is fixed, the position of our eye, as far as it depends upon our will, is completely fixed and cannot be altered.

This law was first satisfactorily proved by Professor Donders, of Utrecht, who, in a very ingenious way, controlled the position of the eye by those ocular spectra which remain in the field of vision after the eye had been fixed steadily during some time upon any brightly coloured object. I have used for this purpose a diagram like fig. 1: the ground is grey paper, and in the middle, along the line *a b*, is placed a narrow strip of red paper on a broader strip of green paper*. The centre of the red strip is marked by two black points. When you look for about a minute steadily and without moving your eye at the centre of the diagram, the image of the

* *Green* is represented in the figure by white; *red* by the central dark stripe.

coloured strips is projected on the nervous membrane of your eye; those parts of this membrane on which the light falls are irritated, and in consequence of this irritation, their irritability is exhausted, they are fatigued

Fig. 1.



and they become less sensitive to that kind of light by which they were excited before. When you cease, therefore, to look at the coloured strips, and turn your eye either to the grey ground of the diagram, or to any other part of the field of vision which is of a uniform feeble degree of illumination, you will see a spectrum of the coloured strips, exhibiting the same apparent magnitude but with colours reversed, a narrow green strip being in the middle of a broader red one. The cause of this appearance is, that those parts of your retina which were excited formerly by green light are less affected by the green rays contained in white or whitish light than by rays of the complementary colour, and white light, therefore, appears to them reddish; to those parts of the nervous membrane, on the other hand, which had been fatigued by red light, white light afterwards appears to be greenish. The nervous membrane of the eye in these cases behaves nearly like the sensitive stratum in a photographic apparatus, which is altered by light during the exposure in such a way that it is impressed differently afterwards by various agents; and the impression of light on the retina may be, perhaps, of the same essential nature as the impression made upon a photographic plate. But the impression made on the living eye does not last so long as that on sensitive compounds of silver; it vanishes very soon if the light be not too strong. Light of great intensity, like that of the sun when directly looked at, can develope very dark ocular spectra, which last a quarter of an hour, or even longer, and disturb the

perception of external objects very much, as is well known. One must be very careful to avoid the use of too strong a light in these experiments, because the nervous apparatus of the eye is easily injured by it; and the brightness of these coloured strips when illuminated by common daylight is quite sufficient for our present purpose.

Now you will perceive easily that these ocular spectra are extremely well adapted to ascertain the position of the eye-ball, because they have a fixed connexion with certain parts of the retina itself. If the eyeball could turn about its visual line as an axis, the ocular spectrum would apparently undergo the same degree of rotation; and hence, when we move about the eye, and at last return to the same direction of the visual line, we can recognize easily and accurately whether the eye has returned into the same position as before, or whether the degree of its rotation round the visual line has been altered. Professor Donders has proved, by using this very delicate test, that *the human eye, in its normal state, returns always into the same position when the visual line is brought into the same direction.* The position and direction of the eye are to be determined in this case in reference to the head of the observer; and I beg you to understand always, when I say that the eye or its visual line is moved upwards or downwards, that it is moved either in the direction of the forehead or in that of the cheek; and when I say it is moved to the left or to the right, you are to understand the left or right side of the head. Therefore, when the head itself is not in its common vertical position, the vertical line here understood is not accordant with the line of the plummet.

Before the researches of Donders, some observers believed they had found a difference in the relative positions of the eye, when the head was brought into different situations. They had used either small brown spots of the iris, or red vessels in the white of the eye, to ascertain the real position of the eyeball; but their apparent results have been shown to be erroneous by the much more trustworthy method of Donders.

In the first place, therefore, we may state that the position of the eyeball depends exclusively upon the direction of the visual line in reference to the position of the head of the observer. But now we must ask, what is the law regulating the position of the eye for every direction of its visual line? In order to define this law, we must first notice that there exists a certain direction of the visual line, which, in relation to the motions of the eye, is distinguished from all other directions of the eye; and we may call it the *central* or *primary direction of the visual line.* This direction is parallel to the median vertical plane of the head; and it is horizontal when the head of the observer, who is standing, is kept in a convenient erect position to look at distant points of the horizon. How this primary direction of the visual line may be determined practically with greater accuracy we shall see afterwards. All other directions of the visual line we may call *secondary directions.*

A plane which passes through the visual line of the eye, I call *a meri-*

dian plane of the eye. Such a plane cuts through the retina in a certain line; and when the eye has been moved, we consider as the same meridian plane that plane which passes through the new direction of the visual line and the same points of the retina as before.

After having given these definitions, we may express the law of the motions of the eye in the following way:—

Whenever the eye is brought into a secondary position, that meridian plane of the eye which goes through the primary direction of the visual line has the same position as it has in the primary direction of the eye.

It follows from this law that the secondary position of the eye may be found also by turning the eye from its primary position round a fixed axis which is normal as well to the primary as to the secondary of the visual line.

[The geometrical relations of these different positions were explained by the lecturer by means of a moveable globe placed on an axis like the common terrestrial globes.]

It would take too long to explain the different ways in which different observers have tried to determine the law of the motions of the eyeball. They have employed complicated apparatus for determining the angles by which the direction and the rotation of the eye were to be measured. But usually two difficulties arise from the use of such instruments containing graduated circles, in the centre of which the eye must be kept steady. In the first place, it is very difficult to fix the head of the observer so firmly that he cannot alter its position during a continuous series of observations, and that he reassumes exactly the same position of the head when he returns to his measurements after a pause,—conditions which must necessarily be fulfilled if the observations are to agree with each other. Secondly, I have found that the eye must not be kept too long a time in a direction which is near to the limits of the field of vision; else its muscles are fatigued, and the positions of the eyeball corresponding to different directions of the visual line are somewhat altered. But if we have to measure angles on graduated circles, it is difficult to avoid keeping the eye too long in directions deviating far from the primary direction.

I think that it depended upon these causes, that the observations carried out by Meissner, Fick, and Wundt agreed very ill with each other and with the law which I have explained above, and which was first stated by Professor Listing of Göttingen, but without any experimental proof. Happily it is possible, as I found out, to prove the validity of this law by a very simple method, which is not subject to those sources of error I have named, and which I may be allowed to explain briefly.

In order to steady the attitude of the head in reference to the direction of the visual line, I have taken a little wooden board, one end of which is hollowed into a curve fitting the arch of the human teeth; the margin of this hollow is covered with sealing-wax, into which, after it had been softened by heat and had been cooled again sufficiently, I inserted both

series of my teeth, so that I kept it firmly between my jaws. The impressions of the teeth remain indented in the sealing-wax; and when I put my teeth afterwards into these impressions, I am sure that the little board is brought exactly into the same position, relatively to my head and my eyes, as it was before. On the other end of that little board, which is kept horizontally between the teeth, a vertical piece of wood is fastened, on which I fix horizontally a little strip of card pointed at each end, so that these two points are situated about five inches before my eyes, one before the right eye, the other before the left. The length of the strip of card must be equal to the distance between the centres of the eyes, which is 68 millimetres for my own eyes. Looking now with the right eye in the direction of the right point of that strip, and with the left eye in the direction of the left point, I am sure to bring the eyes always into the same position relatively to my head, so long as the position of the strip of card on the wooden piece remains unaltered.

As a field of vision I use either a wall covered with a grey paper, in the pattern of which horizontal and vertical lines can be easily perceived, or a drawing-board covered with grey drawing-paper, on which a system of horizontal and vertical lines is drawn, as in fig. 1, and coloured stripes are fastened along the line *ab*.

Now the observer at first must endeavour to find out that position of his eyes which we call the primary position. In order to do this, the observer takes the wooden piece between his teeth, and brings his head into such a position that his right eye looks to the centre of the coloured stripes, in a direction perpendicular to the plane of the drawing. Then he brings his head into such an attitude that the right end of the card-strip appears in the same direction as the centre of the coloured stripe. After having steadily looked for some time to the middle of the coloured stripe, he turns away his gaze to the end of either the vertical or horizontal lines, *ab*, *cd*, which are drawn through the centre of the coloured stripe. There he will see an ocular spectrum of the coloured stripe, and will observe if it coincides with the horizontal lines of the drawing. If not, he must alter the position of the strip of card on the wooden bar to which it is fastened, till he finds that the ocular spectrum of the coloured stripe remains horizontal when any point either of the line *ab* or *cd* is looked at. When he has thus found the primary direction of his visual line for the right eye, he does the same for the left.

The ocular spectra soon vanish, but they are easily renewed by looking again to the centre of the stripes. Care must be taken that the observer looks always in a direction perpendicular to the plane of the drawing whenever he looks to the centre of the coloured stripe, and that he does not move his head. If he should have moved it, he would find it out immediately when he looks back to the strip, because the point of the card-strip would no longer cover the centre of the coloured stripe.

So you see that the primary direction of the visual line is completely

fixed, and that the eye, which wants only to glance for an instant at a peripheral point of the drawing, and then goes back again to the centre, is not fatigued.

This method of finding the primary position of the eye proves at the same time that vertical and horizontal lines keep their vertical or horizontal position in the field of vision when the eye is moved from its primary direction vertically or horizontally; and you see, therefore, that these movements agree with the law which I have enunciated. That is to say, during vertical movements of the eye the vertical meridian plane keeps its vertical position, and during horizontal movements the horizontal meridian.

Now you need only bring either your own head into an inclined position, or the diagram with the lines, and repeat the experiment, putting your head at first into such a position that the centre of the diagram corresponds with the primary direction of the visual line, and moving afterwards the eye along the lines *a b* or *c d*, in either a parallel or perpendicular direction to the coloured line of the diagram, and you will find the ocular spectrum of the coloured line coinciding with those black lines which are parallel with *a b*. In this way, therefore, you can easily prove the law of Listing for every possible direction of the visual line.

I found the results of these experiments in complete agreement with the law of Listing for my own eyes, and for those of several other persons with normal power of vision. The eyes of very short-sighted persons, on the contrary, often show irregularities, which may be caused by the elongation of the posterior part of those eyes.

These motions of our eyes are a peculiar instance of motions which, being quite voluntary, and produced by the action of our will, are nevertheless limited as regards their extent and their combinations. We find similar limitations of motion of the eyes in other cases also. We cannot turn one eye up, the other down; we cannot move both eyes at the same time to the outer angle; we are obliged to combine always a certain degree of accommodation of the eyes to distance, with a certain angle of convergence of their axes. In these latter cases it can be proved that the faculty of producing these motions is given to our will, although our will is commonly not capable of using this faculty. We have come by experience to move our eyes with great dexterity and readiness, so that we see any visible object at the same time single and as accurately as possible; this is the only end which we have learnt to reach by muscular exertion; but we have not learnt to bring our eyes into any given position. In order to move them to the right, we must look to an object situated on our right side, or imagine such an object and search for it with our eyes. We can move them both inwards, but only when we strive to look at the back of our nose, or at an imaginary object situated near that place. But commonly there is no object which could be seen single by turning one eye upwards, the other downwards, or both of them out-

wards, and we are therefore unable to bring our eyes into such positions. But it is a well known fact, that when we look at stereoscopic pictures, and increase the distance of the pictures by degrees, our eyes follow the motion of the pictures, and that we are able to combine them into an apparently single object, although our eyes are obliged to turn into diverging directions. Professor Donders, as well as myself, has found that when we look to a distant object, and put before one of our eyes a prism of glass the refracting angle of which is between 3 and 6 degrees, and turn the prism at first into such a position before the eye that its angle looks to the nose and the visual lines converge, we are able to turn the prism slowly, so that its angle looks upwards or downwards, keeping all this time the object apparently single at which we look. But when we take away the prism, so that the eyes must return to their normal position before they can see the object single, we see the object double for a short time—one image higher than the other. The images approach after some seconds of time and unite at last into one.

By these experiments it is proved that we can move both eyes outward, or one up and the other down, when we use them under such conditions that such a position is required in order that we may see the objects single at which we are looking.

I have sometimes remarked that I saw double images of single objects, when I was sleepy and tried to keep myself awake. Of these images one was sometimes higher than the other, and sometimes they were crossed, one of them being rotated round the visual line. In this state of the brain, therefore, where our will begins to lose its power, and our muscles are left to more involuntary and mechanical impulses, an abnormal rotation of the eye round the visual line is possible. I infer also from this observation, that the rotation of the eye round the visual axis cannot be effected by our will, because we have not learnt by which exertion of our will we are to effect it, and that the inability does not depend on any anatomical structure either of our nerves or of our muscles which limits the combination of motion. We should expect, on the contrary, that, if such an anatomical mechanism existed, it should come out more distinctly when the will has lost its power.

We may ask, therefore, if this peculiar manner of moving the eyes, which is determined by the law of Listing, is produced by practical exercise on account of its affording any advantages to visual perceptions. And I believe that certain advantages are indeed connected with it.

We cannot rotate our eyes in the head, but we can rotate the head with the eyes. When we perform such a motion, looking steadily to the same point, we remark that the visible objects turn apparently a little round the fixed point, and we lose by such a motion of our eye the perception of the steadiness of the objects at which we look. Every position of the visual line is connected with a determined and constant degree of rotation, accord-

ing to the law of Donders; and in altering this rotation we should judge the position of external objects wrongly.

The same will take place when we change the direction of the visual line. Suppose the amplitude of such motions to be infinitely small; then we may consider this part of the field of vision, and the corresponding part of the retina on which it is projected, as plane surfaces. If during any motion of the eye the optic image is displaced so that in its new position it remains parallel to its former position on the retina, we shall have no apparent motions of the objects. When, on the contrary, the optic image of the visible objects is dislocated so that it is not parallel to its former position on the retina, we must expect to perceive an apparent rotation of the objects.

As long as the motions of the eye describe infinitely small angles, the eye can be moved in such a way that the optic image remains always parallel to its first position. For this end the eye must be turned round axes of rotation which are perpendicular to the visual line; and we see indeed that this is done, according to the law of Listing, when the eye is moving near its primary position. But it is not possible to fulfil this condition completely when the eye is moved through a wider area which comprises a larger part of the spherical field of view. For if we were to turn the eye always round an axis perpendicular to the visual line, it would come into very different positions after having been turned through different ways to the same final direction.

The fault, therefore, which we should strive to avoid in the motions of our eye, cannot be completely avoided, but it can be made as small as possible for the whole field of vision.

The problem, to *find such a law for the motions of the eye that the sum of all the rotations round the visual line for all possible infinitely small motions of the eye throughout the whole field of vision becomes a minimum*, is a problem to be solved by the calculus of variations. I have found that the solution for a circular field of vision, which corresponds nearly to the forms of the actual field of vision, gives indeed the law of Listing.

I conclude from these researches, that the actual mode of moving the eye is that mode by which the perception of the steadiness of the objects through the whole field of vision can be kept up the best; and I suppose, therefore, that this mode of motion is produced by experience and exercise, because it is the best suited for accurate perception of the position of external objects.

But in this mode of moving, rotations round the visual line are not completely avoided when the eye is moved in a circular direction round the primary position of the visual line; and it is easy to recognize that in such a case we are subject to optical illusions.

Turn your eyes to a horizontal line situated in the highest part of the field of vision, and let them follow this line from one end to the other.

The line will appear like a curved line, the convexity of which looks downward. When you look to its right extremity, it seems to rise from the left to the right; when you look to the left extremity of the line, the left end seems to rise. In the same way, all straight lines which go through the peripheral parts of the field of vision appear to be curved, and to change their position a little, if you look to their upper or their lower ends.

This explanation relates only to Monocular vision; we have to inquire also how it influences Binocular vision.

Each eye has its field of vision, on which the visible objects appear distributed like the objects of a picture, and the two fields with their images seem to be superimposed. Those points of both fields of view which appear to be superimposed are called *corresponding* (or *identical*) points. If we look at real objects, the accurate perception of the superimposition of two different optical images is hindered by the perception of stereoscopic form and depth; and we unite indeed, as Mr. Wheatstone has shown, two retinal images completely into the perception of one single body, without being able to perceive the duplicity of the images, even if there are very sensible differences of their form and dimensions. To avoid this, and to find those points of both fields of view which correspond with each other, it is necessary to use figures which cannot easily be united into one stereoscopic projection.

In fig. 2 you see such figures, the right of which is drawn with white lines on a black ground, the left with black lines on a white ground. The horizontal lines of both figures are parts of the same straight lines; the vertical lines are not perfectly vertical. The upper end of those of the right figure is inclined to the right, that of the left figure to the left, by about $1\frac{1}{4}$ degree.

Now I beg you to look alternately with the right and with the left eye at these figures. You will find that the angles of the right figure appear to the right eye equal to right angles, and those of the left figure so appear to the left eye; but the angles of the left figure appear to the right eye to deviate much from a right angle, as also do those of the right figure to the left eye.

When you draw on paper a horizontal line, and another line crossing it exactly at right angles, the right superior angle will appear to your right eye too great, to your left eye too small; the other angles show corresponding deviations. To have an apparently right angle, you must make the vertical line incline by an angle of about $1\frac{1}{4}$ degree for it to appear really vertical; and we must distinguish, therefore, the *really vertical* lines and the *apparently* vertical lines in our field of view.

There are several other illusions of the same kind, which I omit because they alter the images of both eyes in the same manner and have no influence upon binocular vision; for example, vertical lines appear always of greater length than horizontal lines having really the same length.

Now combine the two sides of fig. 2 into a stereoscopic combination, either by squinting, or with the help of a stereoscope, and you will see that the white lines of the one coincide exactly with the black lines of the other, as soon as the centres of both the figures coincide, although the vertical lines of the two figures are not parallel to each other.

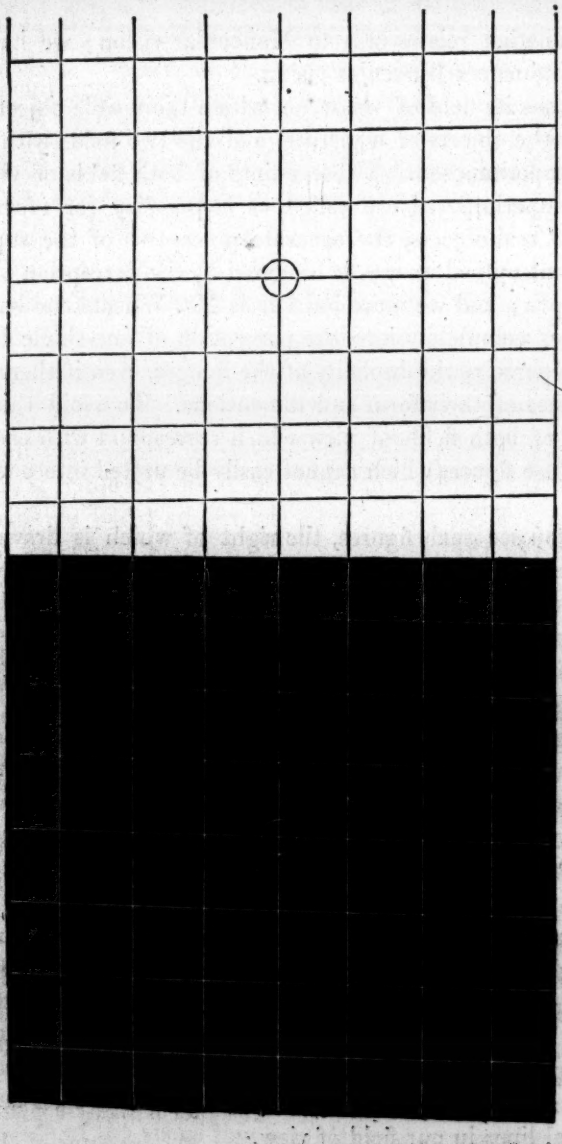


Fig. 2.

Therefore not the *really vertical* meridians of both fields of view correspond, as has been supposed hitherto, but the *apparently* vertical meridians. On the contrary, the horizontal meridians really correspond, at least for normal eyes which are not fatigued. After having kept the eyes a long

time looking down at a near object, as in reading or writing, I found sometimes that the horizontal lines of fig. 2 crossed each other; but they became parallel again when I had looked for some time at distant objects.

In order to define the position of the corresponding points in both fields of vision, let us suppose the observer looking to the centres of the two sides of fig. 2, and uniting both pictures stereoscopically. Then planes may be laid through the horizontal and vertical lines of each picture and the centre of the corresponding eye. The planes laid through the different horizontal lines will include angles between them, which we may call *angles of altitude*; and we may consider as their zero the plane going through the fixed point and the horizontal meridian. The planes going through the vertical lines include other angles, which may be called *angles of longitude*, their zero coinciding also with the fixed point and with the *apparently vertical* meridian. Then the stereoscopic combination of those diagrams shows that *those points correspond which have the same angles of altitude and the same angles of longitude*; and we can use this result of the experiment as a definition of corresponding points.

We are accustomed to call *Horopter* the aggregate of all those points of the space which are projected on corresponding points of the retinæ. After having settled how to define the position of corresponding points, the question, what is the form and situation of the Horopter, is only a geometrical question. With reference to the results I had obtained in regard to the positions of the eye belonging to different directions of the visual lines, I have calculated the form of the Horopter, and found that generally the Horopter is a line of double curvature produced by the intersection of two hyperboloids, and that in some exceptional cases this line of double curvature can be changed into a combination of two plane curves.

That is to say, when the point of convergence is situated in the middle plane of the head, the Horopter is composed of a straight line drawn through the point of convergence, and of a conic section going through the centre of both eyes and intersecting the straight line.

When the point of convergence is situated in the plane which contains the primary directions of both the visual lines, the Horopter is a circle going through that point and through the centres of both eyes and a straight line intersecting the circle.

When the point of convergence is situated as well in the middle plane of the head as in the plane of the primary directions of the visual lines, the Horopter is composed of the circle I have just described, and a straight line going through that point.

There is only one case in which the Horopter is really a plane, as it was supposed to be in every instance by Aguilonius, the inventor of that name,—namely, when the point of convergence is situated in the middle plane of the head and at an infinite distance. Then the Horopter is a plane parallel to the visual lines, and situated beneath them, at a certain distance which depends upon the angle between the *really* and *apparently* vertical

meridians, and which is nearly as great as the distance of the feet of the observer from his eyes when he is standing. Therefore, when we look straight forward to a point of the horizon, the Horopter is a horizontal plane going through our feet—it is the ground upon which we are standing.

Formerly physiologists believed that the Horopter was an infinitely distant plane when we looked to an infinitely distant point. The difference of our present conclusion is consequent upon the difference between the position of the *really* and *apparently* vertical meridians, which they did not know.

When we look, not to an infinitely distant horizon, but to any point of the ground upon which we stand which is equally distant from both our eyes, the Horopter is not a plane; but the straight line which is one of its parts coincides completely with the horizontal plane upon which we are standing.

The form and situation of the Horopter is of great practical importance for the accuracy of our visual perceptions, as I have found.

Take a straight wire—a knitting-needle for instance—and bend it a little in its middle, so that its two halves form an angle of about four degrees. Hold this wire with outstretched arm in a nearly perpendicular position before you, so that both its halves are situated in the middle plane of your head, and the wire appears to both your eyes nearly as a straight line. In this position of the wire you can distinguish whether the angle of the wire is turned towards your face or away from it, by binocular vision only, as in stereoscopic diagrams; and you will find that there is one direction of the wire in which it coincides with the straight line of the Horopter, where the inflexion of the wire is more evident than in other positions. You can test if the wire really coincides with the Horopter, when you look at a point a little more or a little less distant than the wire. Then the wire appears in double images, which are parallel when it is situated in the Horopter line, and are not when the point is not so situated.

Stick three long straight pins into two little wooden boards which can slide one along the other; two pins may be fastened in one of the boards, the third pin in the second. Bring the boards into such a position that the pins are all perpendicular and parallel to each other, and situated nearly in the same plane. Hold them before your eyes and look at them, and strive to recognize if they are really in the same plane, or if their series is bent towards you or from you. You will find that you distinguish this by binocular vision with the greatest degree of certainty and accuracy (and indeed with an astonishing degree of accuracy) when the line of the three pins coincides with the direction of the circle which is a part of the Horopter.

From these observations it follows that the forms and the distances of those objects which are situated in, or very nearly in, the Horopter, are perceived with a greater degree of accuracy than the same forms and distances would be when not situated in the Horopter. If we apply this

result to those cases in which the ground whereon we stand is the plane of the Horopter, it follows that, looking straight forward to the horizon we can distinguish the inequalities and the distances of different parts of the ground better than other objects of the same kind and distance.

This is actually true. We can observe it very conspicuously when we look to a plain and open country with very distant hills, at first in the natural position, and afterwards with the head inclined or inverted, looking under the arm or between our legs, as painters sometimes do in order to distinguish the colours of the landscape better. Comparing the aspect of the distant parts of the ground, you will find that we perceive very well that they are level and stretched out into a great distance in the natural position of your head, but that they seem to ascend to the horizon and to be much shorter and narrower when we look at them with the head inverted: we get the same appearance also when our head remains in its natural position, and we look to the distant objects through two rectangular prisms, the hypothenuses of which are fastened on a horizontal piece of wood, and which show inverted images of the objects. But when we invert our head, and invert at the same time also the landscape by the prisms, we have again the natural view and the accurate perception of distances as in the natural position of our head, because then the apparent situation of the ground is again the plane of the Horopter of our eyes.

The alteration of colour in the distant parts of a landscape when viewed with inverted head, or in an inverted optical image, can be explained, I think, by the defective perception of distance. The alterations of the colour of really distant objects produced by the opacity of the air, are well known to us, and appear as a natural sign of distance; but if the same alterations are found on objects apparently less distant, the alteration of colour appears unusual, and is more easily perceived.

It is evident that this very accurate perception of the form and the distances of the ground, even when viewed indirectly, is a great advantage, because by means of this arrangement of our eyes we are able to look at distant objects, without turning our eyes to the ground, when we walk.

April 21, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read:—

I. "On the Orders and Genera of Quadratic Forms containing more than three Indeterminates." By H. T. Stephen Smith, M.A., F.R.S., Savilian Professor of Geometry in the University of Oxford. Received March 22, 1864.

Let us represent by f_1 a homogeneous form or quantic of any order containing n indeterminates; by $(\alpha^{(1)})$, a square matrix of order n; by

$(\alpha^{(i)})$, its ith derived matrix, *i. e.* the matrix of order $\frac{\lfloor n}{\lfloor i \lfloor n-i} = \mathrm{I}$, the constituents of which are the minor determinants of order i of the matrix $(\alpha^{(1)})$; and lastly, by f_i, a form of any order containing I indeterminates, the coefficients of which depend on the coefficients of f_1. When f_1 is transformed by $(\alpha^{(1)})$, let f_i be transformed by $(\alpha^{(i)})$; if, after division or multiplication by a power of the modulus of transformation, the metamorphic of f_i depends on the metamorphic of f_1, in the same way in which f_i depends on f_1, f_i is said to be a concomitant of the ith species of f_1. Thus: a concomitant of the 1st species is a covariant; a concomitant of the $(n-1)$th species is a contravariant; if $n=2$ there are only covariants; if $n=3$ there are only covariants and contravariants; but if $n>3$, there will exist in general concomitants of the intermediate species.

There is an obvious difference between covariants and contravariants on the one hand, and the intermediate concomitants on the other. The number of indeterminates in a covariant or contravariant is the same as in its primitive; in an intermediate concomitant, the number of indeterminates is always greater than in its primitive. Again, to every metamorphic of a covariant or contravariant, there corresponds a metamorphic of its primitive; whereas, in the case of a concomitant of the intermediate order i, a metamorphic of the primitive will correspond, not to every metamorphic of the concomitant, but only to such metamorphics as result from transformations the matrices of which are the ith derived matrices of matrices of order n.

It is also obvious that, besides the $n-1$ species of concomitance here defined, there are, when n is >3, an infinite number of other species of concomitance of the same general nature. For from any derived matrix we may form another derived matrix, and so on continually; and to every such process of derivation a distinct species of concomitance will correspond.

The notion of intermediate concomitance appears likely to be of use in many researches; in what follows, it is employed to obtain a definition of the ordinal and generic characters of quadratic forms containing more than 3 indeterminates. (The case of quadratic forms containing 3 indeterminates has been considered by Eisenstein in his memoir, "Neue Theoreme des höheren Arithmetik," Crelle, vol. xxxv. pp. 121 and 125.) Let

$$f_1 = \sum_{p=1}^{p=n} \sum_{q=1}^{q=n} \mathrm{A}^{(1)}_{p,q}\, x_p\, x_q$$

represent a quadratic form of n indeterminates; let $(\mathrm{A}^{(1)})$ be the symmetrical matrix of this form, and $(\mathrm{A}^{(i)})$ the ith derived matrix of $(\mathrm{A}^{(1)})$; $(\mathrm{A}^{(i)})$ will also be a symmetrical matrix, and the quadratic form

$$f_i = \sum_{p=1}^{p=\mathrm{I}} \sum_{q=1}^{q=\mathrm{I}} \mathrm{A}^{(i)}_{p,q}\, \mathrm{X}_p\, \mathrm{X}_q \quad \ldots\ldots\ldots \quad (\mathrm{A})$$

will be a concomitant of the ith species of f_1. It is immaterial what

principle of arrangement is adopted in writing the quadratic matrix $(A^{(6)})$, and the transforming matrix $(\alpha^{(6)})$; provided only that the arrangement be the same in the two matrices, and that in each matrix it be the same in height and in depth.

For example, if $f_1 = a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2 + a_4 x_4^2 + 2b_1 x_1 x_2 + 2b_2 x_1 x_3 + 2b_3 x_1 x_4 + 2b_4 x_2 x_3 + 2b_5 x_2 x_4 + 2b_6 x_3 x_4$ be a quadratic form containing four indeterminates, the form $f_2 =$

$$\begin{aligned}
&(b_1^2 - a_1 a_2)\, X_1^2 + (b_2^2 - a_1 a_3)\, X_2^2 + (b_3^2 - a_1 a_4)\, X_3^2 \\
&+ (b_4^2 - a_2 a_3)\, X_4^2 + (b_5^2 - a_2 a_4)\, X_5^2 + (b_6^2 - a_3 a_4)\, X_6^2 \\
&+ 2(b_1 b_2 - a_1 b_4)\, X_1 X_2 + 2(b_1 b_3 - a_1 b_5)\, X_2 X_3 \\
&- 2(b_1 b_4 - a_2 b_2)\, X_1 X_4 - 2(b_1 b_5 - a_2 b_3)\, X_1 X_5 \\
&- 2(b_2 b_5 - b_3 b_4)\, X_1 X_6 + 2(b_2 b_3 - a_1 b_6)\, X_2 X_3 \\
&+ 2(b_2 b_4 - a_3 b_1)\, X_2 X_4 - 2(b_1 b_6 - b_3 b_4)\, X_2 X_5 \\
&- 2(b_2 b_6 - a_3 b_3)\, X_2 X_6 - 2(b_1 b_6 - b_2 b_5)\, X_3 X_4 \\
&+ 2(b_3 b_5 - a_4 b_1)\, X_3 X_5 + 2(b_2 b_6 - a_4 b_2)\, X_3 X_6 \\
&+ 2(b_4 b_5 - a_2 b_6)\, X_4 X_5 - 2(b_4 b_6 - a_3 b_5)\, X_4 X_6 \\
&+ 2(b_5 b_6 - a_4 b_4)\, X_5 X_6
\end{aligned}$$

is the concomitant of the second species of f.

The $\overline{n-1}$ forms defined by the formula (A), of which the first is the form f_1 itself, and the last the contravariant of f_1, we shall term *the fundamental concomitants of* f_1; in contradistinction to those other quadratic concomitants (infinite in number) of which the matrices are the symmetrical matrices that may be derived, by a multiplicate derivation, from $(A^{(1)})$ . . . . Passing to the arithmetical theory of quadratic forms—*i. e.* supposing that the constituents of $(A^{(1)})$ are integral numbers, we shall designate by $\nabla_1, \nabla_2, \ldots \nabla_n$ the greatest common divisors (taken positively) of the minors of different orders of the matrix $(A^{(1)})$, so that, in particular, ∇_1 is the greatest common divisor of its constituents, and ∇_n is the absolute value of its determinant, here supposed to be different from zero. By the primary divisor of a quadratic form we shall understand the greatest common divisor of the coefficients of the squares and double rectangles in the quadratic form; by the secondary divisor we shall understand the greatest common divisor of the coefficients of the squares and of the rectangles; so that the primary divisor is equal to, or is half of, the secondary divisor, according as the quadratic form (to use the phraseology of Gauss) is derived from a form properly or improperly primitive. It will be seen that $\nabla_1, \nabla_2, \ldots . \nabla_{n-1}$ are the primary divisors of the forms $f_1, f_2, \ldots . f_{n-1}$ respectively.

We now consider the totality of arithmetical quadratic forms, containing n indeterminates, and having a given index of inertia, and a given determinant.

If a quadratic form be reduced to a sum of squares by any linear transformation, the number of positive and of negative squares is the same,

whatever be the real transformation by which the reduction is effected. For the index of inertia we may take the number of the positive squares; it is equal to the number of continuations of sign in a series of ascending principal minors of the matrix of the quadratic form, the series commencing with unity, *i. e.* with a minor of order 0, and each minor being so taken as to contain that which precedes it in the series (see Professor Sylvester "On Formulæ connected with Sturm's Theorem," Phil. Trans. vol. cxliii. p. 481). The distribution of these forms into Orders depends on the following principle:—

"Two forms belong to the same order when the primary and secondary divisors of their corresponding concomitants are identical."

Since, as has been just pointed out, there are, beside the fundamental concomitants, an infinite number of other concomitants, it is important to know whether, in order to obtain the distribution into orders, it is, or is not, necessary to consider those other concomitants. With regard to the primary divisors, it can be shown that it is unnecessary to consider any concomitants other than the fundamental ones; *i. e.* it can be shown that the equality of the primary divisors of the corresponding fundamental concomitants of two quadratic forms, implies the equality of the primary divisors of all their corresponding concomitants. And it is probable (but it seems difficult to prove) that the same thing is true for the secondary divisors also.

Confining our attention (in the next place) to the forms contained in any given order, we proceed to indicate the principle from which the subdivision of that order into genera is deducible.

If F_1 be any quadratic form containing r indeterminates, and F_2 be its concomitant of the second species, we have the identical equation

$$\left.\begin{aligned} F_1(x_1, x_2, \ldots x_r) \times F_1(y_1, y_2, \ldots y_r) - \tfrac{1}{4}\left[\sum_{k=1}^{k=r} y^k \frac{dF_1}{dx_k}\right]^2 \\ = F_2\begin{pmatrix} x_1, x_2, \ldots x_r \\ y_1, y_2, \ldots y_r \end{pmatrix} \end{aligned}\right\} \quad . \quad . \quad (B)$$

in which the symbol $F_2\begin{pmatrix} x_1, x_2, \ldots x_r \\ y_1, y_2, \ldots y_r \end{pmatrix}$ indicates that the determinants $\begin{pmatrix} x_1, x_2, \ldots, x_r \\ y_1, y_2, \ldots, y_r \end{pmatrix}$ are to be taken for the indeterminates of F_2, the order in which they are taken being the same as the order in which the determinants of any two horizontal rows of the matrix of F_1 are taken in forming the matrix of F_2. Let $\theta i = \frac{1}{\nabla} f_1$ for every value of i from 1 to $n-1$; it will be found that, if we form the concomitant of the second species of θ, its primary divisor is the quotient $\frac{\nabla i+1}{\nabla i} \div \frac{\nabla i}{\nabla i-1}$, which, as has been shown elsewhere (see Phil. Trans. vol. cli. p. 317) is always an integral number. Let δ_i be any uneven

prime dividing $\frac{\nabla_{i+1}}{\nabla_i} \div \frac{\nabla_i}{\nabla_{i-1}}$; we infer from the identity (B) that the numbers prime to δ_i, which can be represented by θ_i, are either all quadratic residues of δ^i, or all non-quadratic residues of δ_i. In the former case we attribute to f_i the *particular character* $\left(\frac{\theta_i}{\delta_i}\right) = +1$; in the latter the particular character $\left(\frac{\theta_i}{\delta_i}\right) = -1$. If $\nabla_1 = 1$, *i. e.* if the form f_1 itself do not admit of any primary divisor beside unity (which is the only important case), the product $\left(\frac{\nabla_n}{\nabla_{n-1}} \div \frac{\nabla_{n-1}}{\nabla_{n-2}}\right) \times \left(\frac{\nabla_{n-1}}{\nabla_{n-2}} \div \frac{\nabla_{n-2}}{\nabla_{n-3}}\right) \times \ldots$. is equal to $\frac{\nabla_n}{\nabla_{n-1}}$; whence, inasmuch as every prime that divides ∇_n also divides $\frac{\nabla_n}{\nabla_{n-1}}$, it appears that a primitive quadratic form will always have one particular character, at least with respect to every uneven prime dividing its determinant, and will have more than one if the uneven prime divide more than one of the quotients $\frac{\nabla_{i+1}}{\nabla_i} \div \frac{\nabla_i}{\nabla_{i-1}}$.

The subdivision of an order into genera can now be effected by assigning to the same genus all those forms whose particular characters coincide. But it remains to consider whether the above enumeration of particular characters is complete. It is evident that we might apply the theorem (B) to other concomitants besides those included in the fundamental system; and it might appear as if in this manner we could obtain other particular characters besides those which we have given. But it can be shown that such other particular characters are implicitly contained in ours; *i. e.* it can be shown that two quadratic forms, which coincide in respect of the particular characters deducible from their fundamental concomitants, will also coincide in respect of the particular characters deducible from any other concomitant. Again, it will be found that if the determinant be uneven, there are no particular characters with respect to 4 or 8. For this case, therefore, our enumeration is complete. But when the determinant is even, besides the particular characters arising from its uneven prime divisors, there may also be particular characters with regard to 4 or 8. There is no difficulty in enumerating these particular characters; nevertheless we suppress the enumeration here, not only because it would require a detailed distinction of cases, but also because there appears to be some difficulty in showing that the characters with regard to 4 or 8, which may arise from the excluded concomitants, are virtually included in those which arise from the concomitants of the fundamental set.

II. "On some Phenomena exhibited by Gun-cotton and Gunpowder under special conditions of Exposure to Heat." By F. A. ABEL, F.R.S. Received March 29, 1864.

The experiments upon which I have been engaged for some time past, in connexion with the manufacture and properties of gun-cotton, have brought under my notice some interesting points in the behaviour of both gun-cotton and gunpowder, when exposed to high temperatures, under particular conditions. I believe that these phenomena have not been previously observed, at any rate to their full extent, and I therefore venture to lay before the Royal Society a brief account of them.

Being anxious to possess some rapid method of testing the uniformity of products obtained by carrying out General von Lenk's system of manufacture of gun-cotton, I instituted experiments for the purpose of ascertaining whether, by igniting equal weights of gun-cotton of the same composition, by voltaic agency, within a partially exhausted vessel connected with a barometric tube, I could rely upon obtaining a uniform depression of the mercurial column, in different experiments made in atmospheres of uniform rarefaction, and whether slight differences in the composition of the gun-cotton would be indicated, with sufficient accuracy, by a corresponding difference in the volume of gas disengaged, or in the depression of the mercury. I found that, provided the mechanical condition of the gun-cotton, and its position with reference to the source of heat, were in all instances the same, the indications furnished by these experiments were sufficiently accurate for practical purposes. Each experiment was made with fifteen grains of gun-cotton, which were wrapped compactly round the platinum wire; the apparatus was exhausted until the column of mercury was raised to a height varying from 29 inches to 29·5 inches. The flash which accompanied the deflagration of the gun-cotton was apparently similar to that observed upon its ignition in open air; but it was noticed that an interval of time always occurred between the first application of heat (or incandescence of the wire) and the flashing of the gun-cotton, and that during this interval there was a very perceptible fall of the column of mercury. On several occasions, when the gun-cotton, in the form of "roving," or loosely twisted strand, was only laid over the wire, so that it hung down on either side, the red-hot wire simply cut it into two pieces, which fell to the bottom of the exhausted vessel, without continuing to burn. As these results appeared to indicate that the effects of heat upon gun-cotton, in a highly rarefied atmosphere, differed importantly from those observed under ordinary circumstances, or in a very imperfect vacuum, a series of experiments, under variously modified conditions, was instituted, of which the following are the most important.

It was found in numerous experiments, made with proportions of gun-cotton varying from one to two grains, in the form of a loose twist laid double, that in highly rarefied atmospheres (the pressure being varied

from 1 to 8 in inches of mercury) the gun-cotton, when ignited by means of the platinum wire, burned very slowly, presenting by daylight an appearance as if it smouldered, with little or no flame attending the combustion. I was at first led by these results to conjecture that this peculiar kind of combustion of the gun-cotton was determined solely by its ignition in atmospheres rarefied beyond a certain limit; and I was induced, in consequence, to institute a number of experiments with the view of ascertaining what was the most highly rarefied atmosphere in which gun-cotton would burn as in the open air—with a flash, accompanied by a body of bright flame. In order to ensure uniformity in the degree of heat applied to the cotton in these experiments, the platinum wire employed was sufficiently thin to be instantaneously melted on the passage of the voltaic current. About fifty different experiments were made with equal quantities of gun-cotton (0·2 grain), placed always in the same position, so that the platinum wire rested upon the material. A tolerably definite limit of the degree of rarefaction was arrived at, within which the gun-cotton was exploded instantaneously, as in the open air. When the pressure of air in the apparatus was reduced to 8·2 in inches of mercury, the gun-cotton still exploded with a flash, but not quite instantaneously; on reducing the pressure to 8 inches, the cotton underwent the slow kind of combustion in the majority of cases; on a few occasions it exploded with a flash of flame. The same occurred in a succession of experiments, until the pressure was reduced gradually to 7·7 inches, when instances of the rapid explosion of gun-cotton were no longer obtained.

Although these results were moderately definite when the conditions of the experiments were as nearly as possible uniform, it was found that they could be altered by slight modifications of any one particular condition (such as the quantity of gun-cotton, its mechanical condition, its position with reference to the source of heat, the quantity of heat applied, and the duration of its application). In illustration of this, the following results may be quoted.

If the gun-cotton was wrapped round, instead of being simply placed across the wire, its instantaneous combustion was effected in atmospheres considerably more rarefied than with the above experiments.

In employing a small piece of gun-cotton (0·3 of an inch long and weighing 0·3 to 0·4 of a grain) loosely twisted, laid across the wire, or upon a support immediately beneath the latter so that the wire rested upon it, the slow combustion established in it by the heated wire, under greatly diminished atmospheric pressure (amounting to 0·6 inch in this and the following experiments), proceeded uniformly towards each end of the piece of twist, until the whole was transformed into gas. But if a piece of the same twist, of considerably greater length (say 4 inches long and weighing about 2 grains), was exposed to heat in an atmosphere of the same rarefaction, the gun-cotton being laid over the wire and hanging down on either side, it was cut through by the passage of the current, and

the two pieces, falling to the bottom of the vessel, ceased to burn almost immediately. Of a piece of gun-cotton weighing 2·17 grains, there remained unchanged 1·80 gr.; the quantity burned amounted therefore to 0·37 gr., and corresponded closely to the quantity which was completely burned in the preceding experiments. (The depression of the mercurial column in this experiment, by the gases generated from the gun-cotton, amounted to 0·2 inch.)

A piece of the twist, 1½ inch long, was placed across the wire, and supported by a plate of plaster of Paris, fixed immediately beneath it. The current was established to an extent just sufficient to heat the wire to the point of ignition of the gun-cotton, and then interrupted. The twist burned slowly in both directions until about a quarter of an inch was consumed on either side of the wire, when the combustion ceased. The same result was obtained in repetitions of the experiment, the wire being at once raised to a red heat, and thus maintained until the gun-cotton ceased to burn. But upon increasing the battery-power, doubling the thickness of the wire, and maintaining the heat, while a similar piece of twist was burning in both directions, the slow combustion continued until the entire quantity was transformed into gas. The same result was obtained by repeating this experiment with similar and larger quantities of gun-cotton, placed in the same position as before with reference to the wire.

In the next experiment, the mass of cotton exposed at one time to heat was increased by doubling a piece of the twist (4 inches long) and laying it thus doubled across the wire, as before. The current was allowed to pass until the wire was heated just sufficiently to ignite the gun-cotton, and then interrupted. In this case the slow combustion proceeded throughout the entire mass of the cotton. The permanent depression of mercury in this experiment was 0·6 inch. It was particularly noticed on this occasion, that, as the decomposition of the gun-cotton crept slowly along the mass, the burning portions or extremities of twist were surrounded by a beautiful green light, more like a phosphorescence than a flame, and in form something similar to the brush of an electric discharge.

Eight inches of the twist were laid fourfold over the wire, which was heated just sufficiently to ignite the cotton. The decomposition proceeded, as before, gradually throughout the mass of the gun-cotton, but became somewhat more rapid towards the end, when the green glow, observed at first, was superseded by a pale yellowish lambent flame, very different in appearance from the flame which accompanies the combustion of gun-cotton under ordinary conditions. The permanent depression of the column of mercury in this experiment was 1·2 inch.

The various modifications in the nature and extent of combustion which gun-cotton may be made to undergo, as demonstrated by the above experiments, when exposed to heat in highly rarefied atmospheres under variously modified conditions, are evidently due to the same causes which affect the rate of combustion of fuses under different atmospheric pressures, and which

have already been pointed out by Frankland in his interesting paper on the influence of atmospheric pressure upon some of the phenomena of combustion. The heat furnished by an incandescent or melting platinum wire is greatly in excess of that required to induce perfect combustion in gun-cotton which is actually in contact with, or in close proximity to it; and the heat resulting from this combustion, which is contained in the products of the change, will suffice to cause the transformation of the explosion to proceed from particle to particle. But if the pressure of the atmosphere in which the gun-cotton is submitted to the action of heat be reduced, the gases resulting from the combustion of the particles nearest to the source of heat will have a tendency, proportionate to the degree of rarefaction of the air, to pass away into space, and thus to convey away from proximity to the cotton, more or less rapidly and completely, the heat necessary to carry on the combustion established in the first particles. Thus, when the heated wire is enveloped in a considerable body of gun-cotton, the ignition of the entire mass is apparently not instantaneous, if attempted in a highly rarefied atmosphere, because the products of the combustion first established in the centre of the mass of gun-cotton escape rapidly into space, conveying away from the point of combustion the heat essential for its full maintenance; the gun-cotton therefore undergoes at first an imperfect form of combustion, or a kind of metamorphosis different from the normal result of the action of heat upon this material. But the effects of the gradual generation of heated gases from the interior of the mass of cotton are, to impart some of their heat to the material through which they have to escape, as well as gradually to increase the pressure of the atmosphere in the vessel, and thus to diminish the rapidity of their escape; hence a condition of things is in time arrived at when the remainder of the gun-cotton undergoes the ordinary metamorphosis, a result which is accelerated by maintaining the original source of heat. If, however, the gun-cotton be employed in a compact form (in the form of twist or thread), and placed only in contact with the source of heat at one point, the heat will be so effectually conveyed away by the escaping gases, that the material will undergo even what may be termed the secondary combustion or metamorphosis for a limited period only; so that, if a sufficient length of gun-cotton be employed, it will after a short time cease to burn, even imperfectly, because the heat essential for the maintenance of any chemical activity is soon completely abstracted by the escaping gases. These results may obviously be modified in various ways, as shown in the experiments described: thus, by increasing and maintaining the source of heat independent of the burning cotton, the slow combustion may be maintained through a much greater length of the material until the pressure of the atmosphere is increased, by the products disengaged, to an extent sufficient to admit of a more rapid and perfect metamorphosis being established in the remainder of the material; or the same result may be attained, independently of the continued application of external heat, by employing a thicker mass of cotton, or by using the

material in a less compact form. In these cases the maintenance of the chemical change is favoured either by radiation of heat to the cotton, and provision of additional heat, from an external source, to the gases as they escape and expand, or by establishing the change in a greater mass of the material, and thus reducing the rapidity with which the heat will be conveyed away by the escaping gases, or, finally, by allowing the gases, as they escape, to pass to some extent between the fibres of the cotton, and thus favouring the transmission of heat to individual particles of the material.

In the description of the two experiments last referred to above, I have stated that some peculiar phenomena were observed to attend the imperfect kind of combustion induced in the gun-cotton in rarefied atmospheres.

In order to examine these phenomena more closely, I instituted a series of experiments, in a darkened room, with equal quantities of gun-cotton ($\frac{1}{2}$ inch of twist=0·3 gr.) placed always in the same position, across the platinum wire, the only varying element in the experiment being the pressure of the atmosphere in the vessel, which was gradually increased. The following were the results observed :—

Experiment I.—Pressure=0·62 inch. The wire was heated just sufficiently to ignite the material; the current was then interrupted. The gun-cotton burned very slowly in both directions, emitting only the small green phosphorescent flame, or brush, already described.

Exp. II.—Pressure=1 inch. In addition to the green glow which surrounded the burning ends, a very faint yellowish flame was observed hovering over the gun-cotton.

Exp. III.—Pressure=1·5 inch. The cotton burned a little faster, and the faint yellowish flame was of a more decided character; indeed two separate flames were observed, each following up the green light as the cotton burned in the two directions.

Exp. IV.—Pressure=2 inches. The results were the same as in the preceding experiment, excepting that the yellowish flames became more marked.

Exp. V.—Pressure=2·5 inches. The same phenomena, the cotton burning considerably faster.

Exp. VI.—Pressure=3 inches. The same phenomena, the yellow flames increasing in size.

Exp. VII.—Pressure=4 inches. The rapidity of combustion of the cotton increased again considerably; the other phenomena observed were as before.

Exp. VIII.—Pressure=6 inches. The pale yellow flame had increased in size considerably, no longer forming a tongue, as in the preceding experiments, but completely enveloping the burning ends of the gun-cotton. The green glow, though much reduced, was still observed immediately round the burning surfaces.

Exp. IX.—Pressure=8 inches. The green glow was only just perceptible in this instance, and the cotton burned very rapidly, almost with the

ordinary flash; but the flame was still of a pale yellow. In the preceding experiments clouds of white vapour were observed after the decomposition of the gun-cotton; in this and the following experiments this white vapour was produced in much smaller proportion.

Expts. X. *to* XV. *inclusive.*—Pressure=10, 12, 14, 18, 20, 24 inches. The phenomena observed in these experiments did not differ in any important degree from those of Experiment IX.

Exp. XVI.—The same pressure (24 inches) was employed as in the last experiment, but the piece of gun-cotton-twist was laid double across the wire. In this instance the gun-cotton burned with a bright yellow flash, as in open air.

Exp. XVII.—Pressure=26 inches. The gun-cotton was laid singly over the wire, as in all experiments but the last. It burned with a flash o bright light, as in open air.

It appears from these experiments that gun-cotton, when ignited in small quantities in rarefied atmospheres, may exhibit, during its combustion, three distinct luminous phenomena. In the most highly rarefied atmospheres, the only indication of combustion is a beautiful green glow or phosphorescence which surrounds the extremity of the gun-cotton as it is slowly transformed into gases or vapours. When the pressure of the atmosphere is increased to one inch (with the proportion of gun-cotton indicated), a faint yellow flame appears at a short distance from the point of decomposition; and as the pressure is increased this pale yellow flame increases in size, and eventually appears quite to obliterate the green light. Lastly, when the pressure of the atmosphere and consequently proportion of the oxygen in the confined space is considerable, the cotton burns with the ordinary bright yellow flame. There can be no doubt that this final result is due to the almost instantaneous secondary combustion, in the air supplied, of the inflammable gases evolved by the explosion of the gun-cotton. It was thought that the pale yellow flame described might also be due to a combustion (in the air still contained in the vessel) of portions of the gases resulting from the decomposition of the gun-cotton; but a series of experiments, in which nitrogen, instead of air, constituted the rarefied atmosphere, showed that this could not be the case. The results obtained in these experiments corresponded closely to those above described, as far as relates to the production of the green glow and of the pale yellow flame. With rarefied atmospheres of nitrogen ranging down to one inch of pressure, the green flame was alone obtained; and the pale yellow flame, accompanying the green, became very marked at a pressure of 3 inches, as in the experiments with air.

It would seem probable from these results, that the mixture of gaseous products obtained by the peculiar charge which heat effects in gun-cotton in highly rarefied atmospheres, contains not only combustible bodies, such as carbonic oxide, but also a small proportion of oxidizing gas (possibly protoxide of nitrogen, or even oxygen), and that when the pressure of the

atmosphere is sufficiently great this mixture, which has self-combustible properties, retains sufficient heat as it escapes, to burn, more or less completely, according to the degree of rarefaction of the atmosphere.

A series of experiments instituted with gun-cotton in highly rarefied atmospheres of oxygen, showed that the additional proportion of this gas thus introduced into the apparatus, beyond that which would have been contained in it with the employment of air of the same rarefaction, affected in a very important manner the behaviour of the explosion under the influence of heat. If eight or ten grains of gun-cotton are placed round the platinum wire, and the pressure of the atmosphere of oxygen in the vessel be reduced to four or three (in inches of mercury), the cotton explodes instantaneously, with an intensely bright flash, when the wire is heated. In a series of experiments made under gradually diminished pressures, oxygen being used instead of air, it was found that the gun-cotton exploded instantaneously, with a bright flash, until the pressure was reduced to 1·2 inch; from this pressure to that of 0·8 inch it still burned with a flash, but not instantaneously; and at pressures below 0·8 inch it no longer burned with a bright flash, but exhibited the comparatively slow combustion, accompanied by the pale yellow flame, which has been spoken of as observed when gun-cotton was ignited in air rarefied to pressures ranging from 1 inch to 24 inches.

The interesting phenomena exhibited by gun-cotton in highly rarefied atmospheres, induced me to make some experiments of a corresponding nature with gunpowder. The same apparatus was used as in the preceding experiments, but a small glass cup was fixed immediately beneath the platinum wire, so that, by bending the latter in the centre, it was made to dip into the cup, and could be covered by grains of gunpowder.

Two grains' weight of small grain gunpowder were heaped over the wire, and the pressure of air in the apparatus was reduced to 0·65 inch. The wire being heated to redness, three or four grains, in immediate proximity to it, fused in a short time and appeared to boil, evolving yellowish vapours, no doubt of sulphur. After the heat had been continued for eight or ten seconds, those particular grains deflagrated, and the remainder of the powder was scattered by the slight explosion, without being ignited. No appreciable depression of the mercurial column occurred during the evolution of the yellowish vapours; the permanent depression, after the deflagration, was only 0·15 inch.

The experiment was repeated with small-grain gunpowder, amounting to four grains, and the same phenomena were observed, with this difference, that a second slight deflagration followed shortly after the first, probably in consequence of a grain or two of the powder falling back into the cup.

A single piece of gunpowder, weighing 14 grains, so shaped as to remain in good contact with the wire, was placed over the latter, being supported by the cup. The pressure of air in the apparatus was, as before, equal to 0·65 inch of mercury. There was no perceptible effect for a short time

after the wire was first heated to redness; vapours of sulphur were then given off, and slight scintillations were occasionally observed; after a time the wire became deeply buried in the superincumbent mass of gunpowder, which fused, and appeared to boil, where it was in actual contact with the source of heat. After the lapse of three minutes from the commencement of the experiment, the powder deflagrated. The permanent depression of the mercury column amounted to 1·35 inch.

The experiment was repeated with a similar piece of powder, weighing 16 grains; the same phenomena were observed; and five minutes elapsed between the first heating of the wire and the deflagration of the powder.

The experiments were continued with fine-grained gunpowder, and under pressures gradually increased, in successive experiments, from ·07 to 3 in inches of mercury. The same weight of gunpowder (4 grains) was used in all the experiments. In those made under a pressure of 1 inch, the results observed were similar to those obtained in the first experiments; single grains of gunpowder were successively deflagrated, burning very slowly, and scattering but never igniting contiguous grains of powder. Eventually, after the lapse of from ten to twenty seconds, 3 or 4 grains were deflagrated at once, the remainder of the powder being thereby projected from the cup. At a pressure of 1·5 inch, the same phenomena were observed, but the successive deflagrations of fused grains of powder followed more quickly upon each other, and the final ignition of several together occurred in about ten seconds after the wire was first heated. At a pressure of 2 inches, at first only one or two of the fused grains were ignited, singly; and several were deflagrated together after the lapse of five seconds. A larger quantity of the powder was burned, but a portion was projected from the cup as in preceding experiments. At a pressure of 3 inches, no grains were ignited singly; the combustion of the powder was effected after an interval of about four seconds, and the greater portion was burned; the combustion, though it had gradually become more similar to that of gunpowder in open air, was still very slow.

Experiments made with gunpowder in highly rarefied atmospheres of nitrogen furnished results quite similar to those described; nor was any important difference in the character of the phenomena observed when oxygen was substituted for air, except that the scintillations and deflagrations of the powder-grains were in some instances somewhat more brilliant.

The above experiments show that, when gunpowder is in contact with an incandescent wire in a highly rarefied atmosphere, the heat is, in the first instance, abstracted to so great an extent by the volatilization of the sulphur, that the particles of powder cannot be raised to the temperature necessary for their ignition, until at any rate the greater part of that element has been expelled from the mixture, in consequence of which the portions first acted upon by heat will have become less explosive in their character, and require, therefore, a higher temperature for their ignition than

in their original condition. The effect of the continued application of heat to the powder thus changed is, to fuse the saltpetre and to establish chemical action between it and the charcoal, which, however, only gradually and occasionally becomes so energetic as to be accompanied by deflagration, because the gas disengaged by the oxidation of the charcoal continues to convey away much of the heat applied, in escaping into the rarefied space. For the same reason, the grains of unaltered powder which are in actual contact with the deflagrating particles are not ignited by the heat resulting from the combustion, but are simply scattered by the rush of escaping gases, at any rate until the pressure in the vessel has been so far increased by their generation as to diminish the rapidity and extent of their expansion at the moment of their escape. The disengagement, first of sulphur-vapour and then of gaseous products of chemical change, unattended by phenomena of combustion, when gunpowder is maintained in contact with a red-hot wire in very highly rarefied atmospheres, are results quite in harmony with the observations made by Mitchell, Frankland, and Dufour, with regard to the retarding influence of diminished atmospheric pressure upon the combustion of fuses. The phenomena described are most strikingly exhibited by operating upon single masses of gunpowder, of some size, in the manner directed above, when the application of the red-hot wire may be continued from three to five minutes (the gases disengaged during that period depressing the column of mercury from 0·5 to 0·7 inch) before the mass is ignited. There is no doubt that the products of decomposition of the gunpowder, obtained under these circumstances, differ greatly from those which result from its explosion in confined spaces or in the open air under ordinary atmospheric conditions. In all the experiments conducted in the most highly rarefied atmospheres (at pressures of 0·5 to 1·5 in inches of mercury), the contents of the vessel, after the final deflagration of the powder, always possessed a very peculiar odour, similar to that of horse-radish, due to the production of some sulphur-compound; nitrous acid was also very generally observed among the products. It is readily conceivable that the chemical action established between the constituents of gunpowder, under the circumstances described, must be of a very imperfect or partial character, the conditions under which it is established being unfavourable to its energetic development.

In describing the phenomena which accompany the ignition of gun-cotton in atmospheres of different rarefaction, I have pointed out that, at pressures varying from one to twenty-four in inches of mercury, a pale yellow flame was observed, which increased in size with the pressure of the atmosphere; and that a flame of precisely the same character was produced in rarefied atmospheres of nitrogen. The experiments instituted in nitrogen show that the explosion of loose tufts of gun-cotton in atmospheres of that gas, even at normal pressures, was always attended with a pale yellow flash of flame, quite different from the bright flash produced by igniting gun-cotton in air. The same result was observed in atmospheres

of carbonic acid, carbonic oxide, hydrogen, and coal-gas. In operating with pieces of gun-cotton-twist or thread of some length instead of employing the material in loose tufts, the results obtained in the two last-named gases were very different from those observed in atmospheres of nitrogen, carbonic acid, and carbonic oxide. When ignited by means of a platinum wire (across which it is placed) in vessels filled with either of those two gases, and completely closed or open at one end, the piece of twist burned slowly and regularly, the combustion proceeding much more deliberately than if the same piece of gun-cotton had been ignited in the usual manner in air, and being accompanied by only a very small jet or tongue of pale yellow flame, which was thrown out in a line with the burning surface when the gun-cotton was ignited. The same result was obtained in currents of those gases when passed through a long, wide glass tube, along which the gun-cotton twist was laid, one end being allowed to project some distance into the air. The projecting extremity being ignited, as soon as the piece of twist had burnt up to the opening of the tube through which the gas was passing, the character of the combustion of the gun-cotton was changed from the ordinary to the slow form above described. On repeating this form of experiment in currents of hydrogen and of coal-gas, the ignited gun-cotton burned in the slow manner only a very short distance inside the tube, the combustion ceasing altogether when not more than from half an inch to one inch of the twist had burnt in the tube. The same result was observed when the current of gas was interrupted at the moment that the gun-cotton was inflamed. It was at first thought that this extinction of the combustion of gun-cotton by hydrogen and coal-gas might be caused by the very rapid abstraction of heat from the burning surface of gun-cotton in consequence of the diffusive powers of those gases; but when the experiments were made in perfectly closed vessels, the piece of gun-cotton-twist being ignited by means of a platinum wire, the combustion also ceased almost instantaneously. These effects, therefore, can only be ascribed to the high cooling-powers, by convection, of the gases in question. It was found, by a succession of experiments, that when nitrogen was mixed with only one-fifth of its volume of hydrogen the combustion of gun-cotton-twist in the mixture was very slow and uncertain (being arrested after a short time in some instances), and that a mixture of one volume of hydrogen with three of nitrogen prevented its combustion, like coal-gas.

The slow kind of combustion of gun-cotton, in the form of twist, which is determined by its ignition in currents or atmospheres of nitrogen, carbonic acid, &c. may also be obtained in a powerful current of atmospheric air, the thread of cotton being placed in a somewhat narrow glass tube. If, however, the air is at rest, or only passing slowly, the result is uncertain. In employing very narrow tubes into which the gun-cotton fits pretty closely, the combustion passes over into the slow form when it reaches the opening of the tube, and occasionally it will then continue throughout the length of the tube. In that case, while the gun-cotton

burns slowly along the tube, with a very small sharp tongue of pale flame, a jet of flame is obtained at the mouth of the tube, by the burning of the gas evolved by the decomposition of the gun-cotton. Sometimes, and especially when wider tubes are employed, the slow combustion will proceed only for a short distance, and then, in consequence of the ignition of a mixture of the combustible gases and air within the tube, the gun-cotton will explode with great violence, the tube being completely pulverized, and portions of unburnt cotton scattered by the explosion. If still wider tubes are employed, the cotton will flash into flame almost instantaneously throughout the tube directly the flame reaches the opening: in these cases the explosion is not violent; sometimes the tube escapes fracture, and at others is broken in a few places, or torn open longitudinally, a slit being produced in the tube directly over the gun-cotton. By using narrow tubes and gradually shortening the tube through which the gun-cotton was passed, pieces of the twist being allowed to project at both ends, it was found, upon inflaming the material which projected on one side, that the slow form of combustion, induced in it as soon as it burned into the tube, was maintained by that portion which burned in the open air on the other side, when the combustion had proceeded through the tube. Eventually, by the employment of a screen of wood or card-board containing a perforation of the same diameter as that of the gun-cotton-twist, through which the latter was partially drawn, the alteration of the combustion of the material from the ordinary to the slow kind was found to be invariably effected. On the one side of the screen, the gun-cotton burned with the ordinary flame and rapidity, until the combustion extended to the perforation, when the flame was cut off and the material on the opposite side of the screen burned only slowly, emitting the small-pointed tongue of pale yellow flame.

These results indicate that if, even for the briefest space of time, the gases resulting from the first action of heat on gun-cotton upon its ignition in open air are impeded from completely enveloping the burning extremity of the gun-cotton-twist, their ignition is prevented; and as it is the comparatively high temperature produced by their combustion which effects the rapid and more complete combustion of the gun-cotton, the momentary extinction of the gases, and the continuous abstraction of heat by them as they escape from the point of combustion, render it impossible for the gun-cotton to continue to burn otherwise than in the slow and imperfect manner, undergoing a transformation similar in character to destructive distillation.

These facts appear to be fully established by the following additional experimental results:—

1. If, instead of employing in the above experiments a moderately compact gun-cotton-twist, one of more open structure is used, it becomes difficult or even impossible to effect the described change in the nature of the combustion, by the means described, because the gases do not simply burn

at, or escape from, the extremity of the twisted cotton, but pass readily between the separated fibres of the material, rendering it difficult or impossible to divert them all into one direction; and hence they at the same time transmit the combustion from particle to particle, and maintain the heat necessary for their own combustion.

2. If a piece of the compactly twisted gun-cotton, laid upon the table, be inflamed in the ordinary manner, and a jet of air be thrown against the flame, in a line with the piece of cotton, but in a direction opposite to that in which the flame is travelling, the combustion may readily be changed to the slow form, because the flame is prevented from enveloping the burning cotton, and thus becomes extinguished, as in the above experiment.

3. Conversely, if a gentle current of air be so directed against the gun-cotton, when undergoing the slow combustion, that it throws back upon the burning cotton the gases which are escaping, it will very speedily burst into the ordinary kind of combustion. Or, if a piece of the gun-cotton-twist, placed along a board, be made to burn in the imperfect manner, and the end of the board be then gradually raised, as soon as the material is brought into a nearly vertical position, the burning extremity being the lowest, it will burst into flame.

By applying to the extremity of a piece of the compact twist a heated body (the temperature of which may range from 135° C. even up to a red heat), provided the source of heat be not very large in proportion to the surface presented by the extremity of the gun-cotton, the latter may be ignited with certainty in such a manner that the slow form of combustion at once ensues, the heat applied being insufficient to inflame the gases produced by the decomposition of the gun-cotton. By allowing the gun-cotton thus ignited to burn in a moderately wide tube, closed at one end, the inflammable gases produced may be burned at the mouth of the tube, while the gun-cotton is burning in the interior; or they may be ignited and the gun-cotton consequently inflamed, by *approaching* a flame, or a body heated to full redness, to the latter, in the direction in which they are escaping.

It need hardly be stated that these results are regulated by the degree of compactness of the gun-cotton, the size of the twist, and the dimensions of the heated body. Thus a small platinum wire heated to full redness, or the extremity of a piece of smouldering string, will induce the slow combustion in a thin and moderately compact twist; but a larger body, such as a thick rod of iron, heated only to dull redness, will effect the ignition both of the gun-cotton and of the gases evolved by the combustion of the first particles, so that the material will be inflamed in the ordinary manner. Similarly the red-hot platinum wire, or a stout rod heated to redness barely visible in the dark, if they are *maintained* in close proximity to the slowly burning surface of gun-cotton, will eventually cause the gases evolved to burst into flame. The more compact the twist of the gun-cotton, the more superficial is the slow form of combustion induced in

it, and a condition of things is readily attainable, under which the gun-cotton-twist will simply smoulder in open air, leaving a carbonaceous residue; and the heat resulting from this most imperfect combustion will be abstracted by the gases evolved more rapidly than it is generated, so that in a brief space of time the gun-cotton will cease to burn at all in open air *.

The remarkable facility with which the nature of combustion of gun-cotton in air or other gases may be modified, constitutes a most characteristic peculiarity of this substance as an explosive, which is not shared by gunpowder or explosive bodies of that class, and which renders it easily conceivable that this material is susceptible of application to the production of a comparatively great variety of mechanical effects, the nature of which is determined by slight modifications in its physical condition, or by what might at first sight appear very trifling variations of the conditions attending its employment.

There is little doubt that the products of decomposition of gun-cotton vary almost as greatly as the phenomena which attend its exposure to heat under the circumstances described in this paper. A few incidental observations indicative of this variation were made in the course of the experiments. Thus, in the instances of the most imperfect metamorphosis of gun-cotton, the products included a considerable proportion of a white vapour, slowly dissolved by water, as also small quantities of nitrous acid and a very large proportion of nitric oxide. The latter gas is invariably formed on the combustion of gun-cotton in air or other gases; but the quantity produced appears always to be much greater in instances of the imperfect or slow combustion of the material. The odour of the gases produced in combustions of that class is powerfully cyanic, and there is no difficulty in detecting cyanogen among the products. I trust before long to institute a comparative analytical examination of the products resulting from the combustion of gun-cotton under various conditions; meanwhile I have already satisfied myself, by some qualitative experiments, of the very great difference existing between the results of the combustion of gun-cotton in open air, in partially confined spaces, and under conditions precisely similar to those which attend its employment for projectile or destructive purposes. I have, for example, confirmed the correctness of the statement made by Karolyi in his analytical account of the products of decomposition of gun-cotton, that no nitric oxide or higher oxide of nitrogen is eliminated upon the explosion of gun-cotton under considerable pressure, as in shells. Coupling this fact with the invariable production of nitric oxide when gun-cotton is exploded in open air or partially confined spaces, there appears to be very strong reason for the belief that, just as the reduc-

* By enclosing in suitable cases solid cords, made up of two or more strands, and more or less compactly twisted, I have succeeded readily in applying gun-cotton to the production of fuses and slow-matches, the time of burning of which may be accurately regulated.

tion of pressure determines a proportionately imperfect and complicated transformation of the gun-cotton upon its exposure to heat, the results of which are more or less essentially of an intermediate character, so, conversely, the greater the pressure, beyond the normal limits, under which gun-cotton is exploded—that is to say, the greater the pressure exerted by it, or the resistance presented at the first instant of its ignition, the more simple are the products of decomposition, and the greater are the physical effects attending its explosion, because of the greater energy with which the chemical change is effected.

III. "On Magnesium." By Dr. T. L. Phipson, F.C.S. Communicated by Prof. G. G. Stokes, Sec. R.S. Received March 9, 1864.

(Extract.)

Iodine and Sulphur.—I find that iodine can be distilled off magnesium without attacking the metal in the least. In the same manner I distilled several portions of sulphur off magnesium without the metal being at all attacked.

Decomposition of Silicic Acid.—Heated for some time in a porcelain crucible with excess of anhydrous silica, the metal burns vividly if the air has access; and a certain quantity of amorphous silicium is immediately formed. Magnesium is therefore capable of reducing silicic acid at a high temperature. The reason why potassium and sodium cannot effect this is simply because these metals are highly volatile and fly off before the crucible has attained the proper temperature. Magnesium being much less volatile than the alkaline metals, takes oxygen from silica before volatilizing. If the silicic acid be in excess, a silicate of magnesia is formed at the same time; if the metal is in excess, much siliciuret of magnesium is produced. The presence of the latter is immediately detected by throwing a little of the product into water acidulated with sulphuric acid, when the characteristic phosphoric odour of siliciuretted hydrogen is at once perceived.

Decomposition of Boracic Acid.—With boracic acid the phenomena are rather different; the acid melts and covers the metal, so that it does not inflame even when the crucible is left uncovered. A certain quantity of boron is soon liberated, and the product forms a greenish-black mass, which oxidizes and becomes white in contact with water, and *disengages no odoriferous gas* in acidulated water.

Decomposition of Carbonic Acid.—I thought it would be interesting to try a similar experiment with carbonic acid. Accordingly dry carbonate of soda was heated with a little magnesium in a glass tube over a common spirit-lamp; and before the temperature had arrived at a red heat I observed that carbon was liberated abundantly, and magnesia formed.

Action of Alkalies.—A solution of caustic alkali or ammonia has little or no action upon magnesium in the cold.

Precipitation of Metallic Solutions.—Magnesium precipitates nearly all the metals from their neutral solutions. When these are taken in the form of protosalts, even manganese, iron, and zinc are precipitated as black powders. *Aluminium* and *uranium* (and perhaps chrome) are only precipitated as *oxides*.

Alloys of Magnesium.—I have examined only a few alloys of magnesium. Unlike zinc, magnesium *will not unite with mercury at the ordinary temperature* of the air. With tin 85 parts, and magnesium 15 parts, I formed a very curious alloy of a beautiful *lavender-colour*, very hard and brittle, easily pulverized, and decomposing water with considerable rapidity at ordinary temperatures. If the air has access during the formation of this alloy, the mixture takes fire; and if the crucible be then suddenly withdrawn from the lamp, the flame disappears, but a vivid *phosphorescence* ensues, and the unfused mass remains highly luminous for a considerable time. A white powdery mass, containing stannic acid and magnesia, is the result.

[With platinum, according to Mr. Sonstadt, magnesium forms a fusible alloy; so that platinum crucibles can be easily perforated by heating magnesium in them.]

Sodium and potassium unite with magnesium, and form very malleable alloys, which decompose water at the ordinary temperature.

It is probable that an alloy of copper and magnesium, which I have not yet obtained, would differ from *brass*, not only in lightness, but by decomposing water at the ordinary temperature with more or less rapidity.

Uses.—Magnesium will be found a useful metal whenever tenacity and *lightness* are required and tarnish is of no consequence. The light furnished by combustion of the wire has already been utilized in photography at night. In the laboratory it will be found useful to effect decompositions which sodium and potassium cannot effect on account of their greater volatility.

April 28, 1864.

Dr. W. A. MILLER, Treas. & V.P., in the Chair.

The following communications were read:—

I. "On the Magnetic Elements and their Secular Variations at Berlin," as observed by A. Erman. Communicated by General Sabine, P.R.S. Received March 1, 1864.

All observations and results to be mentioned here relate to

Latitude 52° 31′ 55″ North.
Longitude 13° 23′ 20″ E. from Greenwich.

1. *Horizontal Intensity.*

Denoting by $(1800+t)$ the date of observation in tropical years of the

Gregorian epoch, T the absolute value of *horizontal intensity* with millimetre, milligram, and the second of mean time as unities, ω the same in unities of the Gaussian constants; the two values of T for 1805·5 and 1828·31 have been deduced from observed ω, by $T=0{\cdot}00349216\,.\,\omega$.

τ and τ' denote the observed *time of oscillation* of two magnets which, since 1853·523 were carefully guarded from the influence of other magnets; and therefore, marking by C, α, β, C′, α', β' unknown constants, e the basis of hyperbolic logarithms, and taking $t_1=t-53{\cdot}523$, each value of τ and τ' had to fulfil the equations

$$\tau^2=\frac{C}{(1+\alpha e^{-\beta t_1})\,.\,T^2} \qquad \tau'^2=\frac{C'}{(1+\alpha' e^{-\beta' t_1})\,.\,T^2}.$$

In the following list of observed values, the first is due to Humboldt; the twenty-eight following were obtained by Erman:—

Date of observation.	Horizontal intensity, T.		Times of oscillation of			
			Magnet I. τ.		Magnet II. τ'.	
1800+t	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.
1805·5	1·6452	1·6452	.....	.....	.....	.....
1828·31	1·7559	1·7437	.....	.....	.....	.....
1846·13	1·7751	1·7827	.....	.....	.....	.....
1849·59	1·7784	1·7867	.....	.....	.....	.....
1853·89	.....	.....	3·1090	3·1090	.....	.....
1854·59	1·7900	1·7904	3·1072	3·1141	8·0082	8·0056
1856·57	1·7900	1·7913	3·1168	3·1134	8·0954	8·1036
1857·54	1·7879	1·7916	3·1106	3·1131	8·1193	8·1104
1858·58	1·8035	1·7917	3·1158	3·1130	8·1364	8·1126
1859·60	1·7933	1·7912	3·1229	3·1129	8·1223	8·1132
1860·63	.....	.....	3·1043	3·1129	8·0870	8·1135
1861·52	1·7972	1·7917	.....	.....	8·1258	8·1138
1862·52	1·7900	1·7915	.....	.....	8·1100	8·1142
1863·80	1·7929	1·7911	3·1148	3·1135	8·0975	8·1151

The calculated values result from the following most probable expressions for T, and for those values of τ and τ' which agree best with the contemporaneous T.

I. $T=1{\cdot}61892+0{\cdot}0057689\,t-0{\cdot}000048119\,t^2$.

II. $\tau^2=\dfrac{17{\cdot}3633}{\{1+0{\cdot}07392\,.\,e^{-8{\cdot}2988\,.\,t_1}\}\,.\,T^2}$

III. $\tau'^2=\dfrac{117{\cdot}956}{\{1+0{\cdot}09733\,.\,e^{-1{\cdot}16627\,.\,t_1}\}\,.\,T^2}$

} with $t_1=t-53{\cdot}523$.

The expression I. appears liable to the *probable errors*,

in first term, $\pm 0{\cdot}00126$
in coefficient of t, $\pm 0{\cdot}000065$
in coefficient of t^2, $\pm 0{\cdot}00000074$

} of a magnetic unity;

and when brought under the form

(A) $T=1{\cdot}79183-0{\cdot}000048119\{t-59{\cdot}930\}^2$,

identical with I., it shows that the horizontal intensity reached in 1859·930 the maximum of 1·79183.

It ought to be observed, that equal probability has been attributed to the error

$$\pm 1 \text{ in } T,$$

$$\pm\frac{\tau}{2T} \text{ in } \tau,$$

$$\pm\frac{\tau'}{2T} \text{ in } \tau';$$

that is to say, equal errors to an *intensity* determined by each of the three methods,—this supposition being at once the most simple and the most conformable to my experience, by nearly contemporaneous repetitions of each class of observation.

All my determinations of absolute intensity have been obtained either by one of two, or by two magnetometers; the first of which is a Gaussian of large size, by Meyerstein, the second my declination- and transit-instrument by Pistor, completed by the usual graduated holders for deflecting magnets, and perfectly adapted to observations in the open air.

2. *Inclination.*

The values of inclination here employed are taken for 1806·0, 1832·5, and 1836·87, from the observations of Humboldt, Rudberg, and Encke; for the ten other dates since 1825·0, they have been obtained by my own applications of the methods exposed in my 'Reise um die Erde,' Physikal. Beob., tome ii. pp. 8–42, to two different instruments—viz. till 1850 to a large and highly perfect one by Gambey, and since that time to a smaller dip-circle by Robinson. The methods of observation leave no room for any constant error in the resulting inclination, as long as no directive magnetic force is exerted upon the needle by the instrument itself. In order to free my results from any influence from this improbable (but not impossible) source, I compared, in 1860, three full determinations by the last-mentioned apparatus, with an equal number which I obtained under identical circumstances with a most perfect copy of Weber's inductive inclinometer. The result was an agreement of the two kinds of determinations within the limits of accidental error of the first—that is to say, far below *one minute* in the inclination; I venture, therefore, to say hat the following numbers must give the absolute value of the element in question with no less certainty than the rate of its secular variation:

Date of observation.	Inclination, i.	
$1800+t$.	Observed.	Calculated.
1806·0	69° 53′	69° 52′·99
1825·00	68 49·19	68 44·62
1828·29	68 34·55	68 34·17
1832·50	68 18·08	68 21·40
1836·87	68 7·43	68 8·84
1838·75	68 2·04	68 3·66
1846·20	67 43·25	67 44·46
1849·65	67 35·48	67 36·29
1853·78	67 29·81	67 27·09
1856·56	67 20·50	67 24·26
1857·55	67 20·30	67 19·25
1860·60	67 15·75	67 13·31
1862·55	67 7·63	67 9·69

The system of the above calculated values, which best agrees with the observed ones, results from the expression

(B) $\quad i=70°\ 17'·42-4'·1854\,t+0'·018931\,t^2$;

it leaves in each single equation a probable error of $\pm 1'·42$; and accordingly in the expression itself the probable errors appear to be

in the absolute term $\pm 2'·17$;
in the coefficient of t $\pm 0'·1211$;
in the coefficient of t^2 $\pm 0'·001591$.

This expression can be brought under the form

(B*) $\quad i=66°\ 26'·09+(t^2-110·543)^2.0'·018931$,

which would prove that at the place in question the inclination will come, in 1910·543, to a minimum of 66° 26′·09. The aforesaid errors of terms give $\pm 2·27$ years for the uncertainty of the epoch of this minimum, and $\pm 3'·9$ for the uncertainty of its value; but as the expression (B) results from observations between 1806 and 1863, its consequences ought not to be extended as far as 1910.

3. *Declination.*

Four results of observations of this element, made by the late astronomers Kirch in 1731, Bode in 1784 and 1805, and Tralles in 1819, have been added to my own, which extend from 1825 to 1864. These latter were obtained with the declination- and transit-instrument employed in my voyage, which intermediately was frequently compared and found in perfect agreement with a large Gaussian magnetometer, whenever the indications of the latter were duly freed from the torsion of the suspending wires and from the want of parallelism between the normal of the employed speculum and the magnetic axis of the bar. My observations were all made in the open air, with the exception of the two in 1849 and 1850, which, having been executed in a room, were corrected for the influence of local attractions. As the determination of this latter seemed exposed to a somewhat larger

error than the other declinations, in combining the two reduced values with those obtained in the open air, I have given to the two first only a fourth of the weight of the others. A similar allowance for larger probable errors should perhaps have been made in employing the four statements of former observers; but, for want of particulars about the operations they are founded upon, it was more safe to neglect the difference between their weight and that of the others, than to fix it by an arbitrary assumption.

If, for the *moment of observation*, there were marked by $1800+t$, as before, the tropical years elapsed since the Gregorian epoch, m the positive excess of t over the next integer, x the horary angle of *mean Sun*, each observed west declination d' had to be brought under the form

$$d'=\mathrm{D}+f(t)+\phi(m,x),$$

D denoting a constant, and f and ϕ two functions, the first of which was to be determined here.

In order to form $d'-\phi(m,x)=d$ out of each d', I put

$$\phi(m,x)=a+\alpha\,.\cos x+\gamma\,.\cos 2x+\epsilon\,.\cos 3x,$$
$$+\beta\,.\sin x+\delta\,.\sin 2x+\zeta\,.\sin 3x,$$

taking the values of a, α, β, ..., ζ by interpolation according to m, from the following Table, derived from observations in the Russian observatories at St. Petersburg, Catherinbourg, and Barnaoul in the year 1837 and 1838, and well agreeing with my own determinations of $\phi(m,x)$ for the years 1828 to 1830, and at eight places between latitude 50° and 62° North.

m.	a.	α.	β.	γ.	δ.	ϵ.	ζ.
	″	″	″	″	″	″	″
0·042	+ 15	+ 54	+ 15	−27	+ 70	+ 1	+ 5
0·123	− 62	+ 61	+ 46	−28	+104	+33	+12
0·204	− 43	+ 54	+135	+11	+173	+22	+62
0·288	−108	+ 64	+256	+14	+199	+21	+82
0·372	− 88	+104	+264	+65	+182	+42	+42
0·455	+ 14	+107	+290	+71	+184	+64	+45
0·538	+ 77	+ 82	+275	+76	+175	+72	+43
0·623	+ 60	+ 85	+221	+79	+187	+69	+60
0·707	+ 72	+ 91	+139	+51	+156	+57	+34
0·790	+ 63	+ 75	+ 69	+ 2	+137	+29	+ 9
0·884	− 4	+ 64	− 10	−12	+ 76	+36	− 1
0·959	+ 5	+ 82	− 18	−20	+ 68	+14	−11
1·042	+ 15	+ 54	+ 15	−27	+ 70	+ 1	+ 5

When I supposed in this way that the parameters a, α, ..., ζ of the function $\phi(m,x)$, or ϕ as I will call it for abbreviation, are the same for all moments alike situated in different years, I was well aware that this assumption is but approximative, and that all sufficiently extended and direct investigations of ϕ, as chiefly those of General Sabine, have shown a periodicity of about 9·5 years in the total values of this function. But as the laws of such dependence between T and each of the seven para-

meters of ϕ have not yet been perfectly exposed, I preferred in the present to treat the latter as mere functions of m and x. In the following Table of employed mean declinations for the moments t, to each of them is subjoined the value of ϕ by whose subtraction it has resulted from the momentary value furnished by observation. This arrangement will allow us to appreciate (and, if wanted, to correct for) the influence exerted by any periodical variation of ϕ upon the final result of my observations. It may, too, be convenient to observe that for some of the following west declinations (D), as well as for the before-mentioned intensities (T) and inclinations (I), the observations were made in latitude $p-\Delta p$, and longitude $l-\Delta l$ (where p and l mark the corresponding and above alleged values for my ordinary place), and that then the directly obtained results, viz. $d-\Delta d$, $T-\Delta T$, or $i-\Delta i$, have been reduced by

$$\Delta d = -0{\cdot}0940\,.\,\Delta p - 0{\cdot}6103\,.\,\Delta l;$$
$$\Delta T = -0{\cdot}7480\,.\,10^{-3}\,.\,\Delta p + 0{\cdot}2152\,.\,10^{-3}\,.\,\Delta l;$$
$$\Delta i = +0{\cdot}7405\,.\,\Delta p - 0{\cdot}1861\,.\,\Delta l;$$

the minute of arc being the unity for Δp, Δd, Δi, and Δl.

These equations, which result from the Gaussian constants with the given p and l, are sufficiently approximated when, as with us, Δp and Δl do not exceed a few minutes. So then were obtained:

Date of observation. 1800+t.	Momentary declination. Mean declination, ϕ.	Mean declination, d.	
		By observation.	Calculated.
1731·60	0	12° 18′·05	12° 19′·85
1784·00	0	17 59·65	17 46·09
1805·40	0	18 1·35	18 7·86
1819·00	0	17 36·50	17 48·06
1825·79	−1·80	17 24·46	17 28·37
1828·33	−4·08	17 21·35	17 19·34
1834·05	−1·24	17 2·69	16 55·65
*1849·62	+2·74	15 21·55	15 24·39
*1850·63	+2·96	15 20·48	15 20·47
1853·81	+4·32	14 55·17	14 58·26
1854·36	−2·86	15 1·05	14 54·19
1856·58	+1·13	14 38·13	14 37·40
1857·49	−5·95	14 33·88	14 30·29
1858·54	−5·61	14 21·15	14 21·96
1859·58	−4·87	14 14·24	14 13·59
1861·50	+4·47	13 53·70	13 57·63
1862·55	+0·12	13 49·83	13 48·72
1863·79	+4·48	13 36·85	13 37·99

A fourth of the weight of each of the other observed values being given to each of the two marked *, the whole is best represented by

$$d = 18^\circ\ 8'{\cdot}46 + 0'{\cdot}26820\ t - 0'{\cdot}070665\ t^2, \quad . \quad . \quad . \quad \text{(IV)}.$$

which furnishes the above calculated numbers; and by their comparison with the observed ones, the *probable errors* are—

in *the absolute term of d* $\pm 1'\cdot 94$;
in the coefficient of t in d $\pm 0'\cdot 2932$;
in the coefficient of t^2 in $d \pm 0'\cdot 030669$.

If, now, instead of employing the variations $\phi\ (m, x)$, or ϕ according to observations in the years 1837 and 1838, we assume (1) that the periodical dependence between this function and the date t consists in always changing each parameter proportionally to its mean or primitive value, and then (2) that, as General Sabine has proved, the whole function has nearly reached a maximum in all moments marked by $t = 48 \pm n \,.\, 9\cdot 5$, n being an *integer*, and (3) that, according to the same philosopher, the least and the largest amount of corresponding variations are approximately as 1 : 1·4, then, Φ marking the function of t, m, x which in each case must be substituted for ϕ, and C a function of M and X, we shall have

$$\Phi = c\left\{1\cdot 20 + 0\cdot 20 \,.\, \sin\left[\frac{720^\circ}{19}(t - 45\cdot 625)\right]\right\}$$

and

$$\phi = c\left\{1\cdot 20 - 0\cdot 20 \,.\, \sin\frac{720^\circ}{19}(8\cdot 125)\right\} = 1\cdot 35811 \,.\, c.$$

To each of the preceding values of d must therefore be *added*

$$\phi - \Phi = \phi\left\{0\cdot 1163 - 0\cdot 1472 \,.\, \sin\left[\frac{720^\circ}{19}(t - 45\cdot 625)\right]\right\}.$$

By executing this operation, I found that the reduced observations are best represented by

$$(C^*) \qquad d = 18^\circ\ 8\cdot 43 + 0\cdot 26831 \,.\, t - 0\cdot 070652 \,.\, t^2,$$

and that, though scarcely differing from (IV.), this expression is preferable, because the probable error of each of its terms is by nearly $\frac{1}{34}$ of its former value smaller than the corresponding one in (IV.)

As the expression (C*) is identical with

$$(C) \qquad d = 18^\circ\ 8'\cdot 68 - 0'\cdot 070652\{t - 1\cdot 899\}^2,$$

we see that, according to my observations, the west declination at the place in question arrived in 1801·899 at a maximum of 18° 8′·68.

Putting off for a further article some more general observations on the secular changes of terrestrial magnetism, I briefly resume, as results of my nearly forty years' observations, that for

latitude $= 52^\circ\ 31'\ 55''$ North,
longitude $= 13^\circ\ 23'\ 20''$ E. from Greenwich,

there have been—between 1805 and 1864,

$$\textit{Horizontal intensity} = T = 1{\cdot}79183 - 0{\cdot}000048119\ \{t - 59{\cdot}930\}^2;$$

between 1806 and 1863,

$$\textit{Inclination} = i = 66^\circ\ 26'{\cdot}09 + 0'{\cdot}018931\ \{t - 110{\cdot}543\}^2;$$

and between 1731 and 1864,

$$\textit{West declination} = d = 18^\circ\ 8'{\cdot}68 - 0'{\cdot}070652\ \{t - 1{\cdot}899\}^2;$$

all results being meant to be just for the date $1800 + t$ in years of the Gregorian epoch.

N.B. It seems not unworthy of remark, that no evidence of the existence of a *third term* in the expression for any one of the three phenomena results from the above-mentioned observations; and this, though partly due to the inevitable imperfections of the observations, makes it highly probable that a man's lifetime, and even a century is but a very small part of the secular period of terrestrial magnetism.

II. "On the Action of Chlorine upon Methyl." By C. Schorlemmer, Assistant in the Laboratory of Owens College, Manchester. Communicated by Professor Roscoe, F.R.S. Received April 5, 1864.

In a paper published in the Journal of the Chemical Society, New Ser. vol. i. p. 425, I pointed out the great interest which attached to the study of the lower terms of hydrocarbons, known by the name of the "alcohol radicals," inasmuch as the question of the chemical constitution of these bodies requires to be more definitely settled.

Having been aided in these researches by a grant from the Council, I beg to lay before the Royal Society the results of an investigation on the action of chlorine upon methyl, which are as unexpected as they are decisive.

Equal volumes of chlorine and of methyl were exposed in strong well-corked bottles, holding from two to three litres, to diffused daylight in the open air at a temperature of about 5° C. The methyl was prepared according to Kolbe's method, by electrolysis of a concentrated solution of acetate of potassium, and carefully purified by washing with a solution of caustic potash and concentrated sulphuric acid. The colour of the chlorine disappeared rather quickly; colourless oily drops condensed on the the sides of the bottles, and collected after some time on the bottom as a mobile liquid, the greater part of which volatilized again when the bottles were brought into a warm room. Hence it appears that by the action of one volume of chlorine upon one of methyl, substitution-products are formed, consisting chiefly of a volatile liquid, the boiling-point of which lies between 5° and 15° C. In order to collect these products, the bottles were heated till all the liquid had volatilized, and then opened, with the mouth downwards, under a hot concentrated solution of common salt, to which some caustic soda was added in order to quicken the absorption of

the hydrochloric acid, of which half the volume of gas contained in the bottles consisted. The bottles were then taken out of the liquid, placed in an upright position, and the mouth provided with a doubly perforated cork, into one opening of which a siphon fitted. Through this siphon a hot concentrated solution of common salt slowly ran in, whilst the gas thus displaced escaped by a bent tube and was condensed in a small tube receiver, surrounded by a mixture of ice and salt. The liquid thus obtained was left for some hours in contact with a piece of solid caustic potash, in order to remove moisture and the last traces of hydrochloric acid. Subjected to distillation, the liquid began to boil at 11° C., and the boiling-point rose slowly to 30° C., at which temperature two-thirds of the liquid had come over. On continuing the fractional distillation for some time longer, the distillate yielded a few grammes of a colourless mobile liquid, boiling between 11° and 13° C., which, as the following analysis and vapour-density determinations prove, is chloride of ethyl, C_2H_5Cl. The boiling-point of the liquid agrees with that of this compound, and it possesses the strong peculiar smell and the property of burning with a white, luminous, green-bordered flame, characteristic of the chloride of ethyl.

I. Analysis:—

(1) 0·4245 grm. of the substance gave 0·5670 grm. of carbonic acid and 0·3025 of water.

(2) 0·1810 grm. of the substance gave 0·3855 grm. of chloride of silver and 0·0165 grm. of metallic silver.

Calculated for the formula C_2H_5Cl.			Found.
2C	24	37·21	36·43
5H	5	7·75	7·92
Cl	35·5	55·04	55·63
	64·5	100·00	99·98

II. Determination of the vapour-density according to Gay-Lussac's method:—

	Weight of substance employed	0·0893 grm.
	Temperature of air	8° C.
	Height of barometer	739 millims.
(1)	Temperature of vapour	50° C.
	Volume of vapour	46·2 cub. centims.
	Difference of level	140·5 millims.
	Vapour-density calculated from these numbers	2·245.
(2)	Temperature of vapour	70° C.
	Volume of vapour	48·2 cub. centims.
	Difference of level	130·0 millims.
	Vapour-density calculated	2·244.
(3)	Temperature of vapour	80° C.
	Volume of vapour	49·45 cub. centims.
	Difference of level	125·0 millims.

which numbers give the vapour-density 2·235, whilst the theoretical vapour-density of chloride of ethyl is 2·233.

The boiling-point of the residue left after the first distillation rose quickly up to 60° C., whilst nearly the whole distilled over between this temperature and 70° C. By a few more fractional distillations of this latter portion, monochlorinated chloride of ethyl, $C_2H_4Cl_2$, boiling between 62° and 65° C., was isolated.

0·1270 grm. of this compound gave 0·3530 grm. of chloride of silver and 0·0095 grm. of metallic silver, which corresponds to 71·43 per cent. of chlorine, whilst the formula requires 71·71 per cent.

Eight litres of methyl yielded about 8 grammes of the mixed chlorides, or only about one-third of the theoretical quantity of chloride of ethyl which should have been obtained. This is easily explained by the volatilization of the liquid, and its solution in large quantities of water, as well as by the formation of higher substitution-products, in consequence of which a considerable quantity of methyl is left uncombined.

From these results it appears that the lowest term of the series of alcohol radicals behaves with chlorine exactly in the same manner as I have shown in the paper above referred to is the case with its homologues ethyl-amyl, C_7H_{16} (which gives chloride of heptyl, $C_7H_{15}Cl$), and amyl, $C_{10}H_{22}$ (from which chloride of decatyl, $C_{10}H_{21}Cl$, is obtained). If an excess of chlorine is avoided, the principal products consist of the chlorides of monatomic radicals containing the same number of atoms of carbon as the original hydrocarbon contained, whilst at the same time chlorine substitution-products of these chlorides are formed in smaller quantities.

As there is no reason why those terms of the series which are placed between C_2H_6, C_7H_{16}, $C_{10}H_{22}$ should show a different deportment, it becomes obvious that, beginning with marsh-gas, CH_4, the lowest term in the series C_nH_{2n+2}, the most simple of all hydrocarbons, and one which can easily be obtained from its elements, we are now not only in a position to prepare all the members of this series, but likewise to build up by simple synthesis the series of mono-, di-, and polyatomic alcohols, acids, compound ammonias, ethers, &c. &c. of which each of the marsh-gas hydrocarbons forms the starting-point.

III. "On the Calculus of Symbols (Fifth Memoir), with Applications to Linear Partial Differential Equations, and the Calculus of Functions." By W. H. L. Russell, Esq., A.B. Communicated by Professor Stokes, Sec. R.S. Received April 7, 1864.

In applying the calculus of symbols to partial differential equations, we find an extensive class with coefficients involving the independent variables which may in fact, like differential equations with constant coefficients, be

solved by the rules which apply to ordinary algebraical equations; for there are certain functions of the symbols of partial differentiation which combine with certain functions of the independent variables according to the laws of combination of common algebraical quantities. In the first part of this memoir I have investigated the nature of these symbols, and applied them to the solution of partial differential equations. In the second part I have applied the calculus of symbols to the solution of functional equations. For this purpose I have worked out some cases of symbolical division on a modified type, so that the symbols may embrace a greater range. I have then shown how certain functional equations may be expressed in a symbolical form, and have solved them by methods analogous to those already explained.

The Society then adjourned to Thursday, May 12th.

May 12, 1864.

Major-General SABINE, President, in the Chair.

In accordance with the Statutes, the names of the Candidates recommended by the Council for election into the Society were read, as follows:—

Sir Henry Barkly, K.C.B.
William Brinton, M.D.
T. Spencer Cobbold, M.D.
Alexander John Ellis, Esq.
John Evans, Esq.
William Henry Flower, Esq.
Thomas Grubb, Esq.
Sir J. Charles Dalrymple Hay, Bart.
William Jenner, M.D.
Sir Charles Locock, Bart., M.D.
William Sanders, Esq.
Col. William James Smythe, R.A.
Lieut.-Col. Alexander Strange.
Robert Warington, Esq.
Nicholas Wood, Esq.

The following communications were read:—

I. "Second Part of the Supplement to the two Papers on Mortality published in the Philosophical Transactions in 1820 and 1825." By Benjamin Gompertz, F.R.S. Received March 30, 1864.

(Abstract.)

The objects of this paper are various; but the subject appears to the author more especially important in consequence of the state of competition among assurance establishments, which he holds to be injurious to the interest of those valuable establishments, and to those of the assuring population.

The author's purpose in this paper is greatly to extend the modes of calculating valuations, and to improve the methods of calculation hitherto used by actuaries, which are in many cases very laborious, and in some almost impracticable. This part commences with observations on the inge-

nious plan of Barrett, which is shown to be capable of improvement and extension. Parts of the excellent work of the late Mr. David Jones, 'Tables of Life Annuities,' published under the care and suggestion of the Society for the Diffusion of Useful Knowledge, founded on those ideas of Barrett, are by the author here improved and extended so as to give methods easier for common purposes, and capable of extension to almost all difficulties which are likely to occur in the calculation of the value of property.

II. "Investigations of the Specific Heat of Solid and Liquid Bodies." By Hermann Kopp, Ph.D. Communicated by T. Graham, Esq., Master of the Mint. Received April 16, 1864.

(Abstract.)

In the first part the author discusses the earlier investigations on the specific heat of solid bodies, and on the relations of this property to their atomic weight and composition. In this historical report he gives a complete analysis of the various opinions published on the subject.

In the second part the author describes the method he has used for determining the specific heat of solid bodies. This method is based on the method of mixtures. The substance investigated is placed in a glass tube, together with some liquid which does not dissolve it, and the tube is heated in a mercury bath, and then rapidly immersed in a calorimeter containing water. Equalization of temperature takes place rapidly, through the intervention of the liquid in the tube. The thermal effect (increase of temperature in the water of the calorimeter) is determined. Preliminary experiments give the means of allowing for the thermal effect due to the glass and to the liquid in it, and of thereby obtaining the thermal effect produced by the solid substance. The author gives a complete description of the apparatus and of the mode of using it, and also of the means of determining the ancillary magnitudes which require to be taken into account. The entire method is very simple, and it brings the determination of specific heat out of the restricted sphere of the physical cabinet, with its complicated apparatus, within reach of the ordinary appliances of the chemical laboratory. It is also applicable to small quantities, and to such substances as cannot bear a high temperature. The author discusses the possible deficiencies as well as the advantages of this method as compared with those of Neumann and of Regnault.

In the third part the author gives his determinations of a very great number of solid bodies. The specific heat of many of them had been determined by Neumann, or by Regnault; and the almost universal agreement of the numbers found by their methods and by his own proved the comparability of his results with those of other physicists. Where there is a considerable difference, the cause is discussed. By far the greatest

number of the author's experiments are on substances whose specific heat had not been previously determined; they extend to all the more important classes of inorganic compounds, and to a great number of organic compounds.

In the fourth part the author gives a synopsis of the materials at present available and trustworthy for considering the relations between specific heat and atomic weight or composition. That is, he gives for solid bodies of known composition the atomic formula, the atomic weight, the more trustworthy determinations of specific heat, and (corresponding to these) the atomic heats, or products of the specific heats and the atomic weights.

The relations between the atomic heat and the atomic weight or the composition are discussed in the fifth part.

A discussion whether the specific heat of a body varies materially with its different physical conditions forms an introduction to this part. The influence which change of temperature of solid bodies exerts on the specific heat is considered. This difference is inconsiderable, as is also the difference of specific heats found for the same substance, according as it is hammered or annealed, hard or soft. With dimorphous varieties of the same substance, even where the specific gravity is different, the same specific heat is found in most cases. Great difference had been supposed to exist in the specific heat of a substance, according as it was crystalline or amorphous. The author shows that, for a great number of substances, there is no such difference, and that in other cases the apparent differences depend on inaccurate determinations of the specific heat. He shows that three sources of error more especially may give too great a specific heat for a substance, or for one of its various modifications:—

1. When the substance is heated to a temperature at which it begins to soften, and thus to absorb part of its latent heat of fusion.

2. If the substance is heated to a temperature at which it begins to pass into another modification, and this change, with its accompanying development of heat, is continued in the calorimeter.

3. If the substance investigated is porous, and (as was the case in the earlier methods) is directly immersed in the liquid of the calorimeter, in which case the development of heat which accompanies the moistening of porous substances comes into play.

The author arrives at the following result:—From what is at present known with certainty, one and the same body may exhibit small differences with certain physical conditions (temperature, or different degrees of density or porosity); but these differences are never so great as to furnish an explanation of cases in which a body markedly deviates from a regularity which might perhaps have been expected for it—always assuming that the determination of the specific heat, according to which the body in question forms an exception to the regularity, is trustworthy and free from foreign elements.

The author then discusses the applicability of Dalong and Petit's law.

The atomic heats of many elements * are, in accordance with this law, approximately equal; they vary between 6 and 6·8, the average being about 6·4. The explanations attempted why this law only approximately holds good, he considers inadequate. In any case there are individual elements which do not obey this law. The atomic heat of phosphorus, for instance, as deduced from direct determinations of its specific heat in the solid state, is considerably smaller (about 5·4); and still more so are those of silicium (about 4), of boron (about 2·7), and of carbon (1·8 for diamond).

A regularity, to which attention has been already drawn, is, that the quotient obtained by dividing the atomic heat of a compound by the number of elementary atoms in one molecule, is approximately equal to 6·4; equal, that is, to the atomic heat of an element according to Dulong and Petit's law. Thus the atomic heat of the chlorides R Cl and R Cl has been found to be 12·8 on the average, and of the chlorides R Cl_2=18·5. Now $\frac{12·8}{2}$=6·4, and $\frac{18·5}{3}$=6·2. The same regularity is met with in metallic bromides, iodides, and arsenides; and, according to the author's determinations, it is even found in the case of compounds which contain as many as seven, and even of nine elementary atoms. The atomic heat of Zn K_2 Cl_4 is 43·4, and that of Pt K_2 Cl_6 is 55·2; now $\frac{43·4}{7}$=6·2 and $\frac{55·2}{9}$=6·1. But the author shows at the same time that this regularity is far from being general. For the oxides of the metals the quotient is less than six, and is smaller the greater the number of atoms of oxygen in the oxide. (From the average determinations of the atomic heats, it is for the metallic oxides RO_1=$\frac{11·1}{2}$=5·6; for the oxides R_2O_3 and R_2O_3 $\frac{27·2}{5}$ =5·4; for the oxides RO_2=$\frac{13·7}{3}$=4·6.) The quotient is still smaller for compounds which contain boron as well as oxygen (for instance, it is $\frac{16·8}{4}$=4·2 for the borates, R BO_3; it is $\frac{16·6}{5}$=3·3 for boracic acid, B_2O_3), or which contain silicium (for silicic acid, SiO_2, it is $\frac{11·3}{3}$=3·8), or hydrogen (for ice, H_2O, it is $\frac{8·6}{3}$=2·9), or, finally, which contain carbon and hydrogen as well as oxygen (for succinic acid, $C_4H_6O_4$, for instance, it is $\frac{36·9}{14}$ =2·6). It may be stated in a few words, in what cases this quotient approximates to the atomic heat of most of the elements, and in what

* In accordance with recent assumptions for the atomic weights, H=1; Cl=35·5; O=16; S=32; B=10·9; C=12; Si=28. R stands for a monequivalent atom, *e. g.* As=75; Na=23; K=39·1; Ag=100; R signifies a polyequivalent atom, *e. g.* Ca=40; Pb=207; Fe=56; Co=63·4; Cr=52·2; Pt=184, &c.

cases it is less. It is near 6·4 in the case of those compounds which only contain elements whose atomic heats, in accordance with Dulong and Petit's law, are themselves approximately = 6·4. It is less in those compounds containing elements which, as exceptions to Dulong and Petit's law, have a considerably smaller atomic heat than 6·4, and which are found to be exceptions, either directly, by determinations of their specific heat in the solid state, or indirectly, by the method to be subsequently described.

After Dulong and Petit had propounded their law, Neumann showed that a similar regularity existed in the case of compounds, that is, that the atomic heats of analogous compounds are approximately equal. Regnault, as is known, has confirmed Dulong and Petit's, as well as Neumann's law, to a considerably greater extent, and for a larger number of compounds, than had been previously done. And Regnault's researches have more especially shown that the elementary atoms, now regarded as monequivalent, are, as regards the atomic heat of their compounds, comparable with the elementary atoms which are to be considered as polyequivalent. Thus, as regards atomic heat, arsenious acid, $As_2 O_3$, and sesquioxide of iron, $Fe_2 O_3$, or chloride of silver and subchloride of copper, Cu Cl, may be classed together. Of the applicability of Neumann's law, as hitherto investigated and found in the case of chemically analogous compounds, the author's experimental determinations have furnished a number of new examples. But more interest is presented by his results in reference to the applicability of this law to compounds to which it had not hitherto been supposed to apply.

In comparing compounds as regards their atomic heat, their chemical character has been taken into account, as represented by the formulæ hitherto adopted. Sulphates and chromates, for instance, were looked upon as comparable, but they would not have been classed with perchlorates, or with permanganates. According to more recent assumptions for the atomic weights of the elements, the following salts have analogous formulæ, and the adjoined atomic heats have been determined:—

Chromate of lead	Pb Cr O_4	29·0
Sulphate of lead	Pb S O_4	25·8
Permanganate of potass	K Mn O_4	28·3
Perchlorate of potass	K Cl O_4	26·3

The atomic heats of carbonates, R C O_3, of silicates, R Si O_3, of metaphosphates, R P O_3, of nitrates, R N O_3, are also very near.

But not even a common chemical behaviour, such as the bodies in this group possess—that is, a common haloid character—is necessary in order that compounds of analogous atomic composition shall show the same atomic heat. No one would think of considering magnetic oxide of iron as analogous to chromate of potass; and yet both have the same atomic structure, and determinations of their specific heat have given approximately the same atomic heat for both.

Magnetic oxide of iron	$Fe_3 O_4$	37·7
Chromate of potass.....	K_2 Cr O	36·4

And it is not less surprising that arseniate of potass, $K As O_3$, and chlorate of potass have the same atomic heat as sesquioxide of iron, $Fe_2 O_3$, or arsenious acid, $As_2 O_3$: with very different characters these compounds have approximately equal atomic heat.

But comparability of chemical compounds, as regards the atomic heat, is not limited to the cases in which, as far as can be judged, the individual atoms have analogous construction. We do not regard the atom of binoxide of tin or of titanic acid as analogous in construction to the atom of tungstate of lime or of chromate of lime; nor to nitrate of baryta, or metaphosphate of lime. But if the formulæ of those binoxides are doubled or tripled, they may be compared with these salts, and their atomic heats are then approximately equal, as is the case for compounds of analogous chemical character. The atomic heats are for—

Binoxide of tin	$2 Sn O_2 = Sn_2 O_4$	27·6
Titanic acid	$2 Ti O_2 = Ti_2 O_4$	27·3
Tungstate of lime	$Ca W O_4$	27·9
Chromate of lead	$Pb Cr O_4$	29·0
Permanganate of potass	$K Mn O_4$	28·3
Perchlorate of potass	$K Cl O_4$	26·3
Binoxide of tin	$3 Sn O_2 = Sn_3 O_6$	41·4
Titanic acid	$3 Ti O_2 = Ti_3 O_6$	41·0
Nitrate of baryta	$Ba N_2 O_6$	38·9
Metaphosphate of lime	$Ca P_2 O_6$	39·4

These results seem to give to Neumann's law a validity far beyond the limits to which it had hitherto been considered to apply. But, on the other hand, the author's comparisons go to show that neither Neumann's nor Dulong and Petit's law is universally valid.

Neumann's law is only approximate, as is well known. For such analogous compounds as, from what we know at present, are quite comparable and, in accordance with this law, ought to have equal atomic heats, Regnault found the atomic heats differing from each other by $\frac{1}{10}$ to $\frac{1}{9}$. In a few such cases there are even greater differences in the atomic heats, for which an adequate explanation is still wanting.

But there are other differences in the atomic heats of some compounds which might have been expected to have equality of atomic heat in accordance with Neumann's law—differences which occur with regularity, and for which an explanation is possible. Certain elements impress upon all their compounds the common character that their atomic heats are smaller than those of analogous compounds of other elements. This is the case, for instance, with the compounds of boron: the atomic heat of boracic acid is much less than that of the metallic oxides $R_2 O_3$ and $R_2 O_3$; the atomic heat of the borates $R B O_2$ is much less than that of the oxides $R_2 O_2 = (2 R O)$; and the atomic heat of borate of lead, $Pb B_2 O_4$, is far less than that of magnetic oxide of iron, $Fe_3 O_4$. The same is the case with compounds of carbon, if the alkaline carbonates, $R_2 C O_3$, are compared with the metallic oxides $R_3 O_3 = (3 R O)$, or the carbonates $R C O$, with the metallic oxides $R_2 O_3$ and

R_2O_3. It is seen that the compounds of those elements which, in the free state, have themselves a smaller atomic heat than most other elements, are characterized by a smaller atomic heat.

This leads the author to discuss whether it is to be assumed that the elements enter into compounds with the atomic heats which they have in the free state. This assumption is only admissible provided it can be proved that the atomic heat of a compound depends simply on its empirical formula, and not on the chemical character or rational constitution. Much of what has previously been said favours this view of the case. It is also supported by the fact, which the author proves, that similar chemical character in analogous compounds, and even isomorphism, do *not* presuppose equality in the atomic heats, if in one compound an atomic group (a compound radical) stands in the place of an elementary atom of another: for instance, the atomic heat of cyanogen compounds is considerably greater than those of the corresponding chlorine compounds, and those of ammonium materially greater than those of the corresponding potassium compounds. A further support for that assumption is found in the fact that, regardless of the chemical character, the atomic heat of complex compounds is found to be the sum of the atomic heats of simpler atomic groups, the addition of which gives the formulæ of those more complex compounds. A few cases selected from the comparisons of the author may explain this. The atomic heats have been found,—

For the oxides	RO	11·1
For binoxide of tin	SnO_2	13·8
Total for	RRO_3	24·9
For sesquioxide of iron	Fe_2O_3	26·8

Or,

For oxides $2RO$ ..... =	R_2O_2	22·2
For binoxide of tin $3SnO_2$ .. =	R_3O_6	41·4
Total for	R_5O_8	63·6
For arseniate of lead	$Pb_3As_2O_8$	65·4

Finally, the author shows, as supporting that assumption, that (as was already maintained) water is contained in solid compounds with the atomic heat of ice. The various determinations of the specific heat of ice give the atomic heat of H_2O at 8·6 for temperatures distant from 0°, and at 9·1 to 9·8 at temperatures nearer 0°. The atomic heat has been found (to adduce again a few comparisons)

For crystallized chloride of calcium	$CaCl_2+6H_2O$		75·6
For anhydrous chlorides	RCl_2		18·5
Difference for		$6H_2O$	$\frac{57·1}{6}=9·5$
For crystallized gypsum	$CaSO_4+2H_2O$		45·8
For anhydrous sulphates	RSO		26·1
Difference for		$2H_2O$	$\frac{19·7}{2}=9·9$

The opinion that the elements enter into compounds with the atomic heats they have in the free state has been already expressed; but the view has also been defended that the atomic heat of an element may differ in a compound from what it is in the free state, and may be different in different compounds. The author discusses the latter view, and criticises the reasons which may be adduced for it; he comes to the result that it is not proved and is inadmissible.

As the result of all these comparisons and observations, the author arrives at the conclusion, Each element, in the solid state and at an adequate distance from its melting-point, has *one* specific or atomic heat, which may indeed somewhat vary with physical conditions (different temperature, or different density for example), but not so much as to necessitate such variations being taken into account in considering the relation in which the specific or the atomic heat stands to the atomic weight or composition. For each element it is to be assumed that it has essentially the same specific heat or atomic heat in the free state and in compounds. He then passes on to determine what atomic heats are to be assigned to the individual elements. As data for determining this he takes (1) the atomic heats which follow from determinations of the specific heat of the elements in the free, solid state; (2) the atomic heats obtained for an element if, from the atomic heat of one of its compounds, which contains beside it only elements of known atomic heat, the atomic heats corresponding to the latter elements are subtracted; (3) the difference found between the atomic heats of analogous compounds of an element of unknown and of an element of known atomic heat, in which case the difference is taken as being the difference between the atomic heats of these two elements. The author dwells upon the fact that in the indirect deduction of an element by (2) and (3) the result may be uncertain,—first, because the atomic heats of compounds are frequently not known with certainty, as is seen by the circumstance that analogous compounds, for which there is every reason to expect equal atomic heat, are found experimentally to exhibit considerable differences; but secondly, because in such deductions the entire relative uncertainty, in the atomic heats for a compound and for that to be subtracted from its composition, is thrown upon a small number, viz. the residue remaining in the deduction.

The details of the considerations by which the author deduces the atomic heat of the individual elements cannot be gone into; the results simply, which are not all attained with equal certainty, may be adduced. The author adopts the atomic heat 1·8 for C, 2·3 for H, 2·7 for B, 3·7 for Si, 4 for O, 5 for Fl, 5·4 for P and S, 6·4 for the other elements for which or for whose compounds the atomic heat is known in somewhat more trustworthy manner, it being left undecided in the case of the latter elements, whether (in accordance with Dulong and Petit's law) they have the same atomic heats, or whether the differences in the atomic heats cannot at present be shown with certainty.

The author gives for all compounds, whose specific heat has been investigated in a trustworthy manner, a comparison of the specific heats found experimentally with those calculated on the above assumption. The atomic heat of a compound is obtained by adding the atomic heats of the elements in it, and the specific heat by dividing this atomic heat by the atomic weight. The calculated specific heat of chloride of potassium, KCl, is $\frac{6{\cdot}4+6{\cdot}4}{74{\cdot}6}=0{\cdot}172$; of sulphide of lead, Pb S, $\frac{6{\cdot}4+5{\cdot}4}{239}=0{\cdot}0494$; of borate of potass, K B O_2, it is $\frac{6{\cdot}4+2{\cdot}7+(2\times4)}{82}=0{\cdot}209$; of tartaric acid, C_4 H_6 O_6, it is $\frac{(4\times1{\cdot}8)+(6\times2{\cdot}3)+6\times4}{150}=0{\cdot}300$.

The Table, embracing 200 compounds, shows, on the whole, a sufficient agreement between the calculated and the observed specific heats. The author remarks that a closer agreement between calculation and observation cannot be hoped for than that between the observed atomic heats of those compounds for which, from all we know at present, the same atomic heat is to be expected in conformity with Neumann's law, to which in such cases, of course, calculation corresponds. In only a few cases are differences between calculation and observation met with which exceed these limits or exceed the deviation between the results of different observers for the same substance. The author states that he is far from considering the agreement between his calculations and the experimental results as a measure of the accuracy of the latter, since the bases of calculation are too far from being trustworthy. But he hopes that his Table of atomic heats will soon acquire such corrections, and therewith greater trustworthiness, as was the case with the first Table of atomic weights. Here, the data for the Table were at first but little certain, and the differences between the calculated and observed composition of chemical compounds very considerable; but the Table was the means of corrections being introduced by which these differences were diminished.

If calculation of the specific heat does not supersede the necessity of experimental determination in the solid state, and does not give a trustworthy measure for the accuracy of such determinations, it gives a rough control for the experimental determinations, and it indicates sources of error in the experiments which without it would not have been noticed. An instance may be adduced. The author found for sesquichloride of carbon, C_2 Cl_6, which, according to Faraday, melts at 160°, the specific heat between 20° and 50° to be 0·276 in one series of experiments, and 0·265 in another. Hence the number 0·27 might from this be taken to express the specific heat of the compound. But calculation gives $\frac{(2\times1{\cdot}8)+(6\times6{\cdot}4)}{237}=0{\cdot}177$, a very different number. A third series of experiments with substance once more recrystallized, gave for the specific heat between 21° and 49° 0·278, confirming the previous determinations.

It might here appear doubtful whether calculation was not refuted by experiment. The discrepancy was removed by the observation that the substance is distinctly more viscous at 50° than it is at lower temperatures, and by the suspicion that it might at 50° (that is, 100° below its melting-point) already absorb some of its latent heat of vitreous fusion. This was found to be the case; two concordant series of experiments gave as the mean of the specific heat the numbers:

Between 18° and 37°	0·178
Between 18° and 43°	0·194
Between 18° and 50°	0·277

The first two numbers differ so little that it may be supposed the number found for temperatures below 37° is very near the true specific heat of this compound; it also agrees well with the calculated number.

In the sixth part the author enters into considerations on the nature of the chemical elements.

He calls to mind the discrepancy which has prevailed, and still prevails, in reference to certain bodies, between their actual indecomposability, and the considerations, based on analogy, according to which they were held to be compound. Even after Davy had long proclaimed the elementary nature of chlorine, it was maintained that it contained oxygen. In regard both to that substance and to bromine and iodine, the view that they are peroxides of unknown elements still finds defenders. That iodine, by a direct determination of specific heat, and chlorine, by indirect deduction, are found to have an atomic heat in accordance with Dulong and Petit's law, puts out of doubt that iodine and chlorine, if compound at all, are not more so than the other elements to which this law is considered to apply.

According to Dulong and Petit's law, compounds of analogous atomic composition have approximately equal atomic heats. In general, compounds whose atom consists of a larger number of undecomposable atoms, or is of more complex constitution, have greater atomic heat. Especially in those compounds all of whose elements follow Dulong and Petit's law, is the magnitude of the atomic heat a measure of the complication, or of the degree of complication. If Dulong and Petit's law were universally valid, it might be concluded with great certainty that the so-called elements, if they are really compounds of unknown simpler substances, are compounds of the same order. It would be a remarkable result, if the art of chemical decomposition had everywhere reached its limits at such bodies as, if at all compound, have the same degree of composition. Let us imagine the simplest bodies, perhaps as yet unknown to us, the true chemical elements, to form a horizontal layer, and above them to be arranged the more simple and then the more complicated compounds; the general validity of Dulong and Petit's law would include the proof that all the elements at present assumed to be such by chemists lay in the same layer, and that, in admitting hydrogen, oxygen, sulphur, chlorine,

and the various metals as elements, chemistry has penetrated to the same depth in that range of inquiry, and has found at the same depth the limit to its advance.

But with the proof that this law is not universally true, the conclusion to which this result leads loses its authority. If we start from the elements at present assumed in chemistry, we must admit rather that the magnitude of the atomic heat of a body does not depend on the number of elementary atoms contained in a molecule, or on the complication of its composition, but on the atomic heat of the elementary atoms which enter into its composition. It is possible that a decomposable body may have the same atomic heat as an element. Chlorine might certainly be the peroxide of an unknown element which had the atomic heat of hydrogen; the atomic heat of peroxide of hydrogen, $H\Theta$, in the solid state or in solid compounds, must be $=2{\cdot}3+4=6{\cdot}3$, agreeing very nearly with the atomic heats of iodine, chlorine, and the elements which follow Dulong and Petit's law.

In a very great number of compounds the atomic heat gives more or less accurately a measure for the complication of the composition. And this is also the case with those compounds which, from their chemical deportment, are comparable to the undecomposed bodies. If ammonium or cyanogen had not been decomposed, or could not be by the chemical means at present available, the greater atomic heats of the compounds of these bodies, as compared with analogous potassium or chlorine compounds, and the greater atomic heats of ammonium and cyanogen obtained by indirect determination, as compared with those of potassium and chlorine, would indicate the compound nature of those so-called compound radicals. The conclusion appears legitimate, that, for the so-called elements, the directly or indirectly determined atomic heats are a measure for the complication of their composition. Carbon and hydrogen, for example, if not themselves actually simple bodies, are yet simpler compounds of unknown elements than silicium or oxygen; and still more complex are the elements which may be considered as following Dulong and Petit's law.

It may appear surprising, and even improbable, that so-called elements, which can replace each other in compounds, as for instance hydrogen and the metals, or which enter into isomorphous compounds as corresponding elements, like silicium and tin, should possess unequal atomic heats and unequal complication of composition. But this really is not more surprising than that undecomposable bodies and obviously compound bodies, hydrogen and hyponitric acid, or potassium and ammonium, should, without altering the chemical character of the compound, replace one another, or even be present in isomorphous compounds as corresponding constituents.

The author concludes his memoir with the following words:—"I have here expressed opinions, in reference to the nature of the so-called elements, which appear to depend upon allowable conclusions from well-

demonstrated principles. It is of the nature of the case, that with these opinions the certain basis of the actual, and of what can be empirically proved, is left. It must also not be forgotten that these conclusions only give some sort of clue as to which of the present undecomposable bodies are of more complicated, and which of simpler composition, and nothing as to what the simpler substances are which are contained in the more complicated. Consideration of the atomic heats may declare something as to the structure of a compound atom, but can give no information as to the qualitative nature of the simpler substances used in the construction of the compound atoms. But even if these conclusions are not free from uncertainty and imperfection, they appear to me worthy of attention in a subject which is still so shroudedin darkness as the nature of the undecomposed bodies."

III. "On some Foraminifera from the North Atlantic and Arctic Oceans, including Davis Strait and Baffin Bay." By W. Kitchen Parker, F.Z.S., and Professor T. Rupert Jones, F.G.S. Communicated by Professor Huxley. Received April 26, 1864.

(Abstract.)

Having received specimens of sea-bottom, by favour of friends, from Baffin Bay (soundings taken in one of Sir E. Parry's expeditions), from the Hunde Islands in Davis Strait (dredgings by Dr. P. C. Sutherland), from the coast of Norway (dredgings by Messrs. M'Andrew and Barrett), and from the whole width of the North Atlantic (soundings by Commander Dayman), the authors have been enabled to form a tolerably correct estimate of the range and respective abundance of several species of Foraminifera in the Northern seas; and the more perfectly by taking Professor Williamson's and Mr. H. B. Brady's researches in British Foraminifera as supplying the means of estimating the Foraminiferal fauna of the shallower sea-zones at the eastern end of the great "Celtic Province," and the less perfect researches of Professor Bailey on the North American coast, for the opposite, or "Virginian" end,—thus presenting for the first time the whole of a Foraminiferal fauna as a natural-history group, with its internal and external relationships.

The relative abundance or scarcity and the locations of the several species and chief varieties are shown by Tables; and their distribution in other seas (South Atlantic, Pacific, and Indian Oceans, and the Mediterranean and Red Seas) is also tabulated; and in the descriptive part of the memoir notes on their distribution, both in the recent and the fossil state, are carefully given.

In the description of the species and varieties there are observations made on those forms which have been either little understood, hitherto

unknown, or mistaken; and the relationship, by structure or by imitation, of the species and varieties is dwelt upon. For the description of the better-known Foraminifera, the memoir refers to the works of Williamson and Carpenter.

The authors enumerate 109 specific and varietal forms, most of which receive descriptive comment, and all of which are figured in five plates (two for the North Atlantic and three for the Arctic Foraminifera) with upwards of 340 figures.

The relationships of the *Lagenæ* are specially treated of. *Uvigerina*, *Globigerina*, and especially some of the *Rotalinæ* (*Planorbulina*, *Discorbina*, *Rotalia*, *Pulvinulina*) and *Polystomella* (including *Nonionina*) are among those which are well represented in the fauna under description, and have received much attention in the memoir.

The Society then adjourned over the Whitsuntide Recess to Thursday, May 26.

May 26, 1864.

Major-General SABINE, President, in the Chair.

The following communications were read :—

I. "Note on the Variations of Density produced by Heat in Mineral Substances." By Dr. T. L. Phipson, F.C.S., &c. Communicated by Professor Tyndall. Received April 16, 1864.

That any mineral substance, whether crystallized or not, should *diminish* in density by the action of heat might be looked upon as a natural consequence of dilatation being produced in every case and becoming permanent. Such diminution of density occurs with idocrase, Labradorite, felspar, quartz, amphibole, pyroxene, peridote, Samarskite, porcelain, and glass. But Gadolinite, zircons, and yellow obsidians *augment* in density from the same cause. This again may be explained by assuming that under the influence of a powerful heat these substances undergo some permanent molecular change. But in this Note I have to show that this molecular change is *not permanent* but intermittent, at least as regards the species I have examined, and probably with all the others. Such researches, while tending to elucidate certain points of chemical geology, may likewise add something to our present knowledge of the modes of action of heat.

My experiments were undertaken to prove an interesting fact announced formerly by Magnus, namely, that specimens of idocrase after fusion had diminished considerably in density without undergoing any change of composition: before fusion their specific gravity ranged from 3·349 to 3·45, and after fusion only 2·93 to 2·945. Having lately received specimens of this and other minerals brought from Vesuvius in January last by my friend Henry Rutter, Esq., I determined upon repeating this experiment of

Magnus. I found, *first*, that what he stated for idocrase and for a specimen of reddish-brown garnet was also the case with the whole family of garnets as well as the minerals of the idocrase group; *secondly*, that it is not necessary to *melt* the minerals: it is sufficient that they should be heated to redness without fusion, in order to occasion this change of density; *thirdly*, that the diminished density thus produced by the action of a red heat is not a permanent state, but that the specimens, in the course of a month or less, resume their original specific gravities.

These curious results were first obtained by me with a species of lime garnet, in small yellowish crystals, exceedingly brilliant and resinous, almost granular, fusing *with difficulty* to a *black* enamel, accompanied with very little leucite and traces of grossular, and crystallized in the second system.

Specimens weighing some grammes had their specific gravity taken with great care, and by the method described by me in the 'Chemical News' for 1862. They were then perfectly dried and exposed for about a quarter of an hour to a bright red heat. When the whole substance of the specimen was observed to have attained this temperature, without trace of fusion, it was allowed to cool, and when it had arrived at the temperature of the atmosphere, its specific gravity was again taken by the same method as before. The diminution of density being noted, the specimens were carefully dried, enveloped in several folds of filtering paper, and put aside in a box along with other minerals. In the course of a month it occurred to me that it would be interesting to take the specific gravity again, in order to ascertain whether it had not returned to its original figure, when, to my surprise, I found that each specimen had effectively increased in density and had attained its former specific gravity. Thus:—

Lime garnet (from Vesuvius).

Original density.	Density after being heated red-hot for a quarter of an hour and allowed to cool.	Density determined in a month after the experiments.
I. 3·345	2·978	3·344
II. 3·350	2·980	3·350
III. 3·349	2·977	3·345

The same experiments were made with several other minerals belonging to the idocrase and garnet family, and always with similar results. Now I ask, what becomes of the heat that seems to be thus shut up in a mineral substance for the space of a month? The substance of the mineral is dilated, the distance between its molecules is enlarged, but these molecules slowly approach each other again, and in the course of some weeks resume their original positions. What induces the change? or how does it happen that the original specific gravity is not acquired immediately the substance has cooled?* Will the same phenomenon show itself with other families of minerals or with the metallic elements?

* Some minerals, like euclase, that become *electric* by heat, retain that state for a

Such are the points which I propose to examine in the next place; in the mean time the observations I have just alluded to are a proof that bodies can absorb a certain amount of heat not indicated by the thermometer (which becomes *latent*), and that this is effected *without the body undergoing a change of state*; secondly, that they slowly part with this heat again until they have acquired their original densities; thirdly, so many different substances being affected by a change of density when melted or simply heated to redness and allowed to cool, it is probable this property will be found to belong, more or less, to all substances without exception.

II. "On the Spectra of some of the Fixed Stars." By W. Huggins, F.R.A.S., and William A. Miller, M.D., LL.D., Treasurer & V.P.R.S., Professor of Chemistry, King's College, London. Received April 28, 1864.

(Abstract.)

After a few introductory remarks, the authors describe the apparatus which they employ, and their general method of observing the spectra of the fixed stars and planets. The spectroscope contrived for these inquiries was attached to the eye end of a refracting telescope of 10 feet focal length, with an 8-inch achromatic object-glass, the whole mounted equatorially and carried by a clock-movement. In the construction of the spectroscope, a plano-convex cylindrical lens, of 14 inches focal length, was employed to convert the image of the star into a narrow line of light, which was made to fall upon a very fine slit, behind which was placed an achromatic collimating lens. The dispersing portion of the arrangement consisted of two dense flint-glass prisms; and the spectrum was viewed through a small achromatic telescope with a magnifying power of between 5 and 6 diameters. Angular measures of the different parts of the spectrum were obtained by means of a micrometric screw, by which the position of the small telescope was regulated. A reflecting prism was placed over one half of the slit of the spectroscope, and by means of a mirror, suitably adjusted, the spectra of comparison were viewed simultaneously with the stellar spectra. This light was usually obtained from the induction spark taken between electrodes of different metals. The dispersive power of the apparatus was sufficient to enable the observer to see the line *Ni* of Kirchhoff between the two solar lines D; and the three constituents of the magnesium group at *b* are divided still more evidently *. Minute

considerable time. The *increase* of density of Gadolinite and the *decrease* of density of Samarskite by the action of heat are accompanied by a vivid emission of light, as mentioned in my work on 'Phosphorescence' &c., pp. 31 and 32, where H. Rose's ingenious experiment is described.

* Each unit of the scale adopted was about equal to $\frac{1}{1800}$th of the distance between A and H in the solar spectrum. The measures on different occasions of the same line rarely differed by one of these units, and were often identical.

details of the methods adopted for testing the exact coincidence of the corresponding metallic lines with those of the solar and lunar spectrum, are given, and the authors then proceed to give the results of their observations.

Careful examination of the spectrum of the light obtained from various points of the moon's surface failed to show any lines resembling those due to the earth's atmosphere. The planets Venus, Mars, Jupiter, and Saturn were also examined for atmospheric lines, but none such could be discovered, though the characteristic aspect of the solar spectrum was recognized in each case; and several of the principal lines were measured, and found to be exactly coincident with the solar lines.

Between forty and fifty of the fixed stars have been more or less completely examined; and tables of the measures of about 90 lines in Aldebaran, nearly 80 in α Orionis, and 15 in β Pegasi are given, with diagrams of the lines in the two stars first named. These diagrams include the results of the comparison of the spectra of various terrestrial elements with those of the star. In the spectrum of Aldebaran coincidence with nine of the elementary bodies were observed, viz. sodium, magnesium, hydrogen, calcium, iron, bismuth, tellurium, antimony, and mercury; in seven other cases no coincidence was found to occur.

In the spectrum of α Orionis five cases of coincidence were found, viz. sodium, magnesium, calcium, iron, and bismuth, whilst in the case of ten other metals no coincidence with the lines of this stellar spectrum was found.

β Pegasi furnished a spectrum closely resembling that of α Orionis in appearance, but much weaker: only a few of the lines admitted of accurate measurement, for want of light; but the coincidence of sodium and magnesium was ascertained; that of barium, iron, and manganese was doubtful. Four other elements were found not to be coincident. In particular, it was noticed that the lines C and F, corresponding to hydrogen, which are present in nearly all the stars, are wanting in α Orionis and β Pegasi.

The investigation of the stars which follow is less complete, and no details of measurement are given, though several points of much interest have been ascertained.

Sirius gave a spectrum containing five strong lines, and numerous finer lines. The occurrence of sodium, magnesium, hydrogen, and probably of iron, was shown by coincidence of certain lines in the spectra of these metals with those in the star. In α *Lyræ* the occurrence of sodium, magnesium, and hydrogen was also shown by the same means. In *Capella* sodium was shown, and about twenty of the lines in the star were measured. In *Arcturus* the authors have measured about thirty lines, and have observed the coincidence of the sodium line with a double line in the star-spectrum. In *Pollux* they obtained evidence of the presence of sodium,

magnesium, and probably of iron. The presence of sodium was also indicated in *Procyon* and *α Cygni*.

In no single instance have the authors ever observed a star-spectrum in which lines were not discernible, if the light were sufficiently intense, and the atmosphere favourable. Rigel, for instance, which some authors state to be free from lines, is filled with a multitude of fine lines.

Photographs of the spectra of Sirius and Capella were taken upon collodion; but though tolerably sharp, the apparatus employed was not sufficiently perfect to afford any indication of lines in the photograph.

In the concluding portion of their paper, the authors apply the facts observed to an explanation of the colours of the stars. They consider that the difference of colour is to be sought in the difference of the constitution of the investing stellar atmospheres, which act by absorbing particular portions of the light emitted by the incandescent solid or liquid photosphere, the light of which in each case they suppose to be the same in quality originally, as it seems to be independent of the chemical nature of its constituents, so far as observation of the various solid and liquid elementary bodies, when rendered incandescent by terrestrial means, appears to indicate.

III. "A Second Memoir on Skew Surfaces, otherwise Scrolls." By A. Cayley, Esq., F.R.S. Received April 29, 1864.

(Abstract.)

The principal object of the present memoir is to establish the different kinds of skew surfaces of the fourth order, or Quartic Scrolls; but, as preliminary thereto, there are some general researches connected with those in my former memoir "On Skew Surfaces, otherwise Scrolls" (Phil. Trans. vol. 153. 1863, pp. 453, 483), and I also reproduce the theory (which may be considered as a known one) of cubic scrolls; there are also some concluding remarks which relate to the general theory. As regards quartic scrolls, I remark that M. Chasles, in a footnote to his paper, "Description des Courbes de tous les ordres situées sur les surfaces réglées du troisième et du quatrième ordres," Comptes Rendus, t. liii. (1861), see p. 888, states, "les surfaces réglées du quatrième ordre . . . . admettent *quatorze* espèces." This does not agree with my results, since I find only eight species of quartic scrolls; the developable surface or "torse" is perhaps included as a "surface réglée;" but as there is only one species of quartic torse, the deficiency is not to be thus accounted for. My enumeration appears to me complete, but it is possible that there are subforms which M. Chasles has reckoned as distinct species.

IV. "On the Differential Equations which determine the form of the Roots of Algebraic Equations." By GEORGE BOOLE, F.R.S., Professor of Mathematics in Queen's College, Cork. Received April 27, 1864.

(Abstract.)

Mr. Harley* has recently shown that any root of the equation

$$y^n - xy + (n-1)\,x = 0$$

satisfies the differential equation

$$y - \frac{\left(\mathrm{D} - \frac{2n-1}{n}\right)\left(\mathrm{D} - \frac{3n-2}{n}\right) \ldots \left(\mathrm{D} - \frac{n^2-n+1}{n}\right)}{\mathrm{D}\,(\mathrm{D}-1) \ldots (\mathrm{D}-n+1)} e^{(n-1)\theta}\, y = 0, \quad \ldots (1)$$

in which $e^\theta = x$, and $\mathrm{D} = \frac{d}{d\theta}$, provided that n be a positive integer greater than 2. This result, demonstrated for particular values of n, and raised by induction into a general theorem, was subsequently established rigorously by Mr. Cayley by means of Lagrange's theorem.

For the case of $n=2$, the differential equation was found by Mr. Harley to be

$$y - \frac{\mathrm{D} - \frac{3}{2}}{\mathrm{D}} e^\theta y = \tfrac{1}{2} e^\theta \quad \ldots \ldots \ldots \quad (2)$$

Solving these differential equations for the particular cases of $n=2$ and $n=3$, Mr. Harley arrived at the actual expressions of the roots of the given algebraic equation for these cases. That all algebraic equations up to the fifth degree can be reduced to the above trinomial form, is well known.

A solution of (1) by means of definite triple integrals in the case of $n=4$ has been published by Mr. W. H. L. Russell; and I am informed that a general solution of the equation by means of a definite single integral has been obtained by the same analyst.

While the subject seems to be more important with relation to differential than with reference to algebraic equations, the connexion into which the two subjects are brought must itself be considered as a very interesting fact. As respects the former of these subjects, it may be observed that it is a matter of quite fundamental importance to ascertain for what forms of the function $\phi\,(\mathrm{D})$, equations of the type

$$u + \phi\,(\mathrm{D})\, e^{n\theta} u = 0 \quad \ldots \ldots \ldots \quad (3)$$

admit of finite solution. We possess theorems which enable us to deduce from each known integrable form, an infinite number of others. Yet there is every reason to think that the number of really primary forms—of forms the knowledge of which, in combination with such known theorems, would enable us to solve all equations of the above type that are finitely solvable—

* Memoirs of the Literary and Philosophical Society of Manchester.

is extremely small. It will, indeed, be a most remarkable conclusion, should it ultimately prove that the primary solvable forms in question stand in some absolute connexion with a certain class of algebraic equations.

The following paper is a contribution to the general theory under the aspect last mentioned. In endeavouring to solve Mr. Harley's equation by definite integrals, I was led to perceive its relation to a more general equation, and to make this the subject of investigation. The results will be presented in the following order:—

First, I shall show that if u stand for the mth power of any root of the algebraic equation

$$y^n - xy^{n-1} - 1 = 0,$$

then u, considered as a function of x, will satisfy the differential equation

$$[\mathrm{D}]^n u + \left[\frac{n-1}{n}\mathrm{D} + \frac{m}{n} - 1\right]^{n-1} \left(\frac{\mathrm{D}}{n} - \frac{m}{n} - 1\right) e^{n\theta} u = 0,$$

in which $e^\theta = x$, $\mathrm{D} = \frac{d}{d\theta}$, and the notation

$$[a]^b = a\,(a-1)\,(a-2)\,..\,(a-b+1)$$

is adopted.

Secondly, I shall show that for particular values of m, the above equation admits of an immediate first integral, constituting a differential equation of the $n-1$th order, and that the results obtained by Mr. Harley are particular cases of this depressed equation, their difference of form arising from difference of determination of the arbitrary constant.

Thirdly, I shall solve the general differential equation by definite integrals.

Fourthly, I shall determine the arbitrary constants of the solution so as to express the mth power of that real root of the proposed algebraic equation which reduces to 1 when $x=0$.

The differential equation which forms the chief subject of these investigations certainly occupies an important place, if not one of exclusive importance, in the theory of that large class of differential equations of which the type is expressed in (3). At present, I am not aware of the existence of any differential equations of that particular type which admit of finite solution at all otherwise than by an ultimate reduction to the form in question, or by a resolution into linear equations of the first order. It constitutes, in fact, a generalization of the form

$$u + \frac{a(\mathrm{D}-2)^2 \pm n^2}{\mathrm{D}\,(\mathrm{D}-1)} e^{2\theta} u = 0$$

given in my memoir "On a General Method in Analysis" (Philosophical Transactions for 1844, part 2).

V. "A Comparison of the most notable Disturbances of the Magnetic Declination in 1858 and 1859 at Kew and Nertschinsk, preceded by a brief Retrospective View of the Progress of the Investigation into the Laws and Causes of the Magnetic Disturbances." By Major-General Edward Sabine, R.A., President of the Royal Society. Received April 28, 1864.

(Abstract.)

The author commences this paper by taking a retrospective view of the principal facts which have been established regarding the magnetic disturbances, considered as a distinct branch of the magnetic phenomena of the globe, from the time when they were first made the objects of systematic investigation by associations formed for that express purpose, at Berlin in 1828 and at Göttingen in 1834, and dwelling more particularly on the results subsequently obtained by the more complete and extended researches instituted in 1840 by the British Government on the joint recommendation of the Royal Society and of the British Association for the Advancement of Science.

The Berlin Association, formed under the auspices of Baron Alexander von Humboldt, consisted of observers in very distant parts of the European continent, by whom the precise direction of the declination-magnet was recorded simultaneously at hourly intervals of absolute time, at forty-four successive hours at eight concerted periods of the year, which thence obtained the name of "Magnetic Terms." By the comparison of these hourly observations it became known that the declination was subject to very considerable fluctuations, happening on days which seemed to be casual and irregular, but were the same at all the stations, consequently over the continent of Europe generally. This conclusion was confirmed by the Göttingen Association, established at the instance and under the superintendence of MM. Gauss and Weber, by whom the "Term-observations" were extended to six periods in the year, each of twenty-four hours' duration, the records being made at intervals of five minutes. The number of the stations at which these observations were made was about twenty, distributed generally over the continent of Europe, but not extending beyond it. They were continued from 1834 to 1841. The observations themselves, as well as the conclusions drawn from them by MM. Gauss and Weber, were published in the well-known periodical entitled 'Resultate aus der Beobachtungen des magnetischen Vereins.' The synchronous character of the disturbances, over the whole area comprehended by the Association, was thoroughly confirmed: the disturbing action was found to be so considerable as to occasion frequently a partial, and sometimes even a total obliteration of the regular diurnal movements, and to be of such general prevalence over the greater part of Europe, not only in the larger, but in most of the smaller oscillations, as to make it in a very high degree improbable that they could

have either a local or an atmospherical origin. No connexion or correspondence was traceable between the indications of the magnetical and meteorological instruments; nor had the state of the weather any perceptible influence. It happened very frequently that either an extremely quiescent state of the needle or a very regular and uniform progress was preserved during the prevalence of the most violent storm; and as with wind-storms, so with thunder-storms, as even when close at hand they appeared to exercise no perceptible influence on the magnet. At some of the most active of the Göttingen stations the fluctuations of the horizontal force were observed contemporaneously with those of the declination-magnet, by means of the bifilar magnetometer devised by M. Gauss: both elements were generally disturbed on the same days and at the same hours. The magnitude of the disturbances appeared to diminish as their action was traced from north to south, giving rise to the conclusion that the focus whence the most powerful disturbances in the northern hemisphere emanated might perhaps be successfully sought in parts of the globe to the north or north-west of the area comprehended by the stations. The intercomparison of the records obtained at the different stations showed moreover that the same element was very differently affected at the same hours at different stations; and that occasionally the same disturbance showed itself in different elements at different stations. The general conclusion was therefore thus drawn by M. Gauss, that "we are compelled to admit that on the same day and at the same hour various forces are contemporaneously in action, which are probably quite independent of one another and have very different sources, and that the effects of these various forces are intermixed in very dissimilar proportions at various places of observation relatively to the position and distance of these latter; or these effects may pass one into the other, one beginning to act before the other has ceased. The disentanglement of the complications which thus occur in the phenomena at every individual station will undoubtedly prove very difficult. Nevertheless we may confidently hope that these difficulties will not always remain insuperable, when the simultaneous observations shall be much more widely extended. It will be a triumph of science should we at some future time succeed in arranging the manifold intricacies of the phenomena, in separating the individual forces of which they are the compound result, and in assigning the source and measure of each."

In the British investigations, which commenced in 1840, the field of research was extended so as to include the most widely separated localities in both hemispheres, selected chiefly with reference to diversity of geographical circumstances, or to magnetic relations of prominent interest. Suitable instruments were provided for the observation of each of the three magnetic elements; the scheme of research comprehended not alone the casual and irregular fluctuations which had occupied the chief attention of the German associations, but also "the actual distribution of the magnetic influence over the globe at the present epoch in its mean or average state,

together with all that is not permanent in the phenomena, whether it appear in the form of momentary, daily, monthly, semiannual, or annual change, or in progressive changes receiving compensation possibly, either in whole or in part, in cycles of unknown relation and unknown period." The magnetic disturbances to which the notices in the present paper are limited, form a small but important branch of this extensive inquiry, and are referred to in the instructions prepared by the Royal Society in terms which are recalled by the author on the present occasion, because they are explanatory of the principles on which the coordination of the results obtained in such distant parts of the world has been conducted, and the conclusions derived from them established. In pages 2 and 3 of the Report embodying the instructions drawn up by the Royal Society, it is stated that "the investigation of the laws, extent, and mutual relations of the casual and transitory variations is become essential to the successful prosecution of magnetic discovery . . . . because the theory of those transitory changes is in itself one of the most interesting and important points to which the attention of magnetic observers can be turned, as they are no doubt intimately connected with the general causes of terrestrial magnetism, and will probably lead us to a much more perfect knowledge of those causes than we now possess." In the opinion thus expressed, the author, who was himself one of the committee by whom the Report was drawn up, fully concurred; and having been appointed by Her Majesty's Government to superintend the observations made at the British Colonial observatories, and to coordinate and publish their results, he has endeavoured to show in this paper that the methods pursued have been in strict conformity with these instructions, and also that the conclusions derived are in accordance with the anticipations expressed therein.

Inferences regarding the "general causes of terrestrial magnetism" must be based upon the knowledge we possess of the actual distribution of the magnetic influence on the surface of the globe, since that is the only part which is accessible to us. In regard to this distribution, the Report itself refers continually to two works, then recently published, as containing the embodiment of the totality of the known phenomena, viz. a "Memoir on the Variations of the Magnetic Force in different parts of the Earth's Surface," published in 1838 in the Reports of the British Association, and M. Gauss's 'Allgemeine Theorie des Erdmagnetismus,' published in 1839. In both these works the facts, as far as they had been ascertained, were conformable in their main features to the theory, first announced by Dr. Halley in his Papers in the Philosophical Transactions for 1683 and 1693, of a *double* system of magnetic action, the direction and intensity of the magnetic force being, at all points of the earth's surface, the resultants of the two systems. In both these works the Poles, or Points of greatest force (in the northern hemisphere) were traced nearly to the same localities—viz. one in the northern parts of the American continent, and the other in the northern parts of the Europæo-Asiatic continent,—their

geographical positions, as taken from M. Gauss's 'Allgemeine Theorie,' being, in America, lat. 55°, long. 263° E., and in Siberia lat. 71°, long. 116° E. Combining then the expectation expressed in the Report of "a probable connexion existing between the casual and transitory magnetic variations and the general phenomena of terrestrial magnetism," with M. Gauss's conclusion from the Göttingen researches, that "the sources of the magnetic disturbances in Europe might possibly be successfully sought in parts of the globe to the north or to the north-west of the European continent," it seemed reasonable to anticipate that a connexion might be found to exist between the "points of origin" of the disturbances, if these could be more precisely ascertained, and the critical localities of the earth's magnetism above referred to. To put this question to the test, the first step was to ascertain in a more satisfactory way than had been previously attempted, the laws of the disturbances themselves. The process by which a portion of the observations exhibiting the effects of the disturbing action in a very marked degree may be separated from the others, and subjected to a suitable analysis for the determination of their general laws, has been fully described elsewhere. The immediate effect of its application was to show that, casual and irregular as the disturbances might appear to be in the times of their occurrence, they were, in *their mean effects*, strictly periodical phenomena, characterized by laws distinct from those of any other periodical phenomena with which we were then acquainted, and traceable directly to the Sun as their primary source, inasmuch as they were found to be governed everywhere by laws depending upon the solar hours. To those who are familiar with the theory by which the passage of light from the sun to the earth is explained, an analogous transmission of magnetic influences from the sun to the earth may appear to present no particular difficulty. It is when the influences reach the earth that the modes of their reception, distribution, and transmission are less clearly seen and understood; but these are within our own proper terrestrial domain and sphere of research; and accordingly it was to these that the author's attention was directed. Wherever the disturbances had been observed and were analyzed, it was found that those of the declination were occasionally deflections to the east and occasionally deflections to the west of the mean position of the magnet, and those of the horizontal and vertical forces occasionally increased and occasionally diminished the respective forces. The disturbances of each element were therefore separated into two categories, according as they belonged to one or to the other class. Each category was found to present diurnal progressions, of systematic regularity, but quite distinct from one another, and so far in accordance with M. Gauss's inference of the existence of various forces contemporaneously in action, independent of one another, and having different originating sources. Confining our view, for simplicity, to one alone of the elements, viz. the declination, its two categories (of easterly and of westerly deflection) presented, wherever they were examined,

the same distinctive features; the local hours of maximum and minimum varied at different stations, but the same two dissimilar forms were everywhere presented by the curves representing the two diurnal progressions.

Having thus traced apparently two sources in which the disturbances might be supposed to originate, the possible connexion of these with the points of maximum attraction in the two systems of the magnetic terrestrial distribution presented itself as the next object of fitting research. It was inferred that if two stations were selected in nearly the same latitude, but situated one decidedly on the eastern side and the other decidedly on the western side of one of the points referred to, the curve of the easterly deflection at the one station would perhaps be found to correspond with the curve of westerly deflection at the other station at the same hours of absolute time, and *vice versâ*. The Kew photograms in the five years 1858 to 1862 supplied the necessary data for one of the two stations, viz. the one to the west of the point of maximum attraction of one of the two magnetic systems, whilst Pekin, where hourly observations from 1851 to 1855 inclusive are recorded in the 'Annales de l'Observatoire Central Physique de Russie,' might supply a station on its eastern side. As this comparison might be regarded somewhat in the light of a crucial experiment, the reliance to which the Pekin observations were entitled was examined by the very delicate test afforded by rewriting the observations recorded at solar hours in hours of lunar time, and examining the lunar-diurnal variation thence derived. When this is found to come out systematically and well, and similarly in different years, the observations which have furnished it may be safely regarded as trustworthy. The Pekin observations corresponded satisfactorily to this test, and in the Philosophical Transactions for 1863, Art. XII., the comparison was made of the Kew and Pekin disturbance-deflections, the result showing that "the conical form and single maximum which characterize the curve of the easterly deflections at Kew, characterize the curve of the westerly deflections at Pekin at approximately the same hours of absolute time." For a further trial of this important result, a second comparison of the same kind was made, being that of the curves of the disturbance-deflections at Nertschinsk from 1851 to 1857, also recorded in the 'Annales de l'Observatoire,' &c., with those from 1858 to 1862 at Kew. Nertschinsk is about 12° north of Pekin, and is nearly in the same longitude as that station, whilst its latitude is almost identical with that of Kew. The Nertschinsk observations were subjected to the same test in respect to accuracy as those of Pekin, and with a similarly satisfactory result. The comparison of the disturbance-deflections showed a still more perfect accord between the curves representing the easterly deflections at Kew and the westerly at Nertschinsk at approximately the same hours of absolute time.

The present paper contains a further comparison of the nearly synchronous disturbances at Kew and at Nertschinsk on the days of most notable disturbance at both stations in 1858 and 1859, the comparison being

limited to those two years inasmuch as the Kew record did not commence until January 1858, whilst the hourly observations at Nertschinsk for 1860 and subsequent years have not yet reached England. The deflections at Nertschinsk from the normals of the same month and hour, on forty-four days in 1858 and 1859, are given in a Table similar in all respects to the Table, in the Philosophical Transactions for 1863, showing the deflections on the most notable days of disturbance at Kew in the same years. The comparison of the two Tables is discussed in some detail; but it is sufficient to state here that the general conclusions are quite in accordance with those arrived at in the previous comparisons.

The steps by which the author was led to a discovery of the *decennial variation* in the magnetic disturbances, and to its identification in period and epochs with the variation in the magnitude and frequency of the sun-spots resulting from the observations of M. Schwabe since their commencement in 1826, are too well known to need repetition on this occasion. But they furnish the ground on which, in this paper, he has for the first time suggested the possibility that a cosmical connexion of a somewhat similar nature may be hereafter recognized as the origin and source of one of the two magnetic systems which cooperate in producing the general phenomena of the variations of the magnetic direction and force in different parts of the globe. The author's suggestion is, that the one of the two systems which is distinguished by its possessing a systematic and continuous movement of geographical translation, thereby giving rise to the phenomena of the secular change, may be referrible to direct solar influence operating in a cycle of yet unknown duration. The phenomena of the secular change in the earth's magnetism have hitherto received no satisfactory explanation whatsoever; and they have all the characters befitting what we might suppose to be the effects of a cosmical cause. Some of the objections which might have impeded the reception of such an hypothesis before we had learnt to recognize in the sun itself a source of magnetic energy, and to identify magnetic variations observed on the earth with physical changes which manifest themselves to our sight in the photosphere of the sun, are no longer tenable. It is true that we do not yet possess similar ocular evidence of a solar cycle of the much longer duration which would correspond to the secular change in the distribution of terrestrial magnetism. But careful observations of the variable aspects of the solar disk can only be said to be in their commencement, and it would be premature to assume that no visible phenomena will be discovered in the sun which will render the evidence of connexion as complete in the one case as in the other. Such evidence, however, is not a necessary condition of an existing connexion; the decennial period would have been equally true (though not so readily perceived by us) if the sun-spots had been less conspicuous.

"On the degree of uncertainty which Local Attraction, if not allowed for, occasions in the Map of a Country, and in the Mean Figure of the Earth as determined by Geodesy; a Method of obtaining the Mean Figure free from ambiguity by a comparison of the Anglo-Gallic, Russian, and Indian Arcs; and Speculations on the Constitution of the Earth's Crust." By the Venerable J. H. Pratt, Archdeacon of Calcutta. Communicated by Prof. G. G. Stokes, Sec. R.S. Received October 5, 1863*.

1. In former communications to the Royal Society I have shown that Local Attraction, owing to the amount it in some places attains, is a more troublesome element to deal with in geodetical operations than had generally been supposed. The Mountains and the Ocean were shown to combine to make the deviation of the plumb-line as much as 22″·71, 17″·23, 21″·05, 34″·16 (or quantities not differing materially from them) in the four principal stations of the Great Arc of India between Cape Comorin and the Himmalayas—viz. at Punnœ (8° 9′ 31″), Damargida (18° 3′ 15′), Kalianpur (24° 7′ 11″), Kaliana (29° 30′ 48″); and how much these might be increased or lessened by the effect of variations of density in the crust below t was difficult to say. Deviations amounting to at least such quantities as 7″·61 and 7″·87 were shown to exist in the stations of the Indian Arc, arising from this last cause (see Phil. Trans. 1861, p. 593 (4) and (5)).

M. Otto Struve has lately called attention to similarly important deflections caused by local attraction in Russia—and especially to a remarkable difference of deflection at two stations near Moscow, only about eighteen miles apart, amounting to as much as 18″, which is attributed to an invisible unknown cause in the strata below (see Monthly Notices of the Royal Astronomical Society, April 1862).

2. It is therefore an important inquiry, What degree of uncertainty does Local Attraction, if not allowed for, introduce into the two problems of geodesy, viz. (1) obtaining correct Maps of a country, and (2) determining the Mean Figure of the Earth. I have pointed out the effect on mapping in India, as far as determining the latitudes is concerned, in a former paper. I propose now to consider the subject generally with reference to any country, and taking into account the longitudes as well as the latitudes. The effect upon the determination of the mean figure of the earth I discuss at greater length. By a change, I venture to call it a correction, of Bessel's method of applying the principle of least squares to the problem, I obtain formulæ for the semiaxes and ellipticity of the Mean Figure involving expressions for the unknown local deflections of the plumb-line at the standard- or reference-stations of the several arcs made use of in the calculation. These formulæ at once show the great degree of uncertainty which an ignorance of the amount of local attraction must introduce into the determi-

* Read November 26, 1863. See Abstract, vol. xiii. p. 18.

nation of the mean figure. After this I obtain formulæ for the mean figures of the Anglo-Gallic, Russian, and Indian Arcs by the same method, each involving the expression for the unknown local deflection of the plumb-line at the reference-station of the arc concerned. I then show that values of these three unknown deflections can be found which will make the three ellipses which represent the three great arcs almost precisely the same. These deflections are not extravagant quantities, but quite the contrary, being small. I infer, then, that the mean of these three ellipses is in fact the Mean Figure of the Earth, and in this way surmount what was the apparently insuperable difficulty which our ignorance of the amount of local attraction threw in the way of the solution of the problem. The paper concludes with some speculations on the constitution of the earth's crust flowing from the foregoing calculations.

§ 1. *Effect of Local Attraction on Mapping a Country.*

3. In determining differences of latitude and longitude between places by means of the measured lengths which geodesy furnishes, the method of geodesists is to substitute these lengths and the observed middle latitudes in the known trigonometrical formulæ, *using the axes of the* MEAN FIGURE *of the earth.* It might at first sight appear likely that this would lead to incorrect results, as the actual length measured may lie along a curve different to that of the *mean* form. I propose now to show that no sensible error is introduced by following this course, either in latitude or longitude, if the arc does *not exceed twelve degrees and a half* of latitude, or *fifteen degrees* of longitude in extent.

4. *First. An arc of Latitude.*—Suppose an ellipse drawn in the plane of the meridian through the two stations, a and b being its semiaxes; c the chord joining the stations; s the length of the arc; r and θ, r' and θ' polar coordinates to the extremities of the arc from the centre of the ellipse; l and l' their observed latitudes; λ the amplitude of the arc; m its middle latitude: then we have the following formulæ, neglecting the square of the ellipticity (ϵ),

$$s=\frac{1}{2}(a+b)\lambda-\frac{2}{2}(a-b)\sin\lambda\cos 2m,$$

$$r=a(1-\epsilon\sin^2 l),\quad r'=a(1-\epsilon\sin^2 l'),\quad \tan\theta=(1-2\epsilon)\tan l,$$
$$\tan\theta'=(1-2\epsilon)\tan l'.$$

Now

$$c^2=r^2+r'^2-2rr'\cos(\theta-\theta')=2rr'\{1-\cos(\theta-\theta')\}+(r-r')^2$$
$$=2rr'\{1-\cos(\theta-\theta')\}.$$

By expanding the formulæ for $\tan\theta$ and $\tan\theta'$, we have

$$\theta=l-\epsilon\sin 2l,\quad \theta'=l'-\epsilon\sin 2l',$$

$$\therefore\ \theta-\theta'=l-l'-\epsilon(\sin 2l-\sin 2l')=l-l'-2\epsilon\sin(l-l')\cos(l+l')$$
$$=\lambda-2\epsilon\sin\lambda\cos 2m;$$

$$\therefore\ 1-\cos(\theta-\theta')=1-\cos\lambda-2\epsilon\sin^2\lambda\cos 2m$$
$$=2\sin^2\frac{\lambda}{2}\{1-2\epsilon(1+\cos\lambda)\cos 2m\}.$$

Also

$$rr' = a^2\{1-\epsilon(\sin^2 l + \sin^2 l')\} = a^2\left\{1-\frac{\epsilon}{2}(2-\cos 2l-\cos 2l')\right\}$$
$$= a^2\{1-\epsilon(1-\cos\lambda\cos 2m)\};$$

$$\therefore\ c^2 = 4a^2\sin^2\frac{\lambda}{2}\left\{1-\epsilon\{1+(2+\cos\lambda)\cos 2m\}\right\};$$

$$\therefore\quad \sin\frac{\lambda}{2} = \frac{c}{2a}\left\{1+\frac{\epsilon}{2}\{1+(2+\cos\lambda)\cos 2m\}\right\};$$

$$\therefore\qquad \frac{\lambda}{2} = \sin^{-1}\frac{c}{2a} + \frac{\epsilon}{2}\left\{1+(2+\cos\lambda)\cos 2m\right\}\frac{c}{\sqrt{4a^2-c^2}}$$
$$= \sin^{-1}\frac{c}{2a} + \frac{\epsilon}{2}\left\{1+(2+\cos\lambda)\cos 2m\right\}\tan\frac{\lambda}{2}.$$

Hence by the first formula,

$$s = a\left(1-\frac{\epsilon}{2}\right)\lambda - \frac{3}{2}a\epsilon\sin\lambda\cos 2m$$
$$= a(2-\epsilon)\sin^{-1}\frac{c}{2a} + a\epsilon\{1+(2+\cos\lambda)\cos 2m\}\tan\frac{\lambda}{2} - \frac{3}{2}a\epsilon\sin\lambda\cos 2m$$
$$= (a+b)\sin^{-1}\frac{c}{2a} + (a-b)\left\{1+\frac{1}{2}(1-\cos\lambda)\cos 2m\right\}\tan\frac{\lambda}{2}.$$

Taking the variation of s with respect to a and b, considering c as constant, and λ and m also constant, occurring as they do only in small terms, we shall have the difference in length of two arcs joining the stations and belonging to different ellipses, only having their axes parallel. Hence

$$\delta s = (\delta a+\delta b)\sin^{-1}\frac{c}{2a} - \frac{a+b}{a}\frac{c\delta a}{\sqrt{4a^2-c^2}}$$
$$+(\delta a-\delta b)\left\{+\frac{1}{2}(1-\cos\lambda)\cos 2m\right\}\tan\frac{\lambda}{2}.$$

Since the terms are small, we may use the first approximate value for c and b;

$$\therefore\ \delta s = (\delta a+\delta b)\frac{\lambda}{2} - 2\tan\frac{\lambda}{2}\delta a + (\delta a-b)\left\{1+\frac{1}{2}(1-\cos\lambda)\cos 2m\right\}\tan\frac{\lambda}{2}$$
$$= (\delta a+\delta b)\left(\frac{\lambda}{2}-\tan\frac{\lambda}{2}\right) + (\delta a-\delta b)\frac{1}{2}\tan\frac{\lambda}{2}(1-\cos\lambda)\cos 2m$$
$$= (\delta a+\delta b)\mathrm{P} + (\delta a-\delta b)\mathrm{Q}\cos 2m,$$

where

$$\mathrm{P} = \frac{1}{2}\lambda - \tan\frac{1}{2}\lambda, \text{ and } \mathrm{Q} = \frac{1}{2}\tan\frac{1}{2}\lambda(1-\cos\lambda)$$
$$= (\mathrm{P}+\mathrm{Q}\cos 2m)\delta a + (\mathrm{P}-\mathrm{Q}\cos 2m)\delta b.$$

I will find the values of δa and δb which will satisfy this equation and make $\delta a^2+\delta b^2$ a minimum.

$$\delta a^2+\left(\frac{\delta s-(P+Q\cos 2m)\delta a}{P-Q\cos 2m}\right)^2 = \text{a minimum};$$

$$\therefore \{(P-Q\cos 2m)^2+(P+Q\cos 2m)^2\}\delta a=(P+Q\cos 2m)\delta s;$$

$$\therefore \delta a=\frac{P+Q\cos 2m}{P^2+Q^2\cos^2 2m}\frac{\delta s}{2},\quad \delta b=\frac{P-Q\cos 2m}{P^2+Q^2\cos^2 2m}\frac{\delta s}{2},$$

$$\delta a^2+\delta b^2=\frac{1}{P^2+Q^2\cos^2 2m}\frac{\delta s^2}{2}.$$

This is least when $m=0$ and $90°$; then

$$\delta a=\frac{P\pm Q}{P^2+Q^2}\frac{\delta s}{2},\quad \delta b=\frac{P\mp Q}{P^2+Q^2}\frac{\delta s}{2},\quad \delta a\sim\delta b=\frac{Q\delta s}{P^2+Q^2}.$$

Let one of the two ellipses be equal to the mean ellipse of the earth's figure, a and b being the semiaxes, and δa and δb the excess (or defect, if negative) of the semiaxes of the other ellipse. The first ellipse is not necessarily the mean ellipse itself, but is only equal to it in dimensions, and parallel to it in position; for the actual arc may lie above or below the mean ellipse. The result of this is, that the arc of the mean ellipse which corresponds with s of the actual arc will not necessarily have precisely the same middle latitude, although the chord c is of the same length. But as the middle latitude will differ only by a quantity of the order of the ellipticity, this difference will not appear in the result, because we neglect the square of the ellipticity.

I will now make the extravagant supposition that the ellipse to which the arc actually belongs deviates from the form of the mean ellipse so much that $\delta a\sim\delta b=13$ miles, the whole compression of the earth's figure. On this supposition I will find how large the arc may be so as not to produce a difference in length greater than $1''$.

Put $\delta a\sim\delta b=13$, $\delta s=1''=0{\cdot}0193$ mile ($1°$ being $69{\cdot}5$ miles),

$$\therefore (P^2+Q^2)\div Q=0{\cdot}0193\div 13=0{\cdot}0015,$$

or

$$\left(\frac{\lambda}{2}-\tan\frac{\lambda}{2}\right)^2+\frac{1}{4}\tan^2\frac{\lambda}{2}(1-\cos\lambda)^2=0{\cdot}00075\tan\frac{\lambda}{2}(1-\cos\lambda).$$

A slight inspection of this equation shows that λ must be small. Expand in powers of λ; then

$$\left(\frac{1}{9}+1\right)\left(\frac{\lambda}{2}\right)^3=0{\cdot}0015,\text{ or }\left(\frac{\lambda}{2}\right)^3=0{\cdot}00135;$$

$$\therefore \lambda=0{\cdot}22\text{ (in arc)}=0{\cdot}22\times 57°{\cdot}3\text{ (in degrees)}=12°{\cdot}6.$$

This shows that in an arc of meridian as much as twelve degrees and a half in length, it would require a departure from the mean ellipse equal to the whole actual compression of the pole of the earth in order to produce so slight a difference in the length as $1''$. Hence we may conclude that the difference in length between the mean arc and the actual arc, joining any two places on the same meridian, is an insensible quantity, since an extravagant hypothesis regarding the departure of the form from the mean form will not produce a difference in length of more than $1''$. This being the

case, the differences of latitudes calculated from the measured arcs of meridian with the mean axes, as is done in the Survey operations, will come out free from any effects which local attraction can produce, as that attraction can never be capable of causing so great a distortion in the measured arcs as I have supposed for the sake of calculation. The absolute latitude, however, of the station which fixes the arc on the map will be unknown to the extent of the deviation of the plumb-line caused by local attraction at that place.

5. *Second. An Arc of Longitude.*—Let S be the length of the arc, l the latitude, L the longitudinal amplitude or the difference of the longitudes of its extremities, c the chord. Then

$$S=L\cos l\{a+(a-b)\sin^2 l\},\quad c=2\cos l\{a+(a-b)\sin^2 l\}\sin\tfrac{1}{2}L.$$

When a and b vary, c and l remain constant, but S and L vary. Hence

$$\delta S=\delta L\cos l\{a+(a-b)\sin^2 l\}+L\cos l\{\delta a+(\delta a-\delta b)\sin^2 l\}$$

$$0=\{a+(a-b)\sin^2 l\}\cos\tfrac{1}{2}L\delta L+2\{\delta a+(\delta a-\delta b)\sin^2 l\}\sin\tfrac{1}{2}L.$$

By eliminating δL from these,

$$\delta S=\left(L-2\tan\tfrac{1}{2}L\right)\cos l\{\delta a+(\delta a-\delta b)\sin^2 l\};$$

$$\therefore\ \delta a+(\delta a-\delta b)\sin^2 l=\frac{\delta S}{(L-2\tan\frac{1}{2}L)\cos l}=n,\text{ suppose.}$$

I will, as before, find the values of δa and δb which satisfy this equation, and make $\delta a^2+\delta b^2$ a minimum.

$$\sin^4 l\,\delta a^2+\{(1+\sin^2 l)\delta a-n\}^2=\text{a minimum};$$

$$\therefore\ \{\sin^4 l+(1+\sin^2 l)^2\}\delta a=n(1+\sin^2 l);$$

$$\therefore\ \delta a=\frac{(1+\sin^2 l)n}{\sin^4 l+(1+\sin^2 l)^2},\quad \delta b=\frac{-\sin^2 l\,.\,n}{\sin^4 l+(1+\sin^2 l)^2};$$

$$\therefore\ \delta a^2+\delta b^2=\frac{n^2}{\sin^4 l+(1+\sin^2 l)^2}=\frac{\delta S^2}{\cos^2 l\{\sin^4 l+(1+\sin^2 l)^2\}\{L-2\tan\frac{1}{2}L\}^2}$$

This is least when $\cos^2 l\{\sin^4 l+(1+\sin^2 l)^2\}$ is greatest, or when $l=0$; then

$$\delta a=n,\quad \delta b=0,\quad \delta a-\delta b=n=\frac{\delta S}{L-2\tan\frac{1}{2}L}.$$

Now put $\delta a\sim\delta b=13$ miles, $\delta S=$ arc $1''$ of a great circle $=0{\cdot}0193$ mile;

$$\therefore\ L-2\tan\frac{1}{2}L=0{\cdot}0193\div 13=0{\cdot}0015.$$

This shows that L must be small: expanding, we have

$$L^3=0{\cdot}018,\quad L=0{\cdot}262\text{ (in arc)}=0{\cdot}262\times 57°{\cdot}3\text{ (in degrees)}=15°.$$

We can reason from this, as before, that the differences of longitudes will be accurately found by using the measured arcs of longitude and the mean axes, if the arcs are not longer than 15°. Now arcs of this length, and of

the length determined in paragraph 4 for latitudes, are never used in survey operations: the great arcs are always divided into much smaller portions. Hence the maps constructed from geodetic operations will always be relatively correct in themselves; but the precise position of the map on the terrestrial spheroid will be unknown by the amount of the unknown deflection of the plumb-line in latitude and longitude at the place which fixes the map. In India the effect of the Himmalaya Mountains and the Ocean, taken alone, would throw out the map by nearly half a mile. The calculations, however, which I give in the next two sections of this paper, show that the effect of variations in the density of the crust below almost entirely counteracts that of the mountains and ocean at Damargida in latitude 18° 3′ 15″, and the displacement of the map is almost insensible if fixed by that station. If fixed by the observed latitude of any other station, the map will be out of its place by the local deflection of the plumb-line at that station. This, in the Indian Great Arc, does not exceed one-thirteenth of a mile at any of the stations where the latitude has been observed. It appears also from those calculations, that, except in places evidently situated in most disadvantageous positions, the local attraction is rarely of any considerable amount.

§ 2. *Effect of Local Attraction on the Determination of the Mean Figure of the Earth.*

6. The mean radius of the earth is nearly 20890000 feet, the ellipticity is nearly $\frac{1}{300}$, and it is found convenient to put the semiaxes of the earth's figure under the form

$$\left.\begin{aligned} &\frac{a+b}{2} = \left(1-\frac{u}{10000}\right)20890000 = 20890000 - 2089\,u \text{ feet,} \\ &\frac{a-b}{2} = \frac{1}{600}\left(1-\frac{u}{10000}+\frac{v}{50}\right)20890000 = \frac{1}{600}\left(\frac{a+b}{2}+417800\,v\right); \end{aligned}\right\} \quad (1)$$

u and v are quantities to be determined, and the squares and product of these may be neglected.

$$\text{Also, ellipticity} = \frac{a-b}{a} = \frac{1}{300}\left(1+\frac{v}{50}\right).$$

The arcs which are actually measured in geodesy do not necessarily belong to precisely the same ellipse: in fact those arcs may not precisely belong to any ellipse. Suppose one of these measured arcs is laid along the ellipse of which the axes are given above, and that, small corrections x and x' being added to the observed latitudes of its extremities, the arc with its corrected latitudes exactly fits this ellipse. Then $x'-x$ may be expressed in the form $m+\alpha u+\beta v$, where m, α, and β are functions of the measured length, the observed latitudes, and numerical quantities. Let this be done for all the arcs which have been measured and their subdivisions. I shall take the eight arcs used in the chapter on the Figure of the Earth in the Volume of the British Ordnance Survey; viz. the Anglo-Gallic,

Russian, Indian II. (or Great Arc), Indian I., Prussian, Peruvian, Hanoverian, and Danish Arcs. Suppose

$$m_1+\alpha_1 u+\beta_1 v+\ldots . x_1, \quad m'_1+\alpha'_1 u+\beta'_1 v+x_1,$$

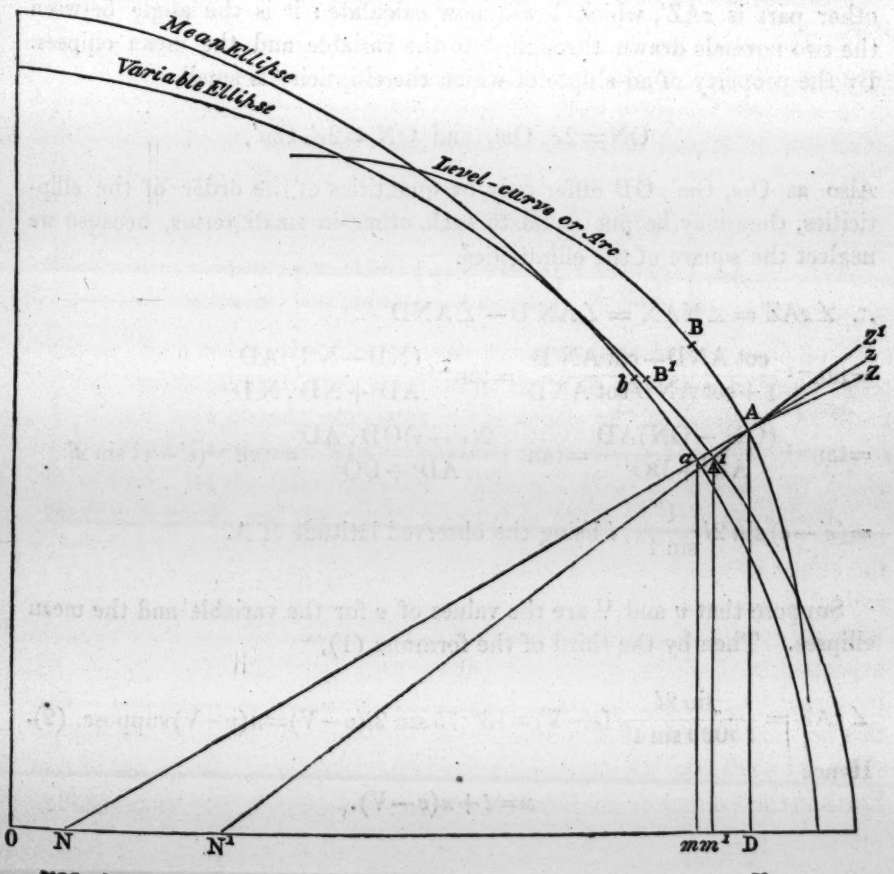
are the corrections of the latitudes of the extremities of the subdivisions of the Anglo-Gallic Arc, x_1 being the correction for the standard or reference station in this Arc. Similarly, let

$$m_2+\alpha_2 u+\beta_2 v+x_2, \quad m'_2+\alpha'_2 u+\beta'_2 v+x_2, \ldots .$$
$$m_3+\alpha_3 u+\beta_3 v+x_3, \quad m'_3+\alpha'_3 u+\beta'_3 v+x_3, \ldots .$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

be the corrections for the divisions of the other Arcs.

Then the values of u and v which give the most likely form are those which make the sum of the squares of all these corrections a minimum. The sum of the squares will involve u and v, and also eight quantities $x_1 \ldots x_8$. The usual course is to regard, not only u and v, but $x_1 \ldots x_8$ as independent variables, and to differentiate the sum of the squares with regard to each of them in succession, and so obtain as many equations as quantities to be determined.

7. This mode of proceeding is, I conceive, erroneous; as I shall now endeavour to show. The corrections $x_1 \ldots . x_8$ are not properly indepen-



dent variables, but are functions of u and v, and of the deflections produced by local attraction. In the preceding diagram the plane of the paper is the plane of the meridian in which the arc, of which A B is one section, has been geodetically measured. A is the reference-station of the several portions of the whole arc. A Z is the vertical at A in which the plumb-line hangs. The two curves, of which A′B′ and $a\,b$ are portions, are a variable ellipse and the mean ellipse having the same centre O and their axes in the same lines, the mean ellipse being what the variable ellipse becomes when the values are substituted for u and v which make the sum of the squares of the errors a minimum: Z′A A′N′ and zA aN are normals through A to these two ellipses; A D, A′m', $a\,m$ are perpendicular to O D.

Now, if the earth had its mean form, a plumb-line at A would hang in the normal zA to the mean ellipse; but it hangs actually in ZA. Hence ZAz is the deflection (northward in the diagram) which the plumb-line suffers from the local attraction arising from the derangement of the figure and mass of the earth from the mean. This angle is some constant but unknown quantity t, t being reckoned positive when the deflection is northward. This quantity t is part of the correction ZAZ′, or w, added to the observed latitude of A before applying the principle of least squares. The other part is zAZ′, which I will now calculate: it is the angle between the two normals drawn through A to the variable and the mean ellipses. By the property of an ellipse of which the ellipticity is small,

$$\mathrm{ON}=2\epsilon\,.\,\mathrm{O}m,\ \text{and}\ \mathrm{ON}'=2\epsilon'\,.\,\mathrm{O}m'.$$

Also as Om, Om', OD differ only by quantities of the order of the ellipticities, they may be put equal to each other in small terms, because we neglect the square of the ellipticities.

$$\therefore\ \angle z\mathrm{AZ}'=\angle\mathrm{NAN}'=\angle\mathrm{AN}'\mathrm{D}-\angle\mathrm{AND}$$

$$=\tan^{-1}\frac{\cot\mathrm{AND}-\cot\mathrm{AN}'\mathrm{D}}{1+\cot\mathrm{AND}\cot\mathrm{AN}'\mathrm{D}}=\tan^{-1}\frac{(\mathrm{ND}-\mathrm{N}'\mathrm{D})\mathrm{AD}}{\mathrm{AD}^2+\mathrm{ND}\,.\,\mathrm{N}'\mathrm{D}}$$

$$=\tan^{-1}\frac{(\mathrm{ON}'-\mathrm{ON})\mathrm{AD}}{\mathrm{AD}^2+\mathrm{DO}^2}=\tan^{-1}\frac{2(\epsilon'-\epsilon)\mathrm{OD}\,.\,\mathrm{AD}}{\mathrm{AD}^2+\mathrm{DO}^2}=\tan^{-1}(\epsilon'-\epsilon)\sin 2l$$

$$=(\epsilon'-\epsilon)\sin 2l\frac{1''}{\sin 1''},\ l\ \text{being the observed latitude of A.}$$

Suppose that v and V are the values of v for the variable and the mean ellipses. Then by the third of the formulæ (1),

$$\angle z\mathrm{AZ}'=\frac{\sin 2l}{15000\sin 1''}(v-\mathrm{V})=13''\cdot 75\sin 2l(v-\mathrm{V})=n(v-\mathrm{V})\ \text{suppose..(2).}$$

Hence

$$x=t+n(v-\mathrm{V}).$$

Therefore the sum of the squares of errors, which is to be differentiated with respect to u and v to obtain a minimum, is

$$(n_1(v-\overline{V})+t_1)^2+(m_1+\alpha_1 u+\beta_1 v+n_1(v-\overline{V})+t_1)^2$$
$$+(m'_1+\alpha'_1 u+\beta'_1 v+n_1(v-\overline{V})+t_1)^2+\ldots\ldots$$
$$(n_2(v-\overline{V})+t_2)^2+(m_2+\alpha_2 u+\beta_2 v+n_2(v-\overline{V})+t_2)^2$$
$$+(m'_2+\alpha'_2 u+\beta'_2 v+n_2(v-\overline{V})+t_2)^2+\ldots.$$
$$+\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad =\text{a minimum.}$$

Let $\overline{U}$ and $\overline{V}$ be the values of u and v which belong to the mean ellipse. These values, then, must be put for u and v in the two equations produced by differentiating the above with respect to u and v, We have

$$\alpha_1(m_1+\alpha_1\overline{U}+\beta_1\overline{V}+t_1)+\alpha'_1(m'_1+\alpha'_1\overline{U}+\beta'_1\overline{V}+t_1)+\ldots$$
$$+\alpha_2(m_2+\alpha_2\overline{U}+\beta_2\overline{V}+t_2)+\alpha'_2(m'_2+\alpha'_2\overline{U}+\beta'_2\overline{V}+t_2)+\ldots$$
$$+\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad =0;$$

and

$$n_1t_1+(\beta_1+n_1)(m_1+\alpha_1\overline{U}+\beta_1\overline{V}+t_1)+(\beta'_1+n_1)(m'_1+\alpha'_1\overline{U}+\beta'_1\overline{V}+t_1)+\ldots$$
$$n_2t_2+(\beta_2+n_2)(m_2+\alpha_2\overline{U}+\beta_2\overline{V}+t_2)+(\beta'_2+n_2)(m'_2+\alpha'_2\overline{U}+\beta'_2\overline{V}+t_2)+\ldots$$
$$+\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad .\quad =0.$$

Let (m) be a symbol representing the sum of all the m's appertaining to the divisions of the same Arc; and let $\Sigma(m)$ represent the sum of all these sums for all the Arcs; and similarly for other quantities besides m. Then the above equations become

$$\Sigma(m\alpha)\ +\Sigma(\alpha^2)\ \overline{U}+\Sigma(\alpha\beta)\ \overline{V}+\Sigma t(\alpha)\ =0$$

and

$$\left.\begin{matrix}\Sigma(m\beta)\\+\Sigma n(m)\end{matrix}\right\}+\left.\begin{matrix}\Sigma(\alpha\beta)\\+\Sigma n(\alpha)\end{matrix}\right\}\overline{U}+\left.\begin{matrix}\Sigma(\beta^2)\\+\Sigma n(\beta)\end{matrix}\right\}\overline{V}+\left.\begin{matrix}\Sigma t(\beta)\\+\Sigma tni\end{matrix}\right\}=0,$$

i being the number of stations on the representative Arc.

The numerical quantities involved in the first two lines of these equations have been already calculated in the article on the Figure of the Earth in the British Ordnance Survey Volume, from which I borrow the results in Table II. on the following page. The quantities involving n are calculated in Table I., and the results inserted in Table II. with the others.

TABLE I., referred to in last page; $\log n = \log 13{\cdot}75 + \log \sin 2 \text{ lat.} = 1{\cdot}138302 + \log \sin 2 \text{ lat.}$

Arcs.	Standard station.	Latitude.	log sin 2 lat.	log n.	n.	log $n(m)$.	$n(m)$.	log $n(\alpha)$.	$n(\alpha)$.	log $n(\beta)$.	$n(\beta)$.	t.	ni.
Anglo-Gallic.	St. Agnes	49° 53′ 33″·93	$\bar{1}$·9936350	1·1319377	13·54995	2·9564540	904·5946	2·6192882	416·1867	2·2251352	−167·9327	34	460·6983
Russian ...	Staro-Nekrassowka	45 20 2·8	$\bar{1}$·9999705	1·1382732	13·74907	2·9804517	955·9864	2·8746163	749·2319	2·3419912	−219·7815	13	178·7379
Indian II. ...	Damargida	18 3 15·29	$\bar{1}$·7703483	0·9086510	8·10310	1·1551496	14·2939	1·1082965	−12·8321	1·3254412	− 21·1564	8	64·8248
Indian I.	Trivandeporum.	11 44 52·59	$\bar{1}$·6006279	0·7389306	5·48189	$\bar{1}$·9403277	0·8716	0·4945768	3·1230	0·4521691	2·8325	2	10·9638
Prussian	Trunz	54 13 11·47	$\bar{1}$·9771092	1·1154119	13·04403	1·4679803	29·3752	0·9733472	9·4047	0·4959849	− 3·1332	3	39·1321
Peruvian	Tarqui, S.	3 4 32·07	$\bar{1}$·0299991	0·1683018	1·47334	$\bar{1}$·3296698	0·2136	0·2184108	1·6535	0·2179462	+ 1·6518	2	2·9467
Hanoverian...	Göttingen	51 31 47·85	$\bar{1}$·9886188	1·1269215	13·39435	1·7802305	60·2879	0·9879777	9·7270	0·3962009	− 2·4900	2	26·7887
Danish	Lauenburg	53 22 17·05	$\bar{1}$·9811875	1·1194902	13·16710	1·2212373	−16·6432	0·8607994	7·2577	0·3519781	− 2·2489	2	26·3342
Totals	.....	.....	.....	.....	.....	.....	1948·9800	.....	1183·7524	.....	−412·2584		

TABLE II., gathered from the Ordnance Survey Volume, and from Table I.

Arcs.	(m).	(α).	(β).	$(m\alpha)$.	$(m\beta)$.	(α^2).	$(\alpha\beta)$.	(β^2).	$n(m)$.	$n(\alpha)$.	$n(\beta)$.	ni.
Anglo-Gallic..	+66·760	+30·7150	−12·3936	+118·9207	− 45·3653	+155·0671	− 35·5339	+11·1906	+ 904·5946	+ 416·1867	−167·9327	460·6983
Russian	69·531	+54·4933	−15·9852	+386·3623	−126·4488	+335·5318	−112·1421	+39·9745	+ 955·9864	+ 749·2319	−219·7815	178·7379
Indian II. ...	1·764	− 1·5836	− 2·6109	− 12·7516	− 10·4936	+ 46·5750	+ 36·6984	+29·3612	+ 14·2939	− 12·8321	− 21·1564	64·8248
Indian I.....	0·159	+ 0·5697	+ 0·5167	+ 0·0906	+ 0·0822	+ 0·3246	+ 0·2944	+ 0·2670	+ 0·8716	+ 3·1230	+ 2·8325	10·9638
Prussian	2·252	+ 0·7210	− 0·2402	+ 1·4630	− 0·4953	+ 0·3267	− 0·1096	+ 0·0368	+ 29·3752	+ 9·4047	− 3·1332	39·1321
Peruvian	0·145	+ 1·1223	+ 1·1211	+ 0·1627	+ 0·1626	+ 1·2596	+ 1·2582	+ 1·2569	+ 0·2136	+ 1·6535	+ 1·6518	2·9467
Hanoverian...	4·501	+ 0·7262	− 0·1859	+ 3·2686	− 0·8367	+ 0·5274	− 0·1350	+ 0·0346	+ 60·2879	+ 9·7270	− 2·4900	26·7887
Danish	− 1·264	+ 0·5512	− 0·1708	− 0·6967	+ 0·2159	+ 0·3038	− 0·0941	+ 0·0292	− 16·6432	+ 7·2577	− 2·2489	26·3342
Totals	.....	.....	.....	+496·8196	−183·1790	+539·9160	−109·7637	+82·1508	+1948·9800	+1183·7524	−412·2584	

TABLE III., containing the Logarithms of the numbers in Table II.

Anglo-Gallic..	1·8245163	1·4873505	−1·0931975	2·0752575	−1·6567238	2·1905197	−1·5506429	1·0488534	2·9564540	2·6192882	−2·2251352	2·6634166
Russian	1·8421785	1·7363431	−1·2037180	2·5869948	−2·1019147	2·5257337	−2·0497687	1·6017831	2·9804517	2·8746163	−2·3419912	2·2522167
Indian II. ...	0·2464986	−0·1996455	−0·4167902	−1·1055647	−1·0209245	1·6681529	1·5646471	1·4677738	1·1551496	−1·1082965	−1·3254412	1·8117412
Indian I.....	$\bar{1}$·2013971	$\bar{1}$·7556462	$\bar{1}$·7132385	$\bar{2}$·9571282	$\bar{2}$·9148718	$\bar{1}$·5113485	$\bar{1}$·4689378	$\bar{1}$·4265113	$\bar{1}$·9403172	0·4945768	0·4521691	1·0399611
Prussian	0·3525684	$\bar{1}$·8579353	−$\bar{1}$·3805730	0·1652443	−$\bar{1}$·6948683	$\bar{1}$·5141491	−$\bar{1}$·0398106	$\bar{2}$·5658478	1·4679803	0·9733472	−0·4959849	1·5925331
Peruvian.....	$\bar{1}$·1613680	0·0501090	0·0496444	$\bar{1}$·2113876	$\bar{1}$·2111205	0·1002327	0·0997497	0·0993007	$\bar{1}$·3296698	0·2184108	0·2179462	0·4693359
Hanoverian...	0·6533090	$\bar{1}$·8610562	−$\bar{1}$·2692794	0·5143618	−$\bar{1}$·9225698	$\bar{1}$·7221401	−$\bar{1}$·1303338	$\bar{2}$·5390761	1·7802305	0·9879777	−0·3962009	1·4279516
Danish	−0·1017471	$\bar{1}$·7413092	−$\bar{1}$·2324879	−$\bar{1}$·8430458	$\bar{1}$·3342526	$\bar{1}$·4825878	−$\bar{2}$·9735896	$\bar{2}$·4653829	−1·2212373	0·8607994	−0·3519781	1·4205201
Totals	.....	.....	.....	2·6961987	−2·2628757	2·7323262	−2·0404588	1·9146118	3·2898074	3·0732608	−2·6151695	

TABLE IV., derived from Table II.

Arcs.	$(m\beta)+n(m)$.	$(\alpha\beta)+n(\alpha)$.	$(\beta^2)+n(\beta)$.	$(\beta)+ni$.	The logarithms of the foregoing numbers.			
Anglo-Gallic	859·2293	380·6528	−156·7421	448·3047	2·9341091	2·5805290	−2·1951856	2·6515733
Russian	829·5376	637·0898	−179·8070	162·7527	2·9188361	2·8042007	−2·2548066	2·2115282
Indian II.	3·8003	23·8663	8·2048	62·2139	0·5798179	1·3777851	0·9140680	1·7938874
Indian I.....	0·9538	3·4174	3·0995	11·4805	$\bar{1}$·9794573	0·5336958	0·4912916	1·0599608
Prussian	28·8799	9·2951	− 3·0964	38·8919	1·4605957	0·9682541	−0·4908571	1·5898592
Peruvian.....	0·3762	2·9117	2·9087	4·0678	$\bar{1}$·5754188	0·4641466	0·4636989	0·6093596
Hanoverian	59·4512	9·5920	− 2·4554	26·6028	1·7741607	0·9819092	−0·3901223	1·4249273
Danish	−16·4273	7·1636	− 2·2197	26·1634	−1·2155662	0·8551313	−0·3462943	1·4176941
Totals.....	1765·8010	1073·9887	−330·1076	.....	3·2469417	3·0309997	−2·5186555	

8. I will now apply the formulæ just obtained to determine the Mean Figure of the Earth from the data afforded by the eight arcs. For convenience I shall use the well-known symbol $(2{\cdot}6961987)$ to mean the number of which $2{\cdot}6961987$ is the logarithm; and so of other numbers. By substitution from the Table, the formulæ give

$$\begin{aligned}&(2{\cdot}6961987)+(2{\cdot}7323262)\mathrm{U}-(2{\cdot}0404588)\mathrm{V}+(1{\cdot}4873505)t_1\\&\quad+(\bar{1}{\cdot}7363431)t_2-(0{\cdot}1996455)t_3+(\bar{1}{\cdot}7556462)t_4+(\bar{1}{\cdot}8579353)t_5\\&\quad+(0{\cdot}0501090)t_6+(\bar{1}{\cdot}8610562)t_7+(\bar{1}{\cdot}7413092)t_8=0,\\&(3{\cdot}2469417)+(3{\cdot}0309997)\mathrm{U}-(2{\cdot}5186555)\mathrm{V}+(2{\cdot}6515733)t_1\\&\quad+(2{\cdot}2115282)t_2+(1{\cdot}7938874)t_3+(1{\cdot}0599608)t_4+(1{\cdot}5898592)t_5\\&\quad+(0{\cdot}6093596)t_6+(1{\cdot}4249273)t_7+(1{\cdot}4176941)t_8=0.\end{aligned}$$

Multiplying by the coefficients of V crosswise, and subtracting so as to eliminate V, we have

$$\begin{aligned}&(5{\cdot}2148542)+(5{\cdot}2509817)\mathrm{U}+(4{\cdot}0060060)t_1+(4{\cdot}2549986)t_2\\&\quad-(2{\cdot}7183010)t_3\\&-(5{\cdot}2874005)-(5{\cdot}0714585)\mathrm{U}-(4{\cdot}6920321)t_1-(4{\cdot}2519870)t_2\\&\quad-(3{\cdot}834362)t_3\\&+(2{\cdot}2743017)t_4+(2{\cdot}2765908)t_5+(2{\cdot}5687645)t_6+(2{\cdot}3797117)t_7\\&\quad+(2{\cdot}2599647)t_8\\&-(3{\cdot}1004196)t_4-(3{\cdot}6303180)t_5-(2{\cdot}6498184)t_6-(3{\cdot}4653861)t_7\\&\quad-(3{\cdot}4581529)t_8=0,\end{aligned}$$

Putting numbers in the place of logarithms,

$$\begin{aligned}&164004+178230\mathrm{U}+\ 10139t_1+17989t_2-\ 523t_3\\&-193821-117885\mathrm{U}-\ 49208t_1-17864t_2-6829t_3\\&\hline\\&=\ 29817+\ 60345\mathrm{U}-\ 39069t_1+\ \ 125t_2-7352t_3\end{aligned}$$

$$\begin{aligned}&+\ 188t_4+\ 238t_5+370t_6+\ 240t_7+\ 182t_8\\&-1260t_4-4269t_5-446t_6-2920t_7-2872t_8\\&\hline\\&-1072t_4-4031t_5-\ 76t_6-2680t_7-2690t_8=0,\end{aligned}$$

Putting logarithms in the place of numbers,

$$\begin{aligned}&-(4{\cdot}4744639)+(4{\cdot}7806413)\mathrm{U}-(4{\cdot}5918323)t_1+(2{\cdot}0969100)t_2\\&\quad-(3{\cdot}8664055)t_3-(3{\cdot}0301948)t_4-(3{\cdot}6054128)t_5-(1{\cdot}8808136)t_6\\&\quad-(3{\cdot}4281348)t_7-(3{\cdot}4297523)t_8=0.\end{aligned}$$

Transposing and dividing by the coefficient of U,

$$\begin{aligned}\mathrm{U}=&(\bar{1}{\cdot}6938226)+(\bar{1}{\cdot}8111910)t_1-(\bar{3}{\cdot}3162687)t_2+(\bar{1}{\cdot}0857642)t_3\\&+(\bar{2}{\cdot}2495535)t_4+(\bar{2}{\cdot}8247715)t_5+(\bar{3}{\cdot}1001723)t_6+(\bar{2}{\cdot}6474933)t_7\\&+(\bar{2}{\cdot}6491110)t\,.\end{aligned}$$

Now $2089 = (3{\cdot}3199384)$,

$$\therefore 2089\,U = (3{\cdot}0137610) + (3{\cdot}1311294)t_1 - (0{\cdot}6362071)t_2 + (2{\cdot}4057026)t_3 + (1{\cdot}5694919)t_4 + (2{\cdot}1447099)t_5 + (0{\cdot}4201107)t_6 + (1{\cdot}9674319)t_7 + (1{\cdot}9690494)t_8$$

$$= 1032{\cdot}2 + 1352{\cdot}4t_1 - 4{\cdot}3t_2 + 254{\cdot}5t_3 + 37{\cdot}1t_4 + 139{\cdot}5t_5 + 2{\cdot}6t_6 + 92{\cdot}8t_7 + 93{\cdot}1t_8.$$

Transposing the term in V in the first equation of this paragraph and dividing by its coefficient, we have

$$V = (0{\cdot}6557399) + (0{\cdot}6918674)U + (\bar{1}{\cdot}4468917)t_1 + (\bar{1}{\cdot}6958843)t_2 - (\bar{2}{\cdot}1591867)t_3 + (\bar{3}{\cdot}7151874)t_4 + (\bar{3}{\cdot}8174765)t_5 + (\bar{2}{\cdot}0096502)t_6 + (3{\cdot}8205974)t_7 + (\bar{3}{\cdot}7008504)t_8$$

$$= (0{\cdot}6557399) + (\bar{1}{\cdot}4468917)t_1 + (\bar{1}{\cdot}6958843)t_2 - (\bar{2}{\cdot}1591867)t_3 + (\bar{3}{\cdot}7151874)t_4$$
$$+ (0{\cdot}3856900) + (0{\cdot}5030584)t_1 - (\bar{2}{\cdot}0081361)t_2 + (\bar{1}{\cdot}7776316)t_3 + (\bar{2}{\cdot}9414209)t_4$$
$$+ (\bar{3}{\cdot}8174765)t_5 + (\bar{2}{\cdot}0096502)t_6 + (\bar{3}{\cdot}8205974)t_7 + (\bar{3}{\cdot}7008504)t_8$$
$$+ (\bar{1}{\cdot}5166389)t_5 + (\bar{3}{\cdot}7920397)t_6 + (\bar{1}{\cdot}3393609)t_7 + (\bar{1}{\cdot}3409784)t_8.$$

Now $417800 = (5{\cdot}6209684)$,

$$\therefore 417800\,V$$
$$= (6{\cdot}2767083) + (5{\cdot}0678601)t_1 + (5{\cdot}3168527)t_2 - (3{\cdot}7801551)t_3 + (3{\cdot}3361558)t_4$$
$$+ (6{\cdot}0066584) + (6{\cdot}1240268)t_1 - (3{\cdot}6291045)t_2 + (5{\cdot}3986000)t_3 + (4{\cdot}5623893)t_4$$
$$+ (3{\cdot}4384449)t_5 + (3{\cdot}6306186)t_6 + (3{\cdot}4415658)t_7 + (3{\cdot}3218188)t_8$$
$$+ (5{\cdot}1376073)t_5 + (3{\cdot}4130081)t_6 + (4{\cdot}9602293)t_7 + (4{\cdot}9619468)t_8$$

$$= 1891073 + 116912t_1 + 207421t_2 - 6028t_3 + 2168t_4$$
$$+ 1015450 + 1330537t_1 - 4257t_2 + 250380t_3 + 36508t_4$$
$$+ 2906523 + 1447449t_1 + 203164t_2 + 244352t_3 + 38676t_4$$
$$+ 2744t_5 + 4272t_6 + 2764t_7 + 2098t_8$$
$$+ 137280t_5 + 2588t_6 + 91249t_7 + 91611t_8$$
$$+ 142024t_5 + 6860t_6 + 94013t_7 + 93709t_8.$$

Substituting the values of 2089 U and 417800 V above deduced in the formulæ (1) of paragraph 6, we have

$$\frac{a+b}{2} = 20888968 - 1352{\cdot}4t_1 + 4{\cdot}3t_2 - 254{\cdot}5t_3 - 37{\cdot}1t_4 - 139{\cdot}5t_5 - 2{\cdot}6t_6 - 92{\cdot}8t_7 - 93{\cdot}1t_8,$$

$$\frac{a-b}{2} = \frac{1}{600}\left\{\frac{a+b}{2} + 417800\,V\right\}$$

$$= \frac{1}{600}\{23795491 + 1446097t_1 + 203168t_2 + 244098t_3 + 38639t_4 + 139884t_5 + 6857t_6 + 93920t_7 + 93616t_8\}$$

$$= 39659 + 2410{\cdot}2t_1 + 338{\cdot}6t_2 + 406{\cdot}8t_3 + 64{\cdot}4t_4 + 233{\cdot}1t_5 + 11{\cdot}4t_6 + 156{\cdot}5t_7 + 156{\cdot}0t_8;$$

$$\therefore\ a = 20928627 + 1057{\cdot}8t_1 + 342{\cdot}9t_2 + 152{\cdot}3t_3 + 27{\cdot}3t_4 + 93{\cdot}6t_5 + 8{\cdot}8t_6 + 63{\cdot}7t_7 + 62{\cdot}9t_8,$$

$$b = 20849309 - 3762{\cdot}6t_1 - 334{\cdot}3t_2 - 661{\cdot}3t_3 - 101{\cdot}5t_4 - 372{\cdot}6t_5 - 14{\cdot}0t_6 - 249{\cdot}3t_7 - 249{\cdot}1t_8.$$

From these we may easily deduce the ellipticity

$$\epsilon = \frac{1}{263{\cdot}9}\{1 + 0{\cdot}0608t_1 + 0{\cdot}0085t_2 + 0{\cdot}0103t_3 + 0{\cdot}0016t_4 + 0{\cdot}0059t_5 + 0{\cdot}0003t_6 + 0{\cdot}0039t_7 + 0{\cdot}001639t_8\}.$$

These formulæ for the semiaxes and ellipticity of the mean figure of the earth show us that the effect of local attraction upon the final numerical results may be very considerable: for example, a deflection of the plumb-line of only 5″ at the standard station (St. Agnes) of the Anglo-Gallic arc would introduce a correction of about one mile to the length of the semi-major axis, and more than three miles to the semi-minor axis. If the deflection at the standard station (Damargida) of the Indian Great Arc be what the mountains and ocean make it (without allowing any compensating effect from variations in density in the crust below, which no doubt exist, but which are altogether unknown), viz. about 17″·24, the semiaxes will be subject to a correction, arising from this cause alone, of half a mile and two miles. This is sufficient to show how great a degree of uncertainty local attraction, if not allowed for, introduces into the determination of the mean figure. As long as we have no means of ascertaining the amount of local attraction at the several standard-stations of the arcs employed in the calculation, this uncertainty regarding the mean figure, as determined by geodesy, must remain.

§ 3. *Comparison of the Anglo-Gallic, Russian, and Indian Arcs, with a view to deduce the Mean Figure of the Earth.*

9. The first three of the eight arcs which have been used in the calculation, viz. the Anglo-Gallic, Russian, and Indian, are of considerable length; and as the *à priori* probability appears to be that the earth nowhere departs much from its mean form, it seems not unlikely that by the following device we may overcome the difficulty pointed out in the last paragraph. I will deduce expressions for the semiaxes of the mean figure of each of these three arcs by the method there given. If reasonable values can be assigned to the expressions for the deflection of the plumb-line from the normals to these three ellipses such as will make the axes the same, we shall have a very strong argument in favour of those being the actual deflections in nature, and of the figure thus deduced, as common to the three arcs, being in fact the mean figure of the earth.

10. In the previous calculation t has represented the angle which the plumb-line makes, in the plane of the meridian, with the normal to the mean ellipse of the earth. I shall now use T as the angle which the plumb-line makes, in the plane of the meridian, with the normal to the mean

ellipse of the particular arc under consideration. I shall begin with the Anglo-Gallic arc. Proceeding precisely as in paragraph 8, we have

$$(2{\cdot}0752575)+(2{\cdot}1905197)U_1-(1{\cdot}5506429)V_1+(1{\cdot}4873505)T_1=0,$$
$$(2{\cdot}9341091)+(2{\cdot}5805290)U_1-(2{\cdot}1951856)V_1+(2{\cdot}6515733)T_1=0,$$

$$(4{\cdot}2704431)+(4{\cdot}3857053)U_1+(3{\cdot}6825361)T_1$$
$$-(4{\cdot}4847520)-(4{\cdot}1311719)U_1-(4{\cdot}2022162)T_1=0,$$

$$18640+24306\,U_1+4814\,T_1$$
$$-30532-13526\,U_1-15930\,T_1$$
$$-11892+10780\,U_1-11116\,T_1=0,$$

or

$$-(4{\cdot}0752549)+(4{\cdot}0326188)U_1-(4{\cdot}0459485)T_1=0;$$
$$\therefore\ U_1=(0{\cdot}0426361)+(0{\cdot}0133297)T_1,\quad 2089=(3{\cdot}3199384),$$
$$2089\,U_1=(3{\cdot}3625745)+(3{\cdot}3332681)T_1=2304{\cdot}5+2154{\cdot}1\,T_1.$$

By the first of the equations in V_1, we have

$$V_1=(0{\cdot}5246146)+(0{\cdot}6398768)U_1+(\bar{1}{\cdot}9367076)T_1$$
$$=(0{\cdot}5246146)+(\bar{1}{\cdot}9367076)T_1$$
$$+(0{\cdot}6825129)+(0{\cdot}6532065)T_1,\quad 417800=(5{\cdot}6209684);$$
$$\therefore 417800\,V_1=(6{\cdot}1455830)+(6{\cdot}3034813)+\{(5{\cdot}5576760)+(6{\cdot}2741749)\}T_1$$
$$=1398244+2011320+\{361140+1880074\}T_1$$
$$=3409564+2241214\,T_1;$$

$$\therefore\ \frac{a_1+b_1}{2}=20887695-2154{\cdot}1\,T_1$$

$$\frac{a_1-b_1}{2}=\frac{1}{600}\{24297259+2239060\,T_1\}=40495+3731{\cdot}8\,T_1;$$

$$\therefore\ a_1=20928190+1577{\cdot}7\,T_1,\quad b_1=20847200-5885{\cdot}9\,T_1,$$

$$\epsilon_1=\frac{1}{258{\cdot}4}(1+0{\cdot}0921\,T_1).$$

11. I proceed to the second, the Russian arc.

$$(2{\cdot}5869948)+(2{\cdot}5257337)U_2-(2{\cdot}0497688)V_2+(1{\cdot}7363431)T_2=0,$$
$$(2{\cdot}9188361)+(2{\cdot}8042007)U_2-(2{\cdot}2548066)V_2+(2{\cdot}2115282)T_2=0,$$

$$(4{\cdot}8418014)+(4{\cdot}7805403)U_2+(3{\cdot}9911497)T_2$$
$$-(4{\cdot}9686049)-(4{\cdot}8539695)U_2-(4{\cdot}2612970)T_2=0,$$

$$69471+60331\,U_2+9798\,T_2$$
$$-93026-71445\,U_2-18251\,T_2$$
$$-23555-11114\,U_2-8453\,T_2=0,$$

or

$$-(4{\cdot}3720831)-(4{\cdot}0458704)\,U_2-(3{\cdot}9270109)T_2=0;$$
$$\therefore\ U_2=-(0{\cdot}3262127)-(\bar{1}{\cdot}8811405)T_2,\quad 2089=(3{\cdot}3199384),$$
$$2089\,U_2=-(3{\cdot}6461511)-(3{\cdot}2010789)T_2=-4427{\cdot}4-1588{\cdot}8\,T_2.$$

By the first of the equations in V_2, we have

$$V_2=(0\cdot5372260)+(0\cdot4759649)U_2+(\bar{1}\cdot6865743)T_2$$
$$=(0\cdot5372260)-(0\cdot8021776)+\{(\bar{1}\cdot6865743)-(0\cdot3571054)\}T_2$$
$$417800=(5\cdot6209684);$$

$$\therefore 417800\,V_2=(6\cdot1581944)-(6\cdot4231460)+\{(5\cdot3075427)-(5\cdot9780738)\}T_2$$
$$=1439443-2649391+\{203022-950766\}T_2$$
$$=-1209948-747744\,T_2;$$

$$\therefore \frac{a_2+b_2}{2}=20894427+1588\cdot8\,T_2,$$

$$\frac{a_2-b_2}{2}=\frac{1}{600}\{19684479-746155\,T_2\}=32807-1243\cdot6\,T_2,$$

$$a_2=20927234+345\cdot2\,T_2,\quad b_2=20861620+2832\cdot4\,T_2,$$

$$e=\frac{1}{318\cdot9}(1-0\cdot0379\,T_2).$$

12. The following is the calculation for the Indian are :—

$$-(1\cdot1055647)+(1\cdot6681529)U_3+(1\cdot5646471)V_3-(0\cdot1996455)T_3=0,$$
$$(0\cdot5798179)+(1\cdot3777851)U_3+(0\cdot9140680)V_3+(1\cdot7938874)T_3=0,$$

$$-(2\cdot0196327)+(2\cdot5822209)U_3-(1\cdot1137135)T_3$$
$$-(2\cdot1444650)-(2\cdot9424322)U_3-(3\cdot3585345)T_3=0,$$

$$-105+382U_3-\ 13\,T_3$$
$$-139-876\,U_3-2283\,T_3$$
$$-244-494\,U_3-2296\,T_3=0,$$

or

$$-(2\cdot3873898)-(2\cdot6937269)U_3-(3\cdot3609719)T_3=0;$$

$$\therefore U_3=-(\bar{1}\cdot6936629)-(0\cdot6672450)T_3,\quad 2089=(3\cdot3199384),$$
$$2089\,U_3=-(3\cdot0136013)-(3\cdot9871834)T_3=-1031\cdot8-9709\cdot2\,T_3.$$

By the first of the equations in V_3, we have

$$V_3=(\bar{1}\cdot5409176)-(\bar{0}\cdot1035058)U_3+(\bar{2}\cdot6349984)T_3$$
$$=(\bar{1}\cdot5409176)+(\bar{1}\cdot7971687)+\{\bar{2}\cdot6349984)+(0\cdot7707508)\}T_3$$
$$417800=(5\cdot6209684),$$
$$417800\,V_3=(5\cdot1618860)+(5\cdot4181371)+\{(4\cdot2559668)+(6\cdot3917192)\}T_3$$
$$=145173+261901+\{18029+2464445\}T_3$$
$$=407074+2482474\,T_3;$$

$$\therefore \frac{a_3+b_3}{2} = 20891032 + 9709{\cdot}2\,T_3,$$

$$\frac{a_3-b_3}{2} = \frac{1}{600}\{21298106 + 2492183\,T_3\} = 35497 + 4153{\cdot}6\,T_3;$$

$$\therefore a_3 = 20926529 + 13862{\cdot}8\,T_3, \quad b_3 = 20855535 + 5555{\cdot}6\,T_3,$$

$$e_3 = \frac{1}{294{\cdot}8}(1 + 0{\cdot}1170\,T_3).$$

13. I have now, if possible, to find values of T_1, T_2, T_3 which will make these three ellipses, which measure the Anglo-Gallic, the Russian, and the Indian arcs, the same; that is, $a_1=a_2=a_3$, $b_1=b_2=b_3$. These give the four following equations :—

$$\left.\begin{aligned} 1577{\cdot}7\,T_1 - 345{\cdot}2\,T_2 + 956 &= 0,\\ 5885{\cdot}9\,T_1 + 2832{\cdot}4\,T_2 + 14420 &= 0,\\ 1577{\cdot}7\,T_1 - 13862{\cdot}8\,T_3 + 1661 &= 0,\\ 5885{\cdot}9\,T_1 + 5555{\cdot}6\,T_3 + 8335 &= 0, \end{aligned}\right\} \text{ or } \left\{\begin{aligned} (3{\cdot}1980244)T_1 - (2{\cdot}5380708)T_2 + (2{\cdot}9804579) &= 0,\\ (3{\cdot}7698129)T_1 + (3{\cdot}4521546)T_2 + (4{\cdot}1589653) &= 0,\\ (3{\cdot}1980244)T_1 - (4{\cdot}1418509)T_3 + (3{\cdot}2203696) &= 0,\\ (3{\cdot}7698129)T_1 + (3{\cdot}7447310)T_3 + (3{\cdot}9209056) &= 0, \end{aligned}\right.$$

The most likely solutions of these four equations connecting the three quantities T_1, T_2, T_3 which we are seeking are found by the method of least squares. This leads to the three following equations :—

$$\left.\begin{aligned} 2(6{\cdot}3960488)\\ +2(7{\cdot}5396258) \end{aligned}\right\} T_1 - (5{\cdot}7360952) \left.\begin{aligned} \\ +(7{\cdot}2219675) \end{aligned}\right\} T_2 - (7{\cdot}3398753) \left.\begin{aligned} \\ +(7{\cdot}5145439) \end{aligned}\right\} T_3 + (6{\cdot}1784823) + (6{\cdot}4183940) + (7{\cdot}9287782) + (7{\cdot}6907185) = 0.$$

$$\left.\begin{aligned} (5{\cdot}7360952)\\ +(7{\cdot}2219675) \end{aligned}\right\} T_1 - (5{\cdot}0761416) \left.\begin{aligned} \\ +(6{\cdot}9043092) \end{aligned}\right\} T_2 + (3{\cdot}5185287) + (7{\cdot}6111199) = 0,$$

$$\left.\begin{aligned} (7{\cdot}3398753)\\ +(7{\cdot}5145439) \end{aligned}\right\} T_1 - (8{\cdot}2837018) \left.\begin{aligned} \\ +(7{\cdot}4894620) \end{aligned}\right\} T_3 + (7{\cdot}3622205) + (7{\cdot}6636366) = 0,$$

or

2×2489	$-\ 545$		-21871	$+\ 1508$	$+\ 2631$	
$+2\times 34644$	$+16671$		$+32700$	$+84875$	$+49059$	
$74266\,T_1$	$+16126\,T_2$		$+10829\,T_3$	$+138063$		$=0$
$(4{\cdot}8707900)$	$(4{\cdot}2075267)$		$(4{\cdot}0345884)$	$(5{\cdot}1400773)$		
545	$-\ 119$	$+\ 330$	21871	-192177	$+23026$	
$+16671$	$+8022$	$+40843$	$+32700$	$+\ 30865$	$+46306$	
$17216\,T_1$	$+7903\,T_2$	$+41173 = 0,$	$54571\,T_1$	$-\ 161312\,T_3$	$+69332 = 0$	
$(4{\cdot}2359323)$	$(3{\cdot}8977920)$	$(4{\cdot}6146125)$	$(4{\cdot}7369619)$	$(5{\cdot}2076667)$	$(4{\cdot}8409387)$;	

$$\therefore \quad T_2 = -(0{\cdot}3381403)T_1 - (0{\cdot}7168205), \qquad T_3 = (\bar{1}{\cdot}5292952)T_1 + (1{\cdot}6332670);$$

$$\therefore 16126\,T_2 = -(4{\cdot}5456670)T_1 - (4{\cdot}9243472), \quad 10829\,T_3 = (3{\cdot}5638836)T_1 + (3{\cdot}6678554),$$

$$= -35129\,T_1 - 84013, \qquad = 3663\,T_1 + 4654;$$

$$\therefore \{74266 - 35129 + 3663\}T_1 + 138063 - 84013 + 4654 = 0,$$

$$42800\,T_1 + 58704 = 0, \quad T_1 = -1''{\cdot}37,$$

$$T_2 = -2{\cdot}18\,T_1 - 5{\cdot}21 = +2{\cdot}99 - 5{\cdot}21 = -2''{\cdot}22,$$

$$T_3 = 0{\cdot}338\,T_1 + 0{\cdot}430 = -0{\cdot}463 + 0{\cdot}430 = -0''{\cdot}033.$$

When these are substituted in the semiaxes, they give

$$a_1 = 20928190 - 2161 = 20926029, \quad a_2 = 20927234 - 766 = 20926468,$$
$$a_3 = 20926529 - 457 = 20926072,$$
$$b_1 = 20847200 + 8064 = 20855264, \quad b_2 = 20861620 - 6288 = 20855332,$$
$$b_3 = 20855535 - 183 = 20855352.$$

These three results are remarkably near each other; they differ from their average, 20926189 and 20855316, in no case by so much as 300 feet, and in most cases by much less. I think, then, that we may safely infer that this average ellipse is in fact the mean figure of the earth. This being the case, T_1, T_2, T_3 are the same as t_1, t_2, t_3; and therefore the deflections of the plumb-line in the meridian at the standard stations of the Anglo-Gallic, Russian, and Indian arcs are $1''{\cdot}37$, $2''{\cdot}22$, $0''{\cdot}033$, all in the southern direction*.

14. The values, then, which I would assign to the semiaxes and ellipticity of the Mean Figure of the Earth are as follows:—

$$a = 20926180, \quad b = 20855316 \text{ feet}, \quad \epsilon = \frac{1}{295{\cdot}3}.$$

If these are substituted in the formulæ (1) of paragraph (6), we have

$$U = -0{\cdot}3581 \text{ and } V = 0{\cdot}8819.$$

§ 4. *Speculations regarding the constitution of the Earth's Crust.*

15. If the reasoning in the last section, which has led to so satisfactory a result, be correct, I think we may draw some useful inferences regarding the constitution of the earth's crust.

By substituting the values of U, V, t_1, t_2, t_3 in the formulæ similar to $m + \alpha U + \beta V + t$ for the fifty-five stations of the eight arcs, which will be found at p. 766 of the Ordnance Survey Volume, every one of the results will be small. These results are the corrections of the latitudes of the stations in referring them to the mean ellipse; that is, they are the deflections of the plumb-line in the meridian at those stations owing to local attraction, or the attraction arising from the departure of the actual figure of the earth from the mean figure.

Fifteen of these formulæ I here select, adding one new one for Dehra about 56 miles to the north of Kaliana, the northern extremity of the Indian arc. They are as follows:—

* The numerical calculations in paragraphs 7 to 13 inclusive have been tested at the Government Trigonometrical Survey Office in Calcutta.

From the Anglo-Gallic Arc.

		Deflections.	Calculated attractions.	Deflections to be accounted for.
(1) Barcelona	+1·440−3·0644 U+0·0553 V−1·37=	+2·22		
(2) Dunkirk	+0·767+0·4115 U−0·0765 V−1·37=	−0·84		
(3) High Port Cliff.	+1·778+0·2532 U−0·0450 V−1·37=	+1·28	+3·29	−2·01
(4) Week Down ...	+1·747+0·2539 U−0·0452 V−1·37=	+0·25	+1·98	−1·73
(5) Boniface Down	+1·967+0·2559 U−0·0455 V−1·37=	+0·46	+2·42	−1·96
(6) Dunnose	−0·499+0·2613 U−0·0466 V−1·37=	−2·00	−0·54	−1·46
(7) Blackdown ...	+4·279+0·2859 U−0·0513 V−1·37=	+2·76		
(8) Burleigh Moor.	−1·814+1·6845 U−0·4137 V−1·37=	−4·15	−4·55	+0·40
(9) Cowhythe	−6·915+2·8048 U−0·8340 V−1·37=	−9·31	−5·50?	−3·81
(10) Ben Hutig	+0·095+3·1173 U−0·9708 V−1·37=	−3·25	−2·01	−1·24
(11) Saxavord	+4·403+3·9370 U−1·3699 V−1·37=	+0·41		
From the Russian Arc.				
(12) Tornea	+11·826+7·3799 U−2·5821 V−2·22=	+4·69		
(13) Fuglenœs	+10·008+9·1231 U−3·8418 V−2·22=	+1·13		
From the Indian Arc.				
(14) Punnœ	+ 0·625−3·5622U−3·1853V−0·033=	−0·94	+22·71	−23·65
(15) Kaliana	+ 0·403+4·1251U+2·7756V−0·033=	+1·34	+34·16	−32·82
(16) Dehra*	+53·796+4·4215U−0·1010V−0·033=	+52·09		

I have inserted the formula of Cowhythe from p. 771 of the Ordnance Survey Volume. I have also added two columns, in one of which are given the deflections of the plumb-line arising from attraction at those of the stations for which it has been calculated. For those of the Anglo-Gallic Arc, I refer to the Ordnance Survey Volume, sect. xi. p. 625; and for those of the Indian Arc to my paper in the Philosophical Transactions for 1861, p. 593. I would observe that not only in the two stations of the Indian Arc, but in those I have selected from the Anglo-Gallic Arc (all of which are near the sea-shore), allowance is made for deficiency of density and attraction of sea-water. In the stations (3), (4), (5), (6) the effect of the sea for about 9 miles south of the coast is taken and estimated at +0″·27 (see Ordnance Survey Volume, p. 631); in station (8) for 36 miles north, and estimated at −0″·39 (p. 642); in station (9) for 50 miles north, and estimated at −0″·70 (p. 664); in station (10) for 50 miles north, and estimated at −0″·64 (p. 662). It is of importance to bear this in mind. For stations (14) and (15) the effect of the sea the whole way to the south pole

* This is calculated by the formulæ at p. 737 of the Ordnance Survey Volume, from the following data obligingly furnished me by Major Walker, Superintendent of the Government Trigonometrical Survey of India, viz.

Astronomical latitude of Dehra 30° 19′ 19″.
Distance of parallels of Dehra and Damargida 4463510·7 feet.
The latitude of Damargida is 18° 3′ 15″.

is taken, and estimated at $+19''\cdot71$ and $+6''\cdot18$, the effect of the mountain mass on the north being $+3''\cdot00$ and $27''\cdot98$.

16. The first thing I observe in the results given in the last paragraph is the very small amount of the resultant deflections at the two extremities of the Indian Arc—Punnœ close to Cape Comorin, and Kaliana the nearest station to the Himmalaya Mountains; whereas the effect of the Ocean and the Mountains has been shown to be very large. This shows that the effect of variations of density in the crust must be very great, in order to bring about this near compensation. In fact the density of the crust beneath the mountains must be less than that below the plains, and still less than that below the ocean-bed. If solidification from the fluid state commenced at the surface, the amount of contraction in the solid parts beneath the mountain-region has been less than in the parts beneath the sea. In fact, it is this unequal contraction which appears to have caused the hollows in the external surface which have become the basins into which the waters have flowed to form the ocean. As the waters flowed into the hollows thus created, the pressure on the ocean-bed would be increased, and the crust, so long as it was sufficiently thin to be influenced by hydrostatic principles of floatation, would so adjust itself that the pressure on any *couche de niveau* of the fluid should remain the same. At the time that the crust first became sufficiently thick to resist fracture under the strain produced by a change in its density—that is, when it first ceased to depend for the elevation or depression of its several parts upon the principles of floatation, the total amount of matter in any vertical prism, drawn down into the fluid below to a given distance from the earth's centre, had been the same through all the previous changes. After this, any further contraction or any expansion in the solid crust would not alter the amount of matter in the vertical prism, except where there was an ocean; in the case of greater contraction under an ocean than elsewhere, the ocean would become deeper and the amount of matter greater, and in case of a less contraction or of an expansion of the crust under an ocean, the ocean would become shallower, or the amount of matter in the vertical prism less than before. It is not likely that expansion and contraction in the solid crust would affect the arrangement of matter in any other way. That changes of level do take place, by the rising and sinking of the surface, is a well-established fact, which rather favours these theoretical considerations. But they receive, I think, great support from the other fact, that the large effect of the ocean at Punnœ and of the mountains at Kaliana almost entirely disappear from the resultant deflections brought out by the calculations. The formulæ of paragraph 15 show that when we get close to the mountain-mass, as at Dehra, which is at the foot of the mountains where they first rise rapidly above the plains, the resultant deflection is very great; the less density of the crust down below the sea-level drawn under the mountain-mass has here a very trifling influence. This is as it should be, if the depth of this less density is considerable;

whereas at Kaliana, and stations still further off, the attraction of the mountain-mass above the sea-level, and the deficiency of attraction from the crust below that level, would nearly counterbalance each other. Thus, if the thickness of the crust below the plains is 100 miles, and the amount of matter in the crust under the plains equals that of the crust and mountains together in the mountain-region, then the deflections at Kaliana, Kalianpur, and Damargida, instead of being 27″·98, 12″·05, 6″·79, arising from the mountains alone, are reduced to 1″·54, −0″·06, −0″·06 (see Philosophical Transactions for 1858, p. 759), which are all insignificant compared with the large deflections caused by the mountains alone.

This theory, that the wide ocean has been collected on parts of the earth's surface where hollows have been made by the contraction and therefore increased density of the crust below, is well illustrated by the existence of a whole hemisphere of water, of which New Zealand is the pole, in stable equilibrium. Were the crust beneath only of the same density as that beneath the surrounding continents, the water would be drawn off by attraction and not allowed to stand in the undisturbed position it now occupies.

17. I have, in what goes before, supposed that, in solidifying, the crust contracts and grows denser, as this appears to be most natural, though, after the solid mass is formed, it may either expand or contract, according as an accession or diminution of heat may take place. If, however, in the process of solidifying, the mass becomes lighter, the same conclusion will follow—the mountains being formed by a greater degree of expansion of the crust beneath them, and not by a less contraction, than in the other parts of the crust. It may seem at first difficult to conceive how a crust could be formed at all, if in the act of solidification it becomes heavier than the fluid on which it rests; for the equilibrium of the heavy crust floating on a lighter fluid would be unstable, and the crust would sooner or later be broken through, and would sink down into the fluid, which would overflow it. If, however, this process went on perpetually, the descending crust, which was originally formed by a loss of heat radiated from the surface into space, would reduce the heat of the fluid into which it sank, and after a time a thicker crust would be formed than before, and the difficulty of its being broken through would become greater every time a new one was formed. Perhaps the tremendous dislocation of stratified rocks in huge masses with which a traveller in the mountains, especially in the interior of the Himmalaya region, is familiar, may have been brought about in this way. The catastrophes, too, which geology seems to teach have at certain epochs destroyed whole species of living creatures, may have been thus caused, at the same time breaking up the strata in which those species had for ages before been deposited as the strata were formed. These phenomena must now long have ceased to occur, at any rate on a very extensive scale, as Mr. Hopkins's investigations on Precession appear

to prove that the crust is very thick, at least 800 or 1000 miles; and this result, I understand, has been recently confirmed by Professor W. Thomson in a paper "On the Rigidity of the Earth."

18. These theoretical considerations receive, I think, some confirmation from an examination of the calculated deflection of the plumb-line at stations near the sea-shore. It is for this reason that I have collected the thirteen examples from the Anglo-Gallic and Russian Arcs in paragraph 15, all of which are near the coast. The evidence they furnish, however, is not to be compared in weight with that of the Indian Arc, already considered. In some instances the local attraction of the surrounding country and of the ocean for a certain distance has been calculated, as already stated. These results I will take into account, except the allowances for the ocean as noted at the end of paragraph 15, which I deduct in the following arrangement of the stations.

The Stations at which the Deflection is towards the Land.

				Deflection.
(1) Barcelona,	lat. 41° 23′	S.E. coast of	Spain,	..... +2″·22
(2) Dunkirk,	„ 51 2,	N.N.W.	France,	..... −0·84
(7) Blackdown,	„ 50 41,	S. „	Dorset,	..... +2·76
(9) Cowhythe,	„ 57 41,	N. „	Banff,	−3·81+0·70=−3·11
(10) Ben Hutig,	„ 58 33,	N. „	Sutherland,	−1·24+0·64=−0·60
(12) Tornea,	„ 65 50,	S. „	Lapland,	..... +4·69

The Stations at which the Deflection is towards the Sea.

				Deflection.
(3) High Port Cliff,	50° 36′,	S. coast of	Isle of Wight,	−2·01−0·27=−2″·28
(4) Week Down,	50 36,	„	„	−1·73−0·27=−2·00
(5) Boniface Down,	50 36,	„	„	−1·96−0·27=−2·23
(6) Dunnose,	50 37,	„	„	−1·46−0·27=−1·73
(8) Burleigh Moor,	54 34,	N. coast of	Yorkshire,	+0·40+0·39=+0·79
(11) Saxavord,	60 50,	N. „	Unst,	..... +0·41
(13) Fuglenœs,	70 40,	N. „	Finmark,	..... +1·13

The theory I have proposed, that contraction of the crust has formed the basins in which the sea has settled, can hardly be expected to apply so completely to such confined sheets of water as the Mediterranean south of Spain, and the Gulf of Bothnia. Here there may be an actual deficiency of attracting matter in the water, not altogether compensated for by increased density of the crust below. These hollows may have been formed during the breaking up of the crust and subsequent removal of portions by currents, and not chiefly by the contraction of the crust. Thus the deflections at the stations (1) and (12) towards the land may be sufficiently accounted for, even if the land about Barcelona and Tornea does not rise sufficiently high to produce them. The deflection at station (2) is small. It seems probable that even if the North Sea has been produced according

to the theory of contraction of the crust, the parts near Dunkirk may have been somewhat hollowed out by the scouring of the tide through the Straits of Dover, so as to give the land, low as it is, every advantage in deflecting the plumb-line south. I have no means of knowing the character of the ground north of station (7) on the coast of Dorset. There is no difficulty, however, in accounting for the north deflection at that place, and even for a greater deflection, if the attraction of the country north of it is as much as the attraction of the land on Burleigh Moor on the north coast of Yorkshire. To this station I shall revert. With regard to stations (9) and (10), I gather the following information from the Ordnance Survey Volume. "At present there are no sufficient data for calculating exactly the disturbance" at Cowhythe (p. 662). It is supposed not to exceed 6″ (p. 664); but the calculation is not made for any part of the mountains further south than 50 miles. The south deflection to be accounted for, viz. −3″·11, may in part be thus explained; or, even if, as before, the North Sea is supposed to have been formed by the contraction of the crust, the confined portion between the north coast of Aberdeen and the Orkney Islands may have been formed by the removal of the superficial strata by currents so as to produce a deficiency of attracting matter. So with respect to the other station, Ben Hutig, the unaccounted-for deflection, which is much smaller, viz. −0·60, may be easily explained, as the effect of the land has not been calculated further off than about 3 miles (pp. 660, 661). Thus, on the whole, the deflections at those coast-stations, where it is towards the land, can be pretty well accounted for, without calling in aid the deficiency of attraction of water and supposing that the crust below the ocean is not condensed.

The seven coast-stations of the second list, where the deflection is towards the sea, seem to bear individual testimony to the truth of the theory, that the crust below the ocean must have undergone greater contraction than other parts of the crust. The four stations (3), (4), (5), (6) on the south coast of the Isle of Wight all have deflections southwards; and their magnitudes diminish in the order that the distances from the sea increase,—that order being (3) High Port Cliff, (5) Boniface Down, (4) Week Down, (6) Dunnose (see the Contour Map of Isle of Wight in the volume of Plates accompanying the Ordnance Survey Volume). The amounts of the deflection seem almost to prove too much for the theory. Still they are all *in the direction* of the ocean, and seem certainly to indicate that there is a redundance of matter, and not a deficiency, in that direction. Blackdown (7) is somewhat further inland than Dunnose is. If, then, the ocean and crust together do really produce the outstanding deflection southward at Dunnose, we shall have to suppose that the north deflection at Blackdown in the first list of coast-stations, arising from the land, is not much less than 2·76+1·73=4·49, which is a little less than the calculated deflection at Burleigh Moor on the coast of Yorkshire, and is therefore not an unlikely amount. The other three coast-stations, (8), (11), (13), all bear out the

theory: though the three deflections are all small, they are towards the sea, the largest of them being at Fuglencæs, which is very near to the North Cape, and has a large expanse of ocean above it.

19. The least that can be gathered from the deflections of these coast-stations is, that they present no obstacle to the theory so remarkably suggested by the facts brought to light in India, viz. that mountain-regions and oceans on a large scale have been produced by the contraction of the materials, as the surface of the earth has passed from a fluid state to a condition of solidity—the amount of contraction beneath the mountain-region having been less than that beneath the ordinary surface, and still less than that beneath the ocean-bed, by which process the hollows have been produced into which the ocean has flowed. In fact the testimony of these coast-stations is in some degree directly in favour of the theory, as they seem to indicate, by *excess* of attraction towards the sea, that the contraction of the crust beneath the ocean has gone on increasing in some instances still further since the crust became too thick to be influenced by the principles of floatation, and that an additional flow of water into the increasing hollow has increased the amount of attraction upon stations on its shores.

Murree, Punjab,
August 20, 1863.

June 2, 1864.

The Annual Meeting for the Election of Fellows was held this day.

Major-General SABINE, President, in the Chair.

The Statutes relating to the Election of Fellows having been read, General Boileau and Sir Andrew Scott Waugh were, with the consent of the Society, nominated Scrutators to assist the Secretaries in examining the lists.

The votes of the Fellows present having been collected, the following gentlemen were declared duly elected into the Society:—

Sir Henry Barkly, K.C.B.
William Brinton, M.D.
T. Spencer Cobbold, M.D.
Alexander John Ellis, Esq.
John Evans, Esq.
William Henry Flower, Esq.
Thomas Grubb, Esq.
Sir John Charles Dalrymple Hay, Bart.
William Jenner, M.D.
Sir Charles Locock, Bart., M.D.
William Sanders, Esq.
Col. William James Smythe, R.A.
Lieut.-Col. Alexander Strange.
Robert Warington, Esq.
Nicholas Wood, Esq.

June 9, 1864.

Major-General SABINE, President, in the Chair.

Mr. W. Sanders; Mr. R. Warington; Dr. Jenner; Mr. J. Evans;

Lieut.-Col. Strange; Mr. W. H. Flower; Dr. Cobbold; Col. W. J. Smythe; Sir J. C. Dalrymple Hay, Bart.; and Mr. A. J. Ellis, were admitted into the Society.

Pursuant to notice given at the last Meeting, MM. Claude Bernard, Jean Bernard Léon Foucault, and Adolph Wurtz, all of Paris, were balloted for and elected Foreign Members of the Society.

The following communication was read:—

"Description of the Cavern of Bruniquel, and its Organic Contents.—Part I. Human Remains." By Professor Richard Owen, F.R.S., &c. Received May 12, 1864.

(Abstract.)

In this communication the author gives an account of the Cavern of Bruniquel, Department of the Tarn and Garonne, France, in the state which it presented when visited by him in January 1864, and a description of the human remains discovered therein by the proprietor, the Vicomte de Lastic St. Jal, in 1863, and subsequently by the author in January 1864.

The circumstances under which these discoveries were made are minutely detailed, and the contemporaneity of the human remains with those of the extinct and other animals with which they are associated, together with the flint and bone implements, is shown by the evidences of the plastic condition of the calcified mud of the breccia at the time of interment, by the chemical constitution of the human bones, corresponding with that of the other animal remains, and by the similarity of their position and relations in the surrounding breccia.

Among the principal remains of the men of the flint-period described are the following:—1st, the hinder portion of the cranium, with several other parts of the same skeleton, which were so situated in their matrix as to indicate that the body had been interred in a crouching posture, and that, after decomposition and dissolution of the soft parts, the skeleton had yielded to the superincumbent weight; 2nd, an almost entire calvarium, which is described and compared with different types of the human skull, shown to be superior in form and capacity to the Australian type, and more closely to correspond with the Celtic type, though proportionally shorter than the modern Celtic, and the form exhibited by the Celtic cranium from Engis, Switzerland; 3rd, jaws and teeth of individuals of different ages.

After noticing other smaller portions of human cranium, the author proceeds to describe minutely the lower jaw and teeth of an adult, and upper and lower jaws of immature individuals, showing the characters of certain deciduous teeth. The proportions of the molars are not those of the Australian, but of other races, and especially those of ancient and modern Europeans. As in most primitive or early races in which mastication was little helped by arts of cookery or by various and refined kinds of food, the crowns of the molars, especially of *m* 1, are worn down beyond

the enamel, flat and smooth to the stumps, exposing there a central tract of osteodentine without any sign of decay.

The paper is illustrated by a view and plans of the cavern, and by figures of the principal human remains, and of two implements of bone on which the Vicomte de Lastic had discovered, on removal of the breccia, outline figures of the head of a reindeer and the head of a horse in profile.

The description of the various remains of the animals killed for food, and of the flint- and bone-implements applied to that and other purposes, will be the subject of a future communication.

June 16, 1864.

Major-General SABINE, President, in the Chair.

Dr. Brinton; Professor Boole; Mr. T. Grubb; Sir Charles Locock, Bart.; and Mr. Nicholas Wood, were admitted into the Society.

The following communications were read:—

I. "On Complex Binary Quadratic Forms." By H. J. Stephen Smith, M.A., F.R.S., Savilian Professor of Geometry in the University of Oxford. Received May 18, 1864.

The purpose of this note is to extend to complex quadratic forms some important investigations of Gauss relating to real quadratic forms. We shall consider in order (I.) the definition of the Genera, (II.) the theory of Composition, (III.) the determination of the number of Ambiguous Classes, (IV.) the representation of forms of the principal genus by ternary quadratic forms of determinant 1. For the comparison of the numbers of classes of different orders, we may refer to a paper by M. Lipschitz (Crelle's Journal, vol. liv. p. 193); and for the principles of the theory of complex numbers and complex quadratic forms, to Lejeune Dirichlet's Memoir, "Recherches sur les formes quadratiques à coefficients et à indéterminées complexes" (Crelle, vol. xxiv. p. 291).

I. *The Definition of the Genera.*

Let $f=(a, b, c)$ be an uneven* primitive form of determinant D, and $m=ax^2+2bxy+cy^2$, $m'=ax'^2+2bx'y'+cy'^2$ two numbers represented by f. The generic characters of f are deducible from the equation

$$(ax^2+2bxy+cy^2)\,(ax'^2+2bx'y'+cy'^2)= (axx'+b[xy'+x'y]+cyy')^2-\mathrm{D}(xy'-x'y)^2,$$

* A primitive form (a, b, c) is uneven, semieven, or even, according as the greatest common divisor of a, $2b$, c is 1, $1+i$, or $(1+i)^2$; *i. e.*, in Lejeune Dirichlet's nomenclature, according as (a, b, c) is of the first, second, or third species. In this paper, when we speak of an uneven, semieven, or even form or class, we shall always suppose the form or class to be primitive. A semieven number is a number divisible by $1+i$, but not by $(1+i)^2$.

or, as we shall write it,

$$mm' = P^2 - DQ^2.$$

Thus, supposing that p is an uneven prime dividing D, and that m and m' are prime to p, the numbers prime to p, which are represented by f, are either all quadratic residues of p, or else all non-quadratic residues of p; in the former case we attribute to f the character $\left[\frac{f}{p}\right] = +1$, in the latter the character $\left[\frac{f}{p}\right] = -1$.

Again, to investigate the supplementary characters relating to powers of the even prime $1+i$, let $m = \mu + i\mu'$ be an uneven number, μ and μ' representing real numbers, and for brevity, let

$$(-1)^{\frac{1}{4}(N.m-1)} = \alpha,$$
$$(-1)^{\frac{1}{8}[(\mu+\gamma\mu')^2-1]} = \beta,$$
$$(-1)^{\mu'} = \gamma.$$

The values of the units, or *characters*, α, β, γ depend on the residue of m for the modulus $(1+i)^5$, as is shown in the following Table.

TABLE I.

$m \equiv$	$\alpha =$	$\beta =$	$\gamma =$
± 1	+1	+1	+1
$\pm i$	+1	+1	−1
± 3	+1	−1	+1
$\pm 3i$	+1	−1	−1
$\pm(1-2i)$ ..	−1	+1	+1
$\pm(2+i)$	−1	+1	−1
$\pm(1+2i)$ ..	−1	−1	+1
$\pm(2-i)$	−1	−1	−1

An inspection of the Table shows that, of the sixteen uneven residues of $(1+i)^5$, eight have the character $\omega = 1$, and eight the character $\omega = -1$, ω representing any one of the seven characters α, β, γ, $\beta\gamma$, $\alpha\gamma$, $\alpha\beta$, $\alpha\beta\gamma$. It will also be seen that any character of a product of two uneven factors

is found by multiplying together the corresponding characters of the factors; so that, conversely, according as any character of a product of two uneven factors is $+1$ or -1, the two factors agree or differ in respect of that character.

The next Table assigns the supplementary characters proper to any given determinant; they depend on the residue of the determinant for the modulus $(1+i)^5$.

TABLE II.

D$\equiv$	Characters.	D$\equiv$	Characters.
$\pm(1+i)$....	β	± 1	γ
$\pm(1-i)$....	$\alpha\beta$	$\pm i$	α
$\pm(3+i)$....	$\alpha\beta\gamma$	± 3	γ
$\pm(3-i)$....	$\beta\gamma$	$\pm 3i$	α
± 2.....	α, γ	$\pm(1-2i)$	γ
$\pm 2i$	γ	$\pm(2+i)$	$\alpha\gamma$
$2(1+i)$	$\alpha\beta, \gamma$	$\pm(1+2i)$	γ
$2(1-i)$	β, γ	$\pm(2-i)$	$\alpha\gamma$
4.....	α, γ		
0.....	α, β, γ		

Of the eighteen propositions contained in this Table, it will suffice to enunciate and demonstrate one.

"If D$\equiv\pm(3+i)$, mod $(1+i)^5$, and f is an uneven form of determinant D, the uneven numbers represented by f, all have the character $\alpha\beta\gamma=+1$, or else all have the character $\alpha\beta\gamma=-1$."

In the equation $P^2-DQ^2=mm'$, let us suppose that m and m' are uneven; then P is uneven because D is semieven; also $Q^2\equiv\pm 1, \pm 2i$, 4 or 0, mod $(1+i)^5$, according as the index of the highest power of $1+i$ dividing Q is 0, 1, 2, or >2. If Q is uneven, $mm'\equiv\pm 3i$ or $\pm(2+i)$, mod $(1+i)^5$; if Q is semieven, $mm'\equiv\pm(1+2i)$, mod $(1+i)^5$; if Q is even, $mm'\equiv\pm 1$, mod $(1+i)^5$; *i. e.* in all three cases mm' has the character $\alpha\beta\gamma=1$, and m and m' both have the character $\alpha\beta\gamma=+1$, or else both have the character $\alpha\beta\gamma=-1$.

We add a third Table for the purpose of distinguishing between the possible and impossible genera. In this Table S^2 is the greatest square dividing D, P is uneven and primary*, I is the index of the highest power of $1+i$ dividing S, ϖ represents an uneven prime dividing P, σ an uneven prime dividing S but not P. For brevity, the symbols ϖ and σ are written instead of $\left[\frac{f}{\varpi}\right]$ and $\left[\frac{f}{\sigma}\right]$.

* By a primary uneven number we understand (with Lejeune Dirichlet) an uneven number $\mu+\mu' i$ satisfying the congruences $\mu\equiv 1$, mod 4, $\mu'\equiv 0$, mod 2.

Table III.

(i) $D=PS^2$, $P\equiv 1$, mod 4.

I=0, 1	ϖ	σ, γ
I=2	ϖ	σ, γ, α
I>2	ϖ	$\sigma, \gamma, \alpha, \beta.$

(ii) $D=PS^2$, $P\equiv 1+2i$, mod 4.

I=0, 1	ϖ, γ	σ
I=2	ϖ, γ	σ, α
I>2	ϖ, γ	$\sigma, \alpha, \beta.$

(iii) $D=iPS^2$, $P\equiv 1$, mod 4.

I=0	ϖ, α	σ
I=1, 2	ϖ, α	σ, γ
I>2	ϖ, α	$\sigma, \gamma, \beta.$

(iv) $D=iPS^2$, $P\equiv 1+2i$, mod 4.

I=0	$\varpi, \alpha\gamma$	σ
I=1, 2	ϖ, α, γ	σ
I>2	ϖ, α, γ	$\sigma, \beta.$

(v) $D=(1+i)PS^2$, $P\equiv 1$, mod 4.

I=0	ϖ, β	σ
I=1	ϖ, β	σ, γ
I>1	ϖ, β	$\sigma, \gamma, \alpha.$

(vi) $D=(1+i)PS^2$, $P\equiv 1+2i$, mod 4.

I=0	$\varpi, \beta\gamma$	σ
I=1	ϖ, β, γ	σ
I>1	ϖ, β, γ	$\sigma, \alpha.$

(vii) $D=i(1+i)PS^2$, $P\equiv 1$, mod 4.

I=0	$\varpi, \alpha\beta$	σ
I=1	$\varpi, \alpha\beta$	σ, γ
I>1	ϖ, α, β	$\sigma, \gamma.$

(viii) $D=i(1+i)PS^2$, $P\equiv 1+2i$, mod 4.

I=0	$\varpi, \alpha\beta\gamma$	σ
I=1	$\varpi, \alpha\beta, \gamma$	σ
I>1	$\varpi, \alpha, \beta, \gamma$	$\sigma.$

The characters preceding the vertical line by which the Table is divided are not independent, but are subject to the condition (arising from the laws of quadratic residues) that their product must be a positive unit. To show that this is so, let $D=i^{\alpha'}(1+i)^{\beta'}PS^2$, where α' and β' are each either 0 or 1; also let $\gamma'=0$, or 1, according as $P\equiv 1$, or $\equiv 1+2i$, mod 4. If m is a number prime to $(1+i)D$ and capable of primitive representation*

* If $m=ax^2+2bxy+cy^2$, the representation of m by (a, b, c) is said to be primitive when the values of the indeterminates are relatively prime.

by f, the congruence $\omega^2 \equiv D$, mod m, is resoluble; and its resolubility implies the condition $\left[\frac{D}{m}\right] = \left[\frac{i^{\alpha'}}{m}\right] \times \left[\frac{(1+i)^{\beta'}}{m}\right] \times \left[\frac{P}{m}\right] = 1$. But, by the laws of quadratic residues, $\left[\frac{i}{m}\right] = \alpha$, $\left[\frac{1+i}{m}\right] = \beta$, $\left[\frac{P}{m}\right] = \gamma^{\gamma'}\left[\frac{m}{P}\right]$; and the condition just written becomes $\alpha^{\alpha'}\beta^{\beta'}\gamma^{\gamma'}\left[\frac{m}{P}\right] = 1$, which is coincident with that indicated in the Table. Thus (as in the real theory) one-half of the whole number of assignable generic characters are impossible*; we shall presently obtain a different proof of this result, and shall also show that the remaining half correspond to actually existing genera.

For the characters of a semieven form f, it is convenient to take the characters of the numbers represented by $\frac{f}{1+i}$; and for the characters of an even form, the characters of the numbers represented by $\frac{f}{2i}$. The following Table will serve to form the complete generic character in each case.

For a semieven form.

(i) $D \equiv PS^2$, $P \equiv 1$, mod 4.

$I = 0 \mid \varpi \mid \sigma$.

(ii) $D \equiv PS^2$, $P \equiv 1 + 2i$, mod 4.

$I = 0 \mid \varpi, \gamma \mid \sigma$.

For an even form.

$I = 0 \mid \varpi \mid \sigma$.

II. *The Theory of Composition.*

The theory of composition given in the 'Disquisitiones Arithmeticæ' is immediately applicable to complex quadratic forms. There are, however, a few points to which we must direct attention.

(1) If m_1, m_2, m_3 are the greatest common divisors of a, $2b$, c; a, $(1+i)b$, c; a, b, c, we have

(i) $m_1 = m_2 = m_3$,

(ii) $m_1 = m_2 = (1+i)m_3$,

(iii) $m_1 = (1+i)m_2 = (1+i)^2 m_3$,

according as (a, b, c) either is, or is derived from, (i) an uneven, (ii) a semieven, (iii) an even primitive. Hence the order of a form is given when m_1 and m_3 are given. Thus, if F is compounded of f and f', and if M_1 M_2 M_3, m_1 m_2 m_3, m'_1 m'_2 m'_3 refer to F, f, f' respectively, the order of F is completely determined by the two theorems, "M_1 is the product of m_1 and

* The determinant is supposed not to be a square.

m_1'." "$\frac{M_1}{M_2}$ is the least common multiple of $\frac{m_1}{m_2}$ and $\frac{m'_1}{m'_2}$." (Gauss's 5th and 6th conclusions, Disq. Arith. art. 235.)

It will be found that Gauss's proof of these theorems can be transferred to the complex theory; only, when f and f' are both semieven, or derived from semieven primitives, the proof of the sixth conclusion is incomplete, and, while showing that F cannot be derived from an uneven primitive, fails to show whether it is derived from a semieven or from an even primitive. But, in the same way in which Gauss has shown that M_1 is divisible by $m_1 \times m'_1$, it can also be shown that M_2 is divisible by $m_2 \times m'_2$*; *i. e.*, in the case which we are considering, M_2 is divisible by M_1, because $m_2 = m_1$, $m'_2 = m'_1$, and $m_1 m'_1 = M_1$. Therefore $M_2 = M_1$, and F is derived from a semieven primitive in accordance with our enunciation of Gauss's sixth conclusion.

(2) In the real theory, when two or more forms are compounded, each form may be taken either directly or inversely; but, however the forms are taken, the determinant of the resulting form is the same. In the complex theory, not only may each of the forms to be compounded be taken in either of two different ways, but also the determinant of the resulting form may receive either of two values, differing, however, only in sign; and it is important to attend to the ambiguities which thus arise. If a complex rational number n be written in the form $i^\lambda(1+i)^\mu \frac{P}{Q}$, where λ is 0, 1, 2, or 3, μ is any positive or negative integer, and P, Q are primary uneven complex integers, we may term i^λ the sign of n. Let F, of which the determinant is D, be transformed into the product $f_1 \times f_2 \times \ldots f_h$, by a substitution [X, Y] linear and homogeneous in respect of h binary sets; we have, as in the real theory, h equations of the type

$$\left(\frac{dX}{dx_k}\frac{dY}{dy_k} - \frac{dX}{dy_k}\frac{dY}{dx}\right)^2 = \frac{d_k}{D} \times \frac{\Pi \cdot f^2}{f^2{}_k},$$

d_k representing the determinant of f_k. Let

$$n_k = \left(\frac{dX}{dx_k}\frac{dY}{dy_k} - \frac{dX}{dy_k}\frac{dY}{dx_k}\right) \div \frac{\Pi \cdot f}{f_k},$$

so that $n^2{}_k = \frac{d_k}{D}$; if i^{λ_k} is the sign of n_k, we shall say that f_k is taken with the sign i^{λ_k}. We can thus enunciate the theorem, "Forms, compounded of the same forms, taken with the same signs, are equivalent." If f_1, $f_2 \ldots f_h$ are given forms which it is required to compound, the signs of $d_1, d_2, \ldots d_h$ must be all real, or else all unreal; and the sign of D will be real or unreal accordingly. The value of D (irrespective of its sign) is ascertained as in the real theory; but it may receive at our option, in the

* Disq. Arith. art. 235. The proof that $2(bb'+\Delta)$ and $2(bb'-\Delta)$ are divisible by $m_1 \times m'_1$, may be employed (*mutatis mutandis*) to show that $(1+i)(bb'+\Delta)$ and $(1+i)(bb'-\Delta)$ are divisible by $m_2 \times m'_2$.

one case, either of the two real signs, and in the other case either of the two unreal signs. And whichever sign we give to D, the form f_k may be taken with either of the two real signs, if the sign of $\frac{d^k}{D}$ is $+1$, and with either of the two unreal signs, if the sign of $\frac{d_k}{D}$ is -1. In the important case in which $d_1, d_2 \ldots$ all have the same sign, we shall always suppose D to have that sign, and $f_1, f_2 \ldots$ to be all taken with the sign $+1$. Adopting this convention, we see that the class compounded of given classes of the same determinant, or of different determinants having the same sign, is defined without ambiguity.

(3) By the general formulæ of M. Arndt (Crelle, vol. lvi. p. 69), which on account of their great utility we transcribe here, we can always obtain a form (A, B, C) compounded in any given manner of two forms (a, b, c) and (a', b', c'), of which the determinants d and d' are to one another as two squares.

$$A = \frac{aa'}{\mu^2}$$

$$\left.\begin{aligned} \frac{an'}{\mu} B &\equiv \frac{ab'}{\mu} \\ \frac{a'n}{\mu} B &\equiv \frac{a'b}{\mu} \\ \frac{bn'+b'n}{\mu} B &\equiv \frac{bb'+Dnn'}{\mu} \end{aligned}\right] \text{mod A}$$

$$C = \frac{B^2 - D}{A}.$$

In these formulæ D is the greatest common divisor of dm'^2 and $d'm^2$, m and m' representing the greatest common divisors of a, $2b$, c, and a', $2b'$ c'; n and n' are the square roots of $\frac{d}{D}$ and $\frac{d'}{D}$; μ is the greatest common divisor of an', $a'n$, and $bn'+b'n$. The signs of D, n, and n' are given, because the manner of the composition is supposed to be given; to μ we may attribute any sign we please, because the forms (A, B, C) and (−A, B, −C) are equivalent.

(4) If F= (A, B, C) is compounded of two primitive forms f and f', and if M is the highest power of $1+i$ dividing A, B, C (so that M is 1, or $1+i$, or $(1+i)^2$), the complete character of the primitive form $\frac{1}{M}$ F is obtained by the following rule:—

"If ω is any character common to f and f', $\frac{1}{M}$ F will have the character $\omega = +1$, or $\omega = -1$, according as f and f' agree or differ in respect of that character."

In comparing the characters of f and f', it is to be observed that if ω and ω' are two supplementary characters of f, and $\omega \times \omega'$ a supplementary character of f', $\omega \times \omega'$ is to be regarded as a character common to f and f'.

(5) Let us represent by (1), (σ), and (Σ)* respectively the principal uneven, semieven, and even classes of determinant D; *i. e.* the classes containing the forms $(1, 0, -D)$, $\left(1+i, 1, -\frac{D-1}{1+i}\right)$, and $\left(2i, i^k, -\frac{D-i^{2k}}{2i}\right)$, the existence of the last two classes implying the congruences $D \equiv 1$, mod 2, $D \equiv i^{2k}$, mod 4, respectively. Employing the formulæ of M. Arndt, we find $(f)\times(1)=(f)$, if (f) is any class of determinant D; $(f)\times(\sigma)=(1+i)(f)$, if f is derived from a semieven or even primitive; $(f)\times(\Sigma)=2i(f)$, if f is derived from an even primitive; and, in particular, $(1)\times(1)=(1)$, $(\sigma)\times(\sigma)=(1+i)(\sigma)$, $(\Sigma)\times(\Sigma)=2i(\Sigma)$. Also, if (f) and (f^{-1}) are two opposite primitive classes, $(f)\times(f)^{-1}=(1)$, or $(1+i)(\sigma)$, or $2i(\Sigma)$, according as f and f^{-1} are uneven, semieven, or even. Hence the three equations $(f_1)\times(\phi)=(f_2)$, $(f_1)\times(\phi)=(1+i)(f_2)$, $(f_1)\times(\phi)=2i(f_2)$, in which (f_1) and (f_2) are given primitive classes, uneven in the first, semieven in the second, and even in the third, are respectively satisfied by the uneven, semieven, and even classes $(\phi)=(f_2)\times(f_1)^{-1}$, $(\phi)=\frac{(f_2)\times(f_1)^{-1}}{1+i}$, $(\phi)=\frac{(f_2)\times(f_1)^{-1}}{2i}$, but by no other classes whatever. Again, let $D=\Delta m^2$ and let the forms (mp, mq, mr), $([1+i]mp, mq, [1+i]mr)$, $(2imp, mq, 2imr)$ represent classes derived by the multiplier m from uneven, semieven, and even primitives of determinant Δ; in all three forms we suppose p prime to 2D; in the second and third we suppose q uneven, and $\Delta \equiv 1$, mod 2; in the third we suppose $\Delta \equiv i^{2k}$, mod 4. The formulæ of M. Arndt will then establish the six equations,—

$$(m, 0, -\Delta m)\times(p, mq, m^2r)=(mp, mq, mr),$$

$$\left([1+i]m, m, -m\frac{\Delta-1}{1+i}\right)\times(p, mq, 2im^2r)=([1+i]mp, mq, [1+i]mr),$$

$$\left(2im, i^k m, -m\frac{\Delta-i^{2k}}{2i}\right)\times(p, mq, -4m^2r)=(2imp, mq, 2imr),$$

$$\left([1+i]m, m, -m\frac{\Delta-1}{1+i}\right)\times([1+i]p, mq, [1+i]m^2r)$$
$$=(1+i)\times([1+i]mp, mq, [1+i]mr),$$

$$\left(2im, i^k m, -m\frac{\Delta-i^{2k}}{2i}\right)\times([1+i]p, mq, 2i[1+i]m^2r)$$
$$=(1+i)\times(2imp, mq, 2imr),$$

$$\left(2im, i^k m, -m\frac{\Delta-i^{2k}}{2i}\right)\times(2ip, mq, 2im^2r)=2i\times(2imp, mq, 2imr).$$

* It is often convenient to symbolize a class by placing within brackets a symbol representing a form contained in the class; thus (f) may be used to symbolize the class containing the form f.

From these equations, which contain a solution (for complex numbers) of the problem solved for real numbers in art. 250 of the 'Disquisitiones Arithmeticæ,' we may infer the following theorems (Disq. Arith. art. 251 and 253):—

"The number ω of classes of any order Ω is a divisor of the number n of uneven classes of the same determinant D; and, given any two classes of order Ω, there are always $\frac{n}{\omega}$ uneven classes which compounded with one of them produce the other."

"If $D \equiv 1$, mod 2, and if the classes of Ω are derived from semieven or even primitives, ω is a divisor of the number n' of semieven classes of determinant D; and, given any two classes of order Ω, there are always $\frac{n'}{\omega}$ semieven classes which compounded with one of them produce $1+i$ times the other."

"If $D \equiv \pm 1$, mod 4, and if the classes of Ω are derived from even primitives, ω is a divisor of the number n'' of even classes of determinant D; and, given any two classes of order Ω, there are always $\frac{n''}{\omega}$ even classes which compounded with one of them produce $2i$ times the other."

III. *Determination of the number of Ambiguous Classes.*

Any form (A, B, C), in which $2B \equiv 0$, mod A, is called by Gauss an ambiguous form; but in the investigation which follows we shall for brevity understand by an ambiguous form an uneven form of one of the four types

(i) (A, 0, C),
(ii) ([1+i] B, B, C),
(iii) (2B, B, C),
(iv) (2i B, B, C).

To determine the number of uneven ambiguous classes of any determinant D supposed not to be a square, we shall determine, first, the number of ambiguous forms of determinant D; and secondly the number of ambiguous forms in each ambiguous class.

(1) Let μ be the number of different uneven primes dividing D. The number of ambiguous forms of the type (i) is $4 \times 2^{\mu}$, or $8 \times 2^{\mu}$, according as D is, or is not, uneven. For we may resolve $-D$ into any two relatively prime factors, and may take one of them (with any sign we please) for A, and the other for C. There are no ambiguous forms of the type (ii), unless $D \equiv i$, mod 2, or $\equiv 0$, mod $(1+i)^3$. For in the equation $D = B\,(B-[1+i]\,C)$, if B is uneven, we have $D \equiv i$, mod 2, because C must be uneven; if B is semieven or even, we have $D \equiv 0$, mod $(1+i)^3$. If $D \equiv i$, mod 2, we resolve D into any two relatively prime factors X and Y, and writing $B = X$, $B-(1+i)\,C = Y$, we find $C = \frac{X-Y}{1+i}$, which is in-

tegral because X and Y are uneven, and uneven because X is not $\equiv$Y, mod 2. Thus if $D \equiv i$, mod 2, there are $4 \times 2^{\mu}$ ambiguous forms of the type (ii). Again, if $D \equiv 0$, mod $(1+i)^3$, we may resolve D in any way we please into two factors having $1+i$ for their greatest common divisor; we find in this way $8 \times 2^{\mu}$ ambiguous forms of the type (ii). There are no ambiguous forms of the types (iii) or (iv), unless $D \equiv 1$, mod 2, or $\equiv 2$, mod 4, or $\equiv 0$, mod $(1+i)^5$. For if in the equation $D = B(B-2C)$, we suppose B uneven, we find $D \equiv 1$, mod 2; if B is semieven, $B^2 \equiv 2i$, and $2BC \equiv 2(1+i)$, mod 4, whence $D \equiv 2$, mod 4; lastly, if B is even, $D \equiv 0$, mod $(1+i)^5$. The same reasoning applies to the equation $D = B(B-2iC)$. If $D \equiv 1$, mod 2, we resolve D in every possible way into the product of two factors relatively prime; let $D = X \times Y$ be such a resolution, then $D = iX \times -iY$ is another; and it will be seen that according as the last coefficient in the two forms

$$\left[2X, X, \frac{X-Y}{2}\right], \quad \left[2iX, X, \frac{X-Y}{2i}\right]$$

is uneven or not uneven, so the last coefficient in the two forms

$$\left[2iX, iX, \frac{iX+iY}{2}\right], \quad \left[-2X, iX, \frac{X+Y}{2}\right],$$

is not, or is, uneven; *i. e.* there are $2 \times 2^{\mu}$ ambiguous forms of each of the types (iii) and (iv). If $D \equiv 2$, mod 4, we resolve D in every possible way into two factors, of which $1+i$ is the greatest common divisor; we thus find $4 \times 2_{\mu}$ uneven forms of each of the types (iii) and (iv). Lastly, if $D \equiv 0$, mod $(1+i)^5$, we resolve D in every possible way into two factors of which $1+i$ is the greatest common divisor, and we obtain 8×2 forms of each of the types (iii) and (iv).

The result of this enumeration is that if D be uneven, or semieven, or $\equiv 2i$, mod 4, there are $8 \times 2^{\mu}$ ambiguous forms; if $D \equiv 2$, mod 4, or $\equiv 0$, mod $(1+i)^3$, but not mod $(1+i)^5$, there are $16 \times 2^{\mu}$; and if $D \equiv 0$, mod $(1+i)^5$, there are $32 \times 2^{\mu}$. On comparing this result with Table III., it will be seen that in every case there are four times as many ambiguous forms as there are assignable generic characters.

(2) Let $f = (a, b, c)$ be any form of an ambiguous class; if $(I) = \begin{vmatrix} \mu, & -\lambda \\ \nu, & -\mu \end{vmatrix}$ is an improper automorphic of f, λ, μ, ν satisfy the equations

$$\mu^2 - \lambda\nu = 1, \quad \ldots\ldots\ldots\ldots \quad (1)$$

$$\lambda a + 2\mu b + \nu c = 0; \quad \ldots\ldots\ldots\ldots \quad (2)$$

and, conversely, if λ, μ, ν satisfy the equations (1) and (2), $(I) = \begin{vmatrix} \mu, & -\lambda \\ \nu, & -\mu \end{vmatrix}$ is an improper automorphic of f. Let α, γ, p, q (of which α and γ are relatively prime) be a system of integral numbers satisfying the equations

$$\left.\begin{aligned} p\alpha &= \lambda, & p\gamma &= \mu - 1, \\ q\alpha &= \mu + 1, & q\gamma &= \nu; \end{aligned}\right\} \quad \ldots\ldots\ldots \quad (3)$$

and let $\theta=0$, $1-i$, 1, or $-i$, according as 0, $1-i$, 1 or $-i$ satisfies the congruences

$$p+\theta\alpha\equiv 0, \text{ mod } 2,$$
$$q+\theta\gamma\equiv 0, \text{ mod } 2,$$

which are simultaneously resoluble, and admit of only one solution, because α and γ are relatively prime, while $q\alpha-p\gamma=2$. Then it will be found that by the proper transformation

$$(\mathrm{J})=\begin{vmatrix}\alpha, & \frac{1}{2}(p+\theta\alpha)\\ \gamma, & \frac{1}{2}(q+\theta\gamma)\end{vmatrix}$$

f is transformed into an ambiguous form ϕ, which will be of the type (i), (ii), (iii), or (iv), according as $\theta=0$, $1-i$, 1, or $-i$. It will also be seen that, subject to the condition that α and γ are relatively prime, there are always four, and only four, solutions of the system (3), represented by the formula

$$i^k\alpha,\quad i^k\gamma,\quad i^{-k}p,\quad i^{-k}q.$$

There are thus four transformations included in the formula (J), two of them transforming f into the same ambiguous form ϕ, and the other two transforming f into the same form taken negatively. The four transformations (J), and the two ambiguous forms ϕ and $-\phi$, we shall term respectively the transformations and the ambiguous forms appertaining to the improper automorphic (I). If we now form the transformations appertaining to every improper automorphic of f, it can be proved (A) that these transformations will all be different, and (B) that they will include every proper transformation of f into an ambiguous form.

(A) As the four transformations appertaining to the same improper automorphic are evidently different, it will be sufficient to show that if (J) and (J′) appertain to the improper automorphics (I) and (I′), the supposition (J)=(J′) implies (I)=(I′). From the equations

$$\alpha=\alpha',\quad \gamma=\gamma',\quad p+\theta\alpha=p'+\theta'\alpha',\quad q+\theta\gamma=q'+\theta'\gamma'$$

(which are equivalent to the symbolic equation (J)=(J′)), combined with the system (3), and with a similar system containing the accented letters, we find

$$(\theta-\theta')\alpha^2=\lambda'-\lambda,\ (\theta-\theta')\alpha\gamma=\mu'-\mu,\ (\theta-\theta')\gamma^2=\nu-\nu';$$

whence again $(\theta-\theta')(a\alpha^2+2b\alpha\gamma+c\gamma^2)=0$, by virtue of equation (2). The coefficient of $\theta-\theta'$ is not zero, for $\mathrm{D}=b^2-ac$ is not a square; therefore $\theta-\theta'=0$; *i.e.* $\lambda=\lambda'$, $\mu=\mu'$, $\nu=\nu'$, or (I)=(I′).

(B) Let $\begin{vmatrix}\alpha, & \beta\\ \gamma, & \delta\end{vmatrix}$ be a proper transformation of f into an ambiguous form ϕ; according as ϕ is of the type (i), (ii), (iii), or (iv), let $\theta=0$, $1-i$, 1, or $-i$; let also $\lambda=2\alpha\beta-\theta\alpha^2$, $\mu=\alpha\delta+\beta\gamma-\theta\alpha\gamma$, $\nu=2\gamma\delta-\theta\gamma^2$; then $\begin{vmatrix}\mu, & -\lambda\\ \nu, & -\mu\end{vmatrix}=$(I) is an improper automorphic of f; for

$$\mu^2-\lambda\nu=(\alpha\delta-\beta\gamma)^2=1,\quad \text{and } \lambda a+2\mu b+\nu c=0,$$

because of the ambiguity of the form into which f is transformed by

$\begin{vmatrix}\alpha, \beta\\ \gamma, \delta\end{vmatrix}$. Also $\begin{vmatrix}\alpha, \beta\\ \gamma, \delta\end{vmatrix}$ appertains to (I); for, writing p and q instead of $2\beta-\theta\alpha$, and $2\delta-\theta\gamma$, we have $\begin{vmatrix}\alpha, \beta\\ \gamma, \delta\end{vmatrix} = \begin{vmatrix}\alpha, \frac{1}{2}(p+\theta\alpha)\\ \gamma, \frac{1}{2}(q+\theta\gamma)\end{vmatrix}$, α, γ, p, q (of which α and γ are relatively prime) being four numbers which satisfy the system (3); *i. e.* $\begin{vmatrix}\alpha, \beta\\ \gamma, \delta\end{vmatrix}$ appertains to (I), an improper automorphic of f.

It follows from (B) that, if we calculate the ambiguous forms ϕ and $-\phi$ appertaining to every improper automorphic of f, we shall obtain all the ambiguous forms to which f is equivalent; it remains to see how many of these ambiguous forms are different from one another. If (I) $= \begin{vmatrix}\mu, -\lambda\\ \nu, -\mu\end{vmatrix}$ is any given improper automorphic of f, all its similar automorphics are contained in the four formulæ

$$(\mathrm{T})^{2k}\times(\mathrm{I}),\ (\mathrm{T})^{2k+1}\times(\mathrm{I}),\ (\mathrm{T})^{2k}\times\begin{vmatrix}-1, 0\\ 0, -1\end{vmatrix}\times(\mathrm{I}),$$

$$(\mathrm{T})^{2k+1}\times\begin{vmatrix}-1, 0\\ 0, -1\end{vmatrix}\times(\mathrm{I}),$$

where k is any positive or negative number, and $(\mathrm{T}) = \begin{vmatrix}t_1 - u_1 b, & -u_1 c\\ u_1 a, & t_1 + u_1 b\end{vmatrix}$, $[t_1, u_1]$ representing a fundamental solution of the equation $t^2 - Du^2 = 1$. Similarly, if (J) represent the four transformations, appertaining to (I), by which f passes into ϕ or $-\phi$, all the proper transformations of f into ϕ or $-\phi$ are included in the formula $(\mathrm{T})^k\times(\mathrm{J})$. We shall now show that the four transformations included in the formula $(\mathrm{T})^k\times(\mathrm{J})$ appertain to the improper automorphic $(\mathrm{T})^{2k}\times(\mathrm{I})$. Writing

$$\alpha_k = (t_k - bu_k)\alpha - cu_k\gamma, \qquad p_k = (t_k - bu_k)p - cu_k q,$$
$$\gamma_k = au_k\alpha + (t_k + bu_k)\gamma, \qquad q_k = au_k p + (t_k + bu_k)q,$$
$$\lambda_{2k} = (t_{2k} - bu_{2k})\lambda - cu_{2k}\mu,$$
$$\mu_{2k} = (t_{2k} - bu_{2k})\mu - cu_{2k}\nu = au_{2k}\lambda + (t_{2k} + bu_{2k})\mu,$$
$$\nu_{2k} = au_{2k}\mu + (t_{2k} + bu_{2k})\nu,$$

we find immediately

$$(\mathrm{T})^k\times(\mathrm{J}) = \begin{vmatrix}\alpha_k, \frac{1}{2}(p_k + \theta\alpha_k)\\ \gamma_k, \frac{1}{2}(q_k + \theta\gamma_k)\end{vmatrix}, \quad (\mathrm{T})^{2k}\times(\mathrm{I}) = \begin{vmatrix}\mu_{2k}, -\lambda_{2k}\\ \nu_{2k}, -\mu_{2k}\end{vmatrix},$$

Also attending to the equations (2) and (3), and to the relations

$$t_{2k} = t^2{}_k - Du^2{}_k, \quad u_{2k} = 2t_k u_k,$$

we obtain, after substitution and reduction,

$$p_k\alpha_k = \lambda_{2k}, \qquad p_k\gamma_{2k} = \mu_{2k} - 1,$$
$$q_k\alpha_k = \mu_{2k} + 1, \quad q_k\gamma_k = \nu_{2k},$$

i. e. $(\mathrm{T})^k\times(\mathrm{J})$ appertains to $(\mathrm{T})^{2k}\times(\mathrm{I})$, if (J) appertains to (I).

It follows from this result that the ambiguous forms appertaining to (I) and to $(\mathrm{T})\times(\mathrm{I})$ are the same; for f is transformed into the same forms by (J) and $(\mathrm{T})\times(\mathrm{J})$; and conversely, if the ambiguous forms appertain-

ing to two different automorphics (I) and (I′) are identical, an equation of the form $(I')=T^{2k}\times(I)$ will subsist; for if (J) and (J′) are the transformations appertaining to (I) and (I′), since by hypothesis (J) and (J′) transform f into the same form, we must have an equation of the form $(J')=(T)^k\times(J)$; but (J′) appertains to (I′), and $(T)^k\times(J)$ to $(T)^{2k}\times(I)$; therefore $(I')=(T)^{2k}\times(I)$, by what has been shown above (A).

If then we calculate the eight ambiguous forms appertaining to the four improper automorphics

$$(I),\ \begin{pmatrix}-1, 0\\ 0, -1\end{pmatrix}\times(I),\ (T)\times(I),\ \begin{pmatrix}-1, 0\\ 0, -1\end{pmatrix}\times(T)\times(I),$$

these eight forms will be the only ambiguous forms equivalent to f. Thus every uneven ambiguous class contains eight ambiguous forms.

Combining this result with the preceding we obtain the Theorem,

"The number of uneven ambiguous classes is one half of the whole number of assignable generic characters."

The number of semieven and even ambiguous classes is determined by the two following Theorems:—

"When $D\equiv\pm1$, mod 4, there are as many even as semieven ambiguous classes."

"When $D\equiv1$, mod 2, there are as many semieven as uneven ambiguous classes, or only half as many, according as there are altogether as many semieven as uneven classes, or only half as many."

To prove the first of these theorems, let $D\equiv i^{2k}$, mod 4, and let

$$\Sigma=\left(2i,\ i^k,\ \frac{i^{2k}-D}{2i}\right);$$

it is evident from the principles of the composition of forms that if (ϕ) is a given semieven ambiguous class, the equation $(\Sigma)\times(\phi)=(1+i)\times(f)$ is satisfied by one and only one even ambiguous class (f); in addition to this we shall now show that, if (f) is a given even ambiguous class, the same equation is satisfied by one and only one semieven ambiguous class (ϕ); from which two things the truth of the theorem is manifest. First, let the whole number of even classes be equal to the whole number of semieven classes*; then the equation

$$(\Sigma)\times(\phi)=(1+i)\times(f)$$

* That if $D\equiv\pm1$, mod 4, there are either as many semieven as even classes, or else three times as many, is a theorem of M. Lipschitz (Crelle, vol. liv. p. 196), of which it is worth while to give a proof here. The number of even classes is to the number of semieven classes, as unity to the number of semieven classes satisfying the equation

$$(\Sigma)\times(\phi)=(1+i)\times(f),$$

f representing any given even form. To investigate the semieven classes satisfying this equation, apply to f a complete system of transformations for the modulus $1+i$, for example, the transformations

$$\begin{vmatrix}1, 0\\ 0, 1+i\end{vmatrix},\quad \begin{vmatrix}1+i, 0\\ 0\ \ 1\end{vmatrix},\quad \begin{vmatrix}1+i, 1\\ 0, 1\end{vmatrix},$$

is satisfied by only one semieven class (ϕ); and this class is ambiguous, for the equation is satisfied by the opposite of (ϕ) as well as by (ϕ) itself; therefore (ϕ) and its opposite are the same class, or (ϕ) is an ambiguous class. Secondly, let the number of semieven classes be three times the number of even classes; then the equation

$$(\Sigma)\times(\phi)=(1+i)\,(f)$$

is satisfied by three and only three different classes (ϕ); but it is also satisfied by the opposites of these classes; therefore one of them is necessarily an ambiguous class. Let that class be (ϕ_0); the other two are defined by the equations

$$(1+i)\,(\phi_1)=(\sigma_1)\times(\phi_0),\quad (1+i)\,(\phi_2)=(\sigma_2)\times(\phi_0),$$

and cannot be ambiguous classes; for by duplication we find

$$(\phi_1)\times(\phi_1)=(1+i)\,(\sigma_2),\quad (\phi_2)\times(\phi_2)=(1+i)\,(\sigma_1);$$

whereas every semieven ambiguous class produces $(1+i)\sigma_0$ by its duplication *.

The second theorem may be proved as follows. Let

$$f=([1+i]p,\ q,\ [1+i]r)$$

be a semieven form of determinant D; and let

$$\sigma_0=\left((1+i),\ 1,\ -\frac{D-1}{1+i}\right);$$

we suppose that p is uneven. The equation $(\sigma_0)\times(\phi)=(f)$ is satisfied by one uneven class (ϕ_0), or by two (ϕ_0) and (ϕ_1), according as the forms $\phi_0=(p,\ q,\ 2ir)$, and $\phi_1=(2ip,\ q,\ r)$, if r is uneven, or the forms $\phi_0=(p,\ q,\ 2ir)$, and $\phi_1=(2ip,\ [1+i]p+q,\ p+[1-i]q+r)$, if r is even, are or are not equivalent†. If any one of the forms f, ϕ_0, ϕ_1 is ambiguous, the others are so too; the same thing is therefore true for the classes (f), (ϕ_0), (ϕ_1). Thus the number of semieven ambiguous classes is equal to or

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and divide the resulting forms by $1+i$; of the quotients, one, or three, will be semieven, according as $D\equiv\pm1$, or ±5, mod $(1+i)^5$. It will be found that each of these semieven forms satisfies the equation $\Sigma\times\phi=(1+i)\times f$; and, conversely, every semieven form ϕ satisfying that equation is equivalent to one of these forms; for, from any transformation of $(1+i)f$ into $\Sigma\times\phi$, we may (by attributing to the indeterminates of Σ the values 1, 0) deduce a transformation of modulus $1+i$ by which f passes into $(1+i)\phi$; *i.e.*, ϕ is equivalent to one of the forms obtained by the preceding process. It only remains to show that when there are three of these forms, they constitute either one or three classes, but never two. For this purpose it is sufficient to consider the three semieven forms $\sigma_0=\left(1+i,\ 1,\ -\frac{D-1}{1+i}\right)$, σ_1, and σ_2, obtained by the preceding process from the form Σ. These forms satisfy the equations $\sigma_0\times\sigma_0=(1+i)\sigma_0$, $\sigma_1\times\sigma_1=(1+i)\sigma_2$, $\sigma_2\times\sigma_2=(1+i)\sigma_1$, $\sigma_1\times\sigma_2=(1+i)\sigma_0$; from which it follows that any one of the suppositions $\sigma_1=\sigma_2$, $\sigma_2=\sigma_0$, $\sigma_0=\sigma_1$ involves the other two.

* For the definition of the classes (σ_0), (σ_1), (σ_2) see the preceding note.

† The forms ϕ_0 and ϕ_1 are obtained by applying to f a complete set of transformations of modulus $1+i$, dividing the resulting forms by $1+i$, and retaining only those quotients which are uneven forms.

is one half of the number of uneven ambiguous classes, according as the classes (ϕ_0) and (ϕ_1) are identical or not; *i. e.*, according as the whole number of semieven classes is equal to or is one-half of the whole number of even classes.

The demonstration in the 'Disquisitiones Arithmeticæ,' that the number of genera of uneven forms of any determinant cannot exceed the number of uneven ambiguous classes of the same determinant, may be transferred without change to the complex theory. We thus obtain a proof (independent of the law of quadratic reciprocity and of the theorems which determine the quadratic characters of i and $1+i$) of the impossibility of one-half of the whole number of assignable generic characters; and from that impossibility, as we shall now show, the quadratic theorems are themselves deducible.

(1) If p is an uneven prime $\equiv 1$, mod 2, there are two genera of uneven forms of determinant p: of these one is the principal genus, and has the complete characters $\left(\frac{f}{p}\right)=1$, $\gamma=1$; the other, containing the form $(i, 0, +ip)$, has the particular character $\gamma=-1$; whence it follows that every uneven form of determinant p, which has the character $\gamma=+1$, is a form of the principal genus, and has the character $\left[\frac{f}{p}\right]=+1$. Again, if $p\equiv 1$, mod 4, the form $\left(2i, i, -\frac{p+1}{2i}\right)$ is an uneven form of determinant p; this form has the particular character $\gamma=-1$, because $-\frac{p+1}{2i}\equiv i$, mod 2; it is therefore not a form of the principal genus; but it has the character $\left(\frac{f}{p}\right)=1$, because $2i$ is a square; therefore, if $p\equiv 1$, mod 4, every uneven form of determinant p has the character $\left[\frac{f}{p}\right]=+1$.

(2) There is but one genus of forms of determinant i, and its complete character is $\alpha=+1$; there is also but one genus of forms of determinant $1+i$, and its complete character is $\beta=+1$.

(3) Let p and q be uneven primes of which the imaginary parts are even; to prove the law of reciprocity, it will suffice to show that if $\left[\frac{p}{q}\right]=1$, then $\left[\frac{q}{p}\right]=1$. The equation $\left[\frac{p}{q}\right]=1$ implies the existence of a congruence of the type $\omega^2-p\equiv 0$, mod q, and consequently of an uneven form of determinant p, and of the type $\left(q, \omega, \frac{\omega^2-p}{q}\right)$. This form has the character $\gamma=+1$, because $q\equiv 1$, mod 2; it therefore has the character $\left[\frac{f}{p}\right]=1$; *i. e.* $\left[\frac{\ }{p}\right]=1$.

(4) To prove the equation $\left[\frac{i}{p}\right]=(-1)^{\frac{1}{4}(Np-1)}$, in which we may sup-

pose that the uneven prime p is primary, it will suffice to show (i) that if $\left[\frac{i}{p}\right]=+1$, then $(-1)^{\frac{1}{4}(Np-1)}=1$; (ii) that if $(-1)^{\frac{1}{4}(Np-1)}=1$, then $\left[\frac{i}{p}\right]=1$. (i) Let $\left[\frac{i}{p}\right]=1$; then, if $\omega^2-i\equiv 0$, mod p, $\left(p, \omega, \frac{\omega^2-i}{p}\right)$ is a form of determinant i; it therefore has the character $\alpha=1$, *i. e.* $(-1)^{\frac{1}{4}(Np-1)}=1$. (ii) Let $(-1)^{\frac{1}{4}(Np-1)}=1$; then $p\equiv 1$, mod 4, and the form $(i, 0, ip)$ is an uneven form of determinant p; it therefore has the character $\left(\frac{f}{p}\right)=+1$; whence $\left[\frac{i}{p}\right]=+1$.

(5) Similarly, if $p=p_0+ip_1$ is an uneven and primary prime, to prove the equation $\left[\frac{1+i}{p}\right]=(-1)^{\frac{(p_0+p_1)^2-1}{8}}$ we shall show, (i) that if $\left[\frac{1+i}{p}\right]=1$, then $(-1)^{\frac{(p_0+p_1)^2-1}{8}}=1$; (ii) that if $(-1)^{\frac{(p_0+p_1)^2-1}{8}}=1$, then $\left[\frac{1+i}{p}\right]=1$. (i) Let $\left[\frac{1+i}{p}\right]=1$; then there is a form of determinant $1+i$ and of the type $\left(p, \omega, \frac{\omega^2-1-i}{p}\right)$; this form has the character $\beta=+1$; therefore $(-1)^{\frac{(p_0+p_1)^2-1}{8}}=+1$. (ii) Let $(-1)^{\frac{(p_0+p_1)^2-1}{8}}=+1$; then p is either $\equiv 1-2i$, or $\equiv 1$, mod $(1+i)^5$; if $p=(1+i)^5k+1-2i$, $([1+i]^3, i, 1-2ki)$ is an uneven form of determinant p; this form has the character $\gamma=+1$, and consequently it also has the character $\left[\frac{f}{p}\right]=+1$; therefore $\left[\frac{1+i}{p}\right]=\left[\frac{(1+i)^3}{p}\right]=+1$; if $p=(1+i)^5k+1$, one or other of the forms $([1+i]^5, 1, -k)$, and $([1+i]^5, 1+[1+i]^3, 1-k)$ is an uneven form of determinant p, having the character $\left[\frac{f}{p}\right]=1$; therefore in this case also $\left[\frac{1+i}{p}\right]=\left[\frac{(1+i)^5}{p}\right]=+1$.

IV. *The representation of Binary Forms of the principal Genus by Ternary Forms of Determinant* 1.

The solution of the general problem, "To find the representations (if any) of a given binary by a given ternary quadratic form," depends, in the case of complex as of real numbers, on the solution of the problem of equivalence for ternary forms. Extending the methods of Gauss to the complex theory, we find the necessary and sufficient condition for the primitive*

* If a matrix of the type $\left|\begin{matrix}\alpha, \beta\\ \alpha', \beta'\\ \alpha'', \beta''\end{matrix}\right|$ transforms a ternary into a binary quadratic form, the representation of the binary by the ternary form is said to be primitive when the three determinants of the matrix are relatively prime.

representation of a binary form f of determinant D by a ternary form of determinant 1 to be, that f should be a form of the principal genus; or, if $D \equiv \pm 1$, mod 4, that f should be a form either of the principal genus, or else of that genus which differs from the principal genus only in having the character $\gamma = -1$, instead of $\gamma = +1$. Again, because the reduction of Lagrange is applicable to complex binary forms, the reduction of Gauss* is applicable to complex ternary forms. It is thus found that the number of classes of such forms of a given determinant is finite; and in particular that every form of determinant 1 is equivalent to one or other of the forms $-x^2-y^2-z^2$ and $x^2+iy^2+iz^2$, of which the former cannot represent numbers $\equiv i$, or $\equiv 1+i$, mod 2; and the latter cannot primitively represent numbers $\equiv 2$, or $\equiv 2(1+i)$, mod 4. The method of reduction itself sup-

* If $F = ax^2+a'y^2+a''z^2+2byz+2b'xz+2b''xy$ is a ternary form of determinant Δ, and $Ax^2+A'y^2+A''z^2+2Byz+2B'xz+2B''xy$ its contravariant, by applying the reduction of Lagrange to the form $ax^2+2b''xy+a'y^2$, we can render $N.a \leqq 2\sqrt{N.A''}$ (Dirichlet in Crelle's Journal, vol. xxiv. p. 348); and by applying the same reduction to the form $A'y^2+2Byz+A''z^2$, we can render $N.A'' \leqq 2\sqrt{N.a\Delta}$. The reduction of Gauss consists in the alternate application of these two reductions until we arrive at a form in which we have simultaneously $N.a \leqq 2\sqrt{N.A''}$, $N.A'' \leqq 2\sqrt{N.a\Delta}$; and consequently $N.a \leqq 4\sqrt[3]{N.\Delta}$, $N.A'' \leqq 4\sqrt[3]{N.\Delta^2}$. If $\Delta = 1$, we have $N.a \leqq 4$, $N.A'' \leqq 4$; whence a and A'' can only have the values $0, \pm 1, \pm i, \pm(1+i), \pm(1-i), \pm 2, \pm 2i$; and it will be found, on an examination of the different cases that can arise, that the reduction can always be continued until a and A'' are either both units, or both zero. In the former case, by applying a further transformation of the type

$$\begin{vmatrix} 1, & \mu'', & \mu' \\ 0, & 1, & \mu \\ 0, & 0, & 1 \end{vmatrix},$$

the coefficients b, b', b'' may be made to disappear; and we obtain a form equivalent to F, and of the type $\epsilon x^2+\epsilon' y^2+\epsilon'' z^2$, ϵ, ϵ', ϵ'' representing units of which the product is -1. In the latter case the form obtained by applying the reduction of Gauss is of the type

$$a'y^2+a''z^2+2byz+2b'xz;$$

whence $a'b'^2 = 1$, so that b' is a unit which we shall call ϵ; and the form $\epsilon^2y^2+a''z^2+2byz+2\epsilon xy$, by a transformation of the type

$$\begin{vmatrix} 1, & 0, & \mu' \\ 0, & 1, & \mu \\ 0, & 0, & 1 \end{vmatrix},$$

is changed into one of the four forms $\epsilon^2y^2+2\epsilon xz$, $\epsilon^2y^2+z^2+2\epsilon xz$, $\epsilon^2y^2+iz^2+2\epsilon xz$, $\epsilon^2y^2+(1+i)z^2+2\epsilon xz$; of which the first two by the transformations

$$\begin{vmatrix} \epsilon^{-1}i, & 0, & \epsilon^{-1} \\ \epsilon i, & \epsilon i, & \epsilon \\ 0, & -i, & -1 \end{vmatrix}, \quad \begin{vmatrix} 0, & 0, & -\epsilon \\ \epsilon^{-1}i, & 0, & 0 \\ 0, & i, & \epsilon^2 \end{vmatrix}$$

are changed into the form $-x^2-y^2-z^2$; the last two by the transformations

$$\begin{vmatrix} 0, & -\epsilon, & 0 \\ -\epsilon^{-1}, & 0, & 0 \\ 0, & -i\epsilon^2, & -1 \end{vmatrix}, \quad \begin{vmatrix} \epsilon^{-1}, & \epsilon^{-1}, & \epsilon^{-1}(1-i) \\ -\epsilon, & -\epsilon, & \epsilon i \\ 0, & -1, & i \end{vmatrix}$$

are changed into $x^2+iy^2+iz^2$. (See Disq. Arith. art. 272-274.)

plies a transformation of any given form of determinant 1 into one or other of those two forms.

If $D \equiv i$, or $1+i$, mod 2, no binary form of determinant D can be represented by $-x^2-y^2-z^2$, because D cannot be represented by the contravariant of that form, *i. e.* by the form $-x^2-y^2-z^2$ itself. Consequently, if $D \equiv i$, or $1+i$, mod 2, the binary forms of its principal genus are certainly capable of primitive representation by $x^2+iy^2+iz^2$.

If $D \equiv 1$, mod 2, no form of the principal genus can be primitively represented by $x^2+iy^2+iz^2$. Let $f=(a, b, c)$ be such a form, and let us suppose, as we may do, that b is even, so that $ac \equiv 1$, mod 2, and $a \equiv c \equiv 1$, mod 2 (the supposition $a \equiv c \equiv i$ is admissible, because f is of the principal genus); if possible, let the prime matrix

$$\left|\begin{matrix} \alpha\ ,\ \beta \\ \alpha',\ \beta' \\ \alpha'',\ \beta'' \end{matrix}\right|$$

(of which A, B, C are the determinants) transform $x^2+iy^2+iz^2$ into f; we have the equations $a=\alpha^2+i\alpha'^2+i\alpha''^2$, $c=\beta^2+i\beta'^2+i\beta''^2$, $D=A^2-iB^2-iC^2$, from which, and from the congruences $D \equiv a \equiv c \equiv 1$, mod 2, we infer the incompatible conditions $\alpha'+i\alpha'' \equiv \beta'+i\beta'' \equiv 0$, mod $1+i$, $A \equiv 1$, mod $1+i$; i. e. f is incapable of primitive representation by $x^2+iy^2+iz^2$. If, therefore, $D \equiv 1$, mod 2, the forms of its principal genus are capable of primitive representation by $-x^2-y^2-z^2$. We may add that when $D \equiv \pm 1$, mod 4, the forms of that genus which differs from the principal genus only in having the character $\gamma=-1$, instead of $\gamma=+1$, are capable of primitive representation by $x^2+iy^2+iz^2$, but not by $-x^2-y^2-z^2$.

Lastly, let $D \equiv 0$, mod 2. If $D \equiv 2$, or $\equiv 2(1+i)$, mod 4, D cannot be primitively represented by $x^2-iy^2-iz^2$, the contravariant of $x^2+iy^2+iz^2$; i.e. no form of determinant D can be primitively represented by $x^2+iy^2+iz^2$; so that forms of the principal genus are certainly capable of primitive representation by $-x^2-y^2-z^2$. But if $D \equiv 2i$, or $\equiv 0$, mod 4, the forms of the principal genus are capable of primitive representation by both the ternary forms $-x^2-y^2-z^2$ and $x^2+iy^2+iz^2$. For if $f=(a, b, c)$ be a form of the principal genus of any even determinant, f can only represent numbers $\equiv 0$, or $\equiv 1$, mod 2; so that a ternary form of determinant 1 and of the type

$$f+p''z^2+2qyz+2q'xz$$

will be equivalent to $-x^2-y^2-z^2$, or to $x^2+iy^2+iz^2$, according as $p'' \equiv 0$, or $\equiv 1$, mod 2, on the one hand, or $p'' \equiv i$, or $\equiv 1+i$, on the other hand. Again, if (k, k') is a value of the expression $\sqrt{(a, -b, c)}$, mod D, (in which we now suppose a uneven and b semieven or even), $\left(k+\frac{D}{1+i}, k'\right)$ is another value of the same expression; and it can be shown* that when

* If $f+p''z^2+2qy^2+2q'xz$ is a ternary form of det. 1, derived from the value (k, k') of the expression $\sqrt{(a, -b, c)}$, mod D, k is the coefficient of yz in the contravariant form. Hence $a=k^2-D(q'^2-ap'')$, or $ap''=q'^2+\frac{a-k^2}{D}$. Observing that $a \equiv 1$, mod 2

$D \equiv 2i$, or 0, mod 4, one of the two forms of determinant 1, and of the type

$$f+p''x^2+2qyz+2q'xz,$$

which are deducible by the method of Gauss from those two values, satisfies the condition $p'' \equiv 0$, or $\equiv 1$, mod 2, while the other satisfies the condition $p''=i$, or $1+i$, mod 2; that is, f is capable of primitive representation by both the forms $-x^2-y^2-z^2$ and $x^2+iy^2+iz^2$.

The preceding theory supplies a solution of the problem, "Given a form of the principal genus of forms of determinant D, to investigate a form from the duplication of which it arises." Let $f=(a, b, c)$ be the given form, and let us suppose (as we may do) that a and c are uneven. When $D \equiv i$, or $1+i$, mod 2, let

$$\begin{vmatrix} \alpha, & \beta \\ \alpha', & \beta' \\ \alpha'', & \beta'' \end{vmatrix}$$

be a prime matrix (of which the determinants are A, B, C) transforming $x^2+iy^2+iz^2$ into $(a, -b, c)$; and let ϕ represent the binary form $(C-iB, A, iC-B)$; then the matrix

$$\begin{pmatrix} \beta'+i\beta'', & \beta, & \beta, & -i(\beta'-i\beta'') \\ \alpha'+i\alpha'', & \alpha, & \alpha, & -i(\alpha'-i\alpha'') \end{pmatrix} \quad \ldots\ldots \quad (Z)$$

transforms f into $\phi \times \phi^*$; and is a prime matrix, for its determinants $C-iB$, $2A$, and $iC-B$ are not simultaneously divisible by any uneven prime (because A, B, and C are relatively prime), and are not simul-

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$q'^2 \equiv 0$, or 1, mod 2, we see that $p'' \equiv 0, 1$, or $\equiv i, 1+i$, mod 2, according as $\frac{a-k^2}{D} \equiv 0, 1$, or $\equiv i, 1+i$, mod 2. But $\frac{a-k^2}{D} - \frac{a-\left(k+\frac{D}{1+i}\right)^2}{D} = (1-i)k+\frac{D}{2i}$; which is congruous to $1+i$, mod 2, if $D \equiv 0$, mod. 4, and to i, mod 2, if $D \equiv 2i$, mod 4, since k is evidently uneven in either case. From this it appears that if $\frac{a-k^2}{D} \equiv 0, 1$, mod 2, then $\frac{a-\left(k+\frac{D}{1+i}\right)^2}{D} = i, 1+i$, mod 2; that is, in one of the two forms $f+p''z^2+2qyz+2q'xz$, $p'' \equiv 0$, or 1, mod 2, and in the other $p'' \equiv i$, or $1+i$, mod 2.

* This assertion may be verified by means of the identity

$$\begin{aligned} &(q_1q_2-q_0q_3)(p_0xx'+p_1xy'+p_2x'y+p_3x'y')^2 \\ &+(q_0p_3+p_0q_3-q_1p_2-p_1q_2)(p_0xx'+p_1xy'+p_2x'y+p_3x'y') \\ &\qquad \times(q_0xx'+q_1xy'+q_2x'y+q_3x'y') \\ &+(p_1p_2-p_0p_3)(q_0xx'+q_1xy'+q_2x'y+q_3x'y')^2 \\ &=[(p_0q_2-p_2q_0)x'^2+(p_0q_3-p_3q_0+p_1q_2-p_2q_1)x'y'+(p_1q_3-p_3q_1)y'^2] \\ &\times[(p_0q_1-p_1q_0)x^2+(p_0q_3-p_3q_0+p_2q_1-p_1q_2)xy+(p_2q_3-p_3q_2)y^2]; \end{aligned}$$

in which we have to replace the quantities

$$\begin{matrix} p_0 & p_1 & p_2 & p_3 \\ q_0 & q_1 & q_2 & q_3 \end{matrix}$$

by the elements of the matrix (Z).

taneously divisible by $1+i$, because (Z) is congruous, for the modulus $1+i$, to the first or second of the matrices

$$\begin{pmatrix} 0, & 1, & 1, & 0 \\ 1, & 0, & 0, & 1 \end{pmatrix} \text{ and } \begin{pmatrix} 1, & 0, & 0, & 1 \\ 0, & 1, & 1, & 0 \end{pmatrix}, \quad \ldots\ldots \quad (Z')$$

according as $a \equiv i$, $c \equiv 1$, or $a \equiv 1$, $c \equiv i$, mod 2. Consequently ϕ is a form the duplication of which produces f. When $D \equiv 1$, or $\equiv 0$, mod 2, let the prime matrix

$$\begin{vmatrix} \alpha\ , & \beta \\ \alpha', & \beta' \\ \alpha'', & \beta'' \end{vmatrix}$$

transform $-x^2-y^2-z^2$ into $(a, -b, c)$. As we cannot have simultaneously $\alpha \equiv \beta$, $\alpha' \equiv \beta'$, $\alpha'' \equiv \beta''$, mod $(1+i)$, we may suppose that α and β are incongruous, mod $(1+i)$. If $\phi = (B+iC, iA, B-iC)$, the matrix

$$\begin{pmatrix} \beta'+i\beta'', & i\beta, & i\beta, & \beta'-i\beta'' \\ \alpha'+i\alpha'', & i\alpha, & i\alpha, & \alpha'-i\alpha'' \end{pmatrix} \quad \ldots\ldots \quad (Z)$$

transforms f into $\phi \times \phi$, and is a prime matrix, being congruous to one or other of the matrices (Z') for the modulus $1+i$, in consequence of the two suppositions that a and c are uneven, and that α and β are incongruous, mod $(1+i)$: so that f arises from the duplication of ϕ.

From the resolubility of this problem we can infer (precisely as Gauss has done in the real theory) that that half of the assignable generic characters which is not impossible corresponds to actually existing genera. We can also deduce a demonstration of the theorem that any form of determinant D can be transformed into any other form of the same genus, by a transformation of which the coefficients are rational fractions having denominators prime to 2D. For every form which arises from the duplication of an uneven primitive form—that is, every form of the principal genus—represents square numbers prime to 2D, and is therefore equivalent to a form of the type $\left(\lambda^2, \mu, \frac{\mu^2-D}{\lambda^2}\right)$. But $(1, 0, -D)$ is transformed into $\left(\lambda^2, \mu, \frac{\mu^2-D}{\lambda^2}\right)$ by $\begin{pmatrix} \lambda, & \frac{\mu}{\lambda} \\ 0, & \frac{1}{\lambda} \end{pmatrix}$; i. e. any two forms of the principal genus can be transformed into one another by transformations of the kind indicated. Again, if f_1, f_2 be two forms of any other genus, a form ϕ of the principal genus exists satisfying the equation $f_2 = \phi \times f_1$. But since ϕ can be transformed into the principal form, we can assign to the indeterminates of ϕ rational values, having denominators prime to 2D, which shall cause ϕ to acquire the value $+1$; and thus, from the transformation of f_2 into $f_1 \times \phi$, we deduce a rational transformation of f_2 into f_1, the coefficients of which have denominators prime to 2D. The truth of the converse proposition, "Two forms which are transformable into one another by rational transformations having denominators prime to 2D belong to the same genus,"

is evident from the definition of the generic characters themselves. The proposition itself is of some importance, as it furnishes a verification of the completeness of the enumeration of generic characters contained in Table III.

II. "Inquiries into the National Dietary." By Dr. E. Smith, F.R.S. Received April 28, 1864.

(Abstract.)

The paper contains an abstract of the scientific results of an inquiry which the author had undertaken for the Government into the exact dietary of large classes of the community, viz. agricultural labourers, cotton operatives, silk-weavers, needlewomen, shoemakers, stocking-weavers, and kid-glovers. The inquiry in reference to the first class was extended to every county in England, to North and South Wales and Anglesea, to the West and North of Ireland, and to the West, North, and part of the South of Scotland, whilst in reference to the other classes it was prosecuted in the towns where they were congregated.

The object of the investigation was to ascertain in the most careful manner the kind and quantity of food which constitutes the ordinary dietary of those populations; and the inquiry was in all cases made at the homes of the operatives.

The number of families included in the inquiry was 691, containing 3016 persons then living and taking food at home. The calculations of the nutritive elements are made upon the basis of an adult, two persons under the age of 10 and one over that age being regarded as an adult, and of the elements, the carbon and nitrogen are calculated in each article of food, whilst the free hydrogen is separately estimated as carbon upon the total quantities.

The author then cites the estimations which in his papers in the Philosophical Transactions for 1859 and 1861 he had made of the quantity of carbon and nitrogen emitted by the body under various conditions, and computes on those bases the amounts of those substances which are required as food by various classes of the community. He then proceeds to state the quantities which have been actually found in the dietaries of the persons included in this investigation, and the great variations which the inquiry had brought to light. He also compares the nutriment with the cost of it in the food, and states the proportion which the nitrogen bears to the carbon in each of the classes and in the different localities.

Each article of food is then considered separately, and the frequency with which, as well as the average quantity in which, it was obtained by these populations is stated.

III. "On some Varieties in Human Myology." By JOHN WOOD, F.R.C.S., Demonstrator of Anatomy in King's College, London. Communicated by Professor HUXLEY, F.R.S. Received May 6, 1864.

(Abstract.)

The paper gives the results of the author's observations on human muscular variations observed during the last fifteen years, and extending over upwards of six hundred subjects. Many of the more striking varieties were drawn by the author from the subject, and form a series of thirty-seven illustrations accompanying the paper. Some of them he has not found placed on record by any authority he has consulted. The author classifies these muscular variations as follows, viz. :—

Variations with redundancy.

1st. Those which have an origin in a development totally independent of any other muscles or tendons.

2nd. Those which consist of extensions or offsets from normal muscles or tendons, and of muscular fibres replacing tendons, and tendinous fibres intersecting muscles.

3rd. Those which are formed by simple areolar separation or segregation of muscles.

These are given in the order of their rarity, and of their comparative value in reference to the muscular anatomy of the lower animals.

Variations with deficiency.

1st. Those produced by total suppression of the germs of muscles.

2nd. Those resulting from amalgamation with neighbouring muscles.

3rd. Those from atrophy or degeneration subsequent to their formation.

All the illustrations belong to the former class, which supply the most fitting subjects for them.

The frequency of varieties of all kinds in the human subject is very great. Few subjects are to be found entirely free from them. Muscular variations are rather more common in the male sex. In them, also, variations with redundancy calculated to increase muscular power, such as are classed in the 2nd division of that section, are more common, but may be also associated in the same individual with anomalies from defect or diminution. The same individual is frequently found subject to more than one irregularity, a muscular irregularity of a marked kind being generally associated with several others. Probably the source is hereditary, as is undoubtedly the case with those which result in deformity. Muscular variations are more common in the arm, back, leg, and head, and least common, as a rule, in the abdomen, the groin excepted. They are generally more or less symmetrical, though often much more evident on one side than the other. Distinct developments are usually found on both sides. Variations by

redundancy more frequent or more developed on the right side; those from deficiency on the left.

Variations by simple reduplication.

The following muscles have been observed double, or in two distinct layers:—Pectoralis, major and minor; gluteus maximus; soleus; pyramidalis abdominis; pyriformis; subclavius.

Variations by deficiency.

The following have been seen *totally* deficient:—Psoas parvus; palmaris longus; superior and inferior gemellus; extensor minimi digiti; pyramidalis abdominis; pyriformis; peronæus tertius; extensor primi internodii pollicis; trapezius; plantaris and palmaris brevis (rarely).

The following have been seen *partly* deficient:—Trapezius; omo-hyoid; sterno-hyoid; serratus magnus; internal oblique and transversalis abdominis; soleus.

Other varieties observed (*all illustrated by drawings*).

1. An offset from the *platysma myoides* below the ear, reaching from the trapezius and occipito-frontalis to the parotid fascia and risorius Santorini (homologue of the panniculus carnosus).

2. Extensive origin of *omo-hyoid* from the clavicle, in addition to its scapular attachment.

3. Double *omo-hyoid*, the upper one digastric, the lower uninterrupted by tendon, and attached to base of coracoid process. Not before recorded.

4. *Levator claviculæ.*—Clavio- or acromio-trachelian, observed in two subjects, on both sides, arising with the *levator anguli scapulæ* from the third and fourth cervical transverse processes, and inserted into the *outer third of the clavicle* under the trapezius. Found in all the Ape-tribe.

5. *Costo-fascialis.*—A broad muscular band arising from the first rib and cartilage, outside the sterno-thyroid, and lost in the fascia of the upper half of the neck. Not before described.

6. A double anterior belly to the *digastric* muscle, with complete decussation across the median line. The latter peculiarity not before recorded.

7. *Sternalis brutorum.*—*Rectus sternalis* vel *thoracicus* connected above with a prolongation downwards of the sternal tendon of the sterno-mastoid, muscular fibres spreading out below over the sternal and epigastric aponeurosis, and attached to the xiphoid and fifth, sixth, and seventh costal cartilages.

8. *Double subclavius.*—The lower fibres distinct from upper, with cephalic vein passing between them, and attached to the coracoid process.

9. Broad slips from *pectoralis major* and *latissimus dorsi*, passing across axillary vessels and nerves, and attached, low down the arm, to the aponeurosis inserted into the inner condyle of humerus and olecranon process. These slips are highly developed in some of the anthropoid Apes;

the former especially in the Gibbon. The same subject, a muscular male, showed also a high and large origin of the *pronator radii teres* in common with the brachialis anticus.

10. *Chondro-coracoid* muscle, passing from the latissimus dorsi at tenth rib to the tip of the coracoid process. Not before recorded. The same subject showed also a low origin of the *pectoralis minor*.

11. *Biceps* with four heads.—Of the two additional heads, the inner arises with the brachialis anticus, and the outer with the supinator longus. In the same arm is a slip from the *coraco-brachialis* to the internal intermuscular septum, passing down to the condyle, and perforated by the brachial artery.

12. *Coraco-capsularis*.—A muscular slip from the tip of the coracoid process to the capsular ligament of the shoulder-joint, inserted between subscapular and triceps muscles.

13. *Brachio-fascialis*, passing from the brachialis anticus to the fascia over the flexor muscles of the forearm, coexistent with the semilunar fascia from the biceps.

14. *Palmaris longus*, with inverted belly and double origin, the additional one (tendinous) from the oblique line of the radius above the flexor sublimis. Given off from it also is the *flexor brevis minimi digiti*. A precisely similar arrangement of this very uncertain muscle not before recorded. A somewhat similar arrangement found in the *Cebus* and *Magot*.

15. Three distinct muscles and tendons connecting,—1, the *flexor pollicis longus* with the indicial tendon of the *flexor profundus digitorum*; 2, the *flexor sublimis* (coronoid origin) with the *flexor pollicis longus*; and 3, the flexor sublimis with the *flexor profundus* (middle part). Segregation also of the indicial portion of the flexor profundus.

16. *Flexor brevis minimi digiti*. Additional long, broad, fleshy origin from the fascia of the forearm, and another from the tendon of *flexor carpi* ulnaris. Aponeurotic slip given off to transverse metacarpal ligament.

17. Striking abnormality seen in two male subjects on both sides. A long tendon with bulky, muscular belly above, arising from the outer condyloid ridge of humerus with the *extensor carpi radialis longior*, and inserted in one case into the base of the first metacarpal bone and origin of the *abductor pollicis*, and in the other passing entirely into the latter muscle. Not before recorded.

18. *Extensor primi internodii pollicis et indicis*.—Arising by a distinct belly above the indicator, going along with that muscle, and giving off two tendons, one to be implanted outside the indicator tendon, and the other to supply the place of the *extensor primi internodii pollicis*. Not before recorded in the human subject. Found in the Dog.

19. *Extensor proprius digiti medii*.—A distinct extensor of the first phalanx of the middle finger, arising from the ulna below the indicator.

20. *Extensor minimi digiti* with double tendon, one going to the fourth finger. Associated with this are three curious slips or displacements of

the *dorsal interossei*, arising from the base of the metacarpal, os magnum and unciforme, and attached to the common expansion of the *extensor* tendons behind the slips from the interossei. These slips are considered by the author to indicate a tendency to the formation of an *extensor brevis digitorum manus*, by posterior displacement of the fibres of the dorsal interossei.

21. A. *Musculus interosseus volaris primus.*—A palmar interosseous going to the dorsal aponeurosis of the thumb. Not usually described, though often present. Mentioned by Henle.

B. In a hand from the subject before given in 9 and 17, all the dorsal interossei were arranged in two portions easily separable. In the first interosseous space the *abductor indicis* was very distinctly divided into a posterior part, arising in the usual manner, and inserted into the base of the first phalanx; and an anterior, arising from the first metacarpal, and inserted partly (by a small slip) into the second metacarpal, but chiefly (by a very distinct tendon) into the dorsal expansion of the common extensor tendon of the index. Not before recorded in the human subject. A similar arrangement found in the Gorilla and other Simiæ.

22. Strong muscular slips seen in the male perineum, arising from the usual attachment of the *transversi perinei* to the ischial tuberosity, and inserted with the front fibres of the *accelerator urinæ* into the dorsal fascia of the penis and corpora cavernosa, in front of the *erector penis*. Not before recorded.

23. Separation of the anterior fibres of the *gluteus minimus* into a distinct muscle homologous with the *scansorius* of Traill, or *invertor femoris* of Owen, found in the Orang and others of the Ape-tribe.

24. *Tensor fasciæ plantaris.*—Arising from the oblique line of the tibia under the soleus, and inserted into the internal annular ligament, near the tuberosity of the os calcis.

25. *Flexor accessorius longus digitorum.*—Fleshy from the aponeurosis halfway up the back of the leg, with a tendon joining the *flexor accessorius* and tendon of the *flexor longus digitorum*.

The *superficial flexor* tendon of the little toe was also, in the same foot, supplied from the outer fibres of the *flexor accessorius*. Not before recorded.

26. *Peroneus quinti digiti.*—In most instances a tendinous, but in one a fleshy offset from the *peroneus brevis*, below the outer ankle-bone, to the expansion of the common extensor tendon of the little toe. Very frequent in the human subject, usual in the Apes.

27. *Tensor fasciæ dorsalis pedis*, from the lower third of fibula to the anterior annular ligament and dorsal fascia. Not before recorded.

28. *Tibialis anticus* tendon divided into three parts, going respectively to the inner cuneiform, base of metatarsal, and first phalangeal bone of the great toe. The last-mentioned offset not before recorded. Similar arrangement in the Quadrumana.

29. *Extensor primi internodii hallucis*, or proper extensor of the first

phalanx of the great toe, arising fleshy from the tibia, inside the *extensor proprius*, and below the *tibialis anticus*. Strong tendon implanted into base of first phalanx inside the short common extensor.

30. The same muscle in another subject, having a different arrangement. Arising from the interosseous ligament *outside* the *extensor proprius*, and inserted by a tendon into that of the short common extensor going to the great toe. Neither of these have been before recorded.

31. A. The *deep flexor* tendon of the little toe supplied by a distinct muscle arising from the inner tubercle of the os calcis. Not before recorded.

B. A very large development of the *flexor accessorius* inserted into both the *long common flexor* and *long flexor of the great toe* tendons.

32. A. *Abductor ossis metatarsi quinti.*—A distinct muscle found by the author in more than one-half of the subjects in which he has looked for it, concealed by the outer part of the plantar fascia and *abductor minimi digiti* muscle, arising from the outer tubercle of the os calcis by a round fleshy belly, and inserted into the base of the fifth metatarsal by a distinct round tendon. Not before observed in the human subject. Found in the Gorilla and Chimpanzee by Huxley and Flower.

B. *Opponens minimi digiti.*—Very commonly found, though not described in anatomical text-books. Arises tendinous from the ligament of the fifth metatarsal and cuboid, and inserted in a bipennate way into the whole length of the fifth metatarsal bone. Found well developed in all the Apes.

IV. "Researches on Isomeric Alkaloids." By C. Greville Williams, F.R.S. Received May 12, 1864.

(Abstract.)

The experiments recorded in the following paper have for their object the determination of two questions:—

1. Whether the base produced by destructive distillation of cinchonine, and having the formula $C^7 H^{9'''} N$, is isomeric or identical with the lutidine from Dippel's oil? and

2. How far the chinoline series of bases, isomeric with the leukoline series, extends beyond lepidine?

In one form or another the observations have been going on for some years, but it is only lately that results have been obtained which appear conclusive.

The first of the above questions is connected with one of the most difficult problems occupying the attention of chemists at the present day, namely that of isomerism. It is true that some isomeric bodies differ so widely in their physical and chemical characters that no difficulty exists in distinguishing them. Thus the members of the aniline and pyridine series have, save their formulæ, few points of resemblance; cespitine and

amylamine could scarcely be confounded with each other. On the other hand, the two modifications of tartaric acid resemble each other so closely in their chemical characters, that it is chiefly by the observation of physical differences that they have been shown to be distinct.

The most difficult cases to solve are where the substances to be studied are so inert as to allow of few reactions by which they may be tested and compared. Thus, up to the present time, no crucial test has been devised by which the alcohol radicals can be distinguished from the isomeric hydrocarbons of Boghead naphtha and American petroleum; and, indeed, a change in opinion appears to be gaining ground, and the possibility of their being identical seems again to be entertained by at least one chemist who has carefully studied the subject*.

Another and scarcely less difficult case presents itself where a substance is supposed to be isomeric with another which at the time has no existence. Thus the constituent of oil of rue, long regarded as capric aldehyde, $C^{10}H^{20}O$†, and which was subsequently shown to have the formula $C^{11}H^{22}O$‡, is by some chemists §, who do not dispute the accuracy of the latter formula, regarded as a ketone isomeric with the aldehyde of euodic alcohol. But if the substance from rue oil be not the aldehyde, then the real one is unknown, and consequently no means exist for a comparative study. The question is, in this instance, complicated by recent researches indicating ketones to be the aldehydes of alcohols formed by the hydration of olefiants ‖.

It is evident that the chances of isomeric bodies being formed in reactions, increase with the complexity of the type from which they are derived; or, in other words, the more replaceable hydrogen exists in the type, the greater the chances of isomerism in the derivatives. The ammonium and ammonia types always, therefore, present the greatest number of instances of isomerism.

In the course of the study of the organic bases produced by destructive distillation, the author very carefully compared the reaction of alkaloids from different sources, and he states that he was almost convinced of there being a true chemical difference between the chinoline and leukoline series before he succeeded in obtaining the decisive reaction resulting in the formation of the blue iodides from the amyl compounds of chinoline and lepidine. That observation led to the following comparative examination of the pyridine series as obtained from cinchonine with the corresponding series from Dippel's-oil and coal naphtha.

In order to submit the alkaloids to a searching examination, it was necessary that they should be in an equally high state of purity, and undergo the same amount of fractionation. The methods of preparation and

* Schorlemmer, Chem. Soc. Journ., xvi. p. 428.

† Gerhardt, Ann. de Chim. et de Phys. [3] xxiv. p. 103.

‡ C. G. W. Phil. Trans. 1858, p. 199. § Hallwachs, Ann. Pharm. cxiii. p. 107.

‖ Berthelot, Comptes Rendus, lvii. 797, 63.

fractionation having, however, been described in his previous papers, the author does not further enter into them.

In a previous investigation it was found that in fractionating the bases from cinchonine, there was a tendency in the fractions boiling below chinoline to accumulate in larger quantities between 160° and 166° C.; and it was found on analysis that the distillate at that point had exactly the percentage composition of lutidine. The results of the analysis of the base and the platinum-salt pointed unmistakeably to the formula

$$C^7 H^9 N.$$

The boiling-point of lutidine, according to Dr. Anderson, is 154°; and it would therefore appear as if the base from cinchonine had a slightly higher boiling-point, but the author states that he does not draw any inference from that or any other isolated fact.

Physical Properties of the Bases.—Both bases, when pure, are colourless refractive oils, lutidine having a density of 0·9467 at 0°, and the cinchonine base (which the author distinguishes by the name of β-lutidine) of 0·9555 at the same temperature. Lutidine boils at 154°, β-lutidine between 163° and 168°.

Lutidine dissolves readily on shaking in three or four times its bulk of water; and, on warming, the liquid immediately becomes milky, and the base separates. *β-lutidine*, on the contrary, requires not less than twenty-five parts of water for solution, and does not present any signs of the very characteristic reaction of lutidine on warming the solution; on the contrary, a mixture of water and β-lutidine, cloudy from excess of base, clears on warming.

Lutidine has a most characteristic smell, resembling that of its lower homologues, but less pungent, and more approaching to aromatic. This smell is never altered in the least by any methods of purification. After boiling with nitric, chromic, or any other acid, the organic smell is given off on neutralization with an alkali. All its salts, however purified, have the same smell on the addition of an alkali.

β-lutidine has also a most characteristic smell, having not the faintest resemblance to that of lutidine. It cannot well be described, but, if anything, reminds one of nicotine; but it is without the peculiar pungency of that base. It is also far more pleasant. No treatment with acids or oxidizing reagents, and no amount of purification of its salt, causes the least alteration in the smell, or makes it approach, however distantly, to that of lutidine.

The vapour-density of lutidine, boiling at 154°, was determined by Dr. Anderson at 201°, and found to be 3·839.

The vapour-density of β-lutidine, boiling between 163° and 168°, was determined, for the purpose of this investigation, at 213°, and found to be 3·787. Theory requires 3·699.

From these numbers some important inferences may be drawn. It will be seen that the vapour-density, by experiment, is lower in the case of β-lutidine than in that of lutidine, although the former has a higher boil-

ing-point. It may be inferred from this that the boiling-point of β-lutidine has not been estimated above the truth, because if the fraction had been taken too high, the vapour-density would have erred considerably in excess. This will be the more evident if we consider that a determination of a vapour-density by the method of Dumas is, in fact, a process of fractional distillation; and if any base of higher formula had been mixed with it, it would have remained in the balloon, and increased the density to an extent proportional to the amount of impurity. The experiment, therefore, tends to show that the true boiling-point of lutidine is, if anything, a little below 154°, and that of β-lutidine is about 165°, inasmuch as the error is not greater than that usually observed with the purest substances obtained by fractional distillation.

The author, after stating that the specimen of lutidine employed in the investigation was prepared by Anderson himself, shows that lutidine retains its colourless state after rectification longer than β-lutidine.

Comparative Study of the Platinum-Salts of Lutidine and β-Lutidine.

The author states that before entering on the comparative examination of the bases, he determined their composition by converting them into platinum-salts. They both gave numbers so closely agreeing with those demanded by calculation from their formulæ, that it was evident that the specimens used were pure.

He then proceeded to ascertain whether any differences existed in their capability of yielding crystalline platinum-salts.

I. *β-Lutidine.*—A solution of hydrochlorate of the base was mixed with a solution of dichloride of platinum, the exact strength of the solutions being known. The mixture solidified instantly to a mass of crystalline plates. After 48 hours' repose their weight was determined and found to be 1·780 gramme.

II. *Lutidine.*—A precisely similar experiment was made with lutidine. No signs of crystallization appeared for 15 minutes. A few crystalline points then began to form. After 48 hours the weight of the crystals which had then formed was found to be 0·816 gramme.

The experiment was then repeated, a known weight of water being added to each solution.

III. *β-Lutidine.*—A large quantity of crystalline plates was deposited instantly. In five minutes the whole had solidified. After 48 hours' repose, the weight of the crystals was found to be 1·66 gramme.

IV. *Lutidine.*—No crystals appeared for 35 minutes. After 48 hours the weight of crystals formed was found to be 0·83 gramme.

The experiments III. and IV., like I. and II., show that β-lutidine, under exactly the same conditions, gives twice as much salt as the Dippel base.

As, according to Dr. Anderson, the presence of hydrochloric acid in excess greatly retards the formation of the lutidine salt, the author made

an experiment to ascertain if the retardation was equal in the case of β-lutidine.

V. *β-Lutidine.*—A mixture, the exact strength of which is given in the paper, containing a large excess of acid, was made. The whole instantly became nearly solid, and in five minutes the beaker could be inverted without any of the solid contents escaping. After twelve hours the weight of crystals formed was 1·70 gramme.

VI. *Lutidine.*—A precisely similar experiment was made with this base. No crystals appeared for thirty minutes. In twelve hours only 0·68 gramme had formed.

From the above experiments, it is evident that there is a vast difference in the tendency of the two bases to form crystalline platinum-salts.

Comparative Experiments upon the Decomposition by boiling of the Platinum-Salts of β-Lutidine and Lutidine.

The author describes in his paper the nature of the apparatus, and the precautions taken to ensure precision in the experiments.

I. *β-Lutidine.*—The salt begins to decompose the instant the solution enters into ebullition, an insoluble powder beginning to deposit. After forty minutes the operation had to be stopped, owing to the large quantity of insoluble sulphur-yellow precipitate which had fallen and rendered the boiling concussive.

The salt formed in this manner was boiled with a fresh quantity of distilled water, and then analyzed. The numbers agreed with the formula

$$\beta\,N\left\{\begin{matrix}C^{7}H^{7}\\Pt''\end{matrix}\right\}2\,HCl,$$

which is that of the bihydrochlorate of platino-lutidine, if regarded as analogous to the so-called bihydrochlorate of platinamine.

II. *Lutidine.*—The solution of the salt required two hours and a half of active boiling before even a trace of the insoluble precipitate began to form. After several hours more boiling, the deposit was excessively small. The author states that the difference in the behaviour of the two salts was so great, that it would be easy at any time to distinguish the two platinum-salts by this test alone.

Platinum Compound of β-Lutidine isomeric with the Bihydrochlorate of Platino-β-Lutidine.

The author states his opinion that Gerhardt's theory of the platinum bases will require some modification to enable it to be applied to the analogous bodies obtained from the tertiary monamines. The bihydrochlorate of platinamine, the empirical formula of which is NH^{3}, $PtCl^{2}$, according to Gerhardt is constituted thus:—

$$N\left\{\begin{matrix}H\\pt\\pt\end{matrix}\right\}2HCl;$$

but the author concludes that this formula is unsuitable to nitryle bases,

which contain no hydrogen capable of replacement by metals or organic radicals. Even assuming platinum in the above instance to be diatomic, there is no reason to suppose that the hydrogen driven out by the platinum has united to the chlorine to form two atoms of hydrochloric acid. The author then endeavours to show that the formula

$$N\left\{\begin{matrix}H\\H\\H\\Pt\end{matrix}\right\}Cl^2$$

would in some respects be preferable, in which case the formula for the bihydrochlorate of platino-β-lutidine would become

$$\beta N\left\{\begin{matrix}C^7H^{9'''}\\Pt\end{matrix}\right\}Cl^2.$$

He states, however, that a formula in which protochloride of platinum is made to replace one atom of hydrogen, would probably more nearly represent the constitution of the substance.

While studying these compounds, the author endeavoured to ascertain if compounds having the same composition as the so-called bihydrochlorates could not be formed by direct combination of the base with dichloride of platinum, instead of removing an equivalent of hydrochloric acid from the platinum-salt.

Dry dichloride of platinum, on being treated with excess of β-lutidine, evolved heat, and yielded an oily fluid, which solidified on cooling. After purification by treatment with dilute hydrochloric acid, a pale-brown amorphous powder resulted, which on analysis yielded the formula

$$\beta C^7H^{9'''}N, PtCl.$$

From its totally different appearance, the author concludes that it is isomeric and not identical with the sulphur-yellow body produced by boiling the platinum-salt.

A third compound, isomeric or identical with one of the last-described bodies, should be produced by treating with chlorine the compound of lutidine, or β-lutidine with protochloride of platinum,

$$C^7H^{9'''}N, PtCl + Cl = C^7H^{9'''}N, PtCl^2.$$

It is evident, even if the existence of bihydrochlorates of these platinum bases should ever be demonstrated, that *all* the compounds having the empirical relation $C^7H^9N, PtCl^2$ cannot be supposed to be constituted like Gerhardt's bihydrochlorate of platinamine. In fact, if we suppose that the product of the action of boiling water upon a platinum-salt yields such a substance, we must make the following admissions:—

1. That an atom of platinum or platinosum (Pt), under the form of two atoms of platinicum (pt^2), may replace two atoms of hydrogen.

2. That a salt containing one atom of platinosum, with the equivalent 99, may, by boiling, become converted into a salt containing two atoms of platinicum with an equivalent 49·5.

3. That a salt containing a highly soluble compound with one equivalent of hydrochloric acid and two of chlorine, may, by loss of hydrochloric acid, become converted into an insoluble bihydrochlorate.

The author states his intention of studying more in detail the substances produced by the union of the primary and tertiary monamines with the chloride and dichloride of platinum, espcially with the view of determining the amount of replaceable hydrogen in them.

Comparative Experiments on the Action of Protochloride of Platinum on β-Lutidine and Lutidine.

I. *β-Lutidine.*—Equal weights of protochloride and base were mixed in an apparatus surrounded with a non-conducting medium. The temperature rose from 16° to 84°, only one gramme of each ingredient being employed. The hard brittle product gave on analysis numbers almost exactly agreeing with the formula

$$C^7 H^{9'''} N, PtCl.$$

According to Gerhardt's views the formula being doubled becomes the chloroplatinate of di-platoso-β-lutidine, and may be written thus:—

$$PtCl^2 H (C^{14} H^{17} Pt) N^2.$$

M. Hugo Schiff has recently presented to the Academy of Sciences* a paper in which he states that chinoline combines with numerous metallic chlorides to form compounds having the general formula

$$N \left\{ \begin{matrix} C^7 H^{9'''} \\ M \end{matrix} \right\} X,$$

X representing an acid radical.

It is evident that this formula satisfies the condition of the compound above described containing β-lutidine. Representing the chlorine by X and the platinum by M, we have

$$N \left\{ \begin{matrix} C^7 H^{9'''} \\ M \end{matrix} \right\} X.$$

But all these bodies are precisely analogous to the compound of chinoline with protochloride of platinum obtained by the author in the course of his researches on that base†.

II. *Lutidine.* When lutidine was treated with protochloride of platinum under exactly the same conditions, the temperature rose two degrees higher. The difference was therefore too small to found any conclusion upon. But the product, instead of becoming a hard brittle mass, remained of the consistence of treacle.

Comparative Examination of the Palladium Salts of β-Lutidine and Lutidine.

I. *β-Lutidine.*—A mixture of known quantities of base, hydrochloric acid, and chloride of palladium was made and put aside. In four days the whole was a nearly solid mass of garnet-red prisms. Collected and dried,

* Comptes Rendus, lvii. p. 837. † Chemical Gazette, September 15, 1858.

they weighed 816 milligrammes. On analysis they yielded numbers agreeing with the formula

$$C^7 H^{9'''} N, HCl, PdCl.$$

Exposed for a long time to a heat of 100°, they become decomposed, one equivalent of hydrochloric acid being expelled from two equivalents of the salt. This was proved by analysis of the substance after leaving for five days, until in fact its weight became constant. The numbers obtained agreed with the formula

$$C^{14} H^{19} N^2 Cl^3 Pd^2 = C^7 H^{9'''} N, HCl, PdCl + C^7 H^{9'''} N Pd, Cl.$$

Chloride of Palladio-β-lutidyl Ammonium.—The author has succeeded in isolating the palladium base existing in the last substance. It is easily procured by acting on chloride of palladium in solution with β-lutidine. It is not readily soluble, and therefore precipitates at once. On analysis it gave numbers almost exactly agreeing with the formula

$$N \left\{ \begin{matrix} C^7 H^{9'''} \\ Pd \end{matrix} \right\} Cl.$$

II. *Lutidine.* An exactly similar mixture of hydrochlorate of lutidine and chloride of palladium was made to the one in which β-lutidine was employed. No crystals, however, were obtained by the time that the contents of the vessel containing the β-lutidine had nearly solidified. After a month's repose, only 232 milligrammes had formed, instead of 816, as in the other experiment.

Comparative Experiments on β-Lutidine and Lutidine with Trichloride of Gold.

I. *β-Lutidine.*—A mixture of the hydrochlorate of the base and solution of trichloride of gold became nearly solid. On heating, a portion dissolved, and on filtration and cooling, gave a beautifully crystalline salt *a*. A large portion, however, melted to a dark oil, which dissolved in boiling dilute hydrochloric acid. On cooling, the salt was deposited in crystals *b*. Both portions on analysis gave numbers agreeing with the formula

$$C^7 H^{9'''} N, HCl, Au Cl^3.$$

II. *Lutidine.*—A similar experiment was made. The precipitate only occupied half the bulk of the liquid; it did not require one-fourth part of the quantity of water to dissolve it that was necessary in the case of the β-lutidine.

The author in his paper gives the results of experiments made with the five bases and solution of trichloride of gold. The differences are fully as great as those observed with the hydrochlorates.

Action of Iodide of Ethyl on β-Lutidine and Lutidine.

I. *β-Lutidine.*—One volume of the base was mixed with two volumes of iodide of ethyl. The mixture was heated in a sealed tube for three minutes to a temperature of 94°. On removing the tube from the water-

bath in which it had been immersed, and plunging it in cold water, the whole solidified at once to a mass of crystals. On analysis, they gave results agreeing with the formula

$$C^{9}H^{14'''}N, I.$$

II. *Lutidine.*—A mixture in similar proportions was heated to the same temperature for the same time. On cooling, it showed no signs of crystallization. It required an hour's digestion at 100° to effect combination. Even then no crystals were obtained on cooling. In twenty-four hours one-half of the product had crystallized. The rest remained in the form of a syrup.

Platinum-Salt of Ethyl β-Lutidine.

This salt was obtained from the iodide in the usual manner. It crystallizes in superb orange-coloured fronds. On analysis it yielded numbers pointing to the formula

$$C^{9}H^{13}N, HCl, PtCl^{2}.$$

This salt, when boiled, undergoes a totally different decomposition from that afforded by the tertiary monamines under the same circumstances. The mixture turns black, and deposits the platinum. After two days' boiling, it yielded 27·5 per cent. of metal. The original salt contained 28·99.

Experiments were also made with the bases and solution of uranium. The results need not be quoted, as they merely confirm the previous ones, and show differences of the same character.

Compound of β-Lutidine with Sulphate of Copper.

When β-lutidine is gradually added to a solution of sulphate of copper, a copious pale green precipitate is formed. It dissolves in excess, forming a rich blue fluid. A small quantity of a pale green residue remains undissolved. The filtered solution soon becomes filled with brilliant blue prisms of considerable size. The air-dried salt retains four equivalents of water. On analysis the results agreed with the formula

$$\left.\begin{matrix} C^{7}H^{9'''}N \\ Cu \\ C^{7}H^{9'''}N \\ Cu \end{matrix}\right\} SO^{4}+4H^{2}O.$$

Dried at 100° it loses two atoms of water, and at 200° it becomes anhydrous.

On the higher Homologues of Chinoline.

In his "Researches on Chinoline and its Homologues"*, the author showed that the distillate from cinchonine yielded a base above chinoline, to which he gave the name of lepidine. He also obtained a base of the same formula from coal-tar, which he subsequently showed to be isomeric and not identical with lepidine. He has recently proposed the name of iridoline† for this base. He also obtained from coal-tar a base having the formula $C^{11}H^{11}N$, to which he gave the name of cryptidine.

* Trans. Roy. Soc. Edinb. xxi. part 3. 377. † Chem. Soc. Journ. New Ser. i. 357.

In this paper he shows that the distillate from cinchonine contains not only a base isomeric with cryptidine, and which he calls dispoline, but also several other homologues of still higher atomic weight.

The separation of these bases is very difficult. They boil at too high a temperature to allow of separation by means of fractional distillation, and, in fact, most of them distil above the range of the mercurial thermometer.

The author gives the details of the methods employed by him to purify the mixtures of bases from resinous and tarry matters; they will not, however, be quoted in this abstract.

After trial of various methods of separation, the author finally adopts fractional precipitation of the platinum-salts. By following out this method, he obtained the platinum-salts of the following bases:—

$C^{11} H^{11} N$ Dispoline.
$C^{12} H^{13} N$
$C^{13} H^{15} N$
$C^{14} H^{17} N$
$C^{15} H^{19} N$
$C^{16} H^{21} N$

He has not given names to the homologues above dispoline.

Summary.—The author concludes from the results of the first portion of the investigation of which the above is an abstract, that the base, of the formula $C^7 H^9 N$, existing in the distillate from cinchonine, is distinct from that obtained from Dippel's oil.

He submits also that the second portion of the investigation shows the chinoline series to consist of no less than eight members, three of them being isomeric with certain bases from coal-oil.

In conclusion he calls attention to the fact that the eighth homologue of chinoline differs only by $C^4 H^4 NO^2$ from cinchonine itself, and he is of opinion that bases free from oxygen exist in the distillate from cinchonine, containing almost, if not quite, as many equivalents of carbon as the cinchona alkaloids themselves.

V. "On the Synchronous Distribution of Temperature over the Earth's Surface." By Henry G. Hennessy, F.R.S., &c. Received May 26, 1864.

(Abstract.)

The results presented in the author's paper, entitled "On the Simultaneous Distribution of Heat throughout the superficial parts of the Earth" *, are confirmed and extended in the present communication.

* June 19, 1862. Proceedings, vol. xii. p. 173.

VI. "Experimental Researches on Spontaneous Generation." By Gilbert W. Child, M.D. Oxon. Communicated by Professor Phillips. Received May 26, 1864.

(Abstract.)

The experiments are twenty in number, and were performed during the summer of 1863. The substances used were in ten experiments milk, and in ten, fragments of meat and water. These were in all cases placed in a bulb of glass about 2½ inches in diameter, and having two narrow and long necks. The experiments are divided into five series of four experiments each. In one series the bulbs were filled with air previously passed through a porcelain tube containing fragments of pumice-stone and heated to vivid redness in a furnace. In the others they were filled respectively with carbonic acid, hydrogen, oxygen, and nitrogen gases. In each series two experiments were made with milk, and two with meat; and each substance was boiled in one case, and not boiled in the other. The joints of the apparatus were formed either by means of non-vulcanized caoutchouc tubing, or india-rubber corks previously boiled in a solution of potash; and in every case, at the end of the experiment, the necks of the bulb were sealed by the lamp. The time of boiling such of the substances as were boiled varied from five to twenty minutes, and the boiling took place in the bulbs, and with the stream of gas or air still passing through. The substances were always allowed to cool in the same stream of gas before the bulbs were sealed. The microscopic examination of the contents of the bulbs took place at various times, from three to four months after their enclosure.

In every case but one in which the substance had not been boiled low organisms were found, apparently irrespective of the kind of gas in which they had to exist. The case in which they were not seen was that of the meat enclosed in a bulb filled with nitrogen. This bulb burst apparently spontaneously, and its doing so may be looked upon as a proof that in it also some change had taken place most likely connected with the development of organic life. Where the substances had been boiled, the results were as follows:—

1. In the carbonic acid experiments, no sign of life.
2. In the hydrogen experiments, no sign of life.
3. In the heated air experiments, organisms found in both cases.
4. In the oxygen experiments, organisms found in the experiments with milk. The bulb containing the oxygen and meat burst spontaneously, therefore probably contained organisms.
5. In the nitrogen experiments, organisms were found where meat was used. None where milk was used.

No definite conclusion can be drawn from so limited a range of experiments; but it is worthy of remark that organisms were found here under the precise circumstances in which M. Pasteur states that they cannot and

do not exist. The very abnormal conditions under which some of these so-called organisms are found, would render it doubtful whether Bacteriums, Vibrios, &c., ought to be considered as independent organisms in any higher sense than are white blood-corpuscles, pollen-grains, mucus-corpuscles, or spermatozoa.

VII. "On a Colloid Acid, a Normal Constituent of Human Urine." By William Marcet, M.D., F.R.S. Received May 28, 1864.

(Abstract.)

The object of the present communication is to describe the mode of extraction and the properties of an acid of a colloid nature which is always present in healthy human urine, and appears destined to become of great importance in Physiological Chemistry.

With the view of separating this acid from the urinary secretion, the fluid is mixed with animal charcoal, concentrated, and filtered, and the filtrate, after precipitation with baryta-water, is dialyzed for about twenty-four hours. The dialyzed liquid, after subsequent filtration and concentration, is mixed with basic acetate of lead, which precipitates the colloid acid as an insoluble lead-salt, along with a little hydrochloric acid and other impurities. The precipitate should be thoroughly washed, decomposed with sulphuretted hydrogen, and again treated with animal charcoal. When the acid is required in a pure state, the hydrochloric acid present is removed with carbonate of silver, the excess of the silver precipitated with sulphuretted hydrogen, and, after boiling to evolve this last substance, basic acetate of lead is again added. The lead-salt perfectly washed may be considered pure, and the pure acid can be obtained from it by decomposition with sulphuretted hydrogen.

The acid is very slow to decompose when exposed to the air. It may be considered to undergo no loss or decomposition by being boiled, as shown by direct experiment. After concentration by heat, its colour darkens and it becomes syrupy, possessing a sharp acid taste, with a slight acrid and astringent after-taste; the taste is perceptible in the solution when very dilute: no crystals of the acid could be obtained in the syrup. Dried at a temperature under 212° F., the acid has the appearance of a transparent varnish; it is very hygroscopic, and dissolves readily in water, though not apparently in alcohol (sp. gr. ·820) or in ether. When burnt, the colloid acid chars, emitting a pungent and irritating smell, and after complete combustion, nothing but the minutest trace of inorganic residue remains. Although strictly a colloid, this acid in the free state passes through a dialyzer, but not so readily as a crystalloid. When under the form of a compound, its property of dialyzing appears much diminished. I could not find that it exerted any action on polarized light*.

* This acid does not precipitate egg-albumen. It precipitates casein, but an excess does not appear to redissolve the precipitate as in the case of acetic acid.

Composition of the Colloid Acid.

The acid was found to consist only of carbon, hydrogen, and oxygen. I have not yet succeeded in establishing its ultimate quantitative composition, but it appears to be very poor in hydrogen and rich in carbon. The atomic weight of the substance was found by the analysis of its insoluble lead-salt, and of its baryta-salt. I determined the lead in the lead compounds from six different samples of urine; the average in 100 parts was—

Oxide of lead	66·3
Acid.....	33·7
	100·0

The analysis of the baryta compound yielded in 100 parts—

Baryta.....	72·2
Acid	27·8
	100·0

Corresponding to the atomic weights

for the lead compound	Oxide of lead	111·5
	Acid	56·7
		168·2
for the baryta compound	Baryta	76·5
	Acid	29·5
		106·0

It is therefore very obvious that the acid forms two salts, an acid and a neutral salt; we shall adopt the number 28·35 $\left(\text{or}\ \frac{56{\cdot}7}{2}\right)$ for the atomic weight of the new acid. The fact of there existing two different compounds of the acid, explains many chemical phenomena exhibited by this substance and its salts.

Compounds of the Colloid Acid of Urine.

The neutral salts are all soluble.

Lead-Salts.—The colloid acid forms two lead-salt—sone which is insoluble in water, and contains two equivalents of acid, and one which is soluble in water, and evidently contains one equivalent of acid.

The insoluble compound is obtained by adding basic acetate of lead to an aqueous solution of the acid or of its neutral salts. An excess of the basic acetate redissolves the precipitate, which reappears on the addition of dilute nitric acid, to be finally redissolved in an excess of the mineral acid. The whole of the colloid acid is not, however, precipitated by basic acetate of lead, principally on account of the formation of a certain quantity of neutral acetate of lead, which I found to have the property of dissolving the insoluble colloid lead-salt. On boiling a mixture of the insoluble lead

compound with neutral acetate of lead, acetic acid was given off, a confirmatory proof of the insoluble lead compound being an acid salt; in this case one equivalent of the colloid acid combines with one equivalent of oxide of lead of the neutral acetate, two equivalents of the neutral lead-salt of the colloid acid being thus formed,

$$PbO, 2(Acid) + PbO, C_4 H_3 O_3 = 2(PbO, Acid) + C_4 H_3 O_3.$$

This shows that it is not possible to estimate with accuracy the amount of the acid in urine by means of basic acetate of lead.

When the acid is boiled with an excess of hydrated oxide of lead, an insoluble compound is formed; if the acid be in excess, a compound soluble in hot water, but precipitating on cooling, is obtained: I have not yet determined the composition of these two lead-salts.

Baryta- and Lime-Salts.—These salts are easily prepared from the carbonates. They contain one equivalent of the acid, are soluble in water, and yield precipitates with basic acetate of lead, nitrate of silver, and protonitrate of mercury and tannic acid; the more concentrated the solution, the more abundant the precipitates. A very slight precipitate occurs by adding neutral acetate of lead to salts of the acid; other reagents fail to yield precipitates.

The acid dissolves silver from the carbonate, but I could not neutralize it perfectly by such means. The lime-salt of the acid cannot be entirely decomposed by boiling it with carbonate of silver.

When the acid is boiled with black oxide of copper, copper is readily dissolved.

Physiological relations of the Colloid Acid of Urine.

I endeavoured to determine approximately the quantity of the colloid in a given bulk of the urinary secretion, and extracted from 8 litres 4·46 grammes of this substance, which, however, must fall short considerably of the real amount of the acid present.

It may be considered as existing in all probability in the blood, where there is little doubt that it acts an important part in the phenomena of the secretion of gastric juice, by displacing the hydrochloric acid from chloride of sodium, combining with the sodium; the soda-salt would remain in the blood, being a colloid compound, while the free hydrochloric acid would find its way into the stomach. An experiment I performed in connexion with this subject bears out the present view.

The formation of the colloid acid appears to result from some transformation of the colloid non-nitrogenous constituent of the liver known as the glucogenic substance. When better acquainted with the chemical composition and physiological relations of the colloid acid of urine, I shall be able to give it an appropriate name.

VIII. "Further observations on the Amyloid Substance met with in the Animal Economy." By ROBERT M'DONNELL, M.D. Communicated by WILLIAM BOWMAN, Esq. Received May 30, 1864.

In the early part of last year I had the honour of making a communication to the Royal Society "On the Amyloid Substance of the Liver, and its ultimate destination in the Animal Economy." The discussion which followed the reading of this paper made it desirable that further observations should be made regarding the natural history of this substance, more particularly with reference to its relations to the tissues of the fœtus. It was not possible to complete these investigations until the spring of the year placed at my disposal fœtal lambs, calves, &c. in various stages of development. This has been the cause of the delay in forwarding the present communication, for which I must apologise to the Society.

The amyloid substance met with in the fœtal tissues is in chemical composition identical with that found in the liver. Absolutely pure specimens, prepared from each of these sources, are represented by the formula $C_{12} . H_{10} . O_{10}$*.

With reference to its optical properties, it was stated in my former communication, on the authority of French observers, that amyloid substance of animal origin, like vegetable dextrine, causes the plane of polarization to deviate to the right; I must now confess that I have not been able to verify this assertion. It is not possible, by any means that I have been able to devise, to obtain a solution of this substance so transparent as to admit of its being submitted to examination in the saccharometer. If a portion of the liver of an adult animal, or of the muscular tissue or lung of a fœtus, be pounded to a pulp in a mortar with silver sand, and the whole afterwards mixed into a paste with animal charcoal and allowed to stand for some hours, and then treated with boiling distilled water and filtered, the liquid thus obtained is too turbid to permit of its rotatory power, as regards polarized light, being investigated. So small a quantity as half a grain of pure amyloid substance dissolved in an ounce of distilled water, produces in the solution a peculiar opalescent appearance. I have proved by experiment that this is not due to fluorescence, but to the fact that the amyloid substance has its particles merely in a state of suspension, not of true solution. No trace of it will pass through a dialyzer without the exercise of pressure, and the liquid thus obtained is not sufficiently translucent for examination by polarized light.

M. Charles Rouget and Professor Claude Bernard have examined the tissues of the fœtus microscopically, so as to determine the presence in

* In my former communication I gave an ultimate analysis of the amyloid substance of the liver, which Professor Apjohn, of Trinity College, Dublin, was good enough to make for me. But the specimen which I had furnished was not absolutely pure, containing a trace of nitrogen. The specimens from which the above formula is deduced were pure.

several of them of amyloid substance in abundance; but neither of these observers has attempted to show, by chemical investigation, at what period of development each of the tissues containing it is found to have it entering most largely into its composition. It must be remembered that the acidulated tincture of iodine is a test of such delicacy for this substance, that it produces its characteristic reaction even when the quantity of amyloid substance present is very minute *; hence, judging from the use of this test under the microscope, one is apt to suppose that the amount present is greater than it really is, or rather that it is equally abundant in tissues which in reality contain it in widely different quantities.

It was possibly owing to this mode of examination that Professor Bernard was led to suppose that this substance continues to exist in muscular tissue during the entire period of intra-uterine life, and that it does not disappear until after birth, when it does so under the influence of the respiratory and muscular movements. I hope to be able to show, however, that the establishment of respiration has little do with the disappearance of the amyloid substance from the tissues of the fœtus, and to prove that, in truth, certain azotized tissues are evolved from a nidus of amylaceous protoplasm, which, after a particular stage of growth, becomes less and less as each of those tissues approaches maturity, and that when maturity is attained the amyloid substance, which once formed so large an ingredient of the growing tissue, has gradually become changed so as to be no longer discoverable, even before respiration has commenced.

M. Rouget has quite correctly pointed out the very early period at which the amyloid substance is found in the cartilaginous tissue: it first shows itself in the cellules of this tissue; in the embryo chick and lamb, at a very early stage of development, it may be demonstrated; it very soon, however, disappears from the cells of cartilage and is too small in amount to be estimated at different stages of growth.

The epithelial cells of the skin are rich in amyloid substance at an early period. The points where cells aggregate themselves together for the commencing development of a feather or a hair, show a great abundance of the amylaceous material. The horny appendages of the skin, the bill of the embryo chick, the claws, hoofs, &c. of other embryos, contain it in large quantities up to a particular period of development. The feet of a fœtal calf of about four months were dried at a heat not exceeding 212°; 7 grains of the horny structure were rasped off, and on examination yielded 1·3 gr. of amyloid substance. An exactly equal quantity from the feet of a nearly full-grown fœtal calf, similarly treated, gave an amount of amyloid substance too minute to be estimated. It almost wholly disappears from the feathers when they become prominent on the surface, and for hairs the same may be said. If one of the large hairs from the eyebrow of a fœtal lamb, shortly before the time of birth, be examined, nothing more than a

* A tenth of a grain of amyloid substance may be readily detected in an ounce of water by the action of acidulated tincture of iodine.

mere trace of amyloid substance can be detected, and that only in the cells of the bulb. If a number of embryo lambs, of various sizes, are placed side by side, and a drop of a weak solution of iodine acidulated be allowed to fall on a corresponding part of each, the peculiar brown stain produced will be seen to increase in intensity up to a certain point. After the appearance of the hair, the stain gradually diminishes in intensity, showing the lessening quantity of amyloid substance in the tissue. If the tails cut off from a series of fœtal lambs are placed in a vessel containing a very weak solution of iodine acidulated, it illustrates (by the colouring) very strikingly the increasing abundance of the amyloid substance in the epidermic tissue up to a particular period (that is, when the hair is fully formed), and then its gradual disappearance. The feet and hoofs similarly treated illustrate the same for the horny tissue of this part. From the time that the fœtus of the sheep is nine inches in length (at which period the amyloid substance seems to be at its maximum), the amyloid substance contained in the horny structure of the hoof gradually diminishes until shortly before birth, when, even after prolonged boiling, scarcely a trace can be extracted from it.

In the tissue of the lung of mammalian embryos, the amyloid substance is at one period present in immense quantity. After the watery part is driven off by evaporation, more than 50 per cent. of the dry residue is found to be nothing else than animal dextrine. As the organ approaches maturity, and the animal is about being born, but before it has yet drawn a single breath, the amyloid substance is found to be reduced to a very small quantity indeed, and in some instances to have absolutely disappeared. The following Table shows the progress of this change in the lung of the embryo of the sheep, and, I believe, very closely represents the corresponding amount of change which takes place in the lung-tissue of other embryos which I have examined, viz. of the rabbit, cat, dog, cow, rat, guinea-pig.

	Size and condition of the embryo.	Amount of amyloid substance contained in 20 grains of the perfectly fresh lung-tissue.
1st.	Not quite 6 inches long, without any vestige of hair	1·9 grain.
2nd.	7 inches long; a trace of hair on the lip.....	2·55 grains.
3rd.	10 inches long; delicate hair about the head	2·8 grains.
4th.	15 inches long; covered with delicate hairs	3·45 grains.
5th.	16½ inches long; well covered with fine hair	2·2 grains.
6th.	Nearly 20 inches long; quite thickly covered with wool, and evidently very near the time of birth	A quantity too small to be estimated.

It would be no very easy matter to attempt to indicate precisely the condition of development of the embryo at which the maximum amount of amyloid substance is to be found in the tissue of voluntary muscle; in embryos of apparently the same age and condition of development it is found

to vary a good deal in amount. But this much may be asserted positively, that for some time before birth it has much diminished in quantity in this tissue, although always existing in it in notable amount up to and after birth. After repeated examination of various embryos, I believe I may state that the following Table, made from examination of the voluntary muscular tissue of fœtal lambs, correctly represents the average quantity of animal dextrine found in this tissue at various periods of its growth* :—

Size and condition of the embryo.		Quantity of fresh muscular tissue examined.	Weight of the foregoing when thoroughly dried.	Amount of amyloid substance in foregoing.
1st.	4 inches long.....	30 grains.	1·7 grain.	·1 grain.
2nd.	7 inches long; hair on lip	60 grains.	5·5 grains.	·5 grain.
3rd.	10 inches long; hair on head very fine	60 grains.	6·2 grains.	1·1 grain.
4th.	15 inches long; covered with delicate hair	60 grains.	7·5 grains.	2 grains.
5th.	16½ inches long; well covered with fine hair	60 grains.	7·8 grains.	2·1 grains.
6th.	Nearly 20 inches long, and almost about to be born	60 grains.	9·5 grains.	1·4 grain.

It appears therefore that, although it exists in a less proportion than at an earlier period, there is in muscular tissue at the period of birth a considerable quantity of amyloid substance; and this does not disappear altogether for some time in lambs, occasionally not for some weeks after birth. The tissue of voluntary muscle cannot be considered to have attained maturity at the time of birth; it has as yet hardly been called upon to exercise its functions. There is, however, a muscular organ the tissue of which commences, of necessity, the active exercise of its functions at an earlier period than that of voluntary muscle. The muscular structure of the heart, so far as its functions and activity are concerned, attains maturity

Size and condition of the embryo.		Weight of the muscular tissue of the heart examined.	Amount of amyloid substance in foregoing.
1st.	7 inches long	20 grains.	1·52 grain.
2nd.	10 inches long	20 grains.	1·60 grain.
3rd.	15 inches long	20 grains.	1·76 grain.
4th.	20 inches long, just before birth	20 grains.	A trace too small to weigh.

* It was stated in my former communication that 3·5 grains of amyloid substance was obtained from 60 grains of muscular tissue of a fœtal calf; but this was not quite pure, the investigation not being made with so much care as in the present instance.

earlier than other muscular tissue. The relation which the amyloid substance bears to it is therefore of much interest. In all embryos, without exception, which I have had an opportunity of examining at a time when they were closely approaching the period of birth, there has been no more than a trace of amyloid substance remaining in the muscular structure of the heart. The preceding Table, drawn up from examination of the heart of the embryo of the sheep, closely represents the corresponding state in other embryos.

The liver, which is the organ destined to form the amyloid matter during adult life, naturally has an increase of this material going on in its tissue up to and after birth: it does not make its appearance in the liver until the embryo is already well advanced in development; it then is found gradually and very slowly to increase in amount, but even at the time of birth is present in comparatively small quantity (2 per cent. in the liver of a lamb 20 inches long).

The rapidly growing horn of a young stag was not found to contain any amyloid substance in the tissues, neither does it exist in the texture of the growing horn of the calf; it is not found in the hair-bulbs of the adult, neither is it to be discovered as a formative material of the newly formed muscular fibres of the uterus when this organ is undergoing its remarkable reconstruction after delivery.

What is the function of this material during fœtal life? It can at least be said it does not change into sugar, neither does it give rise to fat. It seems to be a formative material, which, gradually becoming united with nitrogen, gives origin to the azotized structures.

IX. "Description of a New Mercurial Gasometer and Air-pump." By T. R. Robinson, D.D., LL.D., F.R.S., &c. Received June 2, 1864.

In some experiments on the electric spectra of metal and gases, I felt the want of a mercurial gasometer for working with such of the latter as are absorbable by water. That of Pepys is on too large a scale for my requirements, and it seemed better to contrive one more easily manageable, which I saw could also be made to act as a mercurial air-pump. In this I have succeeded to my satisfaction; and I hope that a description of it may be useful to those who are engaged in similar researches.

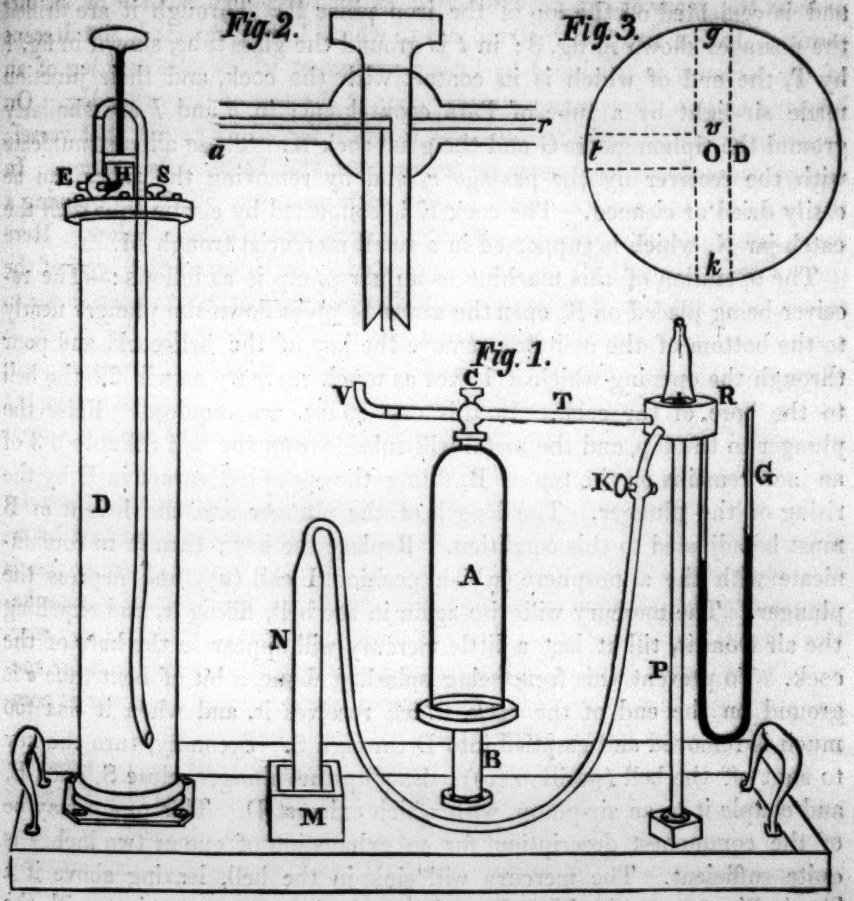
There have been several attempts made to exhaust by means of mercury, the chief of them with which I am acquainted being those of Close (Nicholson's Journal, 4to, iii. p. 264), Edelcrantz (Nicholson, 8vo, vii. p. 188), Traill and Children (Nicholson, xxi. pp. 63 & 161), and that of Geisler, which he uses in preparing the beautiful vacuum-tubes which bear his name. In all the principle is the same. A vessel is filled with mercury, which is made to descend from it, leaving in it a Torricellian

vacuum; this vessel may be made to communicate with a receiver, and abstract from it a portion of the gas which fills it; and by repeating the process the receiver can be exhausted as by successive strokes of an air-pump. In the two first instruments to which I have referred, the descent of the mercury is produced by lifting a plunger which fills one leg of an inverted siphon, the vacuum vessel being at the top of the other leg. On depressing the plunger, the mercury is again forced up to fill that vessel; and of course both legs must be longer than the barometric column. In the two next, the receiver itself is filled with mercury, which, by opening a cock, falls through a tube of sufficient length into a cistern below. Here the stroke (so to call it) cannot be repeated. In Geisler's the bend of the siphon is of vulcanized caoutchouc, so that one leg can be inclined down to a horizontal position, and thus allow the metal to fall from the other, or when raised to the vertical position fill it again. This I believe acts well, but it must be rather unwieldy; and my practical acquaintance with the working of tubes of that material has made me suspicious of their tightness and permanence under such circumstances.

As in all these cases the mercury is supported in the vacuum-vessel by atmospheric pressure, it is obvious that its descent will be produced by removing in any way that pressure; and an effective means of doing this is supplied by the common air-pump; more tedious certainly than the mechanical means above mentioned, but far more manageable; and as any mercurial pump must be slow in its working, while it is only required for special purposes, this defect is not of much importance, and moreover is compensated by some special advantages.

But besides bringing down the mercury, means must be provided for raising it again. My first plan was to do this by condensed air, the same syringe which made the exhaustion having its action reversed by a well-known arrangement. It worked extremely well, was lighter, and required less mercury than the contrivance which I finally adopted; but it is less convenient for gasometric work, as the syringe must be worked while gas is delivered. The machine in its present form is shown in fig. 1. Its base is a stout piece of mahogany, 21 inches by 10·5, with a rim round it 0·5 deep to prevent the loss of any spilled mercury, and handles at the ends by which it can be transported. To this is firmly fixed the iron stand B, 3·5 high, 4 in diameter above; its upper surface is carefully trued to a flanch, in which is cemented the vacuum-bell A, so that when the touching surfaces are lightly smeared with a mixture of lard and wax and screwed together by the six screws (some of which appear in the figure), the joint is air-tight. The bell A is 2 inches in diameter and 6·5 high; it has a tubulure at the top, in which is ground a glass cock C, whose construction is shown in fig. 2. The key of it is pierced from its bottom to a level with the bore, with which this perforation communicates occasionally by a lateral opening. In the position of the figure, it will be seen that the bell communicates with the branch *a*; if the key be turned half round, it is connected with the

branch *r*; and in an intermediate position it is completely shut off. These glass cocks have this great advantage over those of metal, that it can always



be ascertained if they are air-tight; their transparency permits us to see if the key and shell are in optical contact; and the slightest air-way there is at once detected. They should not be lubricated with oil, which *grips*, and may perhaps find its way into the bell and soil its interior. I find the best material to be castor oil with rosin dissolved in it. A hole is drilled down the axis of B, which communicates by a tube (sunk in the wood and therefore not visible in the figure) with the cast-iron cylinder D. This is 13 inches high and 3·2 in internal diameter; its top and bottom are secured to it air-tight by screws; in it works a plunger of boxwood well varnished 10·4 high, and moving so loosely that mercury may pass it easily. The plunger is wrought by a rod passing through the collar of leather H. In the top of the cylinder is a stopcock E, to which is fixed a tube of vulcanized caoutchouc (varnished with a solution of caoutchouc in benzidine), which is shown hanging down; it has a coupling to connect it with an ordinary air-pump. There is also in the top a screw S for admitting air.

One end of the bell's cock communicates with the atmosphere, the other with the receiver-plate R. This is of glass 2 inches in diameter, 0·75 thick, and is cemented on the top of the iron pillar P. Through it are drilled the passages shown in fig. 3; in *t* is ground the glass tube, shown in fig. 1 by T, the end of which is in contact with the cock, and their junction made air-tight by a tube of Para caoutchouc; in *g* and *k* are similarly ground the siphon-gauge G and the glass cock K. These all communicate with the receiver by the passage *v*, and by removing the tubes can be easily dried or cleaned. The cock K is connected by elastic tube with the catch-jar N, which is supported in a small mercurial trough M.

The operation of this machine as an air-pump is as follows:—The receiver being placed on R, open the screw S, press down the plunger nearly to the bottom of the cylinder, remove the key of the bell-cock, and pour through the opening which it leaves as much mercury as will fill the bell to the bore of the cock. In this one 10 lbs. are required. Raise the plunger to the top, and the metal will subside from the bell till only 0·3 of an inch remains on the top of B, filling the space left vacant in D by the rising of the plunger. The length of the plunger and the height of B must be adjusted to this condition. Replace the key; turn it to communicate with the atmosphere (which position I call (*a*)), and depress the plunger. The mercury will rise again in the bell, filling it, and expelling the air from it, till at last a little mercury will appear in the bore of the cock. To prevent this from being splashed about, a bit of bent tube *v* is ground on the end of the cock, which receives it, and when it has too much is removed and emptied into D through S. Secondly, turn the key to shut off the bell (position (*o*)); draw up the plunger, close S, open E, and couple it to an air-pump, with which exhaust D. This pump may be of the commonest description, for an exhaustion of one or two inches is quite sufficient. The mercury will sink in the bell, leaving above it a Torricellian vacuum. Close E, and turn the key to communicate with the receiver (position (*r*)); its air or gas will expand into the bell.

These three operations form the cycle of operation, and must be repeated till the required exhaustion be obtained, with one modification of the first one. In it, at the second and all subsequent strokes, the key is to be at (*o*) and S opened; thus the atmospheric pressure will raise the mercury and do much of the plunger's work; that must then be depressed and the key set at (*a*); the other two steps are as at first.

When the instrument is to be used as a gas-holder, either the receiver must be in its place, or the opening of R must be closed by a piece of flat glass; the bell must be filled by the plunger, and made, by (*r*) and by opening *k*, to communicate with the jar N. The mercury will rise in that to its neck, and sink in A; fill A again, pass gas into N, and, by carefully working the key, draw it into A till that is full. As this gas will be mixed with the air of the vessels and passages, it must be expelled, and A refilled till its purity is certain. If it be noxious, it must be conducted into some

absorbent fluid by an elastic tube, slipped on the *a* end of the cock; which will also convey the gas to any vessel.

If it be required to fill a receiver for experiments in an atmosphere of gas either at common pressure or a less one, it may either be exhausted by an air-pump connected with K, and filled from A, or exhausted by A and filled from N. The former can only be done with gases which have no action on brass.

These operations seem complicated when described with so much detail, but in practice they are very easy, and their result is good. Some precautions, however, are required to ensure it. The bottom of the bell-cock and of its key must be ground, so as to leave no shoulder or hollow in which air may be entangled when the bell is filled. Every part of the metal work must be air-tight; this can only be secured by covering, not only its joints, but its whole surface with several coats of varnish-paint—best of white lead. When the first coat is applied, on exhausting the apparatus, every hole or pore is revealed by an opening in the paint (often almost microscopic), which must be filled up as it forms till all is tight. It is almost needless to mention that the whole must be perfectly dry. If the bell be filled a few times with undried air, enough of moisture will adhere to its walls to prevent an exhaustion of more than 0·1 inch. In such a case it must be dried by drawing air into it through sulphuric acid, and this repeatedly. Moisture also occasionally finds its way into a part still more troublesome, into the passage which connects the bell and cylinder; it is probably condensed there when the mercury is colder than the atmosphere. I remove this by connecting the tube of K with a desiccator; setting C to (*r*), opening K and E, and by working the air-pump drawing a stream of dry air into D, which bubbles up through the mercury in the passage, and at last sweeps away all trace of water and its vapour. In this operation it is necessary to remove a portion of the mercury, as otherwise it would be sucked into the pump; indeed this mischief might occur in ordinary work by some mistake in the manipulation—for instance, by leaving E open with (*a*). To prevent the possibility of this, D is connected with the pump by a mercury trap, easily imagined, which intercepts any of that metal that might come over. And lastly, the interior of the bell must be *perfectly clean* if the highest degree of exhaustion is required. This state is obtained by washing it with strong nitric acid, then with distilled water, and when quite dry wiping it with linen, from which all traces of soap or starch have been removed by boiling it in rain-water. Thus we reduce to a minimum the film of air which adheres to the bell even when filled with mercury, and lessens its vacuum. When all these precautions were taken, I found that with a receiver containing 3·7 inches, the fifth operation brought the gauge (which had been similarly cleaned and *carefully* boiled) down to 0·01. The sixth brought it still lower, but my present means of measurement* are not sufficient to determine the precise amount. In this

* A micrometer microscope put in the place of the telescope of my theodolite.

machine the old air-pump theorem ought to hold, and by it, with the fraction $\frac{3\cdot7}{17}$, I find that the fifth *should* give 0·007, and the sixth 0·0014; so that the presence of adhering air is still sensible, though very slight. So high a power, however, is not long maintained; for by use, and especially with oxygen, which (probably from the presence of ozone) has a peculiar tendency to dirty mercury, the bell becomes soiled; but it continues to give a vacuum of 0·02, which is quite sufficient for ordinary objects. At common pressure and temperature, the electric discharge through the receiver shows no evidence of the presence of mercurial vapour; but at 0·02 it is otherwise; the discharge is greenish white, and the spectrum shows little except the lines of mercury. If the gauge were detached, perhaps this vapour might be absorbed by gold-leaf.

The apparatus acts well as a mercurial gas-holder, and can deliver 18·5 inches. Like all other contrivances for confining gaseous matter by mercury, it is liable to have its contents contaminated with air by diffusion between the metal and the vessel which contains it; but I expected that in this arrangement the defect would be little felt. In order that it may take place, the air must pass a distance of 17·2 inches, of which 14·6 is a tube only 0·125 in diameter, and the rest is in a vertical direction against the pressure of 2·6 inches of mercury. A single experiment will show how far this avails. The bell was filled with dry hydrogen, which was found to contain 0·901 of the pure gas; it was left for ten days exposed to considerable changes of temperature, and was then found to have 0·854; it was therefore contaminated at the rate of 0·005 per day. I am not aware of similar measures with ordinary mercurial apparatus; nor is this amount of error very important; but it may I believe be corrected by a means long since announced by the late Professor Daniell which has been strangely neglected. He proposed it to prevent the infiltration of air into barometers. If the liquid metal adhered to the surface which it touches, as water would, this action could not occur; now it *wets*, if I may use the word, several metals, as copper or silver, but it also dissolves them, and becomes less fluid. Daniell, however, found that it does *wet* platinum without acting on it in any injurious degree; and advised that a ring of platinum wire should be fused round the tube where it dips into its cistern. On inquiring of his friend and fellow-labourer, Dr. W. A. Miller, I learn that it was completely successful, but was not taken up by the opticians, and passed out of memory. It is obvious that if a bit of platinum tube were cemented in the vertical passage below D, it would effectually bar the diffusion. I do not like to undo the joint there, which is now perfectly tight; but I will certainly, when the opportunity offers, try the experiment.

X. "On the Distal Communication of the Blood-vessels with the Lymphatics; and on a Diaplasmatic System of Vessels." By THOMAS ALBERT CARTER, M.D., M.R.C.P., Physician to the Leamington Hospital and Warwick Dispensary. Communicated by W. S. SAVORY, Esq. Received June 2, 1864.

(Abstract.)

In this paper the author has recorded the results at which he has arrived concerning the distal intercommunication of the hæmal with the lymphatic system by means of injections thrown into blood-vessels; he also describes certain minute vessels and networks of vessels which can be shown by the same means to exist in certain mucous membranes and elsewhere. These he has named diaplasmatics.

The author's attention was first particularly called to the relation which the lymphatics bear to the blood-vessels, by observing that when the latter are fully distended with a very penetrating injection, such injection often finds its way into the lymphatics without the occurrence of ordinary extravasation.

He has thus injected the livers of three human beings and of three pigs from the portal and hepatic vessels, the former (vessels) being filled with Turnbull's blue precipitated in gelatine, and the latter with carmine similarly treated; and in each instance he has found that the injection had gained entrance to the superficial lymphatics.

In sections taken from the surface of the pig's liver, these vessels (which may readily be distinguished from the blood-vessels by their knotted irregular appearance and rapid increase and diminution in size) are observed in many instances to surround a lobule, throwing out loops and prolongations towards its centre. A certain number of these prolongations, both in the human liver and in the pig's, when traced are seen to diminish in size so much as to be considerably less in diameter than the capillaries of the organ, in which they appear to lose themselves or rather originate. Their commencements in this part, it is acknowledged, are extremely difficult to determine by simple inspection, on account of the underlying capillaries being filled with injection of the same colour; but in some instances (as, *e. g.*, where the pigment in the capillaries has faded) the author believes that he has seen the actual anastomoses of the two sets of vessels. The circumstance, however, which renders exact microscopic observation so very difficult, is the one which affords the best evidence of the communication of the two systems, viz. that the minutest lymphatics are almost invariably filled with injection of the particular tint seen in the capillaries in close relation to them. Thus if the capillaries be red or blue, or any of the intermediate shades of purple, the smallest lymphatics in the immediate neighbourhood will be of a precisely similar colour; which would appear distinctly to show whence the lymphatics derive their supply of fluid.

A human thyroid body which the author injected with carmine and

gelatine from the blood-vessels, also exhibited a phenomenon similar to that observed in the organs just mentioned.

Sections taken from this gland and examined with the ¼-inch objective, showed that from the capillaries are given off fine processes which break up into a network among the cell-elements of the vesicles, and, furthermore, that this network is in communication with the lymphatics which lie in the intervesicular parts of the gland. In addition to this, however, there are communications between the capillaries and lymphatics in the stroma itself.

The processes which emerge from the capillaries in the stroma of the thyroid as well as in the fibrous tissues of other parts, such as the membrana nictitans of the cat, bear a very strong resemblance to connective-tissue corpuscles; and such the author considers them to be in these parts. But as these tubular processes can be shown by injection to form a plexus in the retina (cat), to be connected with the nuclei of the capillaries, the corpuscles of bone (perch and mouse), and the fusiform bodies found among the fibrillæ of muscle (frog), as well as with the cells of connective tissue and its modifications, it has appeared that the whole of these structures belong to one system of vessels. This system the author has named, provisionally at least, "Diaplasmatic," because, on account of the extreme minuteness of its channels, it can only allow of the passage of the liquor sanguinis.

To designate the whole of these minute vessels lymphatics would, he considers, at the present time be somewhat premature, because those of muscular fibre and of bones, and others which will be mentioned immediately, have not been observed to join recognizable lymphatic trunks; and moreover it would seem by no means improbable that some of them may both commence and terminate in the blood-vessels, thus constituting what might be styled an intercapillary plexus; or they may even have a triple connexion, viz. with the arterial capillaries, the lymphatics, and with the venous capillaries or the veins.

The position in which the diaplasmatic network may be most readily demonstrated, both with and without injection, is in the mucous membrane of the palate of the frog or toad. In this part, when the viscid mucus and the ciliated epithelium have been removed, there may be seen with the ¼-inch objective, a very minute granular nucleated network, in each mesh of which is placed a globular nucleated cell. The membrane consists, therefore, of three layers—of a superficial ciliated layer, next of a granular nucleated plasmatic network, and lower still of a basement layer of globular nucleated cells. The processes of the middle granular plexus extend not only in the horizontal direction, but also upwards between the ciliated cells, and downwards between those of the basement layer, where they become continuous with the blood-vessels lying in the fibrous tissue beneath. This connexion with the blood-vessels, the author says, he has been able to make out by means of injection in the palate and œsophagus of the frog and toad, as well as in the mucous membrane of the eyelid of the latter animal.

He has also demonstrated by injection that a network similar to the one just described is present in the web-membrane of the pectoral fin of the perch. The lung of the toad also exhibits a modification of the plasmatic network in the form of extremely fine hollow processes, which either stretch completely across the mesh from capillary to capillary, or terminate in finely pointed or blunt extremities among the epithelia or nuclei which stud the membrane of the air-vesicle.

In the proper epithelial portion of the skin of batrachians or of mammals, the author has not yet been able to prove distinctly that the plexuses are to be found, but he has been so far successful in this direction as to have displayed them satisfactorily in the follicles and bulbs of the whisker hairs of the mole, mouse, and kitten. From certain observations, however, which cannot here be detailed, he thinks it more than probable, not only that plasma-networks are present in the epithelial layer of the batrachian skin, but also in a corresponding part of the human cutis.

With regard to the offices performed by these networks, the author thinks it probable that all those found in the epidermal or mucous tissues are intimately connected with the function of secretion, and in a minor degree also perhaps with that of absorption; while those situated in the deeper parts of the organism, such as muscle and fibrous tissue, are employed in conveying blood-plasma to, and effete matters from, the tissues through which they pass or with which they may be in contact.

XI. "Aërial Tides." By Pliny Earle Chase, A.M., S.P.A.S. Communicated by Major-General Sabine, Pres. R.S. Received June 16, 1864.

The remarkable coincidence which I have pointed out* between the theoretical effects of rotation and the results of barometrical observations, has led me to extend my researches with a view of defining more precisely some of the most important effects of lunar action on the atmosphere. The popular belief in the influence of the moon on the weather, which antedates all historical records, has received at various times a certain degree of philosophical sanction. Herschel and others have attempted partially to formulate that influence by empirical laws, but the actual character of the lunar wave that is daily rolled over our heads, appears never to have been investigated.

Major-General Sabine has shown that the moon produces a diurnal variation of the barometer, amounting to about ·006 of an inch at St. Helena, which is nearly equivalent to $\frac{1}{10}$ of the average daily variation (Phil. Trans. 1847, Art. V.). This would indicate a tidal wave of rather more than 1 ft. for each mile that we ascend above the earth's surface, or from 3 to 6 ft. near the summits of the principal mountain-chains. It is easy to believe that the rolling of such a wave over the broken surface of the earth may

* See Proceedings of Amer. Philos. Soc. vol. ix. p. 283.

exert a very important influence on the atmospheric and magnetic currents, the deposition of moisture, and other meteorological phenomena. As the height of the wave varies with the changing phases of the moon*, its effects must likewise vary in accordance with mathematical laws, the proper study of which must evidently form an important branch of meteorological science.

Besides this daily wave, there appears to be a much larger, but hitherto undetected, weekly wave. M. Flangergues†, an astronomer at Viviers in France, extended his researches through a whole lunar cycle, from Oct. 19, 1808 to Oct. 18, 1827, and he inferred from his observations—

1. That in a synodical revolution of the moon, the barometer rises regularly from the second octant, when it is the lowest, to the second quadrature, when it is the highest, and then descends to the second octant.

2. That the varying declination of the moon modifies her influence, the barometer being higher in the northern lunistice than in the southern.

The more recent and more complete observations at St. Helena give somewhat different results, which serve to confirm the natural *à priori* conviction that there are two maxima and two minima in each month. The means of three years' hourly observations, indicate the existence of waves which produce in the first quarter a barometric effect of +·004 in., in the second quarter of —·016 in., in the third quarter of +·018 in., and in the fourth quarter of —·006 in.—results which appear to be *precisely* accordant, in their general features, with those which would be naturally anticipated from the combination of the cumulative action of the moon's attraction, with the daily wave of rotation, and the resistance of the æther.

One peculiarity of the lunar-aërial wave deserves attention, for the indirect confirmation that it lends to the rotation theory of the aërobaric tides, and the evidence it furnishes of opposite tidal effects, which require consideration in all investigations of this character. When the daily lunar tides are highest, their pressure is greatest, the lunar influence accumulating the air directly under the meridian, so as to more than compensate for the diminished weight consequent upon its "lift." But in the general aërial fluctuations, as we have seen heretofore, and also in the weekly tides which we are now considering, a high wave is shown by a low barometer, and *vice versâ*. The daily blending of heavy and light waves produces oscillations which are indicated by the alternate rise and fall of the barometer and thermometer at intervals of two or three days.

M. Flangergues's observations at perigee and apogee seem to show that a portion of the movement of the air by the moon is a true lift, which, like the lift of rotation, must probably exert an influence on the barometer. On comparing the *daily* averages at each of the quadratures and syzygies, I found the difference of temperature too slight to warrant any satisfactory inference, but a similar comparison of the *hourly* averages, at hours when

* The height at St. Helena appears to fluctuate between about ·9 and 1·6 ft.

† Bib. Univ., Dec. 1827.

the sun is below the horizon, gave such results as I anticipated; as will be seen by a reference to the following

Table of Barometric and Thermometric Means at the Moon's Changes.

Moon's Phase.	Average Height of Barometer, in inches.	Height of Lunar Weekly Tides.	Height of Lunar Daily Tides.	Daily Height of Thermometer.	Thermometer at 12 P.M.	Thermometer at 4 A.M.
		in.	in.	°	°	°
Full.....	28·270	−·0115	·0054	67·67	60·22	59·787
Third Quarter	28·289	+·0065	·0087	61·68	60·41	59·824
New	28·282	+·0005	·0064	61·65	60·31	59·716
First Quarter	28·286	+·0044	·0047	61·63	60·37	59·823

In obtaining the above averages, I was obliged to interpolate for such changes as took place on Sundays or holidays, when no observations were taken. The interpolation, however, does not change the general result, and on some accounts the Table is more satisfactory than if the observations had been made with special reference to the determination of the lunar influences, accompanied, as such a reference would very likely have been, by a bias to some particular theory.

The thermometric and barometric averages show a general correspondence in the times of the monthly maxima and minima,—the correspondence being most marked and uniform at midnight, when the air is most removed from the direct heat of the sun, and we might therefore reasonably expect to find the strongest evidences of the relations of temperature to lunar attraction.

By taking the difference between the successive weekly tides, we readily obtain the amount of barometric effect in each quarter. The average effect is more than three times as great in the third and fourth quarters as in the remaining half-month,—a fact which suggests interesting inquiries as to the amount of influence attributable to varying centrifugal force, solar conjunction or opposition, temperature, &c.

Although, as in the ocean tides, there are two simultaneous corresponding waves on opposite sides of the earth, those waves are not of equal magnitude, the barometer being uniformly higher when the moon is on the inferior meridian, and its attraction is therefore exerted in the same direction as the earth's, than when it is on the superior meridian, and the two attractions are mutually opposed. Some of the views of those who are not fully satisfied with the prevailing theory of the ocean-tides, derive a partial confirmation from this fact.

I find, therefore, marked evidences of the same lunar action on the atmosphere as on the ocean, the combination of its attraction with that of the sun producing both in the air and water, spring tides at the syzygies,

and neap tides at the quadratures; and I believe that the most important normal atmospheric changes may be explained by the following theory:—

The attraction- and rotation-waves, as will be readily seen, have generally opposite values, the luni-solar wave being

Descending, from 0° to 90°* and from 180° to 270°,

Ascending, from 90° to 180° and from 270° to 0°;

while the rotation-wave is

Ascending, from 330° to 60° and from 150° to 240°,

Descending, from 60° to 150° and from 240° to 330°.

From 60° to 90° and from 240° to 270°, both waves are descending, while from 150° to 180° and from 330° to 360° both are ascending. In consequence of this change of values, besides the principal maxima and minima at the syzygies and quadratures, there should be secondary maxima and minima† at about 60° in advance of those points.

The confirmation of these theoretical inferences by the St. Helena observations appears to me to be quite as remarkable as that of my primary hypothesis. If we arrange those observations in accordance with the moon's position, and take the average daily height of the barometer, we obtain the following

Table of the Lunar Barometric Tides.

Moon's Position.	Mean Daily Height of the Barometer at St. Helena, 28 inches + the numbers in the Table.			
	1844.	1845.	1846.	1844–6. Average.
☽ ☌				
0	·2621	·3020	·2701	·2781
15	·2650	·3058	·2693	·2800
30	·2707	·3153	·2707	·2856
45	·2691	·3165	·2688	·2848
60	·2625	·3077	·2688	·2797
75	·2682	·3093	·2783	·2853
90	·2667	·3184	·2800	·2884
105	·2593	·3170	·2721	·2828
120	·2595	·3124	·2686	·2802
135	·2677	·3099	·2691	·2822
150	·2712	·3118	·2715	·2848
165	·2710	·3104	·2735	·2850
180	·2621‡	·3020	·2701	·2781

This Table shows—

1. That the average of the three years corresponds *precisely* with the theory, except in the secondary maximum, which is one day late.

* Counting θ from either syzygy.

† The secondary maxima and minima should correspond with the daily maxima and minima, which occur at St. Helena at about 4^h and 10^h A.M. and P.M., giving $\theta=60°$ a maximum, and $\theta=150°$ a minimum.

‡ Since the tabular numbers represent the *semiaxes* of the barometric curve, and not the simple *ordinates*, the values for 0° and 180° are the same.

2. That the primary maximum occurred at the quadratures in 1845 and 1846, and one day earlier in 1844.

3. That the primary minimum occurred at the syzygies in 1844 and 1845, and one day later in 1846.

4. That 1846 was a disturbed year; and if it were omitted from the Table, each of the remaining years, as well as the average, would exhibit an entire correspondence with theory, except in the primary maximum of 1844.

5. That 1845 was a normal year, the primary and secondary maxima and minima all corresponding with theory, both in position and relative value.

XII. "On the Microscopical Structure of Meteorites."

By H. C. Sorby, F.R.S., &c. Received June 7, 1864.

For some time past I have endeavoured to apply to the study of meteorites the principles I have made use of in the investigation of terrestrial rocks, as described in my various papers, and especially in that on the microscopical structure of crystals (Quart. Journ. Geol. Soc. 1858, vol. xiv. p. 453). I therein showed that the presence in crystals of "fluid-, glass-, stone-, or gas-cavities" enables us to determine in a very satisfactory manner under what conditions the crystals were formed. There are also other methods of inquiry still requiring much investigation, and a number of experiments must be made which will occupy much time; yet, not wishing to postpone the publication of certain facts, I purpose now to give a short account of them, to be extended and completed on a subsequent occasion *.

In the first place it is important to remark that the olivine of meteorites contains most excellent "glass-cavities," similar to those in the olivine of lavas, thus proving that the material was at one time in a state of igneous fusion. The olivine also contains "gas-cavities," like those so common in volcanic minerals, thus indicating the presence of some gas or vapour (Aussun, Parnallee). To see these cavities distinctly, a carefully prepared thin section and a magnifying power of several hundreds are required. The vitreous substance found in the cavities is also met with outside and amongst the crystals, in such a manner as to show that it is the uncrystalline residue of the material in which they were formed (Mezö-Madaras, Parnallee). It is of a claret or brownish colour, and possesses the characteristic structure and optical properties of artificial glasses. Some isolated portions of meteorites have also a structure very similar to that of stony lavas, where the shape and mutual relations of the crystals to each other prove that they were formed *in situ*, on solidification. Possibly some entire meteorites should be considered to possess this peculiarity (Stannern, New Concord), but the evidence is by no means conclusive, and what crystallization has taken place *in situ* may have been a secondary result; whilst in others the constituent particles have all the characters of broken fragments

* The names given thus (Stannern) indicate what meteorites I more particularly refer to in proof of the various facts previously stated.

(L'Aigle). This sometimes gives rise to a structure remarkably like that of consolidated volcanic ashes, so much, indeed, that I have specimens which, at first sight, might readily be mistaken for sections of meteorites. It would therefore appear that, after the material of the meteorites was melted, a considerable portion was broken up into small fragments, subsequently collected together, and more or less consolidated by mechanical and chemical actions, amongst which must be classed a segregation of iron, either in the metallic state or in combination with other substances. Apparently this breaking up occurred in some cases when the melted matter had become crystalline, but in others the forms of the particles lead me to conclude that it was broken up into detached globules whilst still melted (Mezö-Madaras, Parnallee). This seems to have been the origin of some of the round grains met with in meteorites; for they occasionally still contain a considerable amount of glass, and the crystals which have been formed in it are arranged in groups, radiating from one or more points on the external surface, in such a manner as to indicate that they were developed after the fragments had acquired their present spheroidal shape (Aussun, &c.). In this they differ most characteristically from the general type of concretionary globules found in terrestrial rocks, in which they radiate from the centre; the only case that I know at all analogous being that of certain oolitic grains in the Kelloways rock at Scarborough, which have undergone a secondary crystallization. These facts are all quite independent of the fused black crust.

Some of the minerals in meteorites, usually considered to be the same as those in volcanic rocks, have yet very characteristic differences in structure (Stannern), which I shall describe at greater length on a future occasion. I will then also give a full account of the microscopical structure of meteoric iron as compared with that produced by various artificial processes, showing that under certain conditions the latter may be obtained so as to resemble very closely some varieties of meteoric origin (Newstead, &c.).

There are thus certain peculiarities in physical structure which connect meteorites with volcanic rocks, and at the same time others in which they differ most characteristically,—facts which I think must be borne in mind, not only in forming a conclusion as to the origin of meteorites, but also in attempting to explain volcanic action in general. The discussion of such questions, however, should, I think, be deferred until a more complete account can be given of all the data on which these conclusions are founded.

XIII. "On the Functions of the Cerebellum." By W. H. Dickinson, M.D. Communicated by Dr. Bence Jones. Received June 16, 1864.

This is a revised version of a Paper having the same title which was read on the 7th of April, 1864, and of which an Abstract appeared under that date *.

* See p. 177.

XIV. "On the Properties of Silicic Acid and other analogous Colloidal Substances." By Thomas Graham, F.R.S., Master of the Mint. Received June 16, 1864.

(Preliminary Notice.)

The prevalent notions respecting solubility have been derived chiefly from observations on crystalline salts, and are very imperfectly applicable to the class of colloidal substances. Hydrated silicic acid, for instance, when in the soluble condition, is properly speaking a liquid body, like alcohol, miscible with water in all proportions. We have no degrees of solubility to speak of with respect to silicic acid, like the degrees of solubility of a salt, unless it be with reference to silicic acid in the gelatinous condition, which is usually looked upon as destitute of solubility. The jelly of silicic acid may be more or less rich in combined water, as it is first prepared, and it appears to be soluble in proportion to the extent of its hydration. A jelly containing 1 per cent. of silicic acid, gives with cold water a solution containing about 1 of silicic acid in 5000 water; a jelly containing 5 per cent. of silicic acid, gives a solution containing about 1 part of acid in 10,000 water. A less hydrated jelly than the last mentioned is still less soluble; and finally, when the jelly is rendered anhydrous, it gives gummy-looking white masses, which appear to be absolutely insoluble, like the light dusty silicic acid obtained by drying a jelly charged with salts, in the ordinary analysis of a silicate.

The liquidity of silicic acid is only affected by a change, which is permament (namely, coagulation or pectization), by which the acid is converted into the gelatinous or pectous form, and loses its miscibility with water. The liquidity is permanent in proportion to the degree of dilution of silicic acid, and appears to be favoured by a low temperature. It is opposed, on the contrary, by concentration, and by elevation of temperature. A liquid silicic acid of 10 or 12 per cent. pectizes spontaneously in a few hours at the ordinary temperature, and immediately when heated. A liquid of 5 per cent. may be preserved for five or six days; a liquid of 2 per cent. for two or three months; and a liquid of 1 per cent. has not pectized after two years. Dilute solutions of 0·1 per cent. or less are no doubt practically unalterable by time, and hence the possibility of soluble silicic acid existing in nature. I may add, however, that no solution, weak or strong, of silicic acid in water has shown any disposition to deposit *crystals*, but always appears on drying as a colloidal glassy hyalite. The formation of quartz crystals at a low temperature, of so frequent occurrence in nature, remains still a mystery. I can only imagine that such crystals are formed at an inconceivably slow rate, and from solutions of silicic acid which are extremely dilute. Dilution no doubt weakens the colloidal character of substances, and may therefore allow their crystallizing tendency to gain ground and develope itself, particularly where the crystal once formed is completely insoluble, as with quartz.

The pectization of liquid silicic acid is expedited by contact with solid matter in the form of powder. By contact with pounded graphite, which is chemically inactive, the pectization of a 5 per cent. silicic acid is brought about in an hour or two, and that of a 2 per cent. silicic acid in two days. A rise of temperature of 1°·1 C. was observed during the formation of the 5 per cent. jelly.

The ultimate pectization of silicic acid is preceded by a gradual thickening in the liquid itself. The flow of liquid colloids through a capillary tube is always slow compared with the flow of crystalloid solutions, so that a liquid-transpiration-tube may be employed as a colloidoscope. With a colloidal liquid alterable in viscosity, such as silicic acid, the increased resistance to passage through the colloidoscope is obvious from day to day. Just before gelatinizing, silicic acid flows like an oil.

A dominating quality of colloids is the tendency of their particles to adhere, aggregate, and contract. This idio-attraction is obvious in the gradual thickening of the liquid, and when it advances leads to pectization. In the jelly itself, the specific contraction in question, or *synæresis*, still proceeds, causing separation of water, with the division into a clot and serum; and ending in the production of a hard stony mass, of vitreous structure, which may be anhydrous, or nearly so, when the water is allowed to escape by evaporation. The intense synæresis of isinglass dried in a glass dish over sulphuric acid *in vacuo*, enables the contracting gelatin to tear up the surface of the glass. Glass itself is a colloid, and the adhesion of colloid to colloid appears to be more powerful than that of colloid to crystalloid. The gelatin, when dried in the manner described upon plates of calc-spar and mica, did not adhere to the crystalline surface, but detached itself on drying. Polished plates of glass must not be left in contact, as is well known, owing to the risk of permanent adhesion between their surfaces. The adhesion of broken masses of glacial phosphoric acid to each other is an old illustration of colloidal synæresis.

Bearing in mind that the colloidal phasis of matter is the result of a peculiar attraction and aggregation of molecules, properties never entirely absent from matter but greatly more developed in some substances than in others, it is not surprising that colloidal characters spread on both sides into the liquid and solid conditions. These characters appear in the viscidity of liquids, and in the softness and adhesiveness of certain crystalline substances. Metaphosphate of soda, after fusion by heat, is a true glass or colloid; but when this glass is maintained for a few minutes at a temperature some degrees under its point of fusion, the glass assumes a crystalline structure without losing its transparency. Notwithstanding this change, the low diffusibility of the salt is preserved, with other characters of a colloid. Water in the form of ice has already been represented as a similar intermediate form, both colloid and crystalline, and in the first character adhesive and capable of reunion or "regelation."

It is unnecessary to return here to the fact of the ready pectization of

liquid silicic acid by alkaline salts, including some of very sparing solubility, such as carbonate of lime, beyond stating that the presence of carbonate of lime in water was observed to be incompatible with the coexistence of soluble silicic acid, till the proportion of the latter was reduced to nearly 1 in 10,000 water.

Certain liquid substances differ from the salts in exercising little or no pectizing influence upon liquid silicic acid. But, on the other hand, none of the liquids now referred to appear to conduce to the preservation of the fluidity of the colloid, at least not more than the addition of water would do. Among these inactive diluents of silicic acid are found hydrochloric, nitric, acetic, and tartaric acids, syrup of sugar, glycerine, and alcohol. But all the liquid substances named, and many others, appear to possess an important relation to silicic acid, of a very different nature from the pectizing action of salts. They are capable of displacing the combined water of the silicic acid hydrate, whether that hydrate is in the liquid or gelatinous condition, and give new substitution-products.

A liquid compound of *alcohol* and silicic acid is obtained by adding alcohol to aqueous silicic acid, and then employing proper means to withdraw the water from the mixture. For that purpose the mixture contained in a cup may be placed over dry carbonate of potash or quicklime, within the receiver of an air-pump. Or a dialyzing bag of parchment-paper containing the mixed alcohol and silicic acid may be suspended in a jar of alcohol: the water diffuses away, leaving in the bag a liquid composed of alcohol and silicic acid only. A point to be attended to is, that the silicic acid should never be allowed to form more than 1 per cent. of the alcoholic solution, otherwise it may gelatinize during the experiment. If I may be allowed to distinguish the liquid and gelatinous hydrates of silicic acid by the irregularly formed terms of *hydrosol* and *hydrogel* of silicic acid, the two corresponding alcoholic bodies now introduced may be named the alcosol and alcogel of silicic acid.

The *alcosol* of silicic acid, containing 1 per cent. of the latter, is a colourless liquid, not precipitated by water or salts, nor by contact with insoluable powders, probably from the small proportion of silicic acid present in solution. It may be boiled and evaporated without change, but is gelatinized by a slight concentration. The alcohol is retained less strongly in the alcosol of silicic acid than water is in the hydrosol, but with the same varying force, a small portion of the alcohol being held so strongly as to char when the resulting jelly is rapidly distilled at a high temperature. Not a trace of silicic ether is found in any compound of this class. The jelly burns readily in the air, leaving the whole silicic acid in the form of a white ash.

The *alcogel*, or solid compound, is readily prepared by placing masses of gelatinous silicic acid, containing 8 or 10 per cent. of the dry acid, in absolute alcohol, and changing the latter repeatedly till the water of the hydrogel is fully replaced by alcohol. The alcogel is generally slightly opalescent,

and is similar in aspect to the hydrogel, preserving very nearly its original bulk. The following is the composition of an alcogel carefully prepared from a hydrogel which contained 9·35 per cent. of silicic acid:—

Alcohol	88·13
Water	0·23
Silicic acid	11·64
	100·00

Placed in water, the alcogel is gradually decomposed—alcohol diffusing out and water entering instead, so that a hydrogel is reproduced.

Further, the alcogel may be made the starting-point in the formation of a great variety of other substitution jellies of analogous constitution, the only condition required appearing to be that the new liquid and alcohol should be intermiscible, that is, interdiffusible bodies. Compounds of ether, benzole, and bisulphide of carbon have thus been produced. Again, from *etherogel* another series of silicic acid jellies may be derived, containing fluids soluble in ether, such as the fixed oils.

The preparation of the *glycerine* compound of silicic acid is facilitated by the comparative fixity of that liquid. When hydrated silicic acid is first steeped in glycerine, and then boiled in the same liquid, water distils over, without any change in the appearance of the jelly, except that when formerly opalescent it becomes now entirely colourless, and ceases to be visible when covered by the liquid. But a portion of the silicic acid is dissolved, and a *glycerosol* is produced at the same time as the glycerine jelly. A glycerogel prepared from a hydrate containing 9·35 per cent. of silicic acid, was found by a combustion analysis to be composed of

Glycerine	87·44
Water	3·78
Silicic acid	8·95
	100·17

The glycerogel has somewhat less bulk than the original hydrogel. When a glycerine jelly is distilled by heat, it does not fuse, but the whole of the glycerine comes over, with a slight amount of decomposition towards the end of the process.

The compound of sulphuric acid, *sulphagel*, is also interesting from the facility of its formation, and the complete manner in which the water of the original hydrogel is removed. A mass of hydrated silicic acid may be preserved unbroken if it is first placed in sulphuric acid diluted with two or three volumes of water, and then transferred gradually to stronger acids, till at last it is placed in concentrated oil of vitriol. The sulphagel sinks in the latter fluid, and may be distilled with an excess of it for hours without losing its transparency or gelatinous character. It is always somewhat less in bulk than the primary hydrogel, but not more, to the eye, than one-fifth or one-sixth part of the original volume. This sulphagel is transparent and

colourless. When a sulphagel is heated strongly in an open vessel, the last portions of the monohydrated sulphuric acid in combination are found to require a higher temperature for their expulsion than the boiling-point of the acid. The whole silicic acid remains behind, forming a white, opaque, porous mass, like pumice. A sulphagel placed in water is soon decomposed, and the original hydrogel reproduced. No permanent compound of sulphuric and silicic acids, of the nature of a salt, appears to be formed in any circumstances. A sulphagel placed in alcohol gives ultimately a pure alcogel. Similar jellies of silicic acid may readily be formed with the monohydrates of nitric, acetic, and formic acids, and are all perfectly transparent.

The production of the compounds of silicic acid now described indicates the possession of a wider range of affinity by a colloid than could well be anticipated. The organic colloids are no doubt invested with similar wide powers of combination, which may become of interest to the physiologist. The capacity of a mass of gelatinous silicic acid to assume alcohol, or even oleine, in the place of water of combination, without disintegration or alteration of form, may perhaps afford a clue to the penetration of the albuminous matter of membrane by fatty and other insoluble bodies, which seems to occur in the digestion of food. Still more remarkable and suggestive are the *fluid* compounds of silicic acid. The fluid alcohol-compound favours the possibility of the existence of a compound of the colloid albumen with oleine, soluble also and capable of circulating with the blood.

The feebleness of the force which holds together two substances belonging to different physical classes, one being a colloid and the other a crystalloid, is a subject deserving notice. When such a compound is placed in a fluid, the superior diffusive energy of the crystalloid may cause its separation from the colloid. Thus, of hydrated silicic acid, the combined water (a crystalloid) leaves the acid (a colloid) to diffuse into alcohol; and if the alcohol be repeatedly changed, the entire water is thus removed, alcohol (another crystalloid) at the same time taking the place of water in combination with the silicic acid. The liquid in excess (here the alcohol) gains entire possession of the silicic acid. The process is reversed if an alcogel be placed in a considerable volume of water. Then alcohol separates from combination, in consequence of the opportunity it possesses to diffuse into water; and water, which is now the liquid present in excess, recovers possession of the silicic acid. Such changes illustrate the predominating influence of mass.

Even the compounds of silicic acid with alkalies yield to the decomposing force of diffusion. The compound of silicic acid with 1 or 2 per cent. of soda is a colloidal solution, and, when placed in a dialyzer over water *in vacuo* to exclude carbonic acid, suffers gradual decomposition. The soda diffuses off slowly in the caustic state, and gives the usual brown oxide of silver when tested with the nitrate of that base.

The pectization of liquid silicic acid and many other liquid colloids is effected by contact with minute quantities of salts in a way which is not

understood. On the other hand, the gelatinous acid may again be liquefied and have its energy restored by contact with a very moderate amount of alkali. The latter change is gradual, 1 part of caustic soda, dissolved in 10,000 water, liquefying 200 parts of silicic acid (estimated dry) in 60 minutes at 100° C. Gelatinous stannic acid also is easily liquefied by a small proportion of alkali, even at the ordinary temperature. The alkali, too, after liquefying the gelatinous colloid, may be separated again from it by diffusion into water upon a dialyzer. The solution of these colloids, in such circumstances, may be looked upon as analogous to the solution of insoluble organic colloids witnessed in animal digestion, with the difference that the solvent fluid here is not acid but alkaline. Liquid silicic acid may be represented as the "peptone" of gelatinous silicic acid; and the liquefaction of the latter by a trace of alkali may be spoken of as the peptization of the jelly. The pure jellies of alumina, peroxide of iron, and titanic acid, prepared by dialysis, are assimilated more closely to albumen, being peptized by minute quantities of hydrochloric acid.

Liquid Stannic and Metastannic Acids.—Liquid stannic acid is prepared by dialyzing the bichloride of tin with an addition of alkali, or by dialyzing the stannate of soda with an addition of hydrochloric acid. In both cases a jelly is first formed on the dialyzer; but, as the salts diffuse away, the jelly is again peptized by the small proportion of free alkali remaining: the alkali itself may be removed by continued diffusion, a drop or two of the tincture of iodine facilitating the separation. The liquid stannic acid is converted on heating it into liquid metastannic acid. Both liquid acids are remarkable for the facility with which they are pectized by a minute addition of hydrochloric acid, as well as by salts.

Liquid Titanic Acid is prepared by dissolving gelatinous titanic acid in a small quantity of hydrochloric acid, without heat, and placing the liquid upon a dialyzer for several days. The liquid must not contain more than 1 per cent. of titanic acid, otherwise it spontaneously gelatinizes, but it appears more stable when dilute. Both titanic and the two stannic acids afford the same classes of compounds with alcohol &c. as are obtained with silicic acid.

Liquid Tungstic Acid.—The obscurity which has so long hung over tungstic acid is removed by a dialytic examination. It is in fact a remarkable colloid, of which the pectous form alone has hitherto been known. Liquid tungstic acid is prepared by adding dilute hydrochloric acid carefully to a 5 per cent. solution of tungstate of soda, in sufficient proportion to neutralize the alkali, and then placing the resulting liquid on a dialyser. In about three days the acid is found pure, with the loss of about 20 per cent., the salts having diffused entirely away. It is remarkable that the *purified* acid is not pectized by acids or salts even at the boiling temperature. Evaporated to dryness, it forms vitreous scales, like gum or gelatin, which sometimes adhere so strongly to the surface of the evaporating dish as to detach portions of it. It may be heated to 200° C.

without losing its solubility or passing into the pectous state, but at a temperature near redness it undergoes a molecular change, losing at the same time 2·42 per cent. of water. When water is added to unchanged tungstic acid, it becomes pasty and adhesive like gum; and it forms a liquid with about one-fourth its weight of water, which is so dense as to float glass. The solution effervesces with carbonate of soda, and tungstic acid is evidently associated with silicic and molybdic acids. The taste of tungstic acid dissolved in water is not metallic or acid, but rather bitter and astringent. Solutions of tungstic acid containing 5, 20, 50, 66·5, and 79·8 per cent. of dry acid, possess the following densities at 19°, 1·0475, 1·2168, 1·8001, 2·396, and 3·243. Evaporated *in vacuo* liquid tungstic acid is colourless, but becomes green in air from the deoxidating action of organic matter. Liquid silicic acid is protected from pectizing when mixed with tungstic acid, a circumstance probably connected with the formation of the double compounds of these acids which M. Marignac has lately described.

Molybdic Acid has hitherto been known (like tungstic acid) only in the insoluble form. Crystallized molybdate of soda dissolved in water is decomposed by the gradual addition of hydrochloric acid in excess without any immediate precipitation. The acid liquid thrown upon a dialyzer may gelatinize after a few hours, but again liquefies spontaneously, when the salts diffuse away. After a diffusion of three days, about 60 per cent. of the molybdic acid remains behind in a pure condition. The solution of pure molybdic acid is yellow, astringent to the taste, acid to test-paper, and possesses much stability. The acid may be dried at 100°, and then heated to 200° without losing its solubility. Soluble molybdic acid has the same gummy aspect as soluble tungstic acid, and deliquesces slightly when exposed to damp air. Both acids lose their colloidality when digested with soda for a short time, and give a variety of crystallizable salts.

XV. "Researches on the Colouring-matters derived from Coal-tar.—III. Diphenylamine." By A. W. Hofmann, LL.D., F.R.S., Received June 16, 1864.

In the course of last year* I published an account of some experiments upon the composition of the blue colouring-matter discovered by MM. Girard and De Laire when studying the action of aniline upon rosaniline.

These experiments had established a simple relation between aniline-red and aniline-blue, the latter exhibiting the composition of triphenylated rosaniline,

$$C_{20}H_{19}N_3, H_2O + 3\left.\begin{matrix}C_6H_5\\H_2\end{matrix}\right\}N = 3H_3N + C_{20}\left.\begin{matrix}H_{16}\\(C_6H_5)_3\end{matrix}\right\}N_3, H_2O.$$

The composition of aniline-blue has since been also investigated by M. Schiff†, who, in a paper published shortly after my first communication

* Proceedings of the Royal Society, vol. xii. p. 578; and vol. xiii. p. 9.

† Comptes Rendus, lvi. 1234.

upon this subject, attributes to this compound a formula differing from the expression at which I had arrived. According to M. Schiff, aniline-blue is not a *triamine*, as I had found, but a *tetramine*, which may be looked upon as a combination of rosaniline with triphenylamine,

$$C_{38}H_{34}N_4, H_2O = C_{20}H_{19}N_3, H_2O + (C_6H_5)_3N.$$

This formula is less simple than the one I had given; it attributes to aniline-blue a constitution not supported by analogy, and involves the necessity of assuming, for the formation of this substance, a reaction which ceases to be a simple process of substitution.

M. Schiff's formula is given as the result of an unfinished inquiry, and the author himself appears to have but limited confidence in its correctness. Nevertheless, the publication of his Note imposed upon me the duty of confirming by new experiments the result of my former researches upon this subject.

The material used in these new experiments was likewise furnished to me by Mr. Nicholson; it had been taken from the product of an operation perfectly different from the one which had supplied the first specimen. In the following synopsis the experimental numbers of the former analyses are marked (*a*), in contradistinction to those (*b*) recently performed.

Free Base.

	$C_{38}H_{36}N_4O$ (Schiff).			$C_{38}H_{33}N_3O$ (Hofmann).	
C_{38}	456	80·85	C_{38}	456	83·36
H_{36}	36	6·38	H_{33}	33	6·03
N_4	56	9·93	N_3	42	7·69
O	16	2·84	O	16	2·92
	564	100·00		547	100·00

Analyses (Hofmann).

	a.		*b.*	
Carbon	83·81	83·89	83·13	—
Hydrogen	6·20	6·18	6·04	—
Nitrogen	—	—	—	8·16

Chloride.

	$C_{38}H_{35}N_4Cl$ (Schiff).			$C_{38}H_{32}N_3Cl$ (Hofmann).	
C_{38}	456	78·28	C_{38}	456	80·64
H_{35}	35	6·01	H_{32}	32	5·66
N_4	56	9·61	N_3	42	7·43
Cl	35·5	6·10	Cl	35·5	6·27
	582·5	100·00		565·5	100·00

Analyses (Hofmann).

	a.				*b.*		
Carbon	80·58	80·57	—	—	80·67	—	—
Hydrogen	5·70	5·71	—	—	5·67	—	—
Nitrogen	—	—	—	—	—	7·76	—
Chlorine	—	—	6·10	6·73	—	—	6·12

Bromide.

	$C_{38}H_{35}N_4Br$ (Schiff).		$C_{38}H_{32}N_3Br$ (Hofmann).		
C_{38}	456	72·73	C_{38}	456	74·75
H_{35}	35	5·58	H_{32}	32	5·25
N_4	56	8·93	N_3	42	6·88
Br	80	12·76	Br	80	13·12
	627	100·00		610	100·00

Analyses (Hofmann).

	a.		
Carbon	74·60	—	—
Hydrogen	5·37	—	—
Nitrogen	—	7·59	—
Bromine	—	—	12·59

Nitrate.

	$C_{38}H_{35}N_5O_3$ (Schiff).		$C_{38}H_{32}N_4O_3$ (Hofmann).		
C_{38}	456	74·88	C_{38}	456	77·03
H_{35}	35	5·75	H_{32}	32	5·40
N_5	70	11·49	N_4	56	9·46
O_3	48	7·88	O_3	48	8·11
	609	100·00		592	100·00

Analyses (Hofmann).

Carbon	77·55	77·17
Hydrogen	5·36	5·41

Triphenylic Leucaniline.

	$C_{38}H_{33}N_3$		
C_{38}	456	85·88	85·75
H_{33}	33	6·21	6·15
N_3	42	7·91	—
	531	100·00	

The preceding analyses appear to establish beyond a doubt the composition of aniline-blue; and I do not hesitate to maintain the formula originally established by me as the true expression of the constitution of this compound.

The resumption of this inquiry has led me to some observations which afford an additional support of the composition of aniline-blue.

Rosaniline, when submitted to dry distillation, undergoes an irregular decomposition. Ammonia is evolved, a large amount of liquid bases (from 40 to 50 per cent.) passes over, and a porous mass of charcoal remains in the retort. The principal constituent of the liquid product is *aniline*.

Ethyl-rosaniline, the commercial aniline-violet, already manufactured on a large scale by Messrs. Simpson, Maule, and Nicholson, when distilled, exhibits similar phenomena. There is no difficulty in separating from the liquid product an appreciable amount of *ethylaniline*, the presence of which has been established by the examination of the platinum-salt.

The relation existing between aniline-violet, obtained by the action of iodide of ethyl, and aniline-red, cannot possibly be doubtful. Now, since analysis points out similar relations between aniline-blue and aniline-red, it became perfectly legitimate to anticipate, among the products of distillation of aniline-blue, *i.e.* of *phenylated rosaniline*, the presence of *phenylated aniline*, or *diphenylamine*, a substance which chemists had often, but vainly, endeavoured to prepare. Experiment has verified this anticipation.

Some weeks ago my friend, M. Charles Girard, Director of the Works of the Fuchsine Company at Lyons, had the kindness to transmit to me for examination some basic oils of high boiling-point which he had obtained by the destructive distillation of a considerable quantity of aniline-blue. The product of distillation which he forwarded to me was brown and viscid. When rectified, it furnished a slightly coloured liquid; at 300° the thermometer indicated the distillation of a definite compound.

The distillate which had passed between 280° and 300° solidified on addition of hydrochloric acid, a chloride difficultly soluble (more especially in concentrated hydrochloric acid) being formed. By washing with alcohol, and ultimately by crystallization from this liquid, the chloride was obtained in a state of purity. When treated with ammonia, it furnished colourless oily drops, which, after a few moments, solidified to a white crystalline mass.

The crystals thus obtained possess a peculiar flower-like odour; their taste is aromatic, and afterwards burning. At 45° they fuse to a yellowish oil, distilling constantly, and without alteration, at 312° (corr.). They are almost insoluble in water, easily soluble in alcohol and ether. Neither in aqueous nor in alcoholic solution has the substance any alkaline reaction. When brought into contact with concentrated acids, the crystals are converted into the corresponding salts, which are remarkable for their instability. On the addition of water, the base separates in oily drops, rapidly solidifying to

crystals. The chloride, when washed with water, loses every trace of hydrochloric acid.

The analysis of the base has led to the formula

$$C_{12}H_{11}N.$$

The composition of the chloride, which crystallizes from alcohol in concentrically grouped needles, assuming a bluish tint by contact with the air, is

$$C_{12}H_{11}N, HCl.$$

I do not believe that I am deceived in considering this compound as diphenylamine,

$$C_{12}H_{11}N = \left.\begin{matrix} C_6H_5 \\ C_6H_5 \\ H \end{matrix}\right\} N.$$

It deserves, however, to be stated that the strict experimental demonstration of this mode of viewing the compound is still wanting, the ethylation of the substance presenting difficulties that I have not yet been able to overcome. I regret this gap, since the experience acquired in the study of xenylamine, isomeric with diphenylamine,

$$C_{12}H_{11}N = \left.\begin{matrix} C_{12}H_9 \\ H \\ H \end{matrix}\right\} N = \left.\begin{matrix} C_6H_5 \\ C_6H_5 \\ H \end{matrix}\right\} N,$$

and which for several weeks I regarded as the secondary phenyl-base, points out the necessity of consistently carrying out the process of ethylation in the examination of bases of this kind.

Diphenylamine exhibits a peculiar reaction, which, while it appears to reveal its relation to the colour-generating aniline, enables us to recognize the presence of the new base. In contact with concentrated nitric acid, diphenylamine, as well as its salts, assumes at once a magnificent blue coloration. The reaction succeeds best by pouring concentrated hydrochloric acid upon a crystal of the base, and then adding the nitric acid drop by drop. Immediately the whole liquid becomes intensely indigo-blue. Minute quantities of diphenylamine may in this manner be readily traced. I have thus been enabled to ascertain the presence of this body, or, at all events, of a substance exhibiting this particular reaction, in the products of distillation of rosaniline, leucaniline, and even of melaniline. The last experiment deserves particularly to be noticed, since it affords the general method for the production of the secondary aromatic monamines, which was hitherto wanting.

The substance possessing the blue colour is formed also by the action of other oxidizing agents. On adding chloride of platinum to a solution of the chloride, the liquid at once assumes a deep-blue colour. Only, from

very concentrated solutions, a platinum-salt is deposited exhibiting an undesirably blue tint.

When a mixture of diphenylamine and toluidine is submitted to one of the processes (treatment with chloride of mercury or arsenic acid) which, if phenylamine and toluidine had been employed, would have furnished rosaniline, a mass is formed which dissolves in alcohol with a magnificent blue coloration, and possesses the characters of a true colouring-matter.

An alcoholic solution of diphenylamine furnishes, on addition of bromine, a yellow crystalline precipitate, difficultly soluble in cold alcohol, but crystallizing from boiling alcohol in beautiful needles of a satiny lustre. According to analysis, they contain

$$C_{12}H_7Br_4N,$$

a formula which renders the grouping

$$\left.\begin{matrix}C_6H_3Br_2\\C_6H_3Br_2\\H\end{matrix}\right\}N$$

probable.

A mixture of diphenylamine and chloride of benzoyl, when heated, furnishes a thick oil, which solidifies on cooling. Washed with water and alkali, and recrystallized from boiling alcohol, in which it dissolves with difficulty, the new compound is obtained in beautiful white needles. Analysis has confirmed the theoretical anticipation,

$$C_{19}H_{15}NO=\left.\begin{matrix}C_6H_5\\C_6H_5\\C_7H_5O\end{matrix}\right\}N.$$

This substance has become the starting-point of some experiments which I shall here briefly mention, but to which I intend to return hereafter. On addition of ordinary concentrated nitric acid, the benzoyl-compound liquefies and dissolves. From this solution, water precipitates a light-yellow crystalline compound,

$$C_{19}H_{14}N_2O_3=\left.\begin{matrix}C_6H_5\\C_6H_4(NO_2)\\C_7H_5O\end{matrix}\right\}N,$$

which dissolves in alcoholic soda with a scarlet colour, splitting on ebullition into benzoic acid and reddish-yellow needles of nitro-diphenylamine,

$$C_{12}H_{10}N_2O_2=\left.\begin{matrix}C_6H_5\\C_6H_4(NO_2)\\H\end{matrix}\right\}N.$$

If, instead of ordinary nitric acid, a large excess of the strongest fuming nitric acid be employed, the solution deposits, on addition of water, a crys-

talline compound of a somewhat deeper yellow colour, containing probably

$$C_{19}H_{13}N_3O_5 = \left.\begin{matrix} C_6H_4(NO_2) \\ C_6H_4(NO_2) \\ C_7H_5O \end{matrix}\right\} N.$$

This substance dissolves in alcoholic soda with a most magnificent crimson colour. Addition of water to the boiling liquid furnishes a yellow crystalline deposit, benzoate (?) of sodium remaining in solution.

The yellow powder is dinitro-diphenylamine. From boiling alcohol, it crystallizes in reddish needles, exhibiting a bluish metallic lustre. The analysis of the compound has led to the formula

$$C_{12}H_9N_3O_4 = \left.\begin{matrix} C_6H_4(NO_2) \\ C_6H_4(NO_2) \\ H \end{matrix}\right\} N.$$

The chemical history of these compounds will be the subject of a special communication.

XVI. "A Table of the Mean Declination of the Magnet in each Decade from January 1858 to December 1863, derived from the Observations made at the Magnetic Observatory at Lisbon; showing the Annual Variation, or Semiannual Inequality to which that element is subject." Drawn up by the Superintendent of the Lisbon Observatory, Senhor DA SILVEIRA, and communicated by Major-General SABINE, R.A., President of the Royal Society. Received June 6, 1864.

I have much pleasure in communicating to the Fellows of the Royal Society a copy of a Table which I have received from the Superintendent of the Magnetic Observatory at Lisbon, containing the mean values of the Declination in each Decade from the commencement of 1858 to the close of 1863, with corrections applied for the mean secular change, and showing, in a final column, the difference in each decade of the observed from the mean annual value derived from the 216 decades. This Table is a counterpart of Table VII. in Art. XII. of the Philosophical Transactions for 1863, p. 292, differing only in the substitution in the Lisbon Table of decades for weeks, and the addition of the year 1863.

This general confirmation by the Lisbon Observatory of the annual variation to which the Declination is subject, "the north end of the magnet pointing more towards the East when the sun is north of the Equator, and more towards the West when the sun is south of the Equator," is very satisfactory. In the Lisbon Table the disturbances have not been eliminated.

Means of the West Declination at the Lisbon Observatory, from January 1858 to December 1863.

Months.	Decade.	1858. 21°+	1859. 21°+	1860. 21°+	1861. 21°+	1862. 21°+	1863. 21°+	Means. 21°+	Corrected for Secular Change.	Means corrected. 21°+.	Differences from the means of the six years.
January ...	1.	42 08·0	37 12·5	30 37·2	26 42·0	20 28·8	13 27·0	28 25·9	−2 40·7	25 45·2	+0 37·4
	2.	41 31·5	36 38·7	30 54·6	26 52·8	20 18·6	13 55·8	28 22·0	−2 31·5	25 50·5	+0 42·7
	3.	41 08·3	37 12·4	30 15·6	29 41·4	19 17·4	13 48·0	28 33·8	−2 22·3	26 11·5	+1 03·7
February...	1.	41 14·6	36 4·5	30 10·8	26 53·4	19 18·0	14 9·6	27 58·5	−2 13·1	25 45·4	+0 37·6
	2.	40 17·5	35 38·7	31 8·4	26 19·2	18 22·8	12 32·4	27 23·2	−2 04·0	25 19·2	+0 11·4
	3.	38 11·0	35 40·3	31 34·8	26 40·8	18 27·0	12 10·8	27 7·4	−1 54·8	25 12·6	+0 04·8
March.....	1.	39 56·5	34 20·2	31 16·8	25 45·0	18 55·2	11 31·2	26 57·5	−1 45·6	25 11·9	+0 04·1
	2.	40 31·2	35 2·0	32 45·0	25 47·4	18 27·0	11 52·8	27 24·2	−1 36·4	25 47·8	+0 40·0
	3.	39 17·7	34 48·0	31 36·6	25 10·8	17 19·8	14 48·0	27 10·1	−1 27·3	25 42·8	+0 35·0
April	1.	39 37·2	34 9·0	30 52·2	23 48·6	17 25·2	12 55·8	26 28·0	−1 18·1	25 09·9	+0 02·1
	2.	39 18·7	34 50·4	30 20·4	24 2·4	18 9·6	11 36·6	26 23·0	−1 08·9	25 14·1	+0 06·3
	3.	37 11·5	34 57·0	28 54·0	23 25·8	16 37·2	10 45·6	25 18·5	−0 59·7	24 18·8	−0 49·0
May.....	1.	36 45·5	35 9·6	28 31·2	22 9·0	15 46·2	10 20·4	24 47·0	−0 50·5	23 56·5	−1 11·3
	2.	37 26·2	35 28·8	28 6·6	22 29·4	15 53·4	9 40·2	24 50·8	−0 41·4	24 09·4	−0 58·4
	3.	37 28·8	34 25·8	28 6·0	22 12·0	15 39·6	9 34·8	24 34·5	−0 32·2	24 02·3	−1 05·5

June	1.	36 56·0	32 26·4	28 11·4	21 36·6	15 31·2	9 17·4	23 59·8	−0 23·0	23 36·8	−1 31·0						
	2.	36 28·5	32 25·8	27 57·0	21 28·2	15 30·6	9 10·8	23 50·1	−0 13·8	23 36·3	−1 31·5						
	3.	37 21·1	33 57·6	28 13·8	20 41·4	15 12·6	9 37·2	24 10·6	−0 04·6	24 06·0	−1 01·8						
July	1.	37 40·2	32 56·4	28 45·0	21 15·0	16 13·8	9 51·6	24 27·0	+0 04·6	24 31·6	−0 36·2						
	2.	37 04·5	34 49·2	28 20·4	21 3·6	14 59·4	11 59·4	24 42·7	+0 13·8	24 56·5	−0 11·3						
	3.	38 05·9	34 1·8	28 38·4	21 43·8	15 33·6	9 21·6	24 34·2	+0 23·0	24 57·2	−0 10·6						
August ...	1.	37 18·0	33 25·2	29 50·4	21 40·2	16 9·0	9 50·4	24 42·2	+0 32·2	25 14·4	+0 06·6						
	2.	38 03·0	32 57·6	29 12·0	22 30·0	15 34·8	10 51·6	24 51·5	+0 41·4	25 32·9	+0 25·1						
	3.	37 53·2	32 1·8	27 51·0	21 19·2	15 4·2	10 24·6	24 5·7	+0 50·5	24 56·2	+0 11·6						
September	1.	37 24·7	33 29·4	29 41·4	21 15·6	15 26·4	10 9·0	24 34·4	+0 59·7	25 34·1	+0 26·3						
	2.	38 10·2	33 55·8	27 54·0	21 48·6	14 17·4	11 55·2	24 25·2	+1 08·9	25 34·1	+0 26·3						
	3.	38 03·0	33 33·6	28 43·2	21 40·8	14 50·4	9 44·4	24 25·9	+1 18·1	25 44·0	+0 36·2						
October ...	1.	39 42·2	33 13·8	28 20·4	20 25·8	16 30·6	8 22·2	24 25·8	+1 27·3	25 53·1	+0 45·3						
	2.	37 25·2	34 58·2	27 11·4	21 33·6	14 3·0	8 37·2	23 58·1	+1 36·4	25 34·5	+0 26·7						
	3.	37 58·6	31 44·4	27 8·4	21 1·8	14 19·2	8 4·2	23 22·8	+1 45·6	25 08·4	+0 00·6						
November	1.	38 11·0	31 15·6	27 20·4	21 13·8	14 13·8	8 54·0	23 31·4	+1 54·8	25 26·2	+0 18·4						
	2.	37 05·0	32 54·6	28 13·2	21 34·8	13 45·6	8 54·6	23 44·6	+2 04·0	25 48·6	+0 40·8						
	3.	34 47·2	31 42·6	29 1·2	20 45·6	13 12·0	7 33·6	22 50·4	+2 13·1	25 03·5	−0 04·3						
December	1.	35 17·2	32 49·8	27 39·0	21 42·0	13 13·2	7 58·2	23 6·6	+2 22·3	25 28·9	+0 21·1						
	2.	35 16·1	33 12·6	27 41·4	21 7·2	13 18·0	8 30·0	23 10·9	+2 31·5	25 42·4	+0 34·6						
	3.	35 41·4	32 25·4	26 55·2	20 31·8	13 33·0	7 53·4	22 50·0	+2 40·7	25 30·7	+0 22·9						
Means.		38 06·4	33 59·4	29 1·2	22 57·0	16 8·4	10 34·2	25 07·8									
		4′ 7″·0	4′ 58″·2	6′ 04″·2	6′ 48″·6	5′ 34″·2		5′ 30″·4									
Annual differences.																	

XVII. "On Organic Substances artificially formed from Albumen." By Alfred H. Smee, F.C.S. Communicated by Alfred Smee, Esq., F.R.S. Received May 27, 1864.

In a former paper which I had the honour to submit to the Royal Society, I showed that fibrin was formed by the passage of oxygen through albumen, provided a temperature of 98° F. was maintained. It was then observed that a slightly acid state of the albumen, or the absence of the alkaline salts was found to be most favourable to its formation. I noticed also that ammonia had little effect in preventing the formation of fibrin, but after the lapse of a short time caused it to swell to such a degree that its microscopic characters could no longer be determined. It was observed that albumen acted on by gastric juice and passed through a membrane, still had the capacity to form fibrin in small amount.

Since the publication of that paper, I have conducted the following experiments in addition to those before mentioned. I submitted some of the fluid drawn off from a spina bifida to the action of oxygen and heat in the ordinary manner; after the lapse of a few hours it yielded a substance which, under the microscope, presented all the characters of fibrin.

I tried to obtain fibrin from the urine in two cases in which it was highly albuminous. The urine was so loaded with albumen that it became almost solid by heat. I never have been able to transform this variety of albumen into fibrin, although the experiment was tried in many ways. I expect that on further investigation it will be found that the albumen found in urine (in most cases at least) is a substance not capable of further development.

The next experiment which I have to describe is to my mind one of the most beautiful exemplifications of the artificial formation of organic bodies under physical laws, producing results similar to those which we observe under certain circumstances in disease, the changes being produced by the action of a gas on a second body separated by a membrane, and having to traverse it before the chemical changes can be effected.

I passed a current of oxygen gas through a small portion of perfectly clean intestine, with the peritoneal coat attached. The intestine was placed in an albuminous fluid at a temperature of 98° F.; at the end of twenty-four hours I found the intestine completely invested with minute fibrinous outgrowths, similar to those seen on the intestines of persons who have died at the earliest stage of peritonitis.

It is worth noticing that although these fibrinous outgrowths take place when the peritoneum of the intestine remains, yet if this coat be stripped off they take place to a very limited extent. In many cases no outgrowths appear, even where the conditions of the experiments are equal.

It appears to me that the tendency of fibrin to be deposited on serous membranes, under favourable circumstances, may throw some light on the

frequency with which we find the surfaces of serous membranes (for instance, the pericardium) so often coated with fibrinous outgrowths.

If hydrogen is passed through albumen to which a small quantity of potash has been added sufficient to ensure a slight excess of alkali, after the lapse of some time a dense hard horny mass will be observed, especially at the point where the hydrogen comes into contact with the albumen; in fact the growth of the substance often clogs the tube to such a degree that the hydrogen is prevented from further passing through it. It also has a tendency to grow upon platinized platinum when placed in the albuminous fluid whilst the current of hydrogen is passing. The time required is, as a rule, about four days; a temperature of 98° F. rather favours its formation, but is not absolutely necessary to its production.

The following are the chief chemical and physical reactions of the substance formed by hydrogen.

It is heavier than albumen, always sinking to the bottom of the vessels. It is hard, tough, semitransparent, homogeneous, and slightly elastic. It swells up in cold water, and dissolves to a limited extent. The extent of its solubility is less the longer the time occupied for its formation. It is more soluble in hot water. Peroxide of hydrogen is not decomposed by it.

The watery solution is not coagulated by boiling; it is, however, precipitated by chlorine. Hydrochloric acid does not form a blue solution with excess of that reagent. Bichloride of mercury and bichloride of platinum, after the lapse of some time, precipitate it. Tannic acid, alcohol, acetate of lead, sulphate of the peroxide of iron, and alum also precipitate it from its solution. It is turned yellow by nitric acid and heat. It likewise contains a small quantity of sulphur. Chondrin behaves in a similar manner, in its chemical and physical relations, to the substance thus artificially produced, and hence I propose to call it "artificial chondrin."

In carrying out these experiments, I found that a very nice method of obtaining a constant and equal amount of hydrogen gas was by collecting hydrogen formed at the negative pole of a one-cell battery, and passing the hydrogen thus formed directly into the albumen. The amount of hydrogen required was regulated by increasing or diminishing the size of the negative pole.

This form of apparatus will constantly remain a week or more in action without any appreciable alteration in the quantity of hydrogen evolved.

It may be well to describe the construction of the apparatus used. I first take a common precipitating glass, and place in it a few pieces of zinc with a little mercury to amalgamate it. I then take a tube about $\frac{1}{4}$ inch in diameter, and bent in two places at a right angle; into one end I insert a platinum wire, this end I place in the glass containing the zinc; the other end I place in the vessel containing the albuminous fluid. Dilute sulphuric acid is then added to the zinc. When contact takes place between the platinum wire and the zinc, a constant stream of hydrogen is given off from the platinum wire. The amount of hydrogen required can be regulated by

making a larger or smaller surface of the platinum come in contact with the zinc. The amount of oxygen which is carried over is very limited, provided a tube is used of $\frac{1}{4}$ inch diameter; but when a tube of $1\frac{1}{2}$ inch is used, a quantity might pass sufficient to interfere with the experiment.

The amount of oxygen at times thus carried over when the large tube is used is so great, that a change in the products may take place and fibrin may be formed in the place of the chondrin, provided the albumen is not over alkaline.

As fibrin was formed by oxygen, and this new substance analogous to chondrin by hydrogen, it occurred to me that these two substances might be formed simultaneously by a simple-cell voltaic arrangement. For this purpose I took a tube with one end closed by parchment paper, or sometimes by animal membrane, filled it with albumen which had been made slightly acid by acetic acid, and inserted it into a small vessel containing albumen to which a small quantity of potash or soda had been added. I then connected the two fluids by means of a platinum wire, so that one side might become a positive and the other a negative pole. Considerable action took place after the lapse of some time, when upon examination I found the albumen in the tube was changed, not into the fibrillated fibrin, but into a granular material. The other pole, or rather the the alkaline albumen, was changed into a substance which behaved with various reagents in different ways. In some cases it was a tough, ropy and viscid substance, which was coagulated in water by a solution of acetate of lead, was insoluble in acids and in alcohol, and very slightly soluble in alkali. At other times I have noticed a substance formed having very much the appearance of the expectoration of bronchitis; and at other times the dense hard substance analogous to chondrin in its behaviour with reagents was formed.

The various states of the material into which albumen is converted appear to be influenced by the nature of the alkali employed and by the relative size of the negative pole. The temperature should be as nearly as possible constant during the time the experiment is being conducted. The amount of the surface of membrane interposed appears to have very little influence over the products. When soda was the alkali employed, the viscid and frothy mucus-like product was more frequently obtained.

The amount of water present appears to have a very decided influence on the product formed. When the viscid and frothy material is produced, it appears to form quicker than the hard and dense chondrin. The temperature of 98° F. appears to favour the production of the chondrinous material; but I must admit I have sometimes made all the varieties, the viscid, the frothy, and also the chondrin, at much lower temperatures.

In one case I succeeded after many experiments in obtaining from the acid pole, by keeping it at a temperature of 98° F., fibrin of the fibrillated form, but the greater portion of the albumen at this pole was converted into the granular form. The alkaline pole formed pretty constantly the dense hard artificial chondrin.

When hydrogen was passed through serum, after the lapse of a day or two a tough elastic product was obtained.

In experiments tried by passing hydrogen through albumen greatly diluted with water, I found, after the lapse of a few days, a flocculent deposit very similar in appearance to the deposit of mucus which often takes place when urine is allowed to stand a short time. This point, however, requires further investigation. I tried also the effect of passing hydrogen through a portion of intestine inserted into an albuminous fluid. I have not as yet been able to form either the dense hard or viscid frothy substance by this method. I repeated the experiment for the formation of fibrin from albumen, by decomposing the water of its composition by electricity. I must admit this is the most difficult, troublesome, and unsatisfactory of all the methods I have employed. I find that the great tendency of the poles to form different substances on them, and the great rapidity with which they grow together, lead, without the greatest care, to the belief that two different substances, differing only in density, are formed at one and the same pole, so intimately blended are they together. Thus I was led to believe at first sight that a dense hard substance was formed at the oxygen end, and not until I had repeated the experiment many times did I discover that the substance belonged to the hydrogen and not to the oxygen pole, and had grown across from one pole to the other.

I have obtained on several occasions fibrin and chondrin at the same time by conducting hydrogen and oxygen derived by the decomposition of water by voltaic electricity through separate tubes. The oxygen passed into slightly acid albumen formed fibrin; the hydrogen passed into alkaline albumen formed either the chondrin or else the frothy and viscid material. The temperature was kept up at 98° F. in these experiments. On one occasion, however, I happened accidentally to reverse the current (that is to say, the hydrogen was passed into the acid, and the oxygen into the alkaline albumen), when no chondrin or fibrin was formed.

The following conclusions I have arrived at after the study of the influence which oxygen and hydrogen gases exert upon albumen when submitted to their action separately at a temperature of 98° F., the normal temperature of the living body. Albumen under the action of oxygen forms, after the lapse of a longer or shorter period, fibrin. The fibrin thus artificially produced is of three distinct varieties, viz., 1st, the granular form; 2nd, a form allied to lymph incapable of being unravelled into fibrils; lastly, the true fibrillated fibrin. The law which appears to regulate the state into which the albumen is converted, as far as my observation has gone, is one of molecular aggregation, similar to the electric deposit of metals, as the slower the fibrin is formed the more organized is it in substance.

I have observed that when fibrin is rapidly formed it is almost always produced in the granular state; this is particularly the case with fibrin

formed from albumen by the decomposition of the water of its composition by voltaic means.

Lymph I consider to be imperfectly formed fibrin more highly developed than the preceding or granular form. It is possible for this artificially formed lymph, under favourable circumstances, to assume a more organized appearance.

I have no doubt that the fibrinous outgrowths on the intestine would have become larger and more developed if the experiment had been carried on for a sufficient length of time. In fact almost all the fibrin formed round a platinum wire inserted into albumen is at first covered by outgrowths of a soft structure. These outgrowths, at the earliest period of their formation, do not under the microscope present any appearance of fibrils. After the lapse of some time they appear to undergo condensation, and then to organize to such an extent that it would be difficult at first sight to determine whether the substance might not be a portion of fibrous tissue.

The alkalies, with the exception of ammonia, prevent entirely the formation of fibrin. Ammonia, although it does not retard its formation, dissolves it after the lapse of a short time. The acids and absence of alkaline salts favour its formation. The opposite, however, is the case with the hydrogen products, as an alkaline state favours their production.

The action of hydrogen on albumen, as far as my investigations have as yet proceeded, forms substances analogous to chondrin and mucin. I believe that the organic substances, chondrin and mucin, products formed in a living organism, are very closely allied to one another, if not varieties of the same substance, differing only in their mode of aggregation and stages of development, and the amount of water in their composition.

Of the exact mode in which hydrogen acts on albumen we are at present ignorant. I have noticed that in some experiments sometimes one, sometimes the other product was obtained, even when the same influences were apparently acting on experiments conducted at the same time.

Considering the important physiological part that fibrin, chondrin, and mucin play in the living body, the production artificially of substances analogous in their behaviour with reagents to those products formed in a living organism will, I trust, be taken as a sufficient excuse for submitting to the Royal Society a paper so obviously deficient in many parts, but which, nevertheless, it would require a vast amount of both time and labour to carry one step further.

XVIII. "On the Reduction and Oxidation of the Colouring Matter of the Blood." By G. G. Stokes, M.A., Sec. R.S., Lucasian Professor of Mathematics in the University of Cambridge. Received June 16, 1864.

1. Some time ago my attention was called to a paper by Professor Hoppe*, in which he has pointed out the remarkable spectrum produced by the absorption of light by a very dilute solution of blood, and applied the observation to elucidate the chemical nature of the colouring matter. I had no sooner looked at the spectrum, than the extreme sharpness and beauty of the absorption-bands of blood excited a lively interest in my mind, and I proceeded to try the effect of various reagents. The observation is perfectly simple, since nothing more is required than to place the solution to be tried, which may be contained in a test-tube, behind a slit, and view it through a prism applied to the eye. In this way it is easy to verify Hoppe's statement, that the colouring matter (as may be presumed at least from the retention of its peculiar spectrum) is unaffected by alkaline carbonates and caustic ammonia, but is almost immediately decomposed by acids, and also, but more slowly, by caustic fixed alkalies, the coloured product of decomposition being the hæmatin of Lecanu, which is easily identified by its peculiar spectra. But it seemed to me to be a point of special interest to inquire whether we could imitate the change of colour of arterial into that of venous blood, on the supposition that it arises from reduction.

2. In my experiments I generally employed the blood of sheep or oxen obtained from a butcher; but Hoppe has shown that the blood of animals in general exhibits just the same bands. To obtain the colouring matter in true solution, and at the same time to get rid of a part of the associated matters, I generally allowed the blood to coagulate, cut the clot small, rinsed it well, and extracted it with water. This, however, is not essential, and blood merely diluted with a large quantity of water may be used; but in what follows it is to be understood that the watery extract is used unless the contrary be stated.

3. Since the colouring matter is changed by acids, we must employ reducing agents which are compatible with an alkaline solution. If to a solution of protosulphate of iron enough tartaric acid be added to prevent precipitation by alkalies, and a small quantity of the solution, previously rendered alkaline by either ammonia or carbonate of soda, be added to a solution of blood, the colour is almost instantly changed to a much more purple red as seen in small thicknesses, and a much darker red than before as seen in greater thickness. The change of colour, which recalls the difference between arterial and venous blood, is striking enough, but the change in the absorption spectrum is far more decisive. The two highly

* Virchow's Archiv, vol. xxiii. p. 446 (1862).

characteristic dark bands seen before are now replaced by a *single* band, somewhat broader and less sharply defined at its edges than either of the former, and occupying nearly the position of the bright band separating the dark bands of the original solution. The fluid is more transparent for the blue, and less so for the green than it was before. If the thickness be increased till the whole of the spectrum more refrangible than the red be on the point of disappearing, the last part to remain is *green*, a little beyond the fixed line *b*, in the case of the original solution, and *blue*, some way beyond F, in the case of the modified fluid. Figs. 1 and 2 in the accompanying woodcut represent the bands seen in these two solutions respectively.

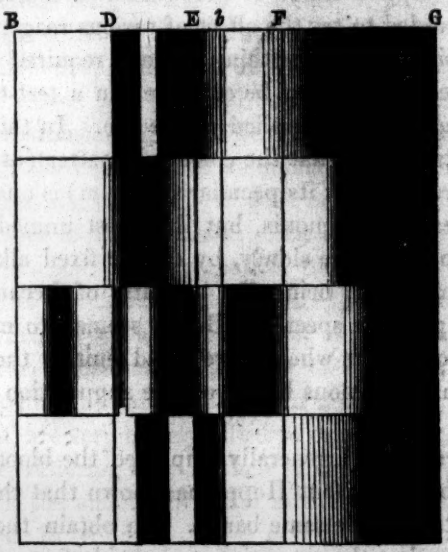


Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

4. If the purple solution be exposed to the air in a shallow vessel, it quickly returns to its original condition, showing the two characteristic bands the same as before; and this change takes place immediately, provided a small quantity only of the reducing agent were employed, when the solution is shaken up with air. If an additional quantity of the reagent be now added, the same effect is produced as at first, and the solution may thus be made to go through its changes any number of times.

5. The change produced by the action of the air (that is, of course, by the absorption of oxygen) may be seen in an instructive form on partly filling a test-tube with a solution of blood suitably diluted, mixing with a little of the reducing agent, and leaving the tube at rest for some time in a vertical position. The upper or oxidized portion of the solution is readily distinguished by its colour; and if the tube be now placed behind a slit and viewed through a prism, a dark band is seen, having the general form of a tuning-fork, like figs. 1 and 2, regarded now as a single figure, the line of separation being supposed removed.

6. Of course it is necessary to assure oneself that the single band in the green is not due to absorption produced merely by the reagent, as is readily done by direct observation of its spectrum, not to mention that in the region of the previous dark bands, or at least the outer portions of it, the solution is actually more transparent than before, which could not be occasioned by an *additional* absorption. Indeed the absorption due to the reagent itself in its different stages of oxidation, unless it be employed in most unnecessary excess, may almost be regarded as evanescent in comparison with the absorption due to the colouring matter; though if the solution be repeatedly put through its changes, the accumulation of the persalt of iron will presently tell on the colour, making it sensibly yellower than at first for small thicknesses of the solution.

7. That the change which the iron salt produces in the spectrum is due to a simple reduction of the colouring matter, and not to the formation of some compound of the colouring matter with the reagent, is shown by the fact that a variety of reducing agents of very different nature produce just the same effect. If protochloride of tin be substituted for protosulphate of iron in the experiment above described, the same changes take place as with the iron salt. The tin solution has the advantage of being colourless, and leaving the visible spectrum quite unaffected, both before and after oxidation, and accordingly of not interfering in the slightest degree with the optical examination of the solutions, but permitting them to be seen with exactly their true tints. The action of this reagent, however, takes some little time at ordinary temperatures, though it is very rapid if previously the solution be gently warmed. Hydrosulphate of ammonia again produces the same change, though a small fraction of the colouring matter is liable to undergo some different modification, as is shown by the occurrence of a slender band in the red, variable in its amount of development, which did not previously exist. In this case, as with the tin salt, the action is somewhat slow, requiring a few minutes unless it be assisted by gentle heat. Other reagents might be mentioned, but these will suffice.

8. We may infer from the facts above mentioned that *the colouring matter of blood*, like indigo, *is capable of existing in two states of oxidation, distinguishable by a difference of colour and a fundamental difference in the action on the spectrum. It may be made to pass from the more to the less oxidized state by the action of suitable reducing agents, and recovers its oxygen by absorption from the air.*

As the term *hæmatin* has been appropriated to a product of decomposition, some other name must be given to the original colouring matter. As it has not been named by Hoppe, I propose to call it *cruorine*, as suggested to me by Dr. Sharpey; and in its two states of oxidation it may conveniently be named *scarlet cruorine* and *purple cruorine* respectively, though the former is slightly purplish at a certain small thickness, and the latter is of a very red purple colour, becoming red at a moderate increase of thickness.

9. When the watery extract from blood-clots is left aside in a corked

bottle, or even in a tall narrow vessel open at the top, it presently changes in colour from a bright to a dark red, decidedly purple in small thicknesses. This change is perceived even before the solution has begun to stink in the least perceptible degree. The tint agrees with that of the purple cruorine obtained immediately by reducing agents; and if a little of the solution be sucked up from the bottom into a quill-tube drawn to a capillary point, and the tube be then placed behind a slit, so as to admit of analyzing the transmitted light without exposing the fluid to the air, the spectrum will be found to agree with that of purple cruorine. On shaking the solution with air it immediately becomes bright red, and now presents the optical characters of scarlet cruorine. It thus appears that scarlet cruorine is capable of being reduced by certain substances, derived from the blood, present in the solution, which must themselves be oxidized at its expense.

10. When the alkaline tartaric solution of protoxide of tin is added in moderate quantity to a solution of scarlet cruorine, the latter is presently reduced. If the solution is now shaken with air, the cruorine is almost instantly oxidized, as is shown by the colour of the solution and its spectrum by transmitted light. On standing for a little time, a couple of minutes or so, the cruorine is again reduced, and the solution may be made to go through these changes a great number of times, though not of course indefinitely, as the tin must at last become completely oxidized. It thus appears that purple cruorine absorbs *free* oxygen with much greater avidity than the tin solution, notwithstanding that the oxidized cruorine is itself reduced by the tin salt. I shall return to this experiment presently.

11. When a little acid, suppose acetic or tartaric acid, which does not produce a precipitate, is added to a solution of blood, the colour is quickly changed from red to brownish red, and in place of the original bands (fig. 1) we have a different system, nearly that of fig. 3. This system is highly characteristic; but in order to bring it out a larger quantity of substance is requisite than in the case of scarlet cruorine. The figure does not exactly correspond to any one thickness, for the bands in the blue are best seen while the band in the red is still rather narrow and ill-defined at its edges, while the narrow inconspicuous band in the yellow hardly comes out till the whole of the blue and violet, and a good part of the green, are absorbed. The difference in the spectra figs. 1 and 3 does not alone prove that the colouring matter is decomposed by the acid (though the fact that the change is not instantaneous favours that supposition), for the one solution is alkaline, though it may be only slightly so, while the other is acid, and the difference of spectra might be due merely to this circumstance. As the direct addition of either ammonia or carbonate of soda to the acid liquid causes a precipitate, it is requisite in the first instance to separate the colouring matter from the substance so precipitated.

This may be easily effected on a small scale by adding to the watery extract from blood-clots about an equal volume of ether, and then some glacial acetic acid, and gently mixing, but not violently shaking for fear

of forming an emulsion. When enough acetic acid has been added, the acid ether rises charged with nearly the whole of the colouring matter, while the substance which caused the precipitate remains in the acid watery layer below*. The acid ether solution shows in perfection the characteristic spectrum fig. 3. When most of the acid is washed out the substance falls, remaining in the ether near the common surface. If after removing the wash-water a solution, even a weak one, of ammonia or carbonate of soda be added, the colouring matter readily dissolves in the alkali. The spectrum of the transmitted light is quite different from that of scarlet cruorine, and by no means so remarkable. It presents a single band of absorption, very obscurely divided into two, the centre of which nearly coincides with the fixed line D, so that the band is decidedly less refrangible than the pair of bands of scarlet cruorine. The relative proportion of the two parts of the band is liable to vary. The presence of alcohol, perhaps even of dissolved ether, seems to favour the first part, and an excess of caustic alkali the second, the fluid at the same time becoming more decidedly dichroitic. The blue end of the spectrum is at the same time absorbed. The band of absorption is by no means so definite at its edges as those of scarlet cruorine, and a far larger quantity of the substance is required to develope it.

This difference of spectra shows that the colouring matter (hæmatin) obtained by acids is a product of the decomposition, or metamorphosis of some kind, of the original colouring matter.

When hæmatin is dissolved in alcohol containing acid, the spectrum nearly agrees with that represented in fig. 3.

12. Hæmatin is capable of reduction and oxidation like cruorine. If it be dissolved in a solution of ammonia or of carbonate of soda, and a little of the iron salt already mentioned, or else of hydrosulphate of ammonia, be added, a pair of very intense bands of absorption is immediately developed (fig. 4). These bands are situated at about the same distance apart as those of scarlet cruorine, and are no less sharp and distinctive. They are a little more refrangible, a clear though narrow interval intervening between the first of them and the line D. They differ much from the bands of cruorine in the relative strength of the first and second band. With cruorine the second band appears almost as soon as the first, on increasing the strength or thickness of the solution from zero onwards, and when both bands are well developed, the second band is decidedly broader than the first. With reduced hæmatin, on the other hand, the first band is already black and intense by the time the second begins to appear; then both bands increase, the first retaining its superiority until the two are on the

* If I may judge from the results obtained with the precipitate given by acetic acid and a neutral salt, a promising mode of separation of the proximate constituents of blood-crystals would be to dissolve the crystals in glacial acetic acid and add ether, which precipitates a white albuminous substance, leaving the hæmatin in solution.

point of merging into one by the absorption of the intervening bright band, when the two appear about equal.

Like cruorine, reduced hæmatin is oxidized by shaking up its solution with air. I have not yet obtained hæmatin in an acid solution in more than one form, that which gives the spectrum fig. 3, and which I have little doubt contains hæmatin in its oxidized form; for when it is withdrawn from acid ether by an alkali, I have not seen any traces of reduced hæmatin, even on taking some precautions against the absorption of oxygen. As the alkaline solution of ordinary hæmatin passes, with increase of thickness, through yellow, green, and brown to red, while that of reduced hæmatin is red throughout, the two kinds may be conveniently distinguished as *brown hæmatin* and *red hæmatin* respectively, the former or oxidized substance being the hæmatin of chemists.

13. Although the spectrum of scarlet cruorine is not affected by the addition to the solution of either ammonia or carbonate of soda, yet if after such addition the solution be either heated or alcohol be added, although there is no precipitation decomposition takes place. The coloured product of decomposition is brown hæmatin, as may be inferred from its spectrum. Since, however, the spectrum of an alkaline solution of brown hæmatin is only moderately distinctive, and is somewhat variable according to the nature of the solvent, it is well to add hydrosulphate of ammonia, which immediately developes the remarkable bands of red hæmatin. This is the easiest way to obtain them; but the less refrangible edge of the first band as obtained in this way is liable to be not quite clean, in consequence of the presence of a small quantity of cruorine which escaped decomposition.

Some very curious reactions are produced in a solution of cruorine by gallic acid combined with other reagents, but these require further study.

14. Hoppe proposed to employ the highly characteristic absorption-bands of scarlet cruorine in forensic inquiries. Since, however, cruorine is very easily decomposed, as by hot water, alcohol, weak acids, &c., the method would often be inapplicable. But as in such cases the coloured product of decomposition is hæmatin, which is a very stable substance, the absorption-bands of red hæmatin in alkaline solution, which in sharpness, distinctive character and sensibility rival those of scarlet cruorine itself, may be employed instead of the latter. The absorption-bands of brown hæmatin dissolved in a mixture of ether and acetic acid, or in acetic acid alone, are hardly less characteristic, but are not quite so sensitive, requiring a somewhat larger quantity of the substance.

15. I have purposely abstained from physiological speculations until I should have finished the chemico-optical part of the subject; but as the facts which have been adduced seem calculated to throw considerable light on the function of cruorine in the animal economy, I may perhaps be permitted to make a few remarks on this subject.

It has been a disputed point whether the oxygen introduced into the blood in its passage through the lungs is simply dissolved or is chemically

combined with some constituent of the blood. The latter and more natural view seems for a time to have given place to the former in consequence of the experiments of Magnus. But Liebig and others have since adduced arguments to show that the oxygen absorbed is, mainly at least, chemically combined, be it only in such a loose way, like a portion of the carbonic acid in bicarbonate of soda, that it is capable of being expelled by indifferent gases. It is known, too, that it is the red corpuscles in which the faculty of absorbing oxygen mainly resides.

Now it has been shown in this paper that we have in cruorine a substance capable of undergoing reduction and oxidation, more especially oxidation, so that if we may assume the presence of purple cruorine in venous blood, we have all that is necessary to account for the absorption and chemical combination of the inspired oxygen.

16. It is stated by Hoppe that venous as well as arterial blood shows the two bands which are characteristic of what has been called in this paper scarlet cruorine. As the precautions taken to prevent the absorption of oxygen are not mentioned, it seemed desirable to repeat the experiment, which Dr. Harley and Dr. Sharpey have kindly done. A pipette adapted to a syringe was filled with water which had been boiled and cooled without exposure to the air, and the point having been introduced into the jugular vein of a live dog, a little blood was drawn into the bulb. Without the water the blood would have been too dark for spectral analysis. The colour did not much differ from that of scarlet cruorine; certainly it was much nearer the scarlet than the purple substance. The spectrum showed the bands of scarlet cruorine.

This, however, does not by any means prove the absence of purple cruorine, but only shows that the colouring matter present was chiefly scarlet cruorine. Indeed the relative proportions of the two present in a mixture of them with one another and with colourless substances, can be better judged of by the tint than by the use of the prism. With the prism the extreme sharpness of the bands of scarlet cruorine is apt to mislead, and to induce the observer greatly to exaggerate the relative proportion of that substance.

Seeing then that the change of colour from arterial to venous blood *as far as it goes* is *in the direction* of the change from scarlet to purple cruorine, that scarlet cruorine is capable of reduction even in the cold by substances present in the blood (§ 9), and that the action of reducing agents upon it is greatly assisted by warmth (§ 7), we have every reason to believe that *a portion* of the cruorine present in venous blood exists in the state of purple cruorine, and is reoxidized in passing through the lungs.

17. That it is only a rather small proportion of the cruorine present in venous blood which exists in the state of purple cruorine under normal conditions of life and health, may be inferred, not only from the colour, but directly from the results of the most recent experiments *. Were it

* Funk's Lehrbuch der Physiologie, 1863, vol. i. § 108.

otherwise, any extensive hæmorrhage could hardly fail to be fatal, if, as there is reason to believe, cruorine be the substance on which the function of respiration mainly depends; nor could chlorotic persons exhale as much carbonic acid as healthy subjects, as is found to be the case.

But after death there is every reason to think that the process of reduction still goes on, especially in the case of warm-blooded animals, while the body is still warm. Hence the blood found in the veins of an animal some time after death can hardly be taken as a fair specimen as to colour of the venous blood in the living animal. Moreover the blood of an animal which has been subjected to abnormal conditions before death is of course liable to be altered thereby. The terms in which Lehmann has described the colour of the blood of frogs which had been slowly asphyxiated by being made to breathe a mixture of air and carbonic acid seem unmistakeably to point to purple cruorine*.

18. The effect of various indifferent reagents in changing the colour of defibrinated blood has been much studied, but not always with due regard to optical principles. The brightening of the colour, as seen by reflexion, produced by the first action of neutral salts, and the darkening caused by the addition of a little water, are, I conceive, easily explained; but I have not seen stated what I feel satisfied is the true explanation. In the former case the corpuscles lose water by exosmose, and become thereby more highly refractive, in consequence of which a more copious reflexion takes place at the common surface of the corpuscles and the surrounding fluid. In the latter case they gain water by endosmose, which makes their refractive power more nearly equal to that of the fluid in which they are contained, and the reflexion is consequently diminished. There is nothing in these cases to indicate any change in the mode in which light is absorbed by the colouring matter, although a change of tint to a certain extent, and not merely a change of intensity, may accompany the change of conditions under which the turbid mixture is seen, as I have elsewhere more fully explained†.

No doubt the form of the corpuscles is changed by the action of the reagents introduced; but to attribute the change of colour to this is, I apprehend, to mistake a concomitant for a cause, and to attribute, moreover, the change of colour to a cause inadequate to produce it.

19. Very different is the effect of carbonic acid. In this case the existence of a fundamental change in the mode of absorption cannot be questioned, especially when the fluid is squeezed thin between two glasses and viewed by transmitted light. I took two portions of defibrinated blood; to one I added a little of the reducing iron solution, and passed carbonic acid into the other, and then compared them. They were as nearly as possible alike. We must not attribute these apparently identical changes to two totally different causes if one will suffice. Now in the case of the iron

* Physiological Chemistry, vol. ii. p. 178.

† Philosophical Transactions, 1852, p. 527.

salt, the change of colour is plainly due to a deoxidation of the cruorine. On the other hand, Magnus removed as much as 10 or 12 per cent. by volume of oxygen from arterialized blood by shaking the blood with carbonic acid. If, as we have reason to believe, this oxygen was for the most part chemically combined, it follows that carbonic acid acts *as if it were* a reducing agent. We are led to regard the change of colour not as a *direct* effect of the *presence of carbonic acid*, but a consequence of the *removal of oxygen*. There is this difference between carbonic acid and the *real* reducing agents, that the former no longer acts on a dilute and comparatively pure solution of scarlet cruorine, while the latter act just as before.

If even in the case of blood exposed to an atmosphere of carbonic acid we are not to attribute the change of colour to the direct presence of the gas, much less should we attempt to account for the darker colour of venous than arterial blood by the small additional percentage of carbonic acid which the former contains. The ascertained properties of cruorine furnish us with a ready explanation, namely that it is due to a partial reduction of scarlet cruorine in supplying the wants of the system.

20. I am indebted to Dr. Akin for calling my attention to a very interesting pamphlet by A. Schmidt on the existence of ozone in the blood*. The author uses throughout the language of the ozone theory. If by ozone be meant the substance, be it allotropic oxygen or teroxide of hydrogen, which is formed by electric discharges in air, there is absolutely nothing to prove its existence in blood; for all attempts to obtain an oxidizing gas from blood failed. But if by ozone be merely meant oxygen in any such state, of combination or otherwise, as to be capable of producing certain oxidizing effects, such as turning guaiacum blue, the experiments of Schmidt have completely established its existence, and have connected it, too, with the colouring matter. Now in cruorine we have a substance admitting of easy oxidation and reduction; and connecting this with Schmidt's results, we may infer that scarlet cruorine is not merely a greedy absorber and a carrier of oxygen, but also an *oxidizing agent*, and that it is by its means that the substances which enter the blood from the food, setting aside those which are either assimilated or excreted by the kidneys, are reduced to the ultimate forms of carbonic acid and water, as if they had been burnt in oxygen.

21. In illustration of the functions of cruorine, I would refer, in conclusion, to the experiment mentioned in § 10. As the purple cruorine in the solution was oxidized almost instantaneously on being presented with free oxygen by shaking with air, while the tin-salt remained in an unoxidized state, so the purple cruorine of the veins is oxidized during the time, brief though it may be, during which it is exposed to air in the lungs, while the substances derived from the food may have little disposition to combine with free oxygen. As the scarlet cruorine is gradually reduced, oxidizing thereby a portion of the tin-salt, so part of the scarlet cruorine is gradually

* Ueber Ozon im Blute. Dorpat, 1862.

reduced in the course of the circulation, oxidizing a portion of the substances derived from the food or of the tissues. The purplish colour now assumed by the solution illustrates the tinge of venous blood, and a fresh shake represents a fresh passage through the lungs.

XIX. "Further Inquiries concerning the Laws and Operation of Electrical Force." By Sir W. Snow Harris, F.R.S., &c. Received June 8, 1864.

(Abstract.)

1. The author first endeavours to definitely express what is meant by *quantity of electricity*, *electrical charge*, and *intensity*.

By *quantity of electricity* he understands the actual amount of the unknown agency constituting electrical force, as represented by some arbitrary quantitative 'electrical' measure. By *electrical charge* he understands the quantity which can be sustained upon a given surface under a given electrometer indication. *Electrical intensity*, on the contrary, is 'the electrometer indication' answering to a given quantity upon a given surface.

2. The experiments of Le Monnier in 1746, of Cavendish in 1770, and the papers of Volta in 1779, are quoted as showing that bodies do not take up electricity in proportion to their surfaces. According to Volta, any plane surface extended in length sustains a greater charge,—a result which this distinguished philosopher attributes to the circumstance that the electrical particles are further apart upon the elongated surface, and consequently further without each other's influence.

3. The author here endeavours to show that, in extending a surface in length, we expose it to a larger amount of inductive action from surrounding matter, by which, on the principles of the condenser, the intensity of the accumulation is diminished, and the charge consequently increased; so that not only are we to take into account the influence of the particles on each other, but likewise their operation upon surrounding matter.

4. No very satisfactory experiments seem to have been instituted showing the relation of quantity to surface. The quantity upon a given surface has been often vaguely estimated without any regard to a constant electrometer indication or intensity. The author thinks we can scarcely infer from the beautiful experiment of Coulomb, in consequence of this omission, that the capacity of a circular plate of twice the diameter of a given sphere is twice the capacity of the sphere, and endeavours to show, in a future part of the paper (Experiment 16), that the charge of the sphere and plate are to each other not really as 1·2, but as $1:\sqrt{2}$, that is, as the square roots of the exposed surfaces; so that we cannot accumulate twice the quantity of electricity upon the plate under the same electrometer indication.

5. On a further investigation of the laws of electrical charge, the quantity which any plane rectangular surface can receive under a given intensity

is found to depend not only on the surface, but also on its linear boundary extension. Thus the linear boundary of 100 square inches of surface under a rectangle 37·5 inches long by 2·66 inches wide, is about 80 inches; whilst the linear boundary of the same 100 square inches of surface under a plate 10 inches square is only 40 inches. Hence the charge of the rectangle is much greater than that of the square, although the surfaces are equal, or nearly so.

6. The author finds, by a rigid experimental examination of this question, that electrical charge depends upon surface and linear extension conjointly. He endeavours to show that there exists in every plane surface what may be termed an electrical boundary, having an important relation to the grouping or disposition of the electrical particles in regard to each other and to surrounding matter. This boundary, in circles or globes, is represented by their circumferences. In plane rectangular surfaces, it is their linear extension or perimeter. If this *boundary* be constant, their electrical charge (1) varies with the square root of the surface. If the *surface* be constant, the charge varies with the square root of the boundary. If the surface and boundary both vary, the charge varies with the square root of the surface multiplied into the square root of the boundary. Thus, calling C the charge S the surface, B the boundary, and μ some arbitrary constant depending on the electrical unit of charge, we have $C=\mu\sqrt{S.B}$, which will be found, with some exceptions, a general law of electrical charge. It follows from this formula, that if when we double the surface we also double the boundary, the charge will be also double. In this case the charge may be said to vary with the surface, since it varies with the square root of the surface, multiplied into the square root of the boundary. If therefore the surface and boundary both increase together, the charge will vary with the square of either quantity. The quantity of electricity therefore which surfaces can sustain under these conditions will be as the surface. If l and b represent respectively the length and breadth of a plane rectangular surface, then the charge of such a surface is expressed by $\mu\sqrt{2lb\,(l+b)}$, which is found to agree perfectly with experiment. We have, however, in all these cases to bear in mind the difference between *electrical charge* and *electrical intensity* (1).

7. The electrical intensity of plane rectangular surfaces is found to vary in an inverse ratio of the boundary multiplied into the surface. If the surface be constant, the intensity is inversely as the boundary. If the boundary be constant, the intensity is inversely as the surface. If both vary alike and together, the intensity is as the square of either quantity; so that if when the surface be doubled the boundary be also doubled, the intensity will be inversely as the square of the surface. The intensity of a plane rectangular surface being given, we may always deduce therefrom its electrical charge under a given greater intensity, since we only require to determine the increased quantity requisite to bring the electrometer indication up to the given required intensity. This is readily deduced, the

intensity being, by a well-established law of electrical force, as the square of the quantity.

8. These laws relating to charge, surface, intensity, &c., apply more especially to continuous surfaces taken as a whole, and not to surfaces divided into separated parts. The author illustrates this by examining the result of an electrical accumulation upon a plane rectangular surface taken as a whole, and the results of the same accumulation upon the same surface divided into two equal and similar portions distant from each other, and endeavours to show, that if as we increase the quantity we also increase the surface and boundary, the intensity does not change. If three or more separated equal spheres, for example, be charged with three or more equal quantities, and be each placed in separate connexion with the electrometer, the intensity of the whole is not greater than the intensity of one of the parts. A similar result ensues in charging any united number of equal and similar electrical jars. A battery of five equal and similar jars, for example, charged with a given quantity $=1$, has the same intensity as a battery of ten equal and similar jars charged with quantity $=2$; so that the intensity of the ten jars taken together is no greater than the intensity of one of the jars taken singly. In accumulating a double quantity upon a given surface divided into two equal and separate parts, the boundaries of each being the same, the intensity varies inversely as the square of the surface. Hence two separate equal parts can receive, taken together under the same electrometer indication, twice the quantity which either can receive alone, in which case the charge varies with the surface. Thus if a given quantity be disposed upon two equal and similar jars instead of upon one of the jars only, the intensity upon the two jars will be only one-fourth the intensity of one of them, since the intensity in this case varies with the square of the surface inversely, whilst the quantity upon the two jars under the same electrometer indication will be double the quantity upon one of them only; in which case the charge varies with the surface, the intensity being constant. If therefore as we increase the number of equal and similar jars we also increase the quantity, the intensity remains the same, and the charge will increase with the number of jars. Taking a given surface therefore in equal and divided parts, as for example four equal and similar electrical jars, the intensity is found to vary with the square of the quantity directly (the number of jars remaining the same), and with the square of the surface inversely (the number of jars being increased or diminished); hence the charge will vary as the square of the quantity divided by the square of the surface; and we have, calling C the charge, Q the quantity, and S the surface, $C=\frac{Q^2}{S^2}$; which formula fully represents the phenomenon of a constant intensity, attendant upon the charging of equal separated surfaces with quantities increasing as the surfaces; as in the case of charging an increasing number of equal electrical jars. Cases, however, may possibly arise in which the intensity varies inversely with the surface, and not in-

versely with the square of the surface. In such cases, of which the author gives some examples, the above formula does not apply.

9. From these inquiries it is evident, as observed by the early electricians, that conducting bodies do not take up electricity in proportion to their surfaces, except under certain relations of surface and boundary. If the breadth of a given surface be indefinitely diminished, and the length indefinitely increased, the surface remaining constant, then, as observed by Volta, the least quantity which can be accumulated under a given electrometer indication is when the given surface is a circular plate, that is to say, when the boundary is a minimum, and the greatest when extended into a right line of small width, that is, when the boundary is a maximum. In the union of two similar surfaces by a boundary contact, as for example two circular plates, two spheres, two rectangular plates, &c., we fail to obtain twice the charge of one of them taken separately. In either case we fail to decrease the intensity (the quantity being constant) or to increase the charge (the intensity being constant), it being evident that whatever decreases the electrometer indication or intensity must increase the charge, that is to say, the quantity which can be accumulated under the given intensity. Conversely, whatever increases the electrometer indication decreases the charge, that is to say, the quantity which can be accumulated under the given intensity.

10. If the grouping or disposition of the electrical particles, in regard to surrounding matter, be such as not to materially influence external induction, then the boundary extension of the surface may be neglected. In all similar figures, for example, such as squares, circles, spheres, &c., the electrical boundary is, in relation to surrounding matter, pretty much the same in each, whatever be the extent of their respective surfaces. In calculating the charge, therefore, of such surfaces, the boundary extensions may be neglected, in which case their relative charges are found to be as the square roots of the surfaces only; thus the charges of circular plates and globes are as their diameters, the charges of square plates are as their sides. In rectangular surfaces also, having the same boundary extensions, the same result ensues, the charges are as the square roots of the surfaces. In cases of hollow cylinders and globes, in which one of the surfaces is shut out from external influences, only one-half the surface may be considered as exposed to external inductive action, and the charge will be as the square root of half the surface, that is to say, as the square root of the exposed surface. If, for example, we suppose a square plate of any given dimensions to be rolled up into an open hollow cylinder, the charge of the cylinder will be to the charge of the plate into which we may suppose it to be expanded as $1:\sqrt{2}$. In like manner, if we take a hollow globe and a circular plate of twice its diameter, the charge of the globe will be to the charge of the plate also as $1:\sqrt{2}$, which is the general relation of the charge of closed to open surfaces of the same extension. The charge of a square plate to the charge of a circular plate of the same diameter was found to be $1 : 1{\cdot}13$;

according to Cavendish it is as 1 : 1·15, which is not far different. It is not unworthy of remark that the electrical relation of a square to a circular plate of the same diameter, as determined by Cavendish nearly a century since, is in near accordance with the formulæ $C = \sqrt{S}$ above deduced.

11. The author enumerates the following formulæ as embracing the general laws of quantity, surface, boundary extension, and intensity, practically useful in deducing the laws of statical electrical force.

Symbols.

Let C= electrical charge; Q= quantity; E= intensity, or electrometer indication; S= surface, B= boundary extension, or perimeter; Δ= direct induction; δ= reflected induction; F= force; D= distance.

Formulæ.

$C \propto S$, when S and B vary together.

$C \propto Q$, E being constant, or equal 1.

$C \propto \sqrt{S}$, B being constant, or equal 1.

$C \propto \sqrt{B}$, S being constant, or equal 1.

$C \propto \sqrt{S.B}$, when S and B vary together.

$E \propto \frac{1}{S.B}$ (Q being constant), for all plane rectangular surfaces.

$E \propto \frac{1}{B}$, S being constant, or equal 1.

$E \propto \frac{1}{S}$, B being constant, or equal 1.

$E \propto \frac{1}{S^2}$, when S and B vary together.

$C \propto \frac{1}{\sqrt{E}}$. . . . . .

$E \propto Q^2$, S being constant, or equal 1.

$C \propto \frac{Q^2}{S^2}$. . . . . .

In square plates, C $\propto$ with side of square.

In circular plates, C $\propto$ with diameter.

In globes, C $\propto$ with diameter.

Δ, or induction $\propto$ S, all other things remaining the same.

The same for δ, or reflected induction.

In circular plates, globes, and closed and open surfaces,

$E \propto \frac{1}{S}$; or as $\frac{1}{\Delta}$.

$F\ (=E) \propto Q^2$.

$F \text{ or } E \propto \frac{1}{D^2}$, S being constant.

Generally we have $F \propto \frac{Q^2}{D^2}$.

12. The author calculates from these laws of charge for circles and globes, a series of circular and globular measures of definite values, taking the circular inch or globular inch as unity, and calling, after Cavendish, a circular plate of an inch in diameter, charged to saturation, a circular inch of electricity; or otherwise charged to any degree short of saturation, a circular inch of electricity under a given intensity. In like manner he designates a globe of an inch in diameter a globular inch of electricity.

In the following Table are given the quantities of electricity contained in circular plates and globes, together with their respective intensities for diameters varying from ·25 to 2 inches; a circular plate of an inch diameter and $\frac{1}{5}$th of an inch thick being taken as unity, and supposed to contain 100 particles or units of charge.

Diameters, or units of charge.	Circle.		Globe.	
	Particles.	Intensity.	Particles.	Intensity.
0·25	25	0·062	35	0·124
0·50	50	0·250	70	0·500
0·75	75	0·560	105	1·120
1·00	100	1·000	140	2·000
1·25	125	1·560	175	3·120
1·40	140	1·960	196	3·920
1·50	150	2·250	210	4·500
1·60	160	2·560	224	5·120
1·75	175	3·060	245	6·120
2·00	200	4·000	280	8·000

13. The experimental investigations upon which these elementary data depend, constitute a second part of this paper. The author here enters upon a brief review of his hydrostatic electrometer, as recently perfected and improved, it being essential to a clear comprehension of the laws and other physical results arrived at.

In this instrument the attractive force between a charged and neutral disk, in connexion with the earth, is hydrostatically counterpoised by a small cylinder of wood accurately weighted, and partially immersed in a vessel of water. The neutral disk and its hydrostatic counterpoise are freely suspended over the circumference of a light wheel of 2·4 inches in diameter, delicately mounted on friction-wheels, so as to have perfectly free motion, and be susceptible of the slightest force added to either side of the balance. Due contrivances are provided for measuring the distance between the attracting disks. The balance-wheel carries a light index of straw reed, moveable over a graduated quadrantal arc, divided into 90° on each side of its centre. The neutral attracting plate of the electrometer is about $1\frac{1}{2}$ inch in diameter, and is suspended from the balance-wheel by a gold thread, over a similar disk, fixed on an insulating rod of glass, placed in connexion with any charged surface the subject of experiment. The least force between the two disks is immediately shown by the movement

of the index over the graduated arc in either direction, and is eventually counterpoised by the elevation or depression in the water of the hydrostatic cylinder suspended from the opposite side of the wheel. The divisions on the graduated quadrant correspond to the addition of small weights to either side of the balance, which stand for or represent the amount of force between the attracting plates at given measured distances, with given measured quantities of electricity. This arrangement is susceptible of very great accuracy of measurement.

The experiment requires an extremely short time for its development, and no calculation is necessary for dissipation. The author carefully describes the manipulation requisite in the use of this instrument, together with its auxiliary appendages. He considers this electrometer, as an instrument of electrical research, quite invaluable, and peculiarly adapted to the measurement of electrical force.

14. Having fully described this electrometer, and the nature of its indications, certain auxiliary instruments of quantitative measure, to be employed in connexion with it, are next adverted to.

First, the construction and use of circular and globular transfer measures given in the preceding Table, by which given measured quantities of electricity may be transferred from an electrical jar (charged through a unit-jar from the conductor of an electrical machine) to any given surface in connexion with the electrometer. The electrical jar he terms a *quantity-jar*, the construction and employment of which is minutely explained; as also the construction and employment of the particular kind of unit-jar he employs.

15. Two experiments (1 and 2) are now given in illustration of this method of investigation.

Experiment 1 developes the law of attractive force as regards quantity; which is found to vary with the square of the number of circular or globular inches of electricity, transferred to a given surface in connexion with the fixed plate of the electrometer, the distance between the attracting surfaces being constant.

Experiment 2 demonstrates the law of force as regards distance between the attracting surfaces, the quantity of electricity being constant; and by which it is seen that the force is in an inverse ratio of the square of the distance between the attracting plates, the plates being susceptible of perfect inductive action. From these two experiments, taken in connexion with each other, we derive the following formula, $F \propto \frac{Q^2}{D^2}$; calling F the force, Q the quantity, and D the distance. It is necessary, however, to observe that this formula only applies to electrical attractive force between a charged and neutral body in connexion with the earth, the two surfaces being susceptible of free electrical induction, both direct and reflected.

16. The author now refers to several experiments (3, 4, 5, and 6), showing that no sensible error arises from the reflected inductive action of the

suspended neutral disk of the electrometer, or from the increased surface attendant on the connexion of the surface under experiment with the fixed plate of the electrometer; as also, that it is of no consequence whether the suspended disk be placed immediately over the fixed attracting plate of the electrometer, or over any point of the attracting surface in connexion with it.

17. Having duly considered these preliminary investigations, the author now proceeds to examine experimentally the laws of surface and boundary as regards plane rectangular surfaces, and to verify the formulæ $C=\sqrt{S.B}$, and $E=\frac{1}{S.B}$; in which C=charge, E=intensity, S=surface, and B=boundary.

For this purpose a series of smoothly-polished plates of copper were employed, varying from 10 inches square to 40 inches long by 2·5 to 6 inches wide, and about $\frac{1}{8}$th of an inch thick, exposing from 100 to 200 square inches of surface.

The charges (1) of these plates were carefully determined under a given electrometer indication, the attracting plates being at a constant distance.

Experiment 7. In this experiment, a copper plate 10 inches square is compared with a rectangular plate 40 inches long by 2·5 inches wide.

In these plates the surfaces are each 100 square inches, whilst the boundaries are 40 and 85 inches. The boundaries may be taken, without sensible error, as 1 : 2, whilst the surfaces are the same.

On examining the charges of these plates, charge of the square plate was found to be 7 circular inches, under an intensity of 10°. Charge of the rectangular plate 10 circular inches nearly, under the same intensity of 10°. The charges therefore were as 7 : 10 nearly, that is, as 1 : 1·4 nearly; being the square roots of the boundaries, that is, as $1 : \sqrt{2}$.

Experiment 8. A rectangular plate 37·5 inches long by 2·7 inches wide, surface 101 square inches, boundary 80·5 inches, compared with a rectangular plate 34·25 inches long by 6 inches wide, surface 205 square inches, boundary 80·5 inches.

Here the boundaries are the same, whilst the surfaces may be taken as 1 : 2.

On determining the charges of these plates, charge of the rectangular plate, surface 101 square inches was found to be 8·5 circular inches under an intensity of 8°. Charge of the plate with double surface =205 square inches, was found to be 12 circular inches under the same intensity of 8°; that is to say, whilst the surfaces are as 1 : 2, the charges are as 8·5 : 12 nearly, or as the square roots of the surfaces, that is, as $1 : \sqrt{2}$.

Experiment 9. A rectangular plate 26·25 inches long by 4 inches wide, surface 105 square inches, boundary 60·5, compared with a rectangular plate 40 inches long by 5 inches wide, surface 200 square inches, boundary 90 inches.

Here the surfaces are as 1 : 2 nearly, whilst their boundaries are as 2 : 3.

Charge of the rectangular plate surface =105 square inches, 7 circular inches under an intensity of 10°. Charge of rectangular plate surface 200 square inches, 12 circular inches, under the same intensity of 10°. The charges therefore are as 7 : 12 nearly, or as 1 : 1·7, being as the square roots of the surfaces multiplied into the square roots of the boundaries very nearly.

Experiment 10. A square plate 10 inches square, surface 100 square inches, boundary 40 inches, compared with a rectangular plate 40 inches long by 5 inches wide, surface 200 square inches, boundary 90 inches.

Here the surfaces are double of each other, and the boundaries also double each other, or so nearly as to admit of their being considered double of each other. Charge of square plate 6 circular inches, under an intensity of 10°. Charge of rectangular plate 12 circular inches, under the same intensity of 10°. The charges, therefore, are as the square roots of the surfaces and boundaries conjointly, according to the formula $C=\sqrt{S.B}$, as also verified in the preceding experiment 9.

A double surface, therefore, having a double boundary, takes a double charge, but not otherwise. Neglecting all considerations of the boundary, therefore, the surface and boundary varying together, the charge in this case will be as the surface directly.

18. The author having verified experimentally the laws of surface and boundary, as regards plane rectangular surfaces, proceeds to consider the charges of square plates, circular plates, spheres, and closed and open surfaces generally.

Experiment 11. Plate 10 inches square, surface 100 square inches, boundary 40 inches, compared with a similar plate 14 inches square, surface 196 square inches, boundary 56 inches. Here the surfaces are as 1 : 2 nearly, whilst the boundaries are as $1 : \sqrt{2}$ nearly.

In this case charge of square plate, surface 100 square inches, was found to be 8 circular inches under an intensity of 10°. Charge of the plate, surface 196 square inches, 11 circular inches, under the same intensity of 10°. Here the charges are as 8 : 11, whilst the surfaces may be taken as 1 : 2, that is to say (neglecting the boundary), the charges are as the square roots of the surfaces, according to the formula $C=\sqrt{S}$.

On examining the intensities of these plates, they were found to be inversely as the surfaces ; thus 8 circular inches upon the plate, surface 100, evinced an intensity of 10° ; 8 circular inches upon the plate, surface 196, evinced an intensity of 5° only, or $\frac{1}{2}$ the former, according to the formula $E=\frac{1}{S}$.

Experiment 12. A circular plate of 9 inches diameter, surface 63·6 square inches, compared with a circular plate of 18 inches, or double that diameter, surface 254 square inches. Here the surfaces are as 1 : 4, whilst the boundaries or circumferences are as 1 : 2.

Charge of 9-inch plate, 6 circular inches, under an intensity of 10°.

Charge of 18-inch plate, 12 circular inches, under the same intensity of 10°. Here the charges are as 1 : 2, whilst the surfaces are as 1 : 4; neglecting the difference of boundary, therefore, the charges, as in the preceding experiments, are as the square roots of the surfaces.

On examining the intensities of these plates, they were found to be inversely as the surfaces; thus 6 circular inches upon the 9-inch plate evinced an intensity of 10°, as just stated; 6 circular inches upon the 18-inch plate had only one-fourth the intensity, or 2°·5; being inversely as the surfaces, according to the formula $E=\frac{1}{S}$.

Experiment 13. A circular plate of 9 inches diameter, surface 63·6 square inches, compared with a circular plate of 12·72 inches diameter, surface 127·2 square inches. Here the surfaces are as 1 : 2.

Charge of 9-inch plate (surface 63·6 square inches), 5 circular inches, under an intensity of 8°. Charge of 12·72-inch plate (surface 127·2 square inches), 7 circular inches, under the same intensity of 8°. The charges here are as 5 : 7, whilst the surfaces are as 1 : 2; that is to say (neglecting the boundaries), the charges are as the square roots of the surfaces.

On examining the intensities of these plates, they were found to be, as in the preceding experiments, inversely as the surfaces.

Experiment 14. Comparison of a sphere of 4·5 inches diameter, surface 63·5 square inches, with a sphere of 9 inches, or double that diameter, surface 254 square inches.

Charge of sphere of 4·5 inches diameter (surface 63·5 square inches), 4 circular inches, under an intensity of 9°. Charge of sphere of 9 inches diameter (surface 254 square inches), 8 circular inches, under the same intensity of 9°. Here the charges are as 1 : 2, whilst the surfaces are as 1 : 4. The charges, therefore, are as the square roots of the surfaces, or as $1 : \sqrt{4}$.

On examining the intensities of these spheres, they were found to be as the surfaces inversely, or very nearly; being as 2°·5 and 9° respectively.

Experiment 15. Circular plate of 9 inches diameter compared with a sphere of the same diameter. Here the actual surfaces are 63·6 square inches for the plate, and 254 square inches for the sphere, being as 1 : 4. We have to observe, however, that one surface of the sphere is closed or shut up, consequently the exposed surfaces, electrically considered, neglecting one-half the surface of the sphere as being closed, are as 1 : 2; and the exposed surface of the plate is exactly one-half the exposed surface of the sphere.

Charge of plate 8 circular inches, under an intensity of 12°. Charge of sphere 11 circular inches, under the same intensity of 12°. The charges, therefore, are as 8 : 11, or sa 1 : 1·4; the exposed surfaces being as 1 : 2. The charges, therefore, are as the square roots of the exposed surfaces.

On examining the intensities of the plate and sphere, they were found

to be in an inverse ratio of the exposed surfaces, as in the former experiments.

Experiment 16. Comparison of a sphere of 7 inches diameter with a circular plate of 14 inches, or double that diameter. In this case the inner and outer surface of the sphere, taken together, are actually the same as the two surfaces of the plate. The inner surface of the sphere being closed, however, as in the last experiment, the surfaces of the sphere and plate, electrically considered, are therefore not equal, and the surface of the plate is twice the surface of the sphere. The surfaces, therefore, open to external induction are as 2 : 1.

On examining the charges of the plate and sphere, they were found to be as 10 : 14, or as 1 : 1·4; charge of sphere being 10 circular inches, under an intensity of 20°, and charge of plate being 14 circular inches, under the same intensity of 20°. The charge of the sphere, therefore, as compared with the charge of the plate, is as $1 : \sqrt{2}$, that is, as the square roots of the exposed surfaces.

On examining the intensities of the sphere and plate, they were found to be, as in the preceding experiments, in an inverse ratio of the exposed surfaces. We cannot, therefore, conclude, as already observed (4), that the capacity of the plate is twice that of the sphere.

19. The following experiments are further adduced in support of the preceding:—

Experiment 17. A copper plate 10 inches square, compared with the same plate rolled up into an open hollow cylinder, 10 inches long by 3·2 inches diameter. Here, as in the last experiments, although the surfaces are actually the same, yet, electrically considered, the plate has twice the surface of the cylinder, one surface of the cylinder being shut up.

On examining the charges of the cylinder and plate, they were found to be, as in the preceding experiments, as $1 : \sqrt{2}$; that is, as the square roots of the exposed surfaces, and the intensities in an inverse ratio of the surfaces, which seems to be a general law for closed and open surfaces.

Experiment 18. A hollow copper cube, side 5·7 inches, surface 195, compared with a hollow copper sphere of diameter equal side of cube, surface 103 square inches nearly.

On examining the charges of the sphere and cube, they were found to be as 9 : 10 nearly; charge of the sphere being 9 circular inches, under an intensity of 10°, and charge of cube being 10 circular inches, under the same intensity of 10°. The charges of a cube, and of a sphere whose diameter equals the side of the cube, approach each other, notwithstanding the differences of the surfaces, owing to the six surfaces of the cube not being in a disjointed or separated state.

20. The author observes, in conclusion, that the numerical results of the foregoing experiments, although not in every instance mathematically exact, yet upon the whole were so nearly accordant as to leave no doubt as to the law in operation. It would be in fact, he observes, assuming too much to

pretend in such delicate experiments to have arrived at nearer approximations than that of a degree or two of the electrometer, or within quantities less than that of ·25 of a circular inch. If the manipulation, however, be skilfully conducted, and the electrical insulations perfect, it is astonishing how rigidly exact the numerical results generally come out.

XX. "On a New Class of Compounds in which Nitrogen is substituted for Hydrogen." By PETER GRIESS, Esq. Communicated by Dr. HOFMANN. Received June 2, 1864.

(Abstract.)

All the bodies in which nitrogen is substituted for hydrogen which I have discovered during the last few years* may be divided into two distinctly different classes. The first class comprises those bodies which are obtained when three atoms of hydrogen in two atoms of an amido-compound are replaced by one of nitrogen from nitrous acid. The members of the second group are formed by the action of nitrous acid upon one equivalent of an amido-compound only. The following equations will best show these various reactions:—

I. $\underbrace{2(C_7H_7NO_2)}_{\text{Amidobenzoic acid.}} + \underbrace{NHO_2}_{\text{Nitrous acid.}} = \underbrace{C_{14}H_{11}N_3O_4}_{\text{Diazoamido-benzoic acid.}} + 2H_2O$

II. $\underbrace{C_6H_5N_3O_5}_{\text{Picramic acid (amidonitrophenylic acid).}} + \underbrace{NHO_2}_{} = \underbrace{C_6H_2N_4O_5}_{\text{Diazo-dinitro-phenylic acid.}} + 2H_2O.$

I have hitherto chiefly examined into the constitution of the bodies that form according to the first equation (diazoamidobenzol, diazoamidobenzoic acid), and have only incidentally explored the field of bodies which equation II. opens up. I have had occasion since to study more closely several representatives of the latter class of compounds, which are derived from aniline (amidobenzol) and analogous organic bases; and since the results which I obtained cannot but excite some interest, I may be permitted to submit them briefly to the Royal Society.

Nitrate of Diazobenzol, $C_6H_4N_2, NHO_3$.

This compound is most readily obtained by passing a rapid current of nitrous acid gas through a solution of nitrate of aniline, saturated in the cold, until aniline ceases to be separated by the addition of solution of caustic potash to the liquid. On diluting the solution then with three times its volume of alcohol, and adding a sufficient quantity of ether, nitrate of diazobenzol separates in long white needles. In order to remove a trace of a colouring substance, the crystals are redissolved in a small quantity of

* Ann. der Chem. und Pharm. vol. cxiii. p. 201; vol. cxvii. p. 1; vol. cxxi. p. 257. Proceedings of the Royal Society, vol. x. p. 591; vol. xi. p. 263; vol. xii. p. 418.

cold dilute alcohol and precipitated by ether. The following equation expresses the reaction:—

$$\underbrace{C_6H_7N, NHO_3}_{\text{Nitrate of aniline.}} + \underbrace{NHO_2}_{\text{Nitrous acid.}} = \underbrace{C_6H_4N_2, NHO_3}_{\text{Nitrate of diazobenzol.}} + 2H_2O.$$

Nitrate of diazobenzol may also be prepared from diazoamidobenzol, a substance described by me on a former occasion*, by treating an etherial solution of the latter with nitrous acid,

$$\underbrace{C_{12}H_{11}N_3}_{\text{Diazoamidobenzol.}} + \underbrace{NHO_2}_{} + \underbrace{2NHO_3}_{} = \underbrace{2(C_6H_4N_2, NHO_3)}_{\text{Nitrate of diazobenzol.}} + 2H_2O.$$

The new compound dissolves very readily in water, more difficultly in alcohol, and is almost insoluble in ether. On heating, the solutions are decomposed with evolution of gas. The dry substance explodes with the greatest violence when gently heated, and it is necessary to observe great precautions whilst working with it. The chemical analysis could not, for the same reason, be performed by the usual methods. Its composition was, however, readily established by studying the products of decomposition to which boiling with water gives rise, according to the equation

$$\underbrace{C_6H_4N_2, NHO_3}_{\text{Nitrate of diazobenzol.}} + \underbrace{H_2O}_{} = \underbrace{C_6H_6O}_{\text{Phenylic acid.}} + \underbrace{N_2}_{\text{Nitrogen.}} + \underbrace{NHO_3}_{\text{Nitric acid.}}.$$

Sulphate of Diazobenzol, $C_6H_4N_2SH_2O_4$.

This salt forms when a highly concentrated aqueous solution of the former compound is treated with a sufficient quantity of cold sulphuric acid diluted with its own bulk of water. The solution is treated, as before, with three times its volume of alcohol, and ether added, which causes the sulphate of diazobenzol to separate in a layer of a very concentrated aqueous solution at the bottom of the vessel. On placing this latter solution over sulphuric acid, crystallization ensues after a short time. The crystals are freed from the mother-liquor by washing with absolute alcohol. In this manner large white prisms, which rapidly deliquesce in moist air, are obtained, and which are decomposed with slight deflagration when heated by themselves.

Hydrobromate of Diazobenzol, $C_6H_4N_2, HBr$.

This compound is obtained in small white soft plates when an etherial solution of diazoamidobenzol is mixed with an etherial solution of bromine,

$$\underbrace{C_{12}H_{11}N_3}_{\text{Diazoamidobenzol.}} + 6Br = \underbrace{C_6H_4N_2, HBr}_{\text{Hydrobromate of diazobenzol.}} + \underbrace{C_6H_4Br_3N}_{\text{Tribromaniline.}} + 2(HBr).$$

* Ann. der Chem. und Pharm. vol. cxxi. p. 258.

Hydrobromate of diazobenzol is very unstable. The beautifully white crystals change so rapidly that in a few moments they acquire a reddish colour, and in a few days the decomposition is almost complete. They explode on heating almost with the same violence as was experienced with nitrate of diazobenzol.

Dibromide of Hydrobromate of Diazobenzol, $C_6H_4N_2$, HBr, Br_2.

On adding excess of bromine-water to an aqueous solution of any one of the compounds previously described, an orange-coloured oil is obtained which rapily solidifies, after the mother-liquor has been removed, to small orange-coloured plates. The crystals of dibromide are obtained in a perfectly pure state by washing with a little alcohol. This compound is rather difficultly soluble in cold alcohol and ether; and the solutions are rapidly decomposed, particularly on the application of heat.

Platinum-salt of the Hydrochlorate of Diazobenzol, $C_6H_4N_2$, HCl, $PtCl_2$.

This salt forms beautiful yellow prisms which are almost insoluble in water. The gold-salt, $C_6H_4N_2$, HCl, $AuCl_3$, can be recrystallized from alcohol, and is obtained in very fine golden-yellow brilliant plates.

It has thus been sufficiently shown that diazobenzol deports itself like an organic base, being capable, like aniline, of forming salts with various acids. It possesses, however, also the property of combining with the hydrates of the metals, thus playing the part of a weak acid.

Compound of Hydrate of Potassium with Diazobenzol, $C_6H_4N_2$, KHO.

This body is obtained when a concentrated aqueous solution of nitrate of diazobenzol is treated with excess of concentrated aqueous potassa. By evaporating on the water-bath, the liquid solidifies, when sufficiently concentrated, to a magma of yellow crystals consisting of nitre and the compound of hydrate of potassa with diazobenzol. The crystalline mass is pressed between porous stones, and thus partly freed from moisture. By dissolving in absolute alcohol and treating with ether, the new compound of hydrate of potassium with diazobenzol is obtained in a pure state, crystallizing in small soft white plates, which rapidly become reddish, especially in the moist condition. It is very readily soluble in water and alcohol; the solutions, however, decompose slowly, and deposit a reddish amorphous body. Heat does not seem to accelerate this decomposition materially.

Compound of Hydrate of Silver with Diazobenzol, $C_6H_4N_2$, AgHO.

This substance is obtained in the form of an almost white precipitate when a solution of silver is added to an aqueous solution of the former compound. It is very stable. Similar compounds are obtained with lead- and zinc-salts.

Diazobenzol, $C_6H_4N_2$.

This substance is obtained when an aqueous solution of the compound of hydrate of potassium with diazobenzol is neutralized with acetic acid. It separates as a thick yellow oil of very little stability. After a few moments an evolution of gas ensues, and the diazobenzol is rapidly converted into a reddish-brown viscid mass. Diazobenzol is soluble in acids, as well as bases, with formation of the saline compounds previously described.

By acting in the cold with aniline upon nitrate of diazobenzol*, the following change takes place:—

$$\underbrace{C_6H_4N_2NHO_3}_{\text{Nitrate of diazobenzol.}}+\underbrace{2C_6H_7N}_{\text{Aniline.}}=\underbrace{C_{12}H_{11}N_3}_{\text{Diazoamidobenzol.}}+\underbrace{C_6H_7N,\,NHO_3}_{\text{Nitrate of aniline.}}.$$

I was formerly of opinion that diazoamidobenzol must be viewed as a double compound of diazobenzol and aniline. The above equation seems to confirm this view.

New compounds analogous to diazobenzol-amidobenzol are obtained by the action of other organic bases upon nitrate of diazobenzol, viz. diazobenzol-amidobromobenzol, $\left\{\begin{matrix} C_6H_4N_2 \\ C_6\begin{pmatrix}H_6\\Br\end{pmatrix}N\end{matrix}\right\}$, by the action of bromaniline.

Naphthalidine and nitrate of diazobenzol combine directly and form nitrate of diazobenzol-amidonaphtol, crystallizing in magnificent large green prisms.

The action of amido-acids upon nitrate of diazobenzol is analogous to that of the aniline; *ex. gr.*,

$$\underbrace{C_6H_4N_2,\,NHO_3}_{\text{Nitrate of diazobenzol.}}+\underbrace{2C_7H_7NO_2}_{\text{Amido-acid.}}=\underbrace{\left\{\begin{matrix}C_6H_4N_2\\C_7H_7NO_2\end{matrix}\right\}}_{\text{Diazobenzol-amidobenzoic acid.}}+\underbrace{C_7H_7NO_2,\,NHO_3}_{\text{Nitrate of amidobenzoic acid.}}.$$

Diazobenzol-amidobenzoic acid separated quickly as a yellow crystalline mass on mixing the aqueous solutions of both substances. It is purified by recrystallization from ether in the form of small yellow plates. It combines with bases and forms saline bodies. Bichloride of platinum precipitates from an alcoholic solution a yellowish-white crystalline platinum-salt of the composition $C_{13}H_{11}N_3O_2,\,2HCl,\,2PtCl_2$.

Similar double acids to the one just described are obtained by the action of amidodracylic acid, amidoanic acid, &c., upon nitrate of diazobenzol.

Imidogen Compounds of Diazobenzol.

These peculiar compounds are formed when aqueous ammonia, as well as certain organic bases, are made to act upon the dibromide of diazobenzol.

* It is self-evident that for this and similar experiments sulphate and hydrobromate of diazobenzol may also be employed.

Diazobenzolimide, $C_6H_4\left.\begin{matrix}N_2\\H\end{matrix}\right\}N$, is obtained according to the equation

$$\underbrace{C_6H_4N_2HBr_3}_{\text{Dibromide.}}+\underbrace{4NH_3}_{\text{Ammonia.}}=\underbrace{C_6H_5N_3}_{\text{Diazobenzolimide.}}+\underbrace{3NH_4Br.}_{\text{Bromide of ammonium.}}$$

It forms a yellowish oil, which must be distilled *in vacuo* with the aid of a current of steam. Exposed by itself to a higher temperature, it decomposes with detonation. It is remarkable for its stupefying ammoniacal-aromatic odour.

Ethyldiazobenzolimide, $C_6\left.\begin{matrix}H_4N_2\\C_2H_5\end{matrix}\right\}N$, is analogous in its properties, and is formed in a similar manner.

Products of Decomposition of Diazobenzol Compounds.

The transformations which the molecule of diazobenzol undergoes under the influence of various reagents are numerous. The products often represent some peculiar classes of entirely new compounds; more frequently, however, they belong to the phenyl- and benzol-group. I will describe a few of them somewhat more fully.

It has already been mentioned that, on boiling with water, nitrate of diazobenzol is broken up into nitrogen, phenylic acid, and nitric acid. Hydrobromate of diazobenzol undergoes an analogous decomposition, viz.,

$$C_6H_4N_2HBr+H_2O=N_2+C_6H_6O+HBr.$$

Treated with alcohol, nitrate of diazobenzol is decomposed in the following manner:—

$$2\underbrace{(C_6H_4N_2,NHO_3)}_{\text{Nitrate of diazobenzol.}}+\underbrace{C_2H_6O}_{\text{Alcohol.}}=\underbrace{C_6H_6}_{\text{Benzol.}}+\underbrace{C_6H_4(NO_2)_2O}_{\text{Dinitrophenylic acid.}}+\underbrace{C_2H_4O}_{\text{Aldehyde.}}+N_4+H_2O.$$

On dissolving sulphate of diazobenzol in a small quantity of concentrated sulphuric acid, it gives rise to the formation of a new sulpho-acid which I propose to call disulphophenylenic acid,

$$\underbrace{C_6H_4N_2SH_2O_4}_{\text{Sulphate of diazobenzol.}}+SH_2O_4=\underbrace{C_6H_4,S_2H_4O_8}_{\text{Disulphophenylic acid.}}+N_2.$$

The excess of sulphuric acid may be removed from the new acid by means of carbonate of barium. The new barium-salt crystallizes in beautiful prisms. Its composition must be expressed by the formula $C_6H_4S_2H_2Ba_2O_8$. The free acid obtained by the addition of sulphuric acid to the barium-salt is deposited in warty masses of radiating crystals which deliquesce in the air. Disulphophenylenic acid is four-basic, and is capable of forming four series of salts. The silver-salt forms, however, an exception, its composition being expressed by the formula $C_6H_4S_2Ag_2O_7$. Disulphophenylenic acid, like phosphoric acid, appears to be capable of existing in different modifications, possessing different powers of basicity.

Diazobenzolimide in alcoholic solutions is decomposed by nascent hydrogen, generated with zinc and sulphuric acid in the following manner:—

$$\underbrace{C_6H_5N_3}_{\text{Diazobenzolimide.}}+8H=\underbrace{C_6H_7N}_{\text{Aniline.}}+2NH_3.$$

On adding to an aqueous solution of nitrate of diazobenzol levigated carbonate of barium, a feeble evolution of gas ensues, which lasts for several days, until the original compound has been completely decomposed. Two new substances are formed, which are very differently soluble in alcohol, and can thus readily be separated. The easily soluble compound, which I will call phenyldiazobenzol, crystallizes from alcohol in yellowish warty masses; from water (in which it dissolves very difficultly) in small rhombic prisms. The difficultly soluble one, which I propose to call phenyldidiazobenzol, crystallizes in reddish-yellow needles. The following equation expresses the formation of these two bodies:—

I. $$2(\underbrace{C_6H_4N_2,NHO_3}_{\text{Nitrate of diazobenzol.}})+H_2O=\underbrace{C_{12}H_{10}N_2O}_{\text{Phenol-diazobenzol.}}+N_2+2NHO_3.$$

II. $$3(C_6H_4N_2,NHO_3)+H_2O=\underbrace{C_{18}H_{14}N_4O}_{\text{Phenoldidiazobenzol.}}+N_2+3NHO_3.$$

On looking at these formulæ, it becomes evident at a glance that both compounds contain phenylic acid and diazobenzol; viz.

$$\underbrace{C_6H_4N_2}_{\text{Diazobenzol.}}+\underbrace{C_6H_6O}_{\text{Phenol.}}=\underbrace{C_{12}H_{10}N_2O}_{\text{Phenol-diazobenzol.}}$$

$$\underbrace{2(C_6H_4N_2)}_{\text{Diazobenzol.}}+\underbrace{C_6H_6O}_{\text{Phenol.}}=\underbrace{C_{18}H_{14}N_4O}_{\text{Phenol-diazobenzol.}}$$

Both compounds are weak acids; the first being capable of forming a well-characterized silver-salt, which is obtained in the form of a blood-red precipitate when an ammoniacal solution of phenoldiazobenzol is treated with nitrate of silver.

On heating the platinum-salt of diazobenzol mixed with carbonate of sodium in a retort, chlorobenzol is obtained, the formation of which may be expressed as follows:—

$$\underbrace{C_6H_4N_2,HCl,PtCl_2}_{\text{Platinum-salt.}}=\underbrace{C_6H_5Cl}_{\text{Chlorobenzol.}}+N_2+PtCl_2.$$

A similar decomposition ensues when the dibromide is heated with carbo-

nate of sodium,

$$\underbrace{C_6H_4N_2HBr_3}_{\text{Dibromide.}} = \underbrace{C_6H_5Br}_{\text{Bromobenzol.}} + N_2 + Br_2.$$

The same change may also be effected by simply boiling an alcoholic solution of the dibromide.

The peculiar and often remarkable properties of the diazobenzole-compounds have induced me to try whether analogous bodies could not be prepared also from bromaniline, nitraniline, dibromaniline, &c. Experiment has fully borne out theory. These analogous diazo-substitutions exhibit, however, so much resemblance to the normal diazobenzol compounds, that I should frequently have to repeat almost literally what has already been said of the latter, were I to describe these compounds in detail. I may be permitted, however, to mention a remarkable and interesting fact which their investigation prominently brought out. There are, as is well known, two isomeric nitranilines—the alphanitraniline of Arppe, and the betanitraniline of Hofmann and Muspratt. This isomerism, I found, extends itself to their respective nitrogen-substitution compounds, and even to their products of decomposition. On heating, *ex. gr.*, the dibromide of alphadiazonitrobenzol with alcohol, the following change takes place:—

$$\underbrace{C_6H_3(NO_2)N_2HBr_3}_{\alpha\text{-dibromide.}} = \underbrace{C_6H_4(NO_2)Br}_{\text{Bromonitrobenzol.}} + N_2 + Br_2.$$

The bromonitrobenzol thus obtained is identical with that prepared by Cooper from benzol derived from coal-tar. It crystallizes in the same form, and fuses, like the latter, at 126° C.; sulphide of ammonium converts it into bromaniline, which crystallizes in octahedra, and is identical with the bromaniline of Hofmann obtained from bromisatine. Bromonitrobenzol, prepared in a perfectly similar manner by the decomposition of the dibromide of betadiazonitrobenzol, possesses, however, widely different properties. The α-bromonitrobenzol just described crystallizes in long needles, whilst the new benzol-derivative, which I will designate by the name of β-bromonitrobenzol, forms well-developed prisms, the fusing-point of which lies at 56° C. Sulphide of ammonium converts it likewise into bromaniline; but this base differs in its physical properties entirely from the bromaniline obtained by Hofmann. It forms a colourless oil, which combines with acids, and gives rise to a series of beautiful salts, which in their turn differ greatly from the ordinary bromaniline salts in their physical properties. I will distinguish this bromaniline by calling it β-bromaniline from that obtained by Hofmann, which I will call α-bromaniline.

It deserves to be mentioned briefly that there exist likewise two isomeric chloronitrobenzols (alpha and beta) obtained by heating the platinum-salts of the respective diazonitrobenzole with carbonate of sodium,

$$\underbrace{C_6H_3(NO_2)N_2,HCl,PtCl_2}_{\alpha,\beta\text{ Platinum-salt.}} = \underbrace{C_6H_4(NO_2)Cl}_{\alpha,\beta\text{ Chloronitrobenzol.}} + N_2 + PtCl_2.$$

Alpha-nitrochlorobenzol furnishes, when reduced by means of sulphide of ammonium, the ordinary (alpha-) chloraniline; beta-chloronitrobenzol yielding a new base of like composition (beta-chloraniline), distinguished from the former by its oily nature.

Corresponding diazo-compounds can readily be prepared from the homologues of aniline and other analogous bases by submitting them to a treatment exactly similar to that which in case of aniline yielded diazobenzol. Thus I have obtained the diazo-compounds from toluidine, naphthalidine, and nitranisidine, $C_7H_7(NO_2)O$. I have abstained from entering more fully into a description of their physical and chemical habitus, as well as the respective products of decomposition to which they give rise, since they offer nothing characteristically new*.

All compounds already described have been derivations from monoatomic amido-bases. I have on a former occasion† had an opportunity of pointing out that the action of nitrous acid upon diatomic bases, such as nitrate of benzidine, is perfectly analogous to that which gives rise to the formation of nitrate of diazobenzol from nitrate of aniline. Whilst, however, in the last-mentioned reaction only one atom of nitrous acid exchanges its nitrogen for three atoms of hydrogen of the original compound, six atoms of hydrogen are exchanged for two atoms of nitrogen when nitrous acid reacts upon nitrate of benzidine. Respecting these compounds I shall only briefly describe a few general properties and a few products of decomposition.

Sulphate of Tetrazodiphenyl, $2C_{12}H_8N_4, 3SH_2O_4$, crystallizes in white or slightly yellowish-coloured needles, which are very soluble in water, and almost insoluble in strong alcohol and ether. On boiling the alcoholic solution, the following decomposition takes place:—

$$\underbrace{2C_{12}H_8N_4.3SH_2O_4}_{\text{Sulphate of tetrazodiphenyl.}}+4H_2O=\underbrace{2C_{12}H_{10}O_2}_{\text{Diphenylenic acid (diphenylene-alcohol).}}+N_8+3SH_2O_4.$$

I have already had occasion to describe diphenylenic acid (diphenylene-alcohol) obtained by decomposition, analogous to that of nitrate of tetrazodiphenyl with water, and I have therefore only to refer to what has been stated on that occasion.

The decomposition which tetrazodiphenyl undergoes on boiling with

* In a former notice (Proceedings, Jan. 22, 1863) I briefly described the formation of nitrate of naphthol, which by its decomposition with water gave rise to the long-sought-for naphthyl-alcohol, $C_{10}H_8O$.

† Proceedings, Jan. 22, 1863.

alcohol differs from the previous one, and takes place according to the equation

$$\underbrace{2C_{12}H_6N_4, 3SH_2O_4}_{\text{Sulphate of tetrazodiphenyl.}} + \underbrace{4C_2H_6O}_{\text{Alcohol.}} = \underbrace{2C_{12}H_{10}}_{\text{Diphenyl.}} + \underbrace{4C_2H_4O}_{\text{Aldehyde.}} + 3SH_2O_4 + 8N.$$

The diphenyl which results from this reaction is identical with the compounds obtained by Fittig from bromobenzol. A comparative examination of the two demonstrates this most unmistakeably.

The transformation which sulphate of tetrazodiphenyl undergoes when it is heated with a small quantity of strong sulphuric acid is likewise of great interest. Two new sulpho-acids are formed, which I shall call tetra- and tri-sulphodiphenylenic acid. The following equation explains their formation in the most natural manner:—

$$\underbrace{C_{12}H_6N_4}_{\text{Tetrazodiphenyl.}} + 4SH_2O_4 = \underbrace{C_{12}H_6, S_4H_8O_{16}}_{\text{Tetrasulphodiphenylenic acid.}} + N_4$$

$$C_{12}H_6N_4 + 3SH_2O_4 = \underbrace{C_{12}H_6, S_3H_6O_{12}}_{\text{Trisulphodiphenylic acid.}} + N_4.$$

The separation of these two acids is based upon the unequal solubility of their barium-salts. The process is, however, somewhat complicated, and I therefore abstain from describing it. Both acids are capable of combining with bases in various proportions. Tetrasulphodiphenylenic acid is octobasic. The lead-salt crystallizes in beautiful needles, and has the composition $C_{12}H_6, S_4Pb_8O_{16}$. Trisulphodiphenylenic acid appears to be hexabasic. I have as yet only prepared the former acid in a free state. It crystallizes in white needles, which are readily soluble in water and alcohol.

Tetrabromide of the Tetrazodiphenyl, $C_{12}H_6N_4, 2HBr, Br_4$.

This compound forms crystals of an orange colour with curved faces. On heating with alcohol, it splits up according to the equation

$$\underbrace{C_{12}H_6N_4, H_2Br_6}_{\text{Tetrabromide.}} = \underbrace{C_{12}H_8Br_2}_{\text{Bromodiphenyl.}} + N_4 + Br_4.$$

Bromodiphenyl crystallizes from alcohol and ether (in which it is rather difficultly soluble) in beautiful prisms which fuse at 164° C. This substance can be distilled without undergoing decomposition. Bromodiphenyl has also been obtained by Fittig (according to a private communication) by the action of bromine upon diphenyl.

The platinum-salt of tetrazodiphenyl, $C_{12}H_6N_4, H_2Cl_2(PtCl_2)_2$, forms small yellow plates, which furnish, when heated with carbonate of sodium, chlorodiphenyl closely resembling the analogous bromine-compound.

Tetrazodiphenyl-amidobenzol, $\left\{\begin{matrix} C_{12}H_6N_4 \\ C_6H_7N \\ C_6H_7N \end{matrix}\right\}$.

This complex body is formed when an aqueous solution of nitrate of tetrazodiphenyl is mixed with aniline. It is deposited in a yellow crystalline mass, which can be recrystallized from alcohol or ether (in which it is but slightly soluble), and is obtained in lancet-like plates. When heated in a dry state, it is decomposed with slight explosion. Its formation is expressed by the equation

$$\underbrace{C_{12}H_6N_4, 2NHO_3}_{\text{Nitrate of tetrazodiphenyl.}} + \underbrace{4C_6H_7N}_{\text{Aniline.}} = \underbrace{C_{24}H_{20}N_6}_{\text{New compound.}} + \underbrace{2(C_6H_7N, NHO_3)}_{\text{Nitrate of aniline.}}$$

Tetrazodiphenylimide, $\left.\begin{matrix} C_{12}H_6 \end{matrix}\right\} \begin{matrix} N_4 \\ H_2 \end{matrix} N_2$.

This body is obtained in the form of slightly yellowish-tinged lustrous plates, which are very difficultly soluble in cold, readily, however, in hot alcohol and in ether. It combines neither with acids nor with bases. Heated by itself it explodes. The following equation explains its formation:—

$$\underbrace{C_{12}H_6N_4H_2Br_6}_{\text{Tetrabromide.}} + 8NH_3 = \underbrace{C_{12}H_8N_6}_{\text{Tetrazodiphenylimide.}} + 6NH_4Br.$$

I have not succeeded in preparing tetrazodiphenyl in a free state, nor have I been able to obtain compounds of tetrazodiphenyl with bases in anything like a well-characterized condition. I pass over the abortive experiments made by me in this direction.

Many of the experiments just described have been carried on in the laboratory of the Royal College of Chemistry, London, others in that of the University of Marburg; and I take this opportunity of returning my thanks to Prof. Hofmann of London, and Prof. Kolbe of Marburg, for allowing me the use of these institutions.

XXI. "New Observations upon the Minute Anatomy of the Papillæ of the Frog's Tongue." By Lionel S. Beale, M.B., F.R.S., F.R.C.P., Professor of Physiology and of General and Morbid Anatomy in King's College, London; Physician to the Hospital, &c. Received June 16, 1864.

(Abstract.)

After alluding to the observations of Axel Key, whose results accord with his own more closely than those of any other observer, the author refers particularly to the drawings of Hartmann, the latest writer upon the structure of the papillæ. According to the author, Dr. Hartmann, owing

to the defective method of preparation he employed, has failed to observe points which had been seen by others who had written before him, and which may now be most positively demonstrated. Hartmann's process consisted in soaking the tissue for three days in solution of bichromate of potash, and afterwards adding solution of caustic soda. It can be shown by experiment that many structures which can be most clearly demonstrated by other modes of investigation, are rendered quite invisible by this process. Hartmann's observations, like those of the author, have been made upon the papillæ of the tongue of the little green tree-frog (*Hyla arborea*).

With reference to the termination of the nerves in the fungiform papillæ of the tongue of the Hyla, the author describes a plexus of very fine nerve-fibres, with nuclei, which has not been demonstrated before. Fibres resulting from the division of the dark-bordered fibres in the axis of the papilla can be traced directly into this plexus. From its upper part fine fibres, which interlace with one another in the most intricate manner, forming a layer which appears perfectly granular, except under a power of 1000 or higher, may be traced into the hemispheroidal mass of epithelium-like cells which surmounts the summit of the papilla. This hemispheroidal mass belongs not to epithelial, but to the nervous tissues. It adheres to the papilla after every epithelial cell has been removed; the so-called cells of which the entire mass consists cannot be separated from one another like epithelial cells; fibres exactly resembling nerve-fibres can often be seen between them; and very fine nerve-fibres may be traced into the mass from the bundle of nerves in the papilla.

The fine nerve-fibres which are distributed to the simple papillæ of the tongue, around the capillary vessels, and to the muscular fibres of these fungiform papillæ, come off *from the very same trunk as that from which the bundle of purely sensitive fibres which terminate in the papillæ are derived.* The fine nucleated nerve-fibres of the capillaries which the author has demonstrated have been traced into undoubted nerve-trunks in many instances, so that it is quite certain that many of the nuclei which have been considered to belong to the connective tissue (connective-tissue corpuscles) are really the nuclei of fine nerve-fibres not to be demonstrated by the processes of investigation usually followed*. These nerve-fibres in the connective tissue around the capillaries are considered by the author to be the afferent fibres of the nerve-centres of which the efferent branches are those distributed to the muscular coat of the small arteries.

The author's observations upon the tissues of the frog convince him that the nervous tissue is distinct in every part of the body from other special tissues. For example, he holds that nerve-fibres never pass by continuity of tissue into the 'nuclei' (germinal matter) of muscular fibres, or into those

* See "On the Structure and Formation of the so-called Apolar, Unipolar, and Bipolar Nerve-cells of the Frog," Phil. Trans. 1863, plate 40, fig. 44.

of tendon, of the cornea, or of epithelium. He advances arguments to show that the epithelium-like tissue upon the summit of the papilla is not epithelium at all, but belongs to the nervous tissues. Hence it follows that nerves do not influence any tissues by reason of continuity of tissue, but solely by the nerve-currents which pass along them*.

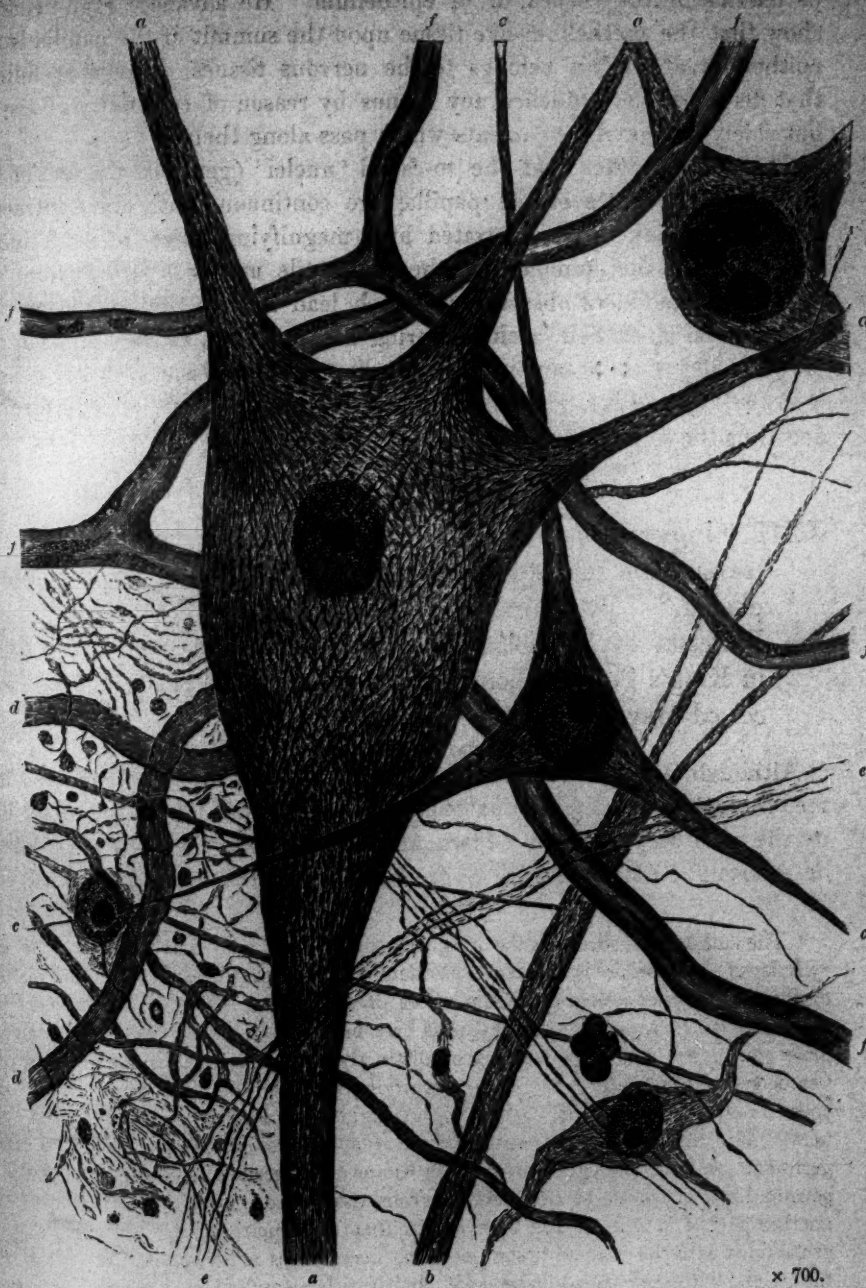
The author states that the so-called 'nuclei' (germinal matter) of the fine muscular fibres of the papillæ are continuous with the contractile material, as may be demonstrated by a magnifying power of 1800 linear; and he holds the opinion that the contractile matter is formed from the nuclei. He adduces observations which lead him to the conclusion that these nuclei alter their position during life, and that, as they move in one or other direction, a narrow line of new muscular tissue (fibrilla) is as it were left behind †. This is added to the muscular tissue already formed, and thus the muscle increases.

XXII. "Indications of the Paths taken by the Nerve-currents as they traverse the caudate Nerve-cells of the Spinal Cord and Encephalon." By Lionel S. Beale, M.B., F.R.S., F.R.C.P., Professor of Physiology and of General and Morbid Anatomy in King's College, London; Physician to the Hospital, &c. Received May 18, 1864.

Although the caudate nerve-vesicles, or cells existing in the spinal cord, medulla oblongata, and in many parts of the brain, have been described by the most distinguished modern anatomists, there yet remains much to be ascertained with reference to their internal structure, connexions, and

* The author feels sure that the conclusions of Kühne, who maintains that the axis cylinder of a nerve-fibre is actually continuous with the 'protoplasm' (germinal matter) of the corneal corpuscle, result from errors of observation. The prolongations of the corneal corpuscles, on the contrary, pass over or under the finest nerve-fibres, but are *never* continuous with them, as may be distinctly proved by examining properly prepared specimens under very high magnifying powers (1000 to 5000 linear). The corneal tissue results from changes occurring in one kind of germinal matter—the nerve-fibres distributed to the corneal tissue from changes occurring in another kind of germinal matter. If the connexion is as Kühne has described, a 'nucleus' or mass of germinal matter would be producing nervous tissue in one part and corneal tissue in another part; and since it has been shown that the 'nuclei' of the corneal tissue are continuous with the corneal tissue itself, the nerve-fibres must be continuous, through the nuclei, with the corneal tissue itself; and if with corneal tissue, probably with every other tissue of the body. But such a view is opposed to many broad facts, and not supported by minute observation. The nuclei of the nerve-fibres are one thing, the nuclei of the corneal tissues another; and the tissues resulting from these nuclei, nerve-tissue, and corneal tissue are distinct in chemical composition, microscopical characters, and properties and actions.

† "New Observations upon the Movements of the Living or Germinal Matter of the Tissues of Man and the higher Animals," Archives, No. XIV. p. 150.



Scale, $\frac{1}{1000}$ of an English Inch [.....] × 700.

Large candate nerve cell, with smaller cells and nerve fibres, from a thin transverse section of the lower part of the grey matter of the medulla oblongata of a young dog. The specimen had been soaked for some weeks in acetic acid and glycerine. The lines of dark granules resulting from the action of the acid are seen passing through the very substance of the cell in very definite directions. Thus the cell is the point where lines from several distant parts intersect (Diagram, Fig. 2). It is probable that each of these lines is but a portion of a complete circuit (see Diagram in Fig. 3). A, A, A, large fibres which leave the cell. B, a fibre from another cell, dividing into finer fibres, exhibiting several lines of granules. C, C, C, fibres from a younger candate nerve vesicle. D, fine and flattened dark-bordered fibres. E, three fine nerve fibres running together in a matrix of connective tissue. F, F, F, capillary vessels.

L. S. B. del. 1863.

mode of development. In this paper I propose to describe some points of interest in connexion with their structure. In the first place, however, I would remark that there are neither '*cells*' nor '*vesicles*' in the ordinary acceptation of these words, for there is no proper investing membrane, neither are there '*cell-contents*' as distinguished from the *membrane* or *capsule*; in fact the so-called cell consists of soft solid matter throughout. The nerve-fibres are not prolonged from the nucleus or from the outer part of the cell, but they are continuous with the very material of which the substance of the 'cell' itself is composed, and they are, chemically speaking, of the same nature. So that in these caudate cells we have but to recognize the so-called '*nucleus*' (*germinal matter*) and *matter around this* (*formed material*) *which passes into the* '*fibres*,' which diverge in various directions from the cell: see Plate III. (fig. 1).

At the outer part of many of these 'cells,' usually collected together in one mass, are a number of granules. These are not usually seen in the young cells, and they probably result from changes taking place in the matter of which the substance of the cell is composed. But it is not proposed to discuss this question in the present paper.

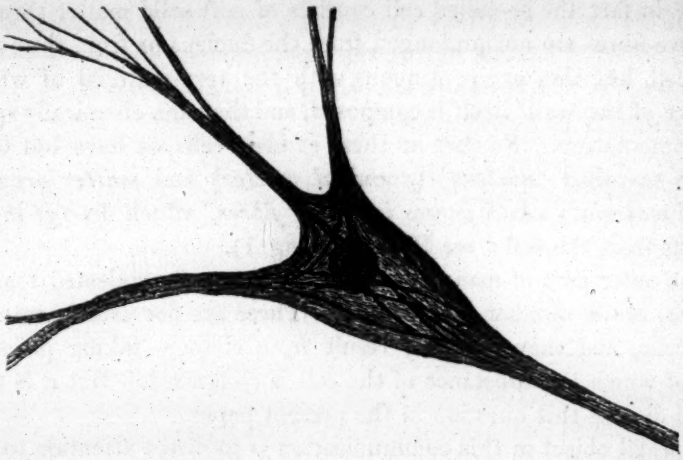
My special object in this communication is to direct attention to a peculiar appearance I have observed in these cells, which enables me to draw some very important inferences with reference to the connexions and action of these very elaborate and most important elements of the nervous mechanism.

In some very thin sections of the cord and medulla oblongata of a young dog, which had been very slowly acted upon by dilute acetic acid, the appearances represented in Plate III. (fig. 1) were observed. Subsequently, similar appearances, though not so distinct, have been demonstrated in the caudate nerve-vesicles of the grey matter of the brain of the dog and cat, as well as of the human subject. I have no doubt that the arrangement is constant, and examination of my specimens will probably satisfy observers that the appearance is not accidental. Each fibre (*a*, *a*, *a*) passing from the cell exhibits in its substance several lines of granules. The appearance is as if the fibre were composed of several very fine fibres imbedded in a soft transparent matrix, which fibres, by being stretched, had been broken transversely at very short intervals. At the point where each large fibre spreads out to form the body of the cell, these lines diverge from one another and pursue different courses through the very substance of the cell, in front of, and behind, in fact around the nucleus. Lines can be traced from each fibre across the cell into every other fibre which passes away from it. The actual appearance is represented in Plate III.; and in the diagram, fig. 2, a plan of a 'cell,' showing the course of a few of the most important of these lines which traverse its substance, is given.

I do not conceive that these lines represent fibres structurally distinct from one another, but I consider the appearance is due to some difference in composition of the material forming the substance of the cell in these

particular lines; and it seems to me that the course which the lines take permits of but one explanation of the appearance. Supposing nerve-currents to be passing along the fibres through the substance of the cell,

Fig. 2.



A diagram of such a cell as that represented in Plate III. (fig. 1), showing the principal lines diverging from the fibres at the point where they become continuous with the substance of the cell. These lines may be traced from one fibre across the cell, and may be followed into *every other fibre* which proceeds from the cell.

they would follow the exact lines here represented; and it must be noticed that these lines are more distinct and more numerous in fully-formed than in young cells. They are, I think, lines which result from the frequent passage of nerve-currents in these definite directions.

Now I have already advanced arguments in favour of the existence of complete nervous circuits, based upon new facts resulting from observations upon *a*, the peripheral arrangement of the nerves in various tissues*; *b*, the course of individual fibres in compound trunks, and the mode of branching and division of nerve-fibres†; and *c*, the structure of ganglion-cells‡. I venture to consider these lines across the substance of the caudate nerve-cells as another remarkable fact in favour of the existence of such circuits; for while the appearance would receive a full and satisfactory explanation upon such an hypothesis, I doubt if it be possible to suggest another explanation which would seem even plausible.

Nor would it, I think, be possible to adduce any arguments which would so completely upset the view that nerve-force passes centrifugally from one

* Papers in the Phil. Trans. for 1860 and 1862. Lectures on the Structure of the Tissues, at the College of Physicians, 1860.

† "On very fine Nerve-fibres, and on Trunks composed of very fine Fibres alone," Archives of Medicine, vol. iv. p. 19. "On the Branching of Nerve-trunks, and of the subdivision of the individual fibres composing them," Archives, vol. iv. p. 127.

‡ Lectures at the College of Physicians. Papers in Phil. Trans. for 1862 and 1863.

cell, as from a centre, towards its peripheral destination, as this fact. So far from the fibres *radiating from one cell*, or from the nucleus as some suppose, in different directions, *all the fibres which reach the cell* are complex, and contain lines which *pass* uninterruptedly *through it into other fibres*. Instead of the cell being the point from which nerve-currents *radiate* in different directions along single fibres, it is the common point where a number of circuits having the most different distribution intersect, cross, or decussate. The so-called *cell is a part of a circuit*, or rather *of a great number of different circuits*.

Fig. 3.

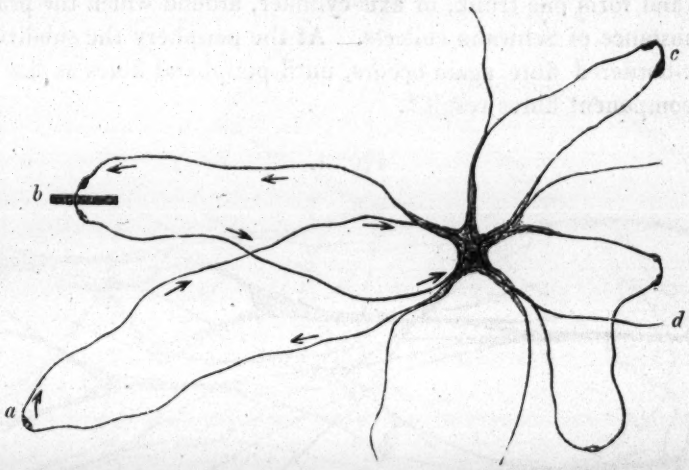


Diagram to show the possible relation to one another of various circuits traversing a single caudate nerve-cell. *a* may be a circuit connecting a peripheral sensitive surface with the cell; *b* may be the path of a motor impulse; *c* and *d* other circuits passing to other cells or other peripheral parts. A current passing along the fibre *a* might induce currents in the three other fibres, *b*, *c*, *d*, which traverse the same cell.

I conclude that at first the formed material of the cell is quite soft and almost homogeneous, but that as currents traverse it in certain definite lines, difference in texture and composition is produced in these lines, and perhaps after a time they become more or less separated from one another, and insulated by the intervening material.

It may perhaps be carrying speculation upon the meaning of minute anatomical facts too far to suggest that a nerve-current traversing one of these numerous paths or channels through the cell may influence all the lines running more or less parallel to it (fig. 3).

I have ascertained that fibres emanating from different caudate nerve-cells situated at a distance from one another (fig. 4, *a*, *a*) at length meet and run on together as a compound fibre (*b*, *b*, *b*), so that I am compelled to conclude (and the inference is in harmony with facts derived from observations of a different kind) that every single nerve-fibre entering into the formation of the trunk of a spinal nerve, or single fibre passing from a

ganglion, really consists of several fibres coming from different and probably very distant parts. In other words, I am led to suppose that a single dark-bordered fibre, or rather its axis-cylinder, is the common channel for the passage of many different nerve-currents having different destinations. It is common to a portion of a great many different circuits. The fibres which result from the subdivision of the large fibre which leaves the cell become exceedingly fine (the $\frac{1}{100,000}$th of an inch in diameter or less), and pursue a very long course before they run parallel with other fibres. As the fibres which have the same destination increase in number, the compound trunk becomes gradually thicker and more distinct. The several individual fibres coalesce and form one trunk, or axis-cylinder, around which the protective white substance of Schwann collects. At the periphery the subdivision of the dark-bordered fibre again occurs, until peripheral fibres as fine as the central component fibres result*.

Fig. 4.

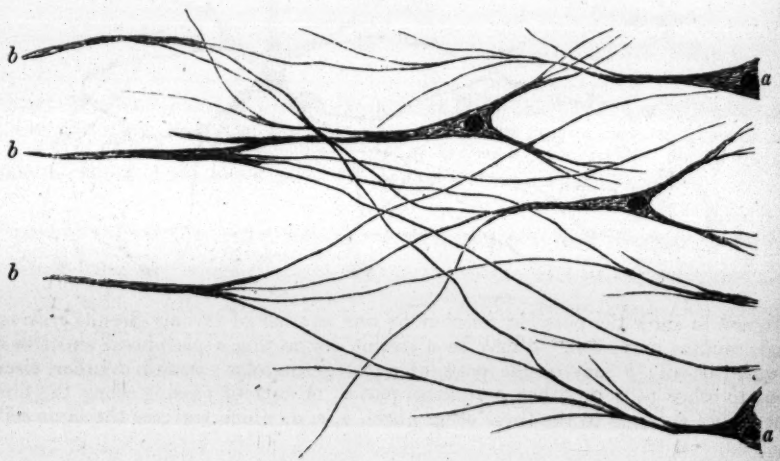


Diagram to show the course of the fibres which leave the caudate nerve-cells. *a*, *a* are parts of two nerve-cells, and two entire cells are also represented. Fibres from several different cells unite to form single nerve-fibres, *b*, *b*, *b*. In passing towards the periphery these fibres divide and subdivide; the resulting subdivisions pass to different destinations. The fine fibres resulting from the subdivision of one of the caudate processes of a nerve-cell may help to form a vast number of dark-bordered nerves, but it is most certain that *no single process ever forms one entire axis-cylinder.*

Although it may be premature to devise diagrams of the actual arrangement, if I permit myself to attempt this, I shall be able to express the inferences to which I have been led up to the present time in a far more intelligible manner than I could by description. But I only offer these schemes as rough suggestions, and feel sure that further observation will

* "General Observations upon the Peripheral Distribution of Nerves," my 'Archives,' iii. p. 234. "Distribution of Nerves to the Bladder of the Frog," p. 243. "Distribution of Nerves to the Mucous Membrane of the Epiglottis of the Human subject," p. 240.

enable me to modify them and render them more exact. The fibres would in nature be infinitely longer than represented in the diagrams. The cell below *c* (fig. 5) may be one of the caudate nerve-cells in the anterior root of a spinal nerve, that above *b* one of the cells of the ganglion upon the posterior root, and *a* the periphery. I will not attempt to describe the course of these fibres until many different observations upon which I am now engaged are further advanced, but I have already demonstrated the passage of the fibres from the ganglion-cell into the dark-bordered fibres as represented in the diagram.

Fig. 5.

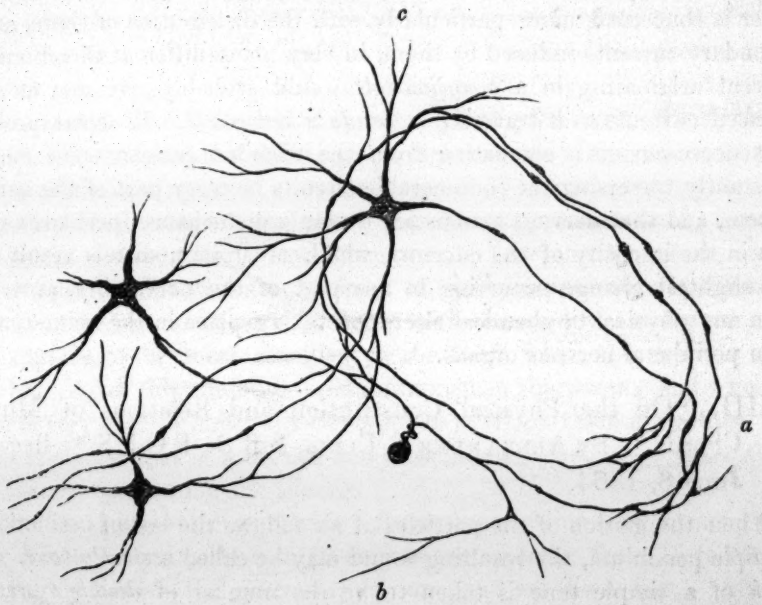


Diagram to show possible relation of fibres from caudate nerve-cells, and fibres from cells in ganglia, as, for example, the ganglia on the posterior roots. *a* is supposed to be the periphery; the cell above *b* one of those in the ganglion. The three caudate cells resemble those in the grey matter of the cord, medulla oblongata, and brain.

The peculiar appearance I have demonstrated in the large caudate cells, taken in connexion with the fact urged by me in several papers, that no true termination or commencement has yet been demonstrated in the case of any nerve, seems to me to favour the conclusion that the action of a nervous apparatus results from varying intensities of continuous currents which are constantly passing along the nerves during life, rather than from the sudden interruption or completion of nerve-currents. So far from any arrangement having been demonstrated in connexion with any nervous structure which would permit the sudden interruption and completion of a current, anatomical observation demonstrates the structural continuity of all nerve-fibres with nerve-cells, and, indirectly through these cells, with one another.

I venture to conclude that the typical anatomical arrangement of a nervous mechanism is not a *cord with two ends—a point of origin and a terminal extremity*, but a cord *without an end—a continuous circuit.*

The peculiar structure of the caudate nerve-cells, which I have described, renders it, I think, very improbable that these cells are *sources* of nervous power, while, on the other hand, the structure, mode of growth, and indeed the whole life-history of the rounded ganglion-cells render it very probable that they perform such an office. These two distinct classes of nerve-cells, in connexion with the nervous system, which are very closely related, and probably, through nerve-fibres, structurally continuous, seem to perform very different functions,—the one *originating* currents, while the other is concerned more particularly with the distribution of these, and of secondary currents induced by them, in very many different directions. A current originating in a *ganglion-cell* would probably give rise to many induced currents as it traversed a *caudate nerve-cell*. It seems probable that nerve-currents emanating from the rounded ganglion-cells may be constantly traversing the innumerable circuits in every part of the nervous system, and that nervous actions are due to a disturbance, perhaps a variation in the intensity of the currents, which must immediately result from the slightest change occurring in any part of the nerve-fibre, as well as from any physical or chemical alteration taking place in the nerve-centres, or in peripheral nervous organs.

XXIII. "On the Physical Constitution and Relations of Musical Chords." By Alexander J. Ellis, F.R.S., F.C.P.S.* Received June 8, 1864.

When the motion of the particles of air follows the law of oscillation of a simple pendulum, the resulting sound may be called a *simple tone*. The *pitch* of a simple tone is taken to be the number of *double* vibrations which the particles of air perform in one second. The greatest elongation of a particle from its position of rest may be termed the *extent* of the tone. The *intensity* or loudness is assumed to vary as the square of the extent. The tone heard when a tuning-fork is held before a proper resonance-box is simple. The tone of wide covered organ-pipes and of flutes is nearly simple.

Professor G. S. Ohm has shown mathematically that all musical tones whatever may be considered as the algebraical sum of a number of simple tones of different intensities, having their pitches in the proportion of the numerical series 1, 2, 3, 4, 5, 6, 7, 8, &c. Professor Helmholtz has established that this mathematical composition corresponds to a fact in nature, that the ear can be taught to hear each one of these simple tones separately, and that the character or *quality* of the tone depends on the law of the intensity of the constituent simple tones.

These constituent simple tones will here be termed indifferently *partial*

* The Tables belonging to this Paper will be found after p. 422.

tones or *harmonics*, and the result of their combination a *compound tone*. By the pitch of a compound tone will be meant the pitch of the lowest partial tone or *primary*.

When two simple tones which are not of the same pitch are sounded together, they will alternately reinforce and enfeeble each other's effect, producing a libration of sound, termed a *beat*. The number of these beats in one second will necessarily be the difference of the pitches of the two simple tones, which may be termed the beat *number*. As for some time the two sets of vibrations concur, and for some time they are nearly opposite, the compound extent will be for some time nearly the sum, and for some time nearly the difference of the two simple extents, and the *intensity* of the beat may be measured by the ratio of the greater intensity to the less.

But the beat will not be audible unless the ratio of the greater to the smaller pitch is less than 6 : 5, according to Professor Helmholtz. This is a convenient limit to fix, but it is probably not quite exact. To try the experiment, I have had two sliding pipes, each stopped at the end, and having each a continuous range of an octave, connected to one mouthpiece. The tones are nearly simple; and when the ratio approaches to 6 : 5, or the interval of a minor third, the beats become faint, finally vanish, and do not reappear. But the exact moment of their disappearance is difficult to fix, and indeed seems to vary, probably with the condition of the ear. The ear appears to be most sensitive to the beats when the ratio is about 16 : 15. After this the beats again diminish in sharpness; and when the ratio is very near to unity, the ear is apt to overlook them altogether. The effect is almost that of a broken line of sound, as —— —— —— ——, the spaces representing the silences.

Slow beats are not disagreeable; for example, when they do not exceed 3 or 4 in a second. At 8 or 10 they become harsh; from 15 to 40 they thoroughly destroy the continuity of tone, and are discordant. After 40 they become less annoying. Professor Helmholtz thinks 33 the beat number of maximum disagreeableness. As the beats become very rapid, from 60 to 80 or 100 in a second, they become almost insensible. Professor Helmholtz considers 132 as the limiting number of beats which can be heard. They are certainly still to be distinguished even at that rate, but become more and more like a scream. Though $f^{6}\sharp$ and g^{6} should give 198 beats in a second if $c=264$, and the interval is that for which the ear is most sensitive, I can detect no beats when these tones are played on two flageolet-fifes. Hence beats from 10 to 70 may be considered as discordant, and as the source of all discord in music. Beyond these limits they produce a certain amount of harshness, but are not properly discordant.

When the extent of the tones is not infinitesimal, Professor Helmholtz has proved that on two simple tones being sounded together, many other tones will be generated. The pitch of the principal and only one of these *combinational* tones necessary to be considered, is the difference of the pitch of its generating tones. It will therefore be termed the *differential*

tone. Its intensity is generally very small, but it becomes distinctly audible in beats. The differential tone is frequently acuter than the lower generator, and hence the ordinary name "grave harmonic" is inapplicable. As its pitch is the beat number of the combination, Dr. T. Young attributed its generation to the beats having become too rapid to be distinguished. This theory is disproved, first, by the existence of differential tones for intervals which do not beat, and secondly, by the simultaneous presence of distinct beats and differential tones, as I have frequently heard on sounding f^4, $f^4\sharp$, or even f^2, $f^2\sharp$ together on the concertina, when the beats form a distinct rattle, and the differential tone is a peculiar penetrating but very deep hum.

The object of this paper is to apply these laws, partly physical and partly physiological, to explain the constitution and relations of musical chords. It is a continuation of my former paper on a *Perfect Musical Scale**, and the Tables are numbered accordingly.

Two simple tones which make a greater interval than 6 : 5, and therefore never beat, will be termed *disjunct*. Simple tones making a smaller interval, and therefore generally beating, will be termed *pulsative*. The unreduced ratio of the pitch of the lower pulsative tone for which the beat number is 70 to that for which it is only 10, will be termed the *range* of the beat. The fraction by which the pitch of the lower pulsative tone must be multiplied to produce the beat number, will be termed the beat *factor*. The ratio of the pitches of the pulsative tones, on which the sharpness of the dissonance depends, will be termed the beat *interval*.

A compound tone will be represented by the absolute pitch of its primary and the relative pitches of its partial tones, as *C* (1, 2, 3, 4, ...). As generally only the relative pitch of two compound tones has to be considered, the pitches will be all reduced accordingly. Thus, if the two primaries are as 2 : 3, the two compound tones will be represented by 2, 4, 6, 8, 10, ..., and 3, 6, 9, 12, 15 . . . . The intensity of the various partial tones differs so much in different cases, that any assumption which can be made respecting them is only approximative. In a well-bowed violin we may assume the extent of the harmonics to vary inversely as the number of their order. Hence, putting the extent and intensity of the primary each equal to 100, we shall have, with sufficient accuracy—

Harmonics...	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Extent...	100,	50,	33,	25,	20,	17,	14,	12,	11,	10.
Intensity...	100,	25,	11,	6,	4,	3,	2,	1,	1,	1.

It will be assumed that this law holds for all combining compound

* Proceedings of the Royal Society, vol. xiii. p. 93. The following misprints require correction:—P. 97, line 7 from bottom, for c^2 read *b*. Table I, p. 105, diminished 5th, example, *read* f : B ; minor 6th, logarithm, *read* ·20412; Pythagorean Major 6th, *read* 27 : 16, $3^3 : 2^4$; Table V., col. VI., last line, *read* †e♯ †g♯ †b♯.

tones, the intensity of the primary in each case being the same. The results will be sufficient to explain the nature of chords on a quartett of bowed instruments, but may be much modified by varying the relative intensities of the combining tones.

On examining a single compound tone, we may separate its partial tones into two groups: the first *disjunct*, which will never beat with each other; the second *pulsative*, which will beat with the neighbouring disjunct tones. Thus

Disjunct..	1,	2,	3,	4,	5,	6,	–,	8,	–,	10,	–,	12,	–,	–,	–,	16,
Pulsative.	–,	–,	–,	–,	–,	–,	7,	–,	9,	–,	11,	–,	13,	14,	15,	–,

Disjunct..	–,	–,	–,	20,	–,	–,	–,	24,	–,	–,	–,	–,	–,	30.		
Pulsative.	17,	18,	19,	–,	21,	22,	23,	–,	25,	26,	27,	28,	29,	–.		

When any compound tone therefore developes any of the harmonics above the 6th, there may, and probably will, be beats, producing various degrees of harshness or shrillness, jarring or tinkling. These, however, are all *natural* qualities of tone, that is, they are produced at once by the natural mode of vibration of the substances employed. But if we were to take a series of *simple* tones having their pitches in the above ratios, and to vary their intensities at pleasure, we should produce a variety of *artificial* qualities of tone, some of which might be coincident with natural qualities, but most of which would be new. This method of producing artificial qualities of tone is difficult to apply, but has been used with success by Professor Helmholtz to imitate vowel-sounds, &c.

If, however, instead of using so many simple tones, we combine a few compound tones, the pitches of which are such that their primaries might be harmonics of some other compound tone, then the two sets of partial tones will necessarily combine into a single set, which may, or rather must be considered by the ear as the partial tones of some new compound tone, having very different intensities from those possessed by the partial tones of either of the combining compound tones. That is, an artificial quality of tone will have been created by the production of these *joint* harmonics. *Such an artificial quality of tone constitutes what is called a musical chord.* The two or more compound tones from which it is built up are its *constituents*. The primary joint harmonic is the real *root* or *fundamental bass* of the chord, which often differs materially from the supposititious root assigned by musicians.

If the primaries of the constituents are disjunct, and all their partial tones are disjunct, then the joint harmonics will be also disjunct, unless some pulsative differential tones have been introduced. If, however, the constituents have pulsative partial tones, the chord will also have them. Such chords, which are generally without beats, and are only exceptionally accompanied by beats, are termed *concords*, and they are *unisonant* or *dissonant* according as the beats are absent or present. Their character therefore consists in having the pitches of their constituents as 1, 3, 5, or as

these numbers multiplied by various powers of 2, that is, as 1, 3, 5, or their octaves.

If any of the constituents is pulsative the chord will generally have beats, but may be exceptionally without beats. Such chords are termed *discords*. Their character consists in having *two* or more of the pitches of their constituents as 1, 3, 5, or their octaves, and at least *one* of them as 7, 9, or some other pulsative tones, or their octaves. What pulsative tones should be selected depends on the sharpness of the dissonance which it is intended to produce, and therefore on the interval of the beat which is created. Thus, since $7:6=1\cdot16667$ and $8:7=1\cdot14286$ are both near the limit $6:5=1\cdot2$, the discord arising from 7 would be slight. Some writers have even considered the chord 1, 3, 5, 7 to be concordant. Again, $9:8=1\cdot125$ is rather rough, but $10:9=1\cdot11111$ is much rougher. Hence, if 9 is introduced, 10 should be avoided, that is, the octave of 5 should be omitted, which generally necessitates the omission of 5 itself, as in the chord 1, 3, 9. But $11:10=1\cdot1$ and $12:11=1\cdot09091$ are both so sharply dissonant, that if 11 is used neither 10 nor 12 should be employed. Now 10 is the octave of 5, and 12 is both the 3rd harmonic of 4 and the 4th harmonic of 3, and would therefore be produced from 3 and 4. Hence the use of 11 would forbid the use of 3, 4, and 5, that is, of the best disjunct tones. Hence 11 cannot be employed at all. Similarly, $13:12=1\cdot08333$ and $14:13=1\cdot07692$ are both extremely harsh. The latter is of no consequence, because 7 can be easily omitted. But even $15:13=1\cdot15384$ is more dissonant than $7:6$. Hence 13 would also beat with the harmonics of 3, 4, and 5. Consequently 13 must be also excluded. All combinations in which the differential tones 11 and 13 are developed will also be extremely harsh. As we therefore suppose that $14:13=1\cdot07692$ never occurs, and as $14:12=7:6$, the mildest of the dissonances, 14 may be used if 15 is absent, and thus $15:14=1\cdot07143$ avoided. When 14 and 15 are developed as harmonics of 7 and 5, and not as the primaries of constituent tones, their intensity will be so much diminished that the discord will not generally be too harsh. When 15 is used as a constituent, 14 and 16 should be avoided; that is, 7, and 1, 2 and 4, of which 14 and 16 are upper harmonics, should be omitted to avoid $15:14=1\cdot07143$ and $16:15=1\cdot06667$, which may be esteemed the maximum dissonance. By omitting 16 and 18, and thus avoiding $17:16=1\cdot0625$ and $18:17=1\cdot05882$ (that is, by not using 4, 8, or 9 as constituent tones), 17 becomes useful; for $17:15=1\cdot13333$ is milder than $9:8=1\cdot125$, which is by no means too rough for occasional use. The other pulsative harmonics, which are represented by prime numbers, are not sufficiently harmonious for use; but those produced from 2, 3, 5 (such as 25, 27, 45) may be sometimes useful, provided that the tones with which they form sharp dissonances are omitted.

The result of the above investigation is that the only pulsative tones suitable for constituents are 7, 9, 15, 17, 25, 27, 45, and their octaves.

The introduction of any one of these tones in conjunction with 1, 3, 5 and their octaves will therefore form a discord, the harshness of which may be frequently much diminished by the omission of 1 and its octaves for the constituents 7, 15, 17, by the omission of 5 for the constituent 9, and by the omission of 24 for the constituents 25, 27, 45.

Using the notation of my former paper, where $\zeta = 63:64$, and putting in addition vij$=84:85$, xj$=33:32$, xiij$=39:40$, $1\zeta = 255:256$, and xvij$=135:136$, the tones 1 to 18 may be represented by the following notes in terms of C^4:—

1,	2,	3,	4,	5,	6,	7,		8,	9,	10,
C^4,	C,	G,	c,	e,	g,	$\zeta b\flat$	or vij $a\sharp$,	c^2,	d^2,	e^2,

11,	12,	13,	14,		15,	16,	17,	
xj f^2,	g^2,	xiij a^2,	$\zeta b^2\flat$	or vij $a^2\sharp$,	b^2,	c^4,	$1\zeta d^4\flat$	or xvij $c^4\sharp$,

18,	20,	24,	25,	27,	45,
d^4,	e^4,	g^4,	$\ddagger g^4\sharp$,	$\dagger a^4$,	$f^4\sharp$.

This notation will show what are the musical names of the constituents of musical chords, and how they may be approximately produced on an organ, harmonium, or pianoforte.

By the *type* of a musical chord is meant the numbers which express the relative pitches of its constituents, after such octaves below them have been taken as to leave only uneven numbers, which are then called the *elements* of the type. By the *form* of the chord is meant the numbers before such reduction. Thus the *type* 1, 3, 5 embraces, among others, the *forms* 1, 3, 5; 1, 2, 3, 5; 2, 3, 5; 4, 3, 5; 3, 8, 10; 6, 10, 16; 2, 5, 6, 8, and so on; hence the types of musical chords consist of groups of the elements 1, 3, 5, 7, 9, 15, 17, 25, 27, 45. The type of a concord is 1, 3, 5, and of a discord 1, 3, 5, P, or 1, 3, 5, P, P', where P, P' are any of the numbers 7, 9, 15, 17, 25, 27, 45. Discords may be divided into *strong* and *weak*, according as those disjunct tones with which the pulsative tones principally beat are retained or omitted. These discords again may be distinguished into those which have one or two pulsative constituents. The chords may also be grouped according to the number of elements in their type, *dyads* containing two, *triads* three, *tetrads* four, and *pentads* five. The number of elements in the type by no means limits the numbers of constituents, as any octaves above any of the elements may be added.

Hence it is possible to classify all the suitable chords of music according to their type, as in Table VI., where the notes corresponding to each type are added in the typical form only. A simple systematic nomenclature is proposed in an adjoining column, and the names by which the true chords or their substitutes are known to musicians are added for reference. Occasionally two forms of substitution are given, as they are of theoretical importance, although confounded on some tempered instruments. A mode of symbolizing the chords is subjoined, in which several types are classed under one family. A capital letter shows the root of major chords, either

complete or imperfect, and of strong discords, and a smaller letter gives the root of weak discords, a number pointing out the family. In the minor triad the characteristic number is omitted; thus *c* is written for 15*c*, meaning the minor triad *g e b*, which is the major tetrad 15 *C*, or *C G E B*, with its root *C* omitted, and is usually called "the minor chord of *e*," a nomenclature which conceals its derivation.

Although chords of the same type have the same general character, this is so much modified by the particular forms which they can assume, that it is necessary to examine these forms in detail. They may be distinguished as *simple* and *duplicated*. In the former the number of constituents is the same as in the type; thus 4, 5, 6; 2, 3, 5 are simple forms of the type 1, 3, 5. In the latter, the number of constituents is increased by the higher octaves of some or all of them; thus 1, 2, 3, 5; 2, 4, 5, 6 are duplicated forms of 1, 3, 5 and 2, 5, 6, as they contain the octaves 1, 2 and 2, 4.

The mode in which the effect of any or all of these combinations may be calculated is shown in Table VII., which consists of two corresponding parts, each commencing with a column containing the "No. of J. H.," or of the joint harmonics resulting from the combination of the harmonics of the constituent compound tones. The next columns are headed by the relative pitch of the constituent tones, and contain their harmonics, never extending beyond the 8th, arranged so that their pitch is opposite to the corresponding number of the joint harmonic. It is thus seen at a glance which harmonics of the constituents are *conjunct* or tend to reinforce each other, and produce a louder joint harmonic, and also which are disjunct and pulsative. In the second part of the Table the extent of each harmonic of each constituent is given on the assumptions already explained. To find the extent of the joint harmonic, we add the extents of the generating conjunct harmonics, and thence find the intensity by squaring and dividing by 100. The differential tones must then be found by subtracting the pitches of the primaries (or in exceptional cases of higher and louder harmonics). The intensity of these differential tones may be called 1 for a single tone, and 4 for two concurrent tones, and this number may be subscribed to the intensity of the corresponding joint harmonic, as 0_1, 25_4.

The beat intervals have next to be noted, and the beat factors, which are usually the reciprocal of the relative pitch of the lower pulsative harmonic. Thus for the dyad 3, 4 the beat interval is $\frac{1}{30}$, and the beat factor $\frac{1}{3}$. From this factor, or $1 : f$, we calculate the range $P : p = 70f : 10f = 210 : 30$ in the present case. This must not be reduced, as it shows that the interval is dissonant when the pitch of the lower tone is between 30 and 210. To find the intensity, we add and subtract the extents of the pulsative joint harmonics; in this case 50 and 33 are the extents of the 8th and 9th joint harmonics, and their sum and difference are 83 and 17. Then we take the ratio of their squares, each divided by 100, which gives 69 : 3. This result must not be reduced, as it gives not only the relative loudness of the swell and fall, but also the loudness of these in relation to the other

joint harmonics. It must be remembered that when there are several disjunct harmonics, their unbroken sound tends to obliterate the action of the beats. There is no sensible silence between the beats unless the tones are simple and the intensities nearly equal. The intensities of the beats between joint harmonics and differential tones cannot be reduced to figures. It is not large. The history of a beat is therefore given by four fractions, which in this case are the interval 9 : 8, the factor 1 : 3, the range 210 : 30, and the intensity 69 : 3.

These calculations have been made for concordant dyads in Table VIII., and for concordant or major triads in Table IX. An attempt has been made to arrange the 13 forms of the first, and the 20 forms of the second in order of sonorousness, by considering the distribution of the intensities among the several joint harmonics, the development of pulsative differential tones, and the nature of the beats, omitting those due to the seventh harmonic of an isolated constituent. It has not been thought necessary to give the history of every beat. The intervals of all the beats are seen at a glance by the list of intensities of the joint harmonics.

By Table VIII. we see that the only unisonant dyad is the octave 1, 2*, which will be as unisonant as the constituents themselves. All other dyads are occasionally dissonant. Thus the fifth itself is decidedly dissonant when the pitch of the lower constituent lies between 20 and 140. On a bass concertina tuned justly, I find the fifth, C^4G^4, quite intolerable, the fifth, $C\,G$, rough, but $D\dagger A$ nearly smooth, and at higher pitches there is no perceptible dissonance. The beat interval of the major third is 16 : 15, and the range of dissonance is much greater. The roughness can be distinctly heard as high as $c\,e$; in the lower octaves CE is quite discordant, and C^4E^4 intolerable. This Table VIII., therefore, establishes the fact that concordance does *not* depend on simplicity of ratio alone; but when the denominator of the beat factor is small the range is lower, and therefore the dissonance less felt. Dissonance also arises from the pulsative differential tones 7 and 11, so that if the relative pitches are expressed in terms high enough to differ by 7 and 11, the combination will be dissonant. The ear is also not satisfied with forms in which great intensities of joint harmonics are widely separated by many small intensities. The four last forms in Table VIII., namely, the minor tenth 5, 12, the eleventh 3, 8, and the two thirteenths 3, 10 and 5, 16, should therefore be treated as discords. The Table also suggests how defects may be remedied by introducing new constituents to fill up gaps, or by duplications.

Similar observations apply to the triads in Table IX. None of them can be unisonant at all pitches. Some of them, as the last seven, are really discordant. The gaps may be generally filled up by duplication. Thus

* That is, within the limits of the Table. Dyads such as 1, 2; 1, 3; 1, 4; 1, 5; 1, 6 are all unisonant; but when the interval is very large, the want of connexion between the tones renders them unpleasant. The dyad 1, 8 which developes the differential tone 7 is dissonant.

1, 3, 5 may be converted into 1, 2, 3, 5, and by thus strengthening the 2, 4, and 8 joint harmonics the finest form of concord is produced. In this way the series of duplications in Table X. was produced. In this Table an example has been added to each form to facilitate trial; but the great imperfection of the major third in the ordinary system of tuning pianos and harmoniums materially deteriorates the effect of the chords, which ought to be taken on some justly tuned instrument.

The discords may be deduced from Tables VII. and VIII., when properly extended, by supposing 7, 9, 15, 17, 25, 27, 45 to be used in the first, and their effect allowed for in the second. The additional discordant effect of 7 will be necessarily least felt where 7 occurs as a differential tone, but these are not the best forms of either triad or tetrad. In the better forms the dissonances 6, 7 and 7, 8 will always be well developed, and as the latter is sharper, the omission of 8, at least as a constituent tone, is suggested. If $7\frac{1}{9}$ is used instead of 7, the omission of 8 becomes more urgent, while 6, $7\frac{1}{9}$ will beat less sharply than 6, 7, and therefore almost inaudibly. The real beats of the constituents 6, $7\frac{1}{9}$ arise from the harmonics $6 \cdot 6$, $5 \cdot 7\frac{1}{9}$, or 36, $35\frac{5}{9}$, which are, however, not so much felt as those of $6 \cdot 6$, $5 \cdot 7$ arising from 6, 7, because $36 : 35\frac{5}{9} = 1{\cdot}0125$ is further from $16 : 15 = 1{\cdot}0667$ than is $36 : 35 = 1{\cdot}0286$. Hence, when 8 is omitted, the dissonance arising from $7\frac{1}{9}$ is less than that arising from 7 itself. When 8 is present, 7 or $7\frac{1}{32}$ is superior to $7\frac{1}{9}$. The use of $17\frac{1}{15}$ for 17 would hardly create any perceptible alteration of roughness when 18 is absent, and when 18 is present $18 : 17\frac{1}{15} = 1{\cdot}0548$ is further from $16 : 15$ than is $18 : 17 = 1{\cdot}0588$, and therefore the roughness is not quite so great.

Of all discords the least dissonant is the minor triad 3, 5, 15, which is formed from the tetrad 1, 3, 5, 15 by omitting the root 1, to avoid the dissonance 15, 16. When the differential tones derived from the primaries of the constituents are deeper than the primaries, and therefore merely indicate the presence of a pulsative tone, which is only faintly realized by the differential tones derived from the upper harmonics of the primaries, and when the dissonant intervals of the minor tenth and major thirteenth 5, 12 and 3, 10 are not present in the constituent tones, this chord may be treated as a concord. But in most positions the minor triad is sensibly dissonant, as shown in Table XI., where an attempt has been made to arrange its 20 forms in order of sonorousness. The pitches of the differential tones are added, and examples subjoined. The effect of the minor chord is very much injured by the usual tuning of harmoniums, &c. A peculiar character of these and other discords, when the pulsative constituent is not the highest, consists in the quality of tone being due to very high joint harmonics, except such as are due to differential tones. The root will be consequently extremely deep when the constituent tones are taken at a moderate absolute pitch. This great depth renders its recognition by the ear difficult. Hence probably the disputes of musicians concerning the roots of certain discords, and their error in considering 5 to be

an octave of the root of the minor triad, so that *e*, *g*, *b* or 10, 12, 15 is considered by them as derived from E^4 instead of C^3.

Chords will evidently be related to each other when one or more of their constituents are identical, and natural qualities of tone will be related which have one or more identical harmonics, or which form parts of related chords. Transitions between related chords and compound tones will be easy and pleasing. Hence, in forming a collection of compound tones for use either as natural qualities of tone (in melody) or as constituents of artificial qualities of tone, that is, chords (in harmony), it is important to select such as will have numerous relations, and will produce the concordant dyads and triads, and the least dissonant discord, the minor triad. Hence, taking the concordant major triad 1, 3, 5 as a basis, we must possess its products by 2, 3, and 5. There must be abundant multiples by 2 in order to take the several forms of the triad and to introduce the duplications. The products by 3 and 5 give 3, 9, 15 and 5, 15, 25. We have then the tones 1, 3, 5, 9, 15, 25, and their octaves. These give three concordant major triads, 1, 3, 5; 3, 9, 15; and 5, 15, 25, each of which has one constituent in common with each of the others. We have also the major pentad 1, 3, 5, 9, 15, the nine-tetrad 1, 3, 5, 9, the major tetrad 1, 3, 5, 15, and the minor tetrad 3, 5, 9, 15, whence, by omissions, result the nine-triad 1, 3, 9 and minor triad 3, 5, 15. Each of these triads is related to two of the three major triads. The minor triad is intimately related to all three major triads by having two constituents in common with each of them. The discords involving 7 and 17 would evidently require 1, 3, 5, 7, 17 to be taken as a basis. Neglecting these discords for the present, the above results show that we should obtain a useful series of tones by multiplying 1, 3, 5 successively by 3, and each product by 5, taking octaves above and below all the tones thus introduced. We thus find

MAJOR.	*Minor.*	MAJOR.	MAJOR.	*Minor.*	MAJOR.
			1, 3, 5	3, 5, 15	5, 15, 25
1, 3, 5			F C A		
3, 9, 15	3, 5, 15	5, 15, 25	C G E	*c a e*	A E ♯C♯
9, 27, 45	9, 15, 45	15, 45, 75	G D B	*g e b*	E B ♯G♯
27, 81, 135	27, 45, 135	45, 135, 225	D †A F♯	*d b f♯*	B F♯ ♯D♯
81, 243, 405			†A †E C♯		

Any of the smaller numbers may be multiplied by 2, 4, 8, 16, 32, 64, 128, 256, in order to compare them with the larger numbers. Such multiplications are presumed to have been made in the columns of notes. Hence

$\dagger A : A = 81 : 5.16$, or $\dagger = 81 : 80$,

$F\sharp : F = 135 : 1.128$, or $\sharp = 135 : 128$,

$\ddagger C\sharp : C = 25 : 3.8$, or $\ddagger\sharp = 25 : 24$, whence $\ddagger = 80 : 81$.

And in the same way the other ratios in 'Proceedings,' vol. xiii. p. 95, are reproduced.

In addition to the chords already noticed, we have now the twenty-seven tetrad, 1, 3, 5, 27, or *F C A D*, and the twenty-seven triad, 1, 5, 27, or *F A D*, and all the discords derived from 1, 3, 5, 9, 15, 25, 27, 45. But for those derived from 7 and 17 substitutes must be employed. These are obtained as follows. The chord 9, 27, 45, 1 . 64 is 9 times 1, 3, 5, $7\frac{1}{9}$, so that *G D B F* approximates to 1, 3, 5, 7 in a manner already tested. Again, 1 . 32, 3 . 32, 5 . 32, 225 is 32 times 1, 3, 5, $7\frac{1}{32}$, whence *F C A* ‡*D*♯ gives the second and closer approximation to 1, 3, 5, 7 already considered. When $7\frac{1}{9}$ is used for 7 it will be better to use 1, 3, 5, $7\frac{1}{9}$, $8\frac{8}{9}$, or one-ninth of 9, 27, 45, 1 . 64, 5 . 16, that is *G B D F A*, in place of 1, 3, 5, $7\frac{1}{9}$, 9 or one-ninth of 9, 27, 44, 1 . 64, 81, that is *G B D F* †*A*, to avoid the dissonance 5 . $7\frac{1}{9}$, 4 . 9, or $35\frac{5}{9}$, 36. This will therefore replace the seven-nine pentad 1, 3, 5, 7, 9.

The chord 45, 135, 225, 5 . 64, 3 . 256 is 45 times 1, 3, 5, $7\frac{1}{9}$, $17\frac{1}{17}$, or *B F*♯ ‡*D*♯ *A C*, and it forms an excellent substitute for the seven-seventeen pentad 1, 3, 5, 7, 17. Again, the chord 3 . 16, 5 . 16, 15 . 16, 135, or 16 times 3, 5, 15, $16\frac{7}{8}$, that is *C A E F*♯, is a sufficiently close approximation to the rough discord 3, 5, 15, 17.

It has already been shown that the alterations in the discords thus produced will be slight, and, under certain circumstances, improvements. The omission of 7, 17 in the base 1, 3, 5 is therefore justified. Their insertion would embarrass the performer and composer by an immense variety of tones very slightly differing from each other, as 64, 63; 135, 136; 255, 256. As it is, the distinction between 81, 80 is the source of much difficulty, and separates chords such as 81, 243, 405, and 5 . 16, 15 . 16, 25 . 16, or 80, 240, 400, that is, †*A* †*E C*♯ and *A E* ‡*C*♯, which composers desire to consider as identical. It was shown in my former paper (Proceedings, vol. xiii. p. 98) that the use of 1, 3, 5 as a basis requires 72 different tones, exclusive of octaves. The introduction of 7 in the base would increase this number by 45, and the introduction of 17 by 30, while the mental effect produced would be very slightly different. On the other hand, if instead of 1, 3, 5 as a base, we took 1, 2*v*, 4*T*, where *v*, *T* are ratios differing slightly from 3 : 2 and 5 : 4, we might avoid the ratio 81 : 80, reduce the number of tones to 27, and greatly increase the relations of chords. How to effect this important result with the least dissonant effect will be considered in the following paper on *Temperament*.

The three major triads 1, 3, 5; 3, 9, 15; 9, 27, 45 are so related as to form two major pentads, 1, 3, 5, 9, 15 and 3 . 1, 3 . 3, 3 . 5, 3 . 9, 3 . 15. Hence the middle triad forming part of both pentads connects the three triads into a whole, closely related to the middle triad, and therefore to its root. These are called the *tonic* chord and *tonic* tone, and the connexion itself is termed *tonality*. If octaves of these tones be taken, thus,

1 . 32,	3 . 8,	5 . 8 or	*F C A*,
3 . 8,	9 . 4,	15 . 2	*C G E*,
9 . 4,	27,	45	*G D B*,

and the results be taken in order of pitch, we find, on supplying the second octave 3 . 16,

24,	27,	30,	32,	36,	40,	45,	48
C,	*D*,	*E*,	*F*,	*G*,	*A*,	*B*,	*c*.

In this series any two consecutive tones, except 40, 45 or *A*, *B*, belong to the same major pentad, and these are therefore eminently adapted for successions of chords. Even 40, 45, or 5 . 8, 45, belong to two related discords; for 1, 3, 5, 9, or *F*, *C*, *A*, *G*, and 1, 3, 5, 27, or *F C A D*, have each two constituents in common with 9, 27, 45, 1 . 64, or *G D B F*. The discord 3, 5, 15, 45, or *C A E B*, contains both the tones in question. These considerations justify the major diatonic scale.

The last discord contains a minor triad, 3, 5, 15. These minor triads, from their relations to three major triads, are evidently peculiarly adapted to introduce successions of harmonies. Taking then the three minor triads and forming their octaves, thus

3 . 64,	5 . 32,	15.8 or c^2 *a e*,
9 . 16,	15 . 8,	45.4 *g e b*,
27 . 8,	45 . 2,	135 d^2 *b* $f\sharp$,

we may extend them into a scale,

120,	135,	144,	160,	180,	192,	216,	240
e,	$f\sharp$,	*g*,	*a*,	*b*,	c^2,	d^2,	e^2,

where the chordal relations are even more intimate than before, and by means of the chord 45, 135, 225, 5 . 64, 3 . 256, or *B* $F\sharp$ $D\sharp$ *a c*, already noticed, the major triad, 45, 135, 225, or *B* $F\sharp$ $D\sharp$, is brought into close connexion with the minor triad, 3, 5, 15, or *c a e*. Practically the use of the minor scale consists of a union of four major triads, 1, 3, 5; 3, 9, 15; 9, 27, 45; 27, 81, 135, forming two major scales, with three other major triads, 5, 15, 25; 15, 45, 75; 45, 135, 225, forming a third major scale, by means of three minor triads, 3, 5, 15; 9, 15, 45; 27, 45, 135, the roots of which, 1, 3, 9, are the same as the roots of the first three major triads. There are therefore seven roots to all the chords introduced, namely 1, 3, 9, 27, and 5, 15, 45, or *F*, *C*, *G*, *D* and *A*, *E*, *B*, and these seven roots form a major diatonic scale. From these relations spring all the others which distinguish the minor scale together with all its various forms and its uncertain tonality, which is generally assumed to be the relation of the chords to 15 or *E*, the tonic of the last three major triads, but which evidently wavers between this and 3, 9 or *C*, *G*, the tonics of the first four major triads, and these three tonics, 3, 9, 15, or *C G E*, form a major triad.

By extending this system of chords up and down, right and left, we arrive at the perfect musical scale in Table V. (Proceedings, vol. xiii. p. 108), which is therefore entirely justified on physical and physiological grounds, without any of those metaphysical assumptions or mystical attributes of numbers which have hitherto disfigured musical science. In that Table the

chords have been arranged in the forms 4, 5, 6 and 10, 12, 15, in accordance with the usual practice of musicians. In the present paper the typical 1, 3, 5 and 3, 5, 15 have, for obvious reasons, been made the basis of the arrangement.

XXIV. "On the Temperament of Musical Instruments with Fixed Tones." By ALEXANDER J. ELLIS, F.R.S., F.C.P.S.* Received June 8, 1864.

In the preceding paper on the Physical Constitution of Musical Chords (Proceedings, vol. xiii. p. 392), of which the present is a continuation, I drew attention to the importance of abolishing the distinction between tones which differ by the comma 81 : 80, on account of the number of fresh relations between chords that would be thus introduced. The contrivances necessary for this purpose have long been known under the name of Temperament. I have shown that the musical scale which introduces the comma consists of tones whose pitch is formed from the numbers 1, 3, 5, by multiplying continually by 2, 3, and 5. Hence to abolish the comma it will be necessary to use other numbers in place of these. But this alteration will necessarily change the physical constitution of musical chords, which will now become approximate, instead of exact representatives of qualities of tone with a precisely defined root. It is also evident that all the conjunct harmonics will be thus rendered pulsative, and that therefore all the concords will be decidedly dissonant at all available pitches. The result would be intolerable if the beats were rapid. Temperament, therefore, only becomes possible because very slow beats are not distressing to the ear. Hence temperament may be defined to consist in slightly altering the perfect ratios of the pitch of the constituents of a chord, for the purpose of increasing the number of relations between chords, and facilitating musical performance and composition by the reduction of the number of tones required for harmonious combinations.

The subject has been frequently treated †, but the laws of beats and

* The Tables belonging to this Paper will be found after p. 422.

† I have consulted the following works and memoirs. *Huyghens*, Cosmotheoreos, lib. i.; Cyclus Harmonicus. *Sauveur*, Mémoires de l'Académie, 1701, 1702, 1707, 1717. *Henfling*, Miscellanea Berolinensia, 1710, vol. i. pp. 265–294. *Smith*, Harmonics, 2nd edit. 1759. *Marpurg*, Anfangsgruende der theoretischen Musik, 1757. *Estève*, Mém. de Math. présentés à l'Acad. par divers Savans, 1755, vol. ii. pp. 113–136. *Cavallo*, Phil. Trans. vol. lxxviii. *Romieu*, Mém. de l'Acad., 1758. *Lambert*, Nouveaux Mém. de l'Acad. de Berlin, 1774, pp. 55–73. *Dr. T. Young*, Phil. Trans. 1800, p. 143; Lectures, xxxiii. *Robison*, Mechanics, vol. iv. p. 412. *Farey*, Philosophical Magazine, 1810, vol. xxxvi. pp. 39 and 374. *Delezenne*, Recueil des Travaux de la Société des Sciences, &c. de Lille, 1826–27. *Woolhouse*, Essay on Musical Intervals, 1835. *De Morgan*, On the Beats of Imperfect Consonances, Cam. Phil. Trans. vol. x. p. 129. *Drobisch*, Ueber musikalische Tonbestimmung und Temperatur, Abhandlungen

composition of tones discovered by Prof. Helmholtz have enabled me to present it in an entirely new form, and to determine with some degree of certainty what is the best possible form of temperament.

Let the compound tones P and Q, of which P is the sharper, form the concordant interval $p:q$. Then $P:Q=p:q$, or $qP=pQ$, that is, the qth harmonic of P and the pth harmonic of Q are conjunct. Now let P be changed into $P.(1+t)$, where t is small, and rarely or never exceeds $\frac{1}{80}=\cdot0125$. Then the qth harmonic of $P.(1+t)$ will be $qP.(1+t)$ and will no longer $=pQ$. The difference between the pitch of these harmonics is $qP.(1+t)-pQ=qt.P=pt.Q$. Hence the number of beats in a second produced by this change in P will be found by multiplying the lower pitch Q by pt, which is therefore the beat factor, and will be positive or negative according as the pitch of P is increased or diminished, or the interval is sharpened or flattened. The other beats which existed between the joint harmonics of the dyad P, Q may be increased or diminished by this change, but in either case so slightly that they may be left out of consideration in comparison with the beats thus introduced. But the differential tone which was $P-Q$ becomes $Pt+P-Q$, and is therefore a tone which is entirely unrelated to the original chord, and which may become prominently dissonant. This is an evil which cannot be avoided by any system of temperament, and is about equally objectionable in all systems. It may therefore be also left out of consideration in selecting a temperament.

The melody will also suffer from the alteration in the perfect ratios. An interval is best measured by the difference of the tabular logarithms of the pitches of the two tones which form it. Hence the *interval error* $\epsilon=\log[P.(1+t)\div Q]-\log[P:Q]=\log(1+t)=\mu t$, if the square and higher powers of t be neglected, and μ be the modulus. Hence the beat factor which $=pt$, will $=p\epsilon\div\mu$, or $\propto p\epsilon$. I call $p\epsilon$ the *beat meter*, and represent it by β.

We may assume that the dissonance created by temperament $\propto\beta^2$. Hence for the same just interval $p:q$, variously represented in different temperaments, the dissonance $\propto\epsilon^2$. That is, the harmony varies inversely as β^2, and the melody varies inversely as ϵ^2. Hence for the same interval the harmony and melody both vary inversely as ϵ^2. The *general* harmony and melody may be assumed to be best when $\Sigma\beta^2$ and $\Sigma\epsilon^2$ are minima, which will not happen simultaneously.

The following contractions for the names of the principal intervals will

der k. Sächsischen Gesellschaft der Wissenschaften, vol. iv. Nachträge zur Theorie der musikalischen Tonverhältnisse, ibid. vol. v. Ueber die wissenschaftliche Bestimmung der musikalischen Temperatur, Poggendorff's Annalen, vol. xc. p. 353. *Naumann*, Ueber die verschiedene Bestimmung der Tonverhältnisse und die Bedeutung des Pythagoreischen oder reinen Quinten-Systems für unsere heutige Musik, 1858. *Helmholtz*, Die Lehre von den Tonempfindungen, 1863. I am most indebted to Smith, Drobisch, and Helmholtz.

be used throughout this paper. See also the last columns in Tables XII. and XIV.

Sign.	Interval.	Example.	Sign.	Interval.	Example.
Ist.	Unison	c c			
IInd.	Major Second	c d	2nd.	Minor Second	e f
IIIrd.	Major Third	c e	3rd.	Minor Third	e g
IVth.	Augmented Fourth	f b	4th.	Perfect Fourth ..	c f
Vth.	Perfect Fifth	c g	5th.	Diminished Fifth..	b f
VIth.	Major Sixth	c a	6th.	Minor Sixth	e c^2
VIIth.	Major Seventh ..	c b	7th.	Minor Seventh ..	g f
VIIIve.	Octave	c c^2			
IXth.	Major Ninth	c d^2	9th.	Minor Ninth	e f
Xth.	Major Tenth	c e^2	10th.	Minor Tenth	e g^2

In no system of temperament will it be possible to interfere with the octave, the only unisonant concord. Hence 2 will remain unchanged. Let the ratios of the tempered IIIrd and Vth be T, v, which will replace 5 : 4 and 3 : 2 throughout the system of chords. Hence if we take four successive perfect major triads in the form 4, 5,6 as $C\,E\,G$, $G\,B\,d$, $d f\sharp \dagger a$, $\dagger a\ c^2\sharp\ \dagger e^2$, and suppose them to be tempered so that the distinction between E and $\dagger E$ no longer exists, but that in each chord the pitch of the second and third tones are T and v times that of the first tone respectively, while the ratio of the octave remains unchanged, the ratio of each of the above tones to C will be as under:—

C,	E,	G,	B,	d,	$f\sharp$,	a,	$c^2\sharp$,	e^2
1,	T,	v,	Tv,	v^2,	Tv^2,	v^3,	Tv^3,	v^4.

Hence, since $e^2=4E$, we have $v^4=4T$ as the first condition of temperament, showing that we shall arrive at the same tone whether we take two VIIIves and a tempered IIIrd, or take four tempered Vths, as in Cc, cc^2, c^2e^2, and $C\,G$, Gd, da, ae^2. In this case the above ratios reduce to

C,	E,	G,	B,	d,	$f\sharp$,	a,	$c^2\sharp$,	e^2
1,	$\frac{1}{4}v^4$,	v,	$\frac{1}{4}v^5$,	v^2,	$\frac{1}{4}v^6$,	v^3,	$\frac{1}{4}v^7$,	v^4.

If we further call the interval of the mean tone m, the limma l, the sharp $\sharp$, the flat $\flat$, and the diesis δ, the above ratios give

$$m=\frac{d}{c}=\frac{d}{2C}=\frac{v^2}{2},$$

$$l=\frac{c}{B}=\frac{2C}{B}=\frac{2^3}{v^3},$$

$$\sharp=\frac{c^2\sharp}{c^2}=\frac{c^3\sharp}{4C}=\frac{v^7}{2^4};\qquad \flat=\frac{1}{\sharp}=\frac{2^4}{v^7},$$

$$\delta=\frac{d^2\flat}{c^2\sharp}=\frac{2v^2\times(2^4\div v^7}{v^7\div 4}=\frac{2^7}{v^{12}}.$$

Whence $m=\sharp l$, $l=\delta\sharp$, $m=\sharp\delta\sharp$, $m^5\,l^2=2$.

Hence all intervals and pitches can be expressed in terms of v. This further appears from arranging the 27 different tones required in tempered scales, in order of Vths, thus

$$\begin{gathered} a\flat\flat,\ e\flat\flat,\ b\flat\flat,\ f\flat,\ c\flat,\ g\flat,\ d\flat,\ a\flat,\ e\flat,\ b\flat, \\ f,\ c,\ g,\ d,\ a,\ e,\ b, \\ f\sharp,\ c\sharp,\ g\sharp,\ d\sharp,\ a\sharp,\ e\sharp,\ b\sharp,\ f\times,\ c\times,\ g\times. \end{gathered}$$

It will be obvious from Table V. (Proceedings, vol. xiii. facing p. 108), when the signs †‡ are omitted, that these 27 tones suffice for all keys from $C\flat$ to $C\sharp$. This also appears from observing that the complete key of C requires 7 naturals, 3 flats and 3 sharps, or 13 tones, and that one flat or sharp is introduced for each additional flat or sharp in the signature of the key. Hence for 7 flats and 7 sharps in the signature 14 additional tones are required, making 27 in all. The rarity of the modulations into $d\flat$, $g\flat$ or $c\flat$ minor enables us generally to dispense with the three tones $a\flat\flat$, $e\flat\flat$, $b\flat\flat$, and thus to reduce all music to 24 tones. The system of writing music usually adopted is only suitable to such a tempered scale, and therefore requires the addition of the acute and grave signs (†‡) to adapt it for a representation of the just scale founded on the numbers 1, 2, 3, 5.

To calculate the value which must be assigned to v so as to fulfil the conditions supposed to produce the least disagreeable system of temperament, it will be most convenient to use logarithms, and to put $\log v = \log\frac{3}{2} - x = \cdot 1760913 - x$. The above arrangement of the requisite 27 tones in order of Vths, therefore, enables us to calculate the logarithms of the ratios of the pitches of all the tones to the pitch of c in terms of x, by continual additions and subtractions of $\log v$, rejecting or adding $\log 2 = \cdot 3010300$, when necessary, to keep all the tones in the same VIIIve. The result is tabulated in Table XII., column T. From this we immediately deduce

$$\begin{aligned} \log m &= \log d - \log c = \cdot 0511526 - 2x \\ \log l &= \log f - \log e = \cdot 0226335 + 5x \\ \log \sharp &= \log f\sharp - \log f = \cdot 0285191 - 7x \\ \log \delta &= \log g\flat - \log f\sharp = -\cdot 0058851 + 12x. \end{aligned}$$

To find the interval errors, the just intervals must be taken for the commonest modulations into the subdominant and dominant keys, as explained in my paper on a Perfect Musical Scale (Proceedings, vol. xiii. p. 97). As the method of determining temperament here supposed makes the errors the same for the same intervals in all keys, that is, makes the temperament *equal*, it is sufficient to determine the interval errors for a single key. Hence the just intervals are calculated in Table XII., column J, for the key of C, and the interval error is given in column ϵ, in terms of x and $k = \log\frac{81}{80}$, the interval of a comma. From these interval errors the beat meters for the six concordant dyads are calculated in column β. To these are added the values of $\Sigma\,\epsilon^2$ and $\Sigma\,\beta^2$, also in terms

of x and k. If for k we put its value ·0053950, these last expressions become

$$\Sigma\epsilon^2 = \cdot 0009314 - 1\cdot 1437400x + 420x^2$$
$$\Sigma\beta^2 = \cdot 00043659 - 5\cdot 8158100x + 1998x^2.$$

Hence Table XII. suffices to give complete information respecting the effect of any system of temperament when x is known. The following are some of the principal conditions on which it has been proposed to found a system of temperament. I shall first determine the value of x and $\log v$ on these conditions, and then compare the results.

A. Harmonic Systems of Equal Temperament.

I. *Systems with two concords perfect.*

No. 1 (45)*. System of perfect 4ths and Vths.

Here $x=0$, $\log v = \cdot 1760913$.

This is the old Greek or Pythagorean system of musical tones, more developed in the modern Arabic scale of 17 tones. No nation using it has shown any appreciation of harmony.

No. 2 (2). System of perfect IIIrds and 6ths.

Here ϵ for III, or $k-4x=0$, $x=\frac{1}{4}k=\cdot 00134875$, $\log v=\cdot 17474255$. Hence $\log m=\log d=\cdot 0484551=\frac{1}{3}\log\frac{5}{4}=\frac{1}{2}(\log\frac{9}{8}+1\ \ \frac{1}{9})$, so that the tempered mean tone is an exact mean between the just major and minor tones. Hence this is known as the System of Mean Tones, or the *Mesotonic System*, as it will be here termed. It was the earliest system of temperament, and is claimed by Zarlino and Salinas. See also Nos. 13 and 19.

No. 3 (23). System of perfect 3rds and VIths.

Here ϵ for 3, or $-k+3x=0$, $x=\frac{1}{3}k=\cdot 0017983$, $\log v=\cdot 1742930$.

II. *Systems in which the harmony of two concords is equal.*

No. 4 (20). The IIIrd and Vth to the same bass; beat equally and in opposite directions†.

Here β for III $+\beta$ for V $=0$, or $(5k-20x)-3x=0$, $x=\frac{5}{23}k=\cdot 0011725$, $\log v=1749188$.

No. 5 (15). The 6th and Vth beat equally, and in the same direction‡.

Here β for $6=\beta$ for V, or $-8k+32x=-3x$, $x=\frac{8}{35}k=\cdot 0012331$, $\log v=\cdot 1748582$.

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* The number preceded by No. points out the order of the system in the present classification. The number in a parenthesis shows the position of the system in the comparative Table XV., which is explained hereafter (p. 418).

† That is, one interval is too great, or "beats sharp," and the other too small, or "beats flat."

‡ That is, both "beat sharp" or both "beat flat."

No. 6 (21). The IIIrd and 4th beat equally, and in the same direction.

Here β for III$=\beta$ for 4, or $5k-20x=4x$, $x=\frac{5}{24}k=\cdot0011239$, log $v=\cdot1749674$.

No. 7 (18). The 6th and 4th beat equally, and in opposite directions.

Here β for $6+\beta$ for $4=0$, or $(-8k+32x)+4x=0$, $x=\frac{2}{9}k=\cdot0011989$, log $v=1748924$.

No. 8 (16). The 3rd and Vth beat equally, and in the same direction.

Here β for $3=\beta$ for V, or $-6k+18x=-3x$, $x=\frac{3}{7}x=\cdot0015414$, log $v=\cdot1745199$. See No. 20.

No. 9 (13). The VIth and Vth beat equally, and in opposite directions.

Here β for VI$+\beta$ for V$=0$, or $(5k-15x)-3x=0$, $x=\frac{5}{18}k=\cdot0014986$, log $v=\cdot1745927$.

This coincides with Dr. Smith's system of equal harmony, as contained in the Table facing p. 224 of his 'Harmonics,' 2nd ed.

No. 10 (9). The 3rd and 4th beat equally, and in opposite directions.

Here β for $3+\beta$ for $4=0$, or $(-6k+18x)+4x=0$, $x=\frac{3}{11}k=\cdot0014713$, log $v=\cdot1746200$.

No. 11 (2). The VIth and 4th beat equally, and in the same direction.

Here β for VI$=\beta$ for 4, or $5k-15x=4x$, $x=\frac{5}{19}k=\cdot0014197$, log $v=\cdot1746716$.

III. *Systems in which the harmony of two concords is in a given ratio.*

No. 12 (24). The beats of the IIIrd and Vth are as 5 : 3, but in opposite directions.

Here β for III : β for V$=-5:3$, or $15k-60x=15x$, $x=\frac{1}{5}k=\cdot0010790$, log $v=\cdot1750123$.

M. Romieu gives this temperament under the title of "système tempéré de $\frac{1}{5}$ comma," Mém. de l'Acad. 1758. See No. 18.

No. 13 (2). The beats of the 3rd and Vth are as 2 : 1, and in the same direction.

Here β for 3 : β for V$=2$, or $-6k+18x=-6x$, $x=\frac{1}{4}k$, as in No. 2.

No. 14 (12). The beats of the 3rd and Vth are as 5 : 2, and in the same direction.

Here β for 3 : β for V$=5:2$, or $-12k+36x=-15x$, $x=\frac{4}{17}k=\cdot0012694$, log $v=\cdot1748219$. See No. 29.

IV. *Systems of least harmonic errors.*

No. 15 (7). The harmonic errors of all the harmonic intervals conjointly are a minimum.

This is determined by putting the sum of the squares of the beat meters,

or (by Table XII.) $150k^2-1078kx+1998x^2$=a minimum, which gives
$x=\frac{539}{1998}k$=·0014554, log v=·1746359.

If we had used the sum of the squares of the beat *factors*, we should have obtained an equation of 16 dimensions in v, which gives log v=·1746387. The difference between the two values of log v is not appreciable to the ear.

No. 16 (14). The harmonic errors of the 3rd, IIIrd, and Vth conjointly are a minimum.

Here (β for 3)^2+(β for III)^2+(β for V)^2, or
$(6k-18x)^2+(5k-20x)^2+9x^2$=a minimum, which gives
$x=\frac{208}{733}k$=·0015309, log v=·1744404.

No. 17 (6). The harmonic errors of the Vth and IIIrd conjointly are a minimum.

Here (β for V)^2+(β for III)^2, or $9x^2+(5k-20x)^2$=a minimum, $x=\frac{100}{409}k$=·0013190, log v=·1747723.

B. Melodic Systems of Equal Temperament.

V. *Systems of equal or equal and opposite interval errors.*

No. 18 (24). The interval errors of the IIIrd and Vth are equal and opposite.

Here ϵ for III+ ϵ for V=0, or $k-4x=x$, $x=\frac{1}{5}k$, as in No. 12.

No. 19 (2). The interval errors of the 3rd and Vth are equal.

Here ϵ for 3 =ϵ for V, or $-k+3x=-x$, $x=\frac{1}{4}k$, as in No. 2,

No. 20 (16). The interval errors of the IIIrd and 3rd are equal.

Here ϵ for III=ϵ for 3, or $k-4x=-k+3x$, $x=\frac{2}{7}k$, as in No. 8.

VI. *Systems in which the interval errors of two intervals are in a given ratio.*

No. 21 (17). The errors of the IIIrd and Vth are as 5 : 3, but in opposite directions.

Here ϵ for III : ϵ for V =$-5:3$, or $3k-12x=5x$, $x=\frac{3}{17}k$=·0015750, log v=·1745163.

This is the theoretical determination of M. Romieu's anacratic temperament (Mém. de l'Acad. 1758, p. 510), to which, however, he has in practice preferred No. 22.

No. 22 (29). The errors of the IIIrd and Vth are as 2 : 1, but in opposite directions.

Here ϵ for III : ϵ for V=-2, or $k-4x=2x$, $x=\frac{1}{6}k$=·0008975, log v=·1751938.

This is M. Romieu's anacratic temperament. See No. 21.

No. 23 (26). The errors of the IIIrd and Vth are as 1·94 : 1, and in opposite directions.

Here ϵ for III : ϵ for V $=-1{\cdot}94$, or $k-4x=1{\cdot}94x$, $x=\frac{100}{594}k=\cdot0009683$, $\log v=\cdot1751830$.

This is the temperament calculated by Drobisch (Nachträge, § 10) from Delezenne's conclusion (Rec. Soc. Lille, 1826–27, pp. 9 and 10), that the ear can detect an error of $\cdot284k$ in the IIIrd, and $\cdot146k$ in the Vth, which gives the comparative sensibility as $\cdot284 : \cdot146 = 1{\cdot}94$.

No. 24 (20). The errors of the IIIrd and 3rd are as 2 : 5, but in opposite directions.

Here ϵ for III : for ϵ 3 $=-2 : 5$, or $5k-20x=2k-6x$, $x=\frac{3}{14}k$ $=\cdot0011561$, $\log v=\cdot1749352$. See No. 27.

No. 25 (46). The errors of the 3rd and IIIrd are as 2 : 1, but in opposite directions, or the errors of the Vth and 3rd are equal and opposite.

Here ϵ for 3 : ϵ for III $=-2$, or $2k-6x=k-4x$, or else $x=-k+3x$; both give $x=\frac{1}{2}k=\cdot0026975$, $\log v=\cdot1733938$.

Here the error of the Vth reaches the utmost limit of endurance.

VII. *Systems of least melodic errors.*

No. 26 (1). The interval errors of all the melodic intervals conjointly are a minimum.

Here the sum of the squares of the 23 interval errors in Table XII., or $32k^2-212kx+420x^2=$ a minimum, $x=\frac{53}{210}k=\cdot0013616$, $\log v=\cdot1747297$.

No. 27 (20). The melodic errors of the IInd, IIIrd, 4th, Vth, VIth, and VIIth conjointly, are a minimum.

Here (ϵ for II)^2 + (ϵ for III)^2 + (ϵ for 4)^2 + (ϵ for V)^2 + (ϵ for VI)^2 + (ϵ for VII)^2, or $4x^2+(k-4x)^2+2x^2+(k-3x)^2+(k-5x)^2=$ a minimum, $x=\frac{3}{14}k$, as in No. 24.

This is Drobisch's "most perfect possible" (*möglich reinste*) temperament (Poggendorff's Annalen, vol. xc. p. 353, as corrected in Nachträge, § 7). It is only the "most perfect possible" for the major scale.

No. 28 (5). The melodic errors of the 3rd, IIIrd, and 4th conjointly are a minimum.

Here (ϵ for 3)^2 + (ϵ for III)^2 + (ϵ for 4)^2, or $(-k+3x)^2+(k-4x^2)$ $+x^2=$ a minimum, $x=\frac{7}{26}k=\cdot0014525$, $\log v=\cdot1746388$.

This is Woolhouse's Equal Harmony (Essay on Musical Intervals, p. 45).

No. 29 (12). The melodic errors of the IIIrd and Vth conjointly are a minimum.

Here (ϵ for III)^2 + (ϵ for V)^2, or $x^2+(k-4x)^2=$ a minimum, $x=\frac{4}{17}k$, as in No. 14.

This is given by Drobisch (Nachträge, § 8) as "the simplest solution of the problem."

C. Combined Systems of Equal Temperament.

No. 30 (4). The combined harmonic and melodic errors are a minimum.

By combining the equations of No. 15 and No. 26, we have $(539+106)k=(1998+420)x$, or $x=\frac{645}{2418}k=\cdot001444$, $\log v=\cdot1746439$.

No. 31 (32). The tones are a mean between those of No. 1 and No. 2.

Here $x=\frac{1}{2}$ (sum of the two values of x in No. 1 and No. 2)$=\cdot0006744$, $\log v=\cdot1754169$.

This is proposed by Drobisch (Nachträge, § 9).

No. 32 (42). The errors occasioned by using the tempered c, d, f, $f_\sharp$, g, $b_\flat\flat$, $c^2\flat$, c^2 for the just c, d, e, f, g, a, b, c^2 are a minimum.

Using s for $\cdot0004901$, and forming the values of these errors by Table XII., we have $4x^2+(s-8x)^2+x^2+(s-9x)^2+(s-7x)^2=$ a minimum, $x=\frac{24}{199}s=\cdot000059084$, $\log v=\cdot1760322$.

This is proposed by Drobisch as a system of temperament adapted to bowed instruments (Mus. Tonbestim. § 57), allowing them to use a system of perfect fifths, and yet play the perfect scale very nearly by substitution. Such a system would be more complicated than the just scale for any instrument, and would require many more than 27 tones. It is, therefore, unnecessary for the violin, and impossible on instruments with fixed tones.

D. Cyclic Systems of Equal Temperament.

When it was supposed that the number of just tones required would be infinite, importance was attached to cycles of tones which by a limited number expressed all possible tones. Hence Huyghens's celebrated Cyclus Harmonicus, which he proposed to employ for an instrument with 31 strings, struck by levers and acted upon by a moveable finger-board (*abacus mobilis*), acting like a shifting piano or harmonium. The condition of forming a cycle is not properly harmonic or melodic; it is rather arithmetic. If $\log v : \log 2$ be converted into a continued fraction for any of the preceding values of $\log v$, and $y:z$ be any of the convergents, then, putting $\log 2 = z\,.\,h$, we shall have $\log v = yh$, which is commensurable with $\log 2$, and consequently the logarithms of all the intervals will be multiples of h, and therefore commensurable with $\log 2$. A cycle of z tones to the octave will thus be formed. If z is less than 27, the number of tones otherwise necessary, the cycle may be useful, otherwise it can only be judged by its merits as an equal temperament. As an historical interest attaches to several of these cycles, I subjoin a new method for deducing them all, without reference to previous calculations of $\log v$.

Since $\log v = y\,.\,h$, and $\log \delta = 7\log 2 - 12\log v = (7z-12y)\,.\,h$, we have only to put $7z-12y=\ldots -2,-1,0,1,2,\ldots$ and find all the positive integral solutions of the resulting equations. This gives for

$$7z-12y=-2, \qquad \frac{y}{z}=\frac{13}{22},\frac{27}{46},\frac{41}{70},\frac{55}{94},\frac{69}{118},\ldots$$

$7z-12y=-1$, $\frac{y}{z}=\frac{3}{5},\frac{10}{17},\frac{17}{29},\frac{24}{41},\frac{31}{53},\frac{38}{65},\frac{45}{77},\frac{52}{89},\cdots$

$7z-12y=0$, $\frac{y}{z}=\frac{7}{12}$.

$7z-12y=1$, $\frac{y}{z}=\frac{4}{7},\frac{11}{19},\frac{18}{31},\frac{25}{43},\frac{32}{55},\frac{39}{67},\frac{46}{79},\frac{53}{91},\cdots$

$7z-12y=2$, $\frac{y}{z}=\frac{1}{2},\frac{15}{26},\frac{29}{50},\frac{43}{74},\frac{57}{98},\cdots$

$7z-12y=3$, $\frac{y}{z}=\frac{5}{9},\frac{12}{21},\frac{19}{33},\frac{26}{45},\frac{33}{57},\frac{40}{69},\frac{47}{81},\frac{54}{93},\cdots$

Many of these cycles are quite useless. The following selection is arranged in order of magnitude, from the greatest to the smallest cycle.

No. 33 (38). Cycle of 118; $h=\cdot0025511$, $\log v=69h=\cdot1760259$.
This is Drobisch's cycle (Mus. Ton. § 58) representing No. 32.

No. 34 (8). Cycle of 93; $h=\cdot0032368$, $\log v=54h=\cdot1747872$.
This may represent No. 2.

No. 35 (3). Cycle of 81; $h=\cdot0037164$, $\log v=47h=\cdot1746708$.
This may represent No. 11 (2).

No. 36 (39). Cycle of 77; $h=\cdot0039095$, $\log v=45h=\cdot1759275$.
This is the same as No. 52.

No. 37 (19). Cycle of 74; $h=\cdot004068$, $\log v=43h=\cdot1749200$.
This is another of Drobisch's cycles (Nachträge, § 7) representing No. 27.

No. 38 (22). Cycle of 69; $h=\cdot004363$, $\log v=40h=\cdot1745200$.

No. 39 (28). Cycle of 67; $h=\cdot004493$, $\log v=39h=\cdot1752270$.

No. 40 (40). Cycle of 65; $h=\cdot0046123$, $\log v=38h=\cdot17598674$.

No. 41 (27). Cycle of 57; $h=\cdot0052812$, $\log v=33h=\cdot1742796$.

No. 42 (30). Cycle of 55; $h=\cdot0054733$, $\log v=32h=\cdot1751456$.
This is mentioned by Sauveur (Mém. de l'Acad. 1707) as the commonly received cycle in his time. Estève (*loc. cit.* p. 135) calls it the Musicians' Cycle.

No. 43 (11). Cycle of 50; $h=\cdot0060206$, $\log v=29h=\cdot1745974$.
This is Henfling's cycle (*loc. cit.* p. 281), and is used by Dr. Smith to represent No. 9.

No. 44 (43). Cycle of 53; $h=\cdot0056798$, $\log v=31h=\cdot1760800$.
This is the cycle employed by Nicholas Mercator (as reported by Holder, 'Treatise on Harmony,' p. 79) to represent approximately the just scale. He did not propose it as a system of temperament as has been recently done by Drobisch (Musik. Tonbestim. Einleit.). It was the foundation of

the division into degrees and sixteenths adopted in my previous paper Proceedings, vol. xiii. p. 96.

No. 45 (37). Cycle of 45; $h = \cdot 006689$, $\log v = 26h = \cdot 1738940$.

No. 46 (25). Cycle of 43; $h = \cdot 0070007$, $\log v = 25h = \cdot 1750175$.

This is Sauveur's cycle, defended in Mém. de l'Acad. for 1701, 1702, 1707, and 1711.

No. 47 (10). Cycle of 31; $h = \cdot 009711$, $\log v = 18h = \cdot 1747900$.

This is Huyghens's Cyclus Harmonicus, which nearly represents No. 2 (2). It was adopted, apparently without acknowledgment, by Galin (Delezenne, *loc. cit.* p. 19).

No. 48 (44). Cycle of 26; $h = \cdot 011578$, $\log v = 15h = \cdot 1736700$.

No. 49 (25). Cycle of 19; $h = \cdot 0158437$, $\log v = 11h = \cdot 1742807$.

This is the cycle adopted by Mr. Woolhouse (Essay on Beats, p. 50) as most convenient for organs and pianos. It may therefore go by his name, although it is frequently mentioned by older writers. It is almost exactly the same as No. 3 (23).

No. 50 (35). Cycle of 12; $h = \cdot 0250858$, $\log v = 7h = \cdot 1756008$.

As this is a cycle of twelve equal semitones, it may be termed the *Hemitonic* temperament. It is the one most advocated at the present day, and generally spoken of as "equal temperament" without any qualification, as if there were no other. It was consequently referred to by that name only in my former paper (Proceedings, vol. xiii. p. 95). For its harmonic character see No. 53.

E. Defective Systems of Equal Temperament.

It has been from the earliest times customary to have only twelve fixed tones to the octave, on the organ, harpsichord, piano, &c., and to play the other fifteen by substitution, as shown below, where the tones tuned, arranged in dominative order, occupy the middle line, and the tones for which they are used as substitutes are placed in the outer lines, and are bracketed.

$$[A\flat\flat,\ E\flat\flat,\ B\flat\flat,\ F\flat,\ C\flat,\ G\flat,\ D\flat,\ A\flat]$$
$$E\flat,\ B\flat,\ F,\ C,\ G,\ D,\ A,\ E,\ B,\ F\sharp,\ C\sharp,\ G\sharp$$
$$[D\sharp,\ A\sharp,\ E\sharp,\ B\sharp,\ F\times,\ C\times,\ G\times].$$

The consequence was, that while the Vths in the middle line were uniform, the Vths and 4ths produced in passing from one line to the other (as $G\sharp E\flat$ for $A\flat E\flat$ or $G\sharp D\sharp$) were strikingly different. Similar errors arose in the other concordant intervals. It is evident that the interval error thus produced must be the usual interval error of the system increased or diminished by the logarithm of the diesis, where $\log \delta = \log g\flat - \log f\sharp =$

$-\cdot0058851+12x=-k-s+12x$, where $s=\cdot0004901$ *. Such interval errors are termed *wolves*, from their howling discordance. In Table XIII. will be found an enumeration of all the wolves, with a notation for them, and an expression of their interval errors and beat meters in terms of k, s, and x.

No. 51 (33). System of least wolf melodic errors.

The sum of the squares of the wolf interval errors, or

$$2k^2+2ks+6s^2-4\,(11k+28s)\,x+266x^2,$$

is a minimum. Hence $22k+56s=266x$, or $x=\cdot0005495$, $\log v=\cdot1755418$.

No. 52 (39). System of least wolf harmonic errors.

The sum of the squares of the wolf beat meters, or

$$25k^2+175s^2+50ks-(550k+3072s)\,x+13662x^2,$$

is a minimum. Hence $275k+1536s=13662x$, or $x=\cdot0001638$, $\log v=\cdot1759275$, as in No. 36 (39).

No. 53 (35). The wolf interval errors are equal to the usual interval errors, that is, there are no wolves, or there are none but wolves.

In this case $\log\delta=0$, or, since $\delta=\mathit{gb} : f\sharp=2^7 : v^{12}$, $7\log 2=12\log v$. Hence this system is the cycle of 12, No. 50. When δ is greater than 1, gb is *sharper* than $f\sharp$, and $\log v$ is less than $\frac{7}{12}\log 2$, or $\cdot1756008$. But if δ is less than 1, gb is *flatter* than $f\sharp$, and $\log v$ is less than $\frac{7}{12}\log 2$, or $\cdot1756008$. The latter case is, according to Drobisch, indispensable for musical theory and violin practice (Musik. Tonbestim. Einleit.). Since this temperament thus forms the boundary of the two other classes, distinguished by gb being flatter or sharper than $f\sharp$, Drobisch terms it the "mean" temperament (*ibid.* § 51). It is this property of making $\mathit{gb}=f\sharp$ which renders this temperament so popular, as the ear is never distressed by the occurrence of intervals different from those expected, and the whole number of tones is reduced to 12.

No. 54 (31). The wolf interval error of the IIIrd is to its usual interval error as 14 : 5.

This gives $-s+8x : k-4x=14 : 5$, or $96x=14k+5s$, $x=\cdot0008123$, $\log v=\cdot1752790$. This is Marsh's system of temperament; see Phil. Mag. vol. xxxvi. p. 437, and p. 39 *seqq.* Schol. 8.

No. 55 (36). The wolf errors of the IIIrd and Vth conjointly are a minimum.

Here $(-s+8x)^2+(-k-s+11x)^2$ is a minimum, whence $11k+19s=185x$, $x=\cdot0003712$, $\log v=\cdot1757201$.

No. 56 (37). The wolf errors of the Vth and IIIrd are equal and opposite.

Here $-k-s+11x=s-8x$, $19x=k+2s$, $x=\cdot0003356$, $\log v=\cdot1757557$.

No. 57 (34). There is no Vth wolf.

Here $-k-s+11x=0$, $x=\cdot0005351$, $\log v=\cdot1755562$.

* It appears from Proceedings, vol. xiii. p. 95, that s must be nearly the logarithm of the schisma or $\log\P$. Actual calculation shows that s and $\log\P$ agree to 14 places of decimals.

No. 58 (41). There is no IIIrd wolf.

Here $-s+8x=0$, $x=\cdot0000613$, $\log v=\cdot1760300$. This is almost exactly No. 32 (42).

No. 59 (42). There is no 3rd wolf.

Here $s-9x=0$, $x=\cdot0000545$, $\log v=\cdot1760368$.

F. Systems of Unequal Temperament.

In a defective equal temperament the same just concordance is represented by two different discordances. As performers limited themselves to twelve tones to the octave, those who found the Hemitonic temperament No. 50 (35) too rough, accepted this variety of representatives of the same concordance as the basis of a temperament, hoping to have better IIIrds in the usual chords, without the wolves of the defective temperament. Others conceived that an advantage would be gained by altering the character of the different keys. Thus arose *unequal temperament*, properly so called, which must be carefully distinguished from any defective equal temperament with which it is popularly confused.

Arrange the twelve unequally tempered chords as follows, where the identical numbers indicate identical chords with different names:—

1. *C E G.*	7. $F\sharp$ $A\sharp$ $c\sharp$.	7. $G\flat$ $B\flat$ $d\flat$.
2. *G B d.*	8. $C\sharp$ $E\sharp$ $G\sharp$.	8. $D\flat$ *F* $A\flat$.
3. *D* $F\sharp$ *A.*	9. $G\sharp$ $B\sharp$ $d\sharp$.	9. $A\flat$ *c* $e\flat$.
4. *A* $c\sharp$ *e.*	10. $D\sharp$ $F\times$ $A\sharp$.	10. $E\flat$ *G* $B\flat$.
5. *E* $G\sharp$ *B.*	11. $A\sharp$ $c\times$ $e\sharp$.	11. $B\flat$ *d f.*
6. *B* $d\sharp$ $f\sharp$.	12. $E\sharp$ $G\times$ $B\sharp$.	12. *F A c.*

Let T_n, t_n, v_n be the ratios of the IIIrd, 3rd, and Vth in the nth chord, so that, for example, in the 6th chord $d\sharp=T_6 . B$, $f\sharp=t_6 . d\sharp$, $f\sharp=v_6 . E$. Then it is evident from the above scheme that there exist 12 pairs of equations between these 36 ratios, of the form

$$T_n . t_n = v_n \text{ and } 4T_n = v_n . v_{n+1} . v_{n+2} . v_{n+3}$$

(where, when the subscript numbers exceed 12, they must be diminished by 12), and one condition,

$$v_1 . v_2 . v_3 . v_4 . v_5 . v_6 . v_7 . v_8 . v_9 . v_{10} . v_{11} . v_{12} = 2^7.$$

Put $\log T_n = \log \frac{5}{4} + y_n$, $\log t_n = \log \frac{6}{5} - z_n$, $\log v_n = \log \frac{3}{2} - x_n$, then the above equations become

$$z_n = x_n + y_n,$$
$$y_n = k - (x_n + x_{n+1} + x_{n+2} + x_{n+3}),$$
$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} = \cdot0058851,$$

which represent 25 equations, where the second set of 12 may be replaced by the following, which are readily deduced from them and the last condition:—

$$y_1+y_5+y_9=y_2+y_6+y_{10}=y_3+y_7+y_{11}=y_4+y_8+y_{12}=\cdot0103000\ldots\ldots(a)$$

$$x_5=x_1+y_1-y_2, \qquad x_9=x_1+y_{10}-y_9,$$
$$x_6=x_2+y_2-y_3, \qquad x_{10}=x_2+y_{11}-y_{10},$$
$$x_7=x_3+y_3-y_4, \qquad x_{11}=x_3+y_{12}-y_{11},$$
$$x_8=x_4+y_4-y_5, \qquad x_{12}=x_4+y_1-y_{12}.$$

A system of unequal temperament may therefore be determined by arbitrarily selecting eleven different Vths, or else eight different IIIrds and three Vths. The equations (*a*) show that if the temperament is not equal (in which case all the *y*'s are equal, and the interval error of the IIIrd is $\frac{1}{3}\times\cdot0103000=\cdot0034333$, as in the Hemitonic temperament), at least four IIIrds must have their interval errors greater than $\frac{1}{3}\times\cdot0103000$, that is, there must be at least four IIIrds in every unequal temperament which are inferior to the very bad IIIrds of the Hemitonic system. Kirnberger, Dr. T. Young, and Lord Stanhope*, in the unequal systems they propose, have each seven IIIrds sharper, and therefore worse than the Hemitonic IIIrds. In one of Prof. de Morgan's unequal temperaments, six IIIrds are sharper than the Hemitonic; in another four are sharper and four the same; in a third all are the same, but the Vths differ†. Hence nothing is gained over the Hemitonic system in the way of harmony, while the uniformity in the representation of the uniformity of just intonation is entirely lost.

In selecting a temperament, therefore, we may dismiss all unequal temperaments, as they must be inferior to the Hemitonic in both harmony and melody, and will have no advantage over it in the relations of chords or the number of tones required. Also, if it is considered necessary to play in all keys with only twelve tones, any system of defective equal temperament will be inferior to the Hemitonic, on account of the various and distressing wolf intervals which occur when the music is not confined to the six major scales of *E♭*, *F*, *C*, *G*, *D*, *A*, and the three minor scales of *g*, *d*, *a*. Hence the two conditions of having only twelve tones (exclusive of octaves) and of playing in all keys, at once exclude all temperaments but the Hemitonic. As, however, organs, harpsichords, and pianofortes with 14, 16, 17, 19, 21, 22, and 24 tones to the octave have been actually constructed and used ‡, as Mr. Liston used 59§, Mr. Poole used 50, and Gen. T. Perronet Thomp-

* *Kirnberger*, Kunst des reinen Satzes in der Musik. *Dr. T. Young*, loc. cit. *Charles Earl Stanhope*, Principles of the Science of Tuning, 1806.

† *De Morgan*, loc. cit. p. 129, temperaments *Q*, *R*, *S*.

‡ Mr. Farey (Phil. Mag. vol. xxxix. p. 416) gives the particulars of their scales, builders, and localities.

§ The following account of Mr. Liston's organ is deduced from the data of Mr. Farey (Phil. Mag. vol. xxxix. p. 418). Scale: *c* †*c* ‡*c♯* *c♯* *d♭* †*c♯* †*d♭* ‡*c*× ‡*d* *c*× *d* †*d* ‡*d♯* *e♭* *d♯* †*e♭* ††*e♭* *e* *f♭* †*e* †*f♭* ††*f♭* ‡*e♯* *e♯* *f* †*e♯* †*f* ‡*f♯* *f♯* *g♭* †*f♯* †*g♭* ‡*f*× ‡*g* *f*× *g* †*g* ‡*g♯* *a♭* *g♯* †*a♭* ††*a♭* *a* †*a* *b♭♭* †*b♭♭* ‡*a♯* *a♯* *b♭* †*a♯* †*b♭* ‡*b* *b* *c♭* †*b* †*c♭* ‡*b♯* ‡*c* *b♯*. Chords: Table V. col. III., lines 4 to 18;

son now uses 40 tones to the octave on their justly intoned organs, the condition of having twelve tones and no more, does not seem to be inevitable. It will therefore be necessary to determine what would be the best system of temperament for the complete equally tempered scale of 27 tones, and how great a sacrifice of musical effect is required by the use of the Hemitonic system.

In Table XV. I have calculated for each of the 59 (reducing to 51) systems of equal temperament already named, the interval errors of the Vths, IIIrds, and 3rds, and the sums of the squares of the 23 interval errors and the 6 beat meters of Table XII. I have then arranged the temperaments in order according to each of these five results, and numbered the order. Finally, I have added the five order numbers together and arranged the whole in the order of these sums. The smallest number would therefore clearly give the best temperament, supposing that all the five points of comparison were of equal value. Now the first and second temperament on the list, or No. 26 and No. 2, only differ from each other in the fifth, sixth, or seventh place of decimals with respect to these five results, a difference which no human ear, however finely constituted by nature or assisted by art, could be taught to detect. As No. 2, or the Mesotonic system, is determined in the simplest manner, I consider it as the real head of the list. There is, however, little to choose between it and any one of the ten or twelve systems which follow, except in simplicity of construction and comparative ease in realization. The Hemitonic system, however, comes 35th in the list, and the old Pythagorean, recently defended by Drobisch and Naumann (*op. cit.*), and asserted to be the system actually used by violinists, is the 45th. No one who has heard any harmonies played on the Pythagorean system will dispute the correctness of the position here assigned to it, which fully explains the absence of all feeling for harmony among the nations which use it—the ancient and modern Greeks, the old Chinese, the Gaels, the Arabs, Persians, and Turks. No modern quartett players could be listened to who adopted it.

The contest lies, therefore, between the Mesotonic and the Hemitonic systems. The Mesotonic is that known as "the old organ-tuning," or, since it was generally used as a defective twelve-toned system, as the "unequal temperament." Within the limits of the nine scales already named, the superiority of the Mesotonic to the Hemitonic system has long been practically acknowledged. But the extremely disagreeable effect of the wolves (more especially to the performer himself) has finally expelled the system from Germany altogether, and from England in great measure. On the pianoforte

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IV., 5 to 14; V., 5 to 16; VI., 6 to 15; VII., 7 to 16; VIII., 7 to 17; IX., 9 to 13; X., 9 to 13. Tones not forming part of any chord and required chiefly by the system of tuning: †$d\flat$ ††$e\flat$ †$f\flat$ ††$f\flat$ †$g\flat$ ††$a\flat$ $b\flat\flat$ †$b\flat\flat$ †$c\flat$. Complete keys: *F*, *C*, *G*, *D*, †*A*; *E*, *B*, *F*♯. The keys of $E\flat$, $B\flat$ had their synonymous, and †*E*, †*B* their relative minors perfect.

the Hemitonic system is universally adopted in intention. It is, however, so difficult to realize by the ordinary methods of tuning, that "equal temperament," as the Hemitonic system is usually called, has probably never been attained in this country, with any approach to mathematical precision.

In Table XIV. I have given a detailed comparison of the Mesotonic and Hemitonic temperaments with each other and with just intonation, for the system of *C* (Proceedings, vol. xiii. p. 98), from which the great superiority of the Mesotonic over the Hemitonic both in melody and harmony becomes apparent. But this comparison rests upon the preceding calculations, which were founded upon the beats that arise from rendering the conjunct harmonics pulsative. It was therefore assumed that the qualities of tone employed were such as to develope these beats. The result will consequently be materially modified when the requisite harmonics either do not exist or are very faint. Now

for the Vth the conjunct harmonics are 2 and 3,
„ 4th „ „ 3 and 4,
„ VIth „ „ 3 and 5,
„ IIIrd „ „ 4 and 5,
„ 3rd „ „ 5 and 6,
„ 6th „ „ 5 and 8.

If then only simple tones are used, as in the wide covered pipes of organs, or such qualities as develope the second harmonic only, such as tuning-forks, to which we may add flutes, which have almost simple tones, no beats will be heard, and *any* system of temperament may be used in which the ear can tolerate the interval errors. Now Delezenne's experiments show (*loc. cit.*) that a good ear distinguishes

in the unison an interval error of $0{\cdot}2807k$,
„ VIIIve „ „ $0{\cdot}31k$,
„ Vth „ „ $0{\cdot}1461k$,
„ IIIrd „ „ $0{\cdot}284k$,
„ VIth „ „ $0{\cdot}299k$,

and an indifferent ear perceives an error of $0{\cdot}561k$ in the VIIIve, and $0{\cdot}292k$ in the Vth. We may say, therefore, generally that the ear just perceives an interval error of $\frac{1}{4}k$ in the Vth, and $\frac{1}{3}k$ in the other intervals. Now in the Mesotonic system the interval error of the Vth is $-\frac{1}{4}k$, and therefore just perceptible, but in scarcely any other interval does it exceed $\frac{1}{4}k$. Thus it is $-\frac{1}{4}k$ in the VIIth, 0 in the IIIrd, and $+\frac{1}{4}k$ in the VIth, and it is therefore in those intervals imperceptible. In the Hemitonic system the error of the Vth is $-\frac{1}{11}k$, and hence quite imperceptible, but the errors of the VIIth, IIIrd, and VIth are respectively $\frac{6}{11}k$, $\frac{7}{11}k$, and $\frac{8}{11}k$, and therefore perfectly appreciable. It is only in the VIIth that this error is at all agreeable. The sharpness of the IIIrd and VIth is universally disliked. Hence in those qualities of tone which are most favourable to the Hemitonic system, it is much inferior to the Mesotonic. In Table XV.

the Mesotonic stands 2nd in order of melody, inappreciably different from the 1st, and the Hemitonic 39th.

If the 3rd harmonic only is developed in the qualities of tone combined, the beats of the Vth are heard, but those of the other intervals are not perceived. The beats of the IIIrd and VIth, which are so faulty on the Hemitonic system, will not be perceived at all unless the 5th harmonic be developed, and will not be much perceived unless it be strongly developed. Now the 5th harmonic is comparatively weak on all organ pipes and on pianofortes, and hence the errors are not so violently offensive on these instruments. If, however, the 'mixture stops,' which strengthen the upper harmonics by additional pipes, are employed on the organ, the effect is unmistakeably bad, unless drowned by din or dimmed by distance. On the pianoforte, however, these intervals, and even the still worse 3rd and 6th, depending on the 6th and 8th harmonics, which are undeveloped on pianoforte strings, are quite endurable.

Hence the Hemitonic system, except as regards melody, will not be greatly inferior to the Mesotonic on a pianoforte and on soft stops of organs, but will only become offensive on loud stops. But for harmoniums and concertinas, violins and voices, where harmonics up to the 8th, and even higher, are well developed, the Hemitonic temperament is offensive. The roughness of harmoniums is almost entirely due to this mode of tuning. The beats of the VIth, IIIrd, and 3rd are distinctly heard, and the development of differential tones is so strong as frequently to form an unintelligibly inharmonious accompaniment*. Concertinas having 14 tones to the octave are indeed generally tuned mesotonically (or intentionally so), thus $c\ c^{\sharp}_{\flat}$, $d\ d^{\sharp}_{\flat}$, $e\ e\flat$, $f\ f^{\sharp}_{\flat}$, $g\ g^{\sharp}_{\flat}$, $a\ a\flat$, $b\ b\flat$. They are, however, occasionally tuned hemitonically (or intentionally so) to accompany pianofortes, thus $c\ c^{\sharp}_{\flat}$, $d\ d^{\sharp}_{\flat}$

* The three recognized forms of the common major triad 4, 5, 6; 5, 6, 8; 3, 4, 5, or CEG, EGc, Gce, have the pitches of their tones as $4n, 5n, 6n$; $5n, 6n, 8n$, and $3n, 4n, 5n$ respectively. They produce, therefore, the differential tones $n, n, 2n$; $n, 2n, 3n$, and $n, n, 2n$ respectively. If the chords are tempered, the altered unisons n, n become pulsative, and the other tones disjunct. Now if in Table XII. we put $x=\log(1+t)$ and neglect t^2, we shall have very nearly $E=\frac{81}{64}.(1-4t).C$; $G=\frac{3}{2}.(1-t).C$; $c=2C$, $e=\frac{81}{32}.(1-4t).C$; $g=3.(1-t).C$. The pairs of pulsative differential tones are therefore $E-C=(\frac{17}{14}-\frac{81}{16}t).C$, $G-C=(\frac{15}{64}+\frac{57}{16}t).C$, and $c-G=(\frac{1}{2}+\frac{3}{2}t).C$, $e-c=(\frac{17}{7}-\frac{81}{8}t).C$. The numbers of beats are the absolute value of the differences of these pairs of numbers, or of $(-\frac{1}{32}+\frac{69}{8}t).C$, and $(-\frac{1}{32}+\frac{93}{8}t).C$. The squares of these expressions, and the sum of their squares, will be minima respectively for $t=\frac{1}{276}$, $x=\cdot00157070$, $\log v=\cdot1745206$, which is nearly No. 38 (22); $t=\frac{1}{372}$, $x=\cdot0011658$, $\log v=\cdot1749255$, which is nearly No. 24 (20); and $t=\frac{9}{2980}$, $x=\cdot0013096$, $\log v=\cdot1747817$, which is nearly No. 34 (8). These beats, though perfectly distinct in some octaves, do not appear to be sufficiently prominent to serve as a criterion of the relative value of different systems of temperament, or to form the basis of a system, and they have consequently not been introduced into the text. They were noticed and used by H. Scheibler (Der physikalische und musikalische Tonmesser, p. 15).

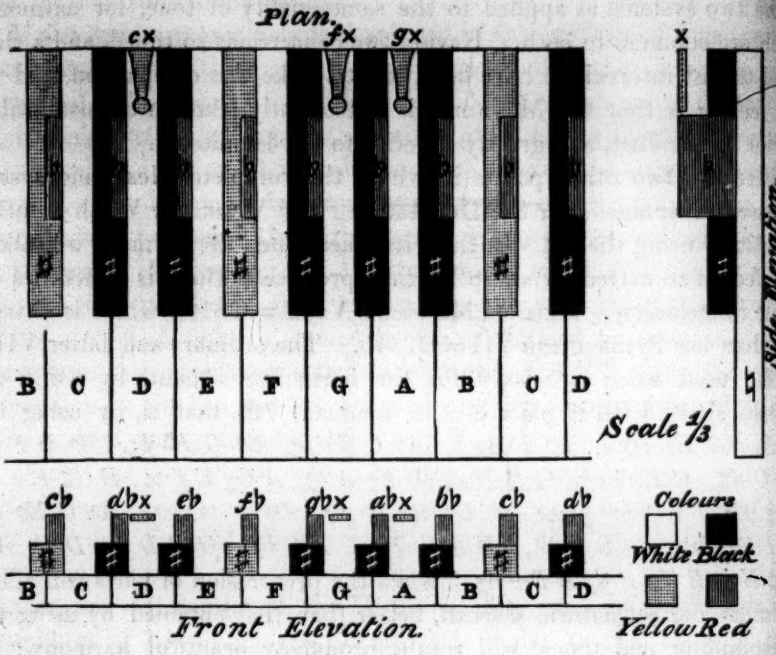
e $d\sharp$, *f* $f\sharp$, *g* $g\sharp$, *a* $g\sharp$, *b* $a\sharp$. Hence it is easy to compare the different effects of the two systems as applied to the same quality of tone, for harmonies which are common to both. Having two concertinas so tuned, and a third tuned to just intervals, I have been able to make this comparison, and my own feeling is that the Mesotonic is but slightly, though unmistakeably, inferior to the Just, and greatly superior to the Hemitonic.

There are two other points in which the complete Mesotonic system possesses advantages over the Hemitonic. The Mesotonic VIIth is rather flat, but by using the flat VIIIth in its place, when the harmony will allow, the effect of an extremely sharp VIIth is produced, which is sometimes desirable in melodies. Thus log Mesotonic VIII$\flat$ = ·28195, which is sharper even than log Pythagorean VII = ·27840. The ordinary and flatter VIIth can be used when necessary for the harmony. Again, by using the German sharp VIth in place of the dominant 7th, that is, by using the chords *G* $B\flat$ *D* $\flat$ *e*, *D* $\flat$ *F A* $\flat$ *b*, *A* $\flat$ *C E* $\flat$ $f\sharp$, *E* $\flat$ *G B* $\flat$ $c\sharp$, *B* $\flat$ *D F* $g\sharp$, *F A C* $d\sharp$, *C E G* $a\sharp$, *G B D* $e\sharp$, *D* $F\sharp$ *A* $b\sharp$, *A* $C\sharp$ *E f* ×, *E* $G\sharp$ *B c* ×, *B* $D\sharp$ $F\sharp$ *g* ×, in place of *G* $B\flat$ *D* $\flat$ $f\flat$, *D* $\flat$ *F A* $\flat$ $c\flat$, *A* $\flat$ *C E* $\flat$ $g\flat$, *E* $\flat$ *G B* $\flat$ $d\flat$, *B* $\flat$ *D F* $a\flat$, *F A C* $e\flat$, *C E G* $b\flat$, *G B D f*, *D* $F\sharp$ *A c*, *A* $C\sharp$ *E g*, *E* $G\sharp$ *B d*, *B* $D\sharp$ $F\sharp$ *a*, when the progression of parts will allow, an almost perfect natural seventh, better than that obtained by using the corresponding just tones, will result, producing beautiful harmony; for log Mesotonic VI$\sharp$ = ·24228, log $\frac{7}{4}$ = ·24304, and log Just VI$\sharp$ = ·24497. The ordinary sharper 7th can be used when necessary. Neither of these effective substitutions is possible on the Hemitonic system.

Considering that singers and violinists naturally intone justly (Delezenne, *loc. cit.*), and that the interval errors of the Mesotonic system seldom exceed the natural errors of intonation which may be expected from the inability of the ear to appreciate minute distinctions of pitch, it appears desirable to tune harmoniums at least, and perhaps organs, mesotonically. Except as an instrument for practising singers, however (for which purpose it would be superseded by a Mesotonic harmonium), it would be unnecessary to alter the Hemitonic tuning and arrangement of the piano. But it would be best to teach the Mesotonic intonation on the violin in preference to the Hemitonic, as proposed by Spohr *. As, however, it would be useless to tune mesotonically with only 12 tones to the octave, it is necessary to have some practical arrangement for 27, 24, or 21 tones at least. I propose the following plan for 24 tones, and as these are exactly twice as many as on pianos, &c. of the usual construction, I call my arrangement the

* "Unter reiner Intonation wird natürlich die der gleichschwebenden [Hemitonic] Temperatur verstanden, da es für moderne Musik keine andere giebt. Der angehende Geiger braucht auch nur diese eine zu kennen; es ist deshalb in dieser Schule von einer ungleichschwebenden [defective equal, or unequal] Temperatur eben so wenig die Rede, wie von kleinen und grossen halben Tönen [*c* $c\sharp$ & *c* $d\flat$ = *B c*, that is, $\sharp = I$], weil durch beides die Lehre von der völlig gleichen Grösse aller 12 halben Töne nur in Verwirrung gebracht wird."—Violinschule, p. 8.

DUPLEX FINGER-BOARD.



Let the black and white manuals remain as at present, and let a yellow manual, of the same form as the black, be introduced between *B* and *C*, and *E* and *F*. Cut out about the middle third of each black and yellow manual, up to half its width, on the right side only, and introduce a thin red manual rising as high above the black or yellow as these do above the white. Over *G*, *A*, and *D*, each of which lies between two black manuals, introduce three yellow metal manuals (lacquered or aluminium-bronze) shaped like flute keys, and standing at the height of a red manual above the white one, which can therefore, when necessary, be reached below it. The 7 white manuals are the 7 naturals; the 5 black manuals are the 5 usual sharps, $c\sharp$ $d\sharp$ $f\sharp$ $g\sharp$ $a\sharp$; the 2 long yellow manuals are the unusual sharps $e\sharp$ $b\sharp$, and the 3 metal yellow manuals are the double sharps $f\times$ $g\times$ $c\times$; and the 7 thin red manuals are the 7 flats, $c\flat$, $d\flat$, $e\flat$, $f\flat$, $g\flat$, $a\flat$, $b\flat$. The shapes of the red and metal manuals were suggested by those of General T. Perronet Thompson's *quarrils* and *flutals*. The 24 levers opening the valves on the organ or harmonium would lie side by side, being made half the width of those now in use, and metallic, if required for strength. The organ pipes or harmonium reeds would be arranged in two ranks of 12 for each octave, the first rank containing the 7 naturals and 5 usual sharps, and the back rank containing the 7 flats, 2 unusual and 3 double sharps. The use of this finger-board is accurately pointed out by the ordinary musical notation which distinguishes the sharps from the flats, and is therefore in no respect adapted to the Hemitonic fusion of sharps and flats into mean semitones.

Table VI.—Classification of Musical Chords. (See p. 397.)

Type.	Example.	Symbol.	Syst. Name.	Ordinary Name.
		I. Concords.		
1, 1	C^4 C^4	C	1 Dyad	Unison (Octave 1, 2).
1, 3	C^4 G	C	3 Dyad	Twelfth (Fifth 2, 3; Fourth 3, 4).
1, 5	C^4 e	C	5 Dyad	Major 17th (Ma. 10th 2, 5; Ma. 3rd 4, 5; Mi. 6th 5, 8).
3, 5	G e	c	3, 5 Dyad	Ma. 6th (Mi. 3rd 5, 6).
1, 3, 5	C^4 G e	C	Major Triad	Common Major Chord.
		II. Strong Discords.		
		1. One Pulsative Constituent.		
1, 7	C^4 $\flat_2\flat$	7C	7 Dyad	Perfect 7th 4, 7; Extended tone 7, 8.
3, 7	G $\flat_2\flat$	7C	3, 7 Dyad	Contracted 3rd 6, 7; Ext. 6th 7, 12.
5, 7	e $\flat_2\flat$	7C	5, 7 Dyad	Contracted 5th 5, 7.
1, 3, 7	C^4 G $\flat_2\flat$	7C	3, 7 Triad	Imp. Ch. of Dominant 7th.
1, 5, 7	C^4 e $\flat_2\flat$	7C	5, 7 Triad	" " "
1, 5, 7	C^4 e vij a♯	7′C	5, 7′ Triad	Chord of the Italian 6th.
1, 5, 9	C^4 e d	9C	5, 9 Triad	Imp. Ch. of the 9th.
1, 3, 15	C^4 G b^2	15C	3, 15 Triad	Imp. Ch. of the Mi. 6th, Minor mode.
1, 5, 15	C^4 e b^2	15C	5, 15 Triad	
1, 3, 17	C^4 G $1\flat_2 d^4\flat$	17C	3, 17 Triad	Imp. Ch. of the Minor 9th.
1, 5, 17	C^4 e $1\flat_2 d^4\flat$	17C	5, 17 Triad	
1, 3, 5, 7	C^4 G e $\flat_2\flat$	7C	7 Tetrad	Ch. of the Dominant 7th.
1, 3, 5, 7	C^4 G e vij a♯	7′C	7′ Tetrad	" " German 6th.
1, 3, 5, 9	C^4 G e d^2	9C	9 Tetrad	" " 9th.

Side Elevation.

Table VI.—Classification of Musical Chords. (See p. 397.)

Type.	Example.	Symbol.	Syst. Name.	Ordinary Name.
		I. Concords.		
1, 1	C^4 C^4	C	1 Dyad	Unison (Octave 1, 2).
1, 3	C^4 G	C	3 Dyad	Twelfth (Fifth 2, 3; Fourth 3, 4).
1, 5	C^4 e	C	5 Dyad	Major 17th (Ma. 10th 2, 5; Ma. 3rd 4, 5; Mi. 6th 5, 8).
3, 5	G e	c	3, 5 Dyad	Ma. 6th (Mi. 3rd 5, 6).
1, 3, 5	C^4 G e	C	Major Triad	Common Major Chord.
		II. Strong Discords.		
		1. One Pulsative Constituent.		
1, 7	C^4 $2^{\flat\flat}$	7C	7 Dyad	Perfect 7th 4, 7; Extended tone 7, 8.
3, 7	G $2^{\flat\flat}$	7C	3, 7 Dyad	Contracted 3rd 6, 7; Ext. 6th 7, 12.
5, 7	e $2^{\flat\flat}$	7C	5, 7 Dyad	Contracted 5th 5, 7.
1, 3, 7	C^4 G $2^{\flat\flat}$	7C	3, 7 Triad	Imp. Ch. of Dominant 7th.
1, 5, 7	C^4 e $2^{\flat\flat}$	7C	5, 7 Triad	" " "
1, 5, 7	C^4 e vij $a^{\sharp}$	7′C	5, 7′Triad	Chord of the Italian 6th.
1, 5, 9	C^4 e d	9C	5, 9 Triad	Imp. Ch. of the 9th.
1, 3, 15	C^4 G b^2	15C	3, 15 Triad	Imp. Ch. of the Mi. 6th, Minor mode.
1, 5, 15	C^4 e b^2	15C	5, 15 Triad	
1, 3, 17	C^4 G $1^2d^{4\flat}$	17C	3, 17 Triad	Imp. Ch. of the Minor 9th.
1, 5, 17	C^4 e $1^2d^{4\flat}$	17C	5, 17 Triad	
1, 3, 5, 7	C^4 G e $2^{\flat\flat}$	7C	7 Tetrad	Ch. of the Dominant 7th.
1, 3, 5, 7	C^4 G e vij $a^{\sharp}$	7′C	7′Tetrad	" " German 6th.
1, 3, 5, 9	C^4 G e d^2	9C	9 Tetrad	" " 9th.
1, 3, 5, 15	C^4 G e b^2	15C	Major Tetrad	" " Ma. 7th.
1, 3, 5, 17	C^4 G e $1^2d^{4\flat}$	17C	17 Tetrad	" " Mi. 9th (imp.)

1, 3, 5, 7	C⁴ G e ♭7 ♭♭	7C	7 Tetrad	Ch. of the Dominant 7th.
1, 3, 5, 7	C⁴ G e vij a♯	7′C	7′Tetrad	„ „ German 6th.
1, 3, 5, 9	C⁴ G e d²	9C	9 Tetrad	„ „ 9th.
1, 3, 5, 15	C⁴ G e b²	15C	Major Tetrad	„ „ Ma. 7th.
1, 3, 5, 17	C⁴ G e l♭d⁴♭	17C	17 Tetrad	„ „ Mi. 9th (imp.)
1, 3, 5, 27	C⁴ G e †a⁴	27C	27 Tetrad	„ „ add. 6th, ma. m.

2. Two Pulsative Constituents.

1, 3, 5, 7, 9	C⁴ G e ♭7 ♭♭ d²	′9C	7, 9 Pentad	Ch. of the added 9th.
1, 3, 5, 7, 17	C⁴ G e ♭7 ♭♭ l♭d⁴♭	′17C	7, 17 Pentad	„ „ Mi. 9th.
1, 3, 5, 9, 15	C⁴ G e d² b⁴	′15C	Major Pentad	„ „ Ma. 9th.
1, 3, 5, 15, 17	C⁴ G e b² xvij c⁴♯	15, 17C	15, 17 Pentad	„ „ augmented 8th.

III. Weak Discords.

1. One Pulsative Constituent.

3, 5, 7	G e ♭7 ♭♭	7c	7 Triad	Ch. of the Diminished 5th.
1, 3, 9	C⁴ G d	9c	9 Triad	„ „ 9th (imp.).
3, 5, 15	G e b	c	Minor Triad	Common Minor Chord.
3, 5, 17	G e l♭d⁴♭	17c	17 Triad	Ch. of the Dim. 7th (imp.).
3, 5, 17	G e xvij c⁴♯	17′c	17′Triad	„ „ Minor added 6th (imperfect).
1, 5, 25	C e †g♯	25c	25 Triad	Superfluous Triad.
1, 5, 27	C⁴ e †a	27c	27 Triad	Ch. of the Ma. added 6th (imperfect).

2. Two Pulsative Constituents.

3, 5, 7, 9	G e ♭7 ♭♭ d²	′9c	7, 9 Tetrad	Ch. of the Mi. 7th, ma. m.
3, 5, 7, 17	G e ♭7 ♭♭ l♭d⁴♭	′17c	7, 17 Tetrad	„ „ Diminished 7th.
3, 5, 9, 15	G e d b	′c	Minor Tetrad	„ „ Mi. 7th (mi. m.).
3, 5, 15, 17	G e b xvij c⁴♯	15, 17c	15, 17 Tetrad	„ „ added 6th (mi. mode).
1, 5, 15, 25	C e b †g♯	25c	25 Tetrad	„ „ augmented 5th.
3, 5, 15, 45	G e b f⁴♯	′45c	45 Tetrad	„ „ added 9th (mi. mode).

Table VII.—Construction of Musical Chords from Compound Tones. (See p. 398.)

No. of J. H.	Harmonics of the Constituent Tones.														
	1	2	3	4	5	6	7	8	9	10	12	14	15	16	17
1	1														
2	2	2													
3	3	..	3												
4	4	4	..	4											
5	5	..	..	..	5										
6	6	6	6	..	..	6									
7	7	..	..	..	..	..	7								
8	8	8	..	8	..	..	..	8							
9	..	..	9	..	..	..	..	..	9						
10	..	10	..	..	10	..	..	..	..	10					
11	..	..	..	..	..	..	..	..	..	..					
12	..	12	12	12	..	12	..	..	..	..	12				
13	..	..	..	..	..	..	..	..	..	..	..				
14	..	14	..	..	..	..	14	..	..	..	..	14			
15	..	..	15	..	15	..	..	..	..	..	..	..	15		
16	..	16	..	16	..	..	..	16	..	..	..	..	..	16	
17	..	..	..	..	..	..	..	..	..	..	..	..	..	..	17
18	..	..	18	..	..	18	..	..	18	..	..	..	..	..	..
19	..	..	..	..	..	..	..	..	..	..	..	..	..	..	..
20	..	..	..	20	20	..	..	..	..	20	..	..	..	..	..
21	..	..	21	..	..	..	21	..	..	..	..	..	..	..	..
24	..	..	24	24	..	24	..	24	..	..	24	..	..	..	..

No. of J. H.	Extent of the Harmonics of the Constituent Tones.														
	1	2	3	4	5	6	7	8	9	10	12	14	15	16	17
1	100														
2	50	100													
3	33	....	100												
4	25	50	....	100											
5	20	....	....	....	100										
6	17	33	50	....	....	100									
7	14	....	....	....	....	....	100								
8	12	25	....	50	....	....	....	100							
9	....	....	33	....	....	....	....	....	100						
10	....	20	....	....	50	....	....	....	....	100					
11	....	....	....	....	....	....	....	....	....	....					
12	....	17	25	33	....	50	....	....	....	....	100				
13	....	....	....	....	....	....	....	....	....	....	....				
14	....	14	....	....	....	....	50	....	....	....	....	100			
15	....	....	20	....	33	....	....	....	....	....	....	....	100		
16	....	12	....	25	....	....	....	50	....	....	....	....	....	100	
17	....	....	....	....	....	....	....	....	....	....	....	....	....	....	100
18	....	....	17	....	....	33	....	....	50	....	....	....	....	....	....
19	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
20	....	....	....	20	25	....	....	....	....	50	....	....	....	....	....
21	....	....	14	....	....	....	33	....	....	....	....	....	....	....	....
24	....	....	12	17	....	25	....	33	....	....	50	....	....	....	....

Table VIII.—Qualities of Concordant Dyads. (See p. 399.)

Harmonics.	1	2	3	4	5	6	7.	8	9	10	11	12	13	No.
	2 1	3 2	5 2	3 1	4 3	5 3	5 4	6 5	8 5	12 5	8 3	10 3	16 5	Form.
Index.														

| No. of Joint Harmonics. | | | | | | | | | | | | | | | Intensities of the Joint Harmonics. | | | | | | | | | | | | | | |

TABLE IX.—Qualities of Concordant

No. of J. H.	1	2	3	4	5	6	7	8	9	10	11	12	13
	5 3 2	5 3 1	5 4 3	6 5 2	8 5 3	6 5 4	8 6 5	10 3 1	6 5 1	10 4 3	10 3 2	12 5 2	10 8 3
1	0_1	100	0_4	0_1	....	0_4	0_1	100_1	100_1	0_1	0_1	....	....
2	100_1	25_4	0_1	100	0_1	0_1	0_1	25_1	25	....	100	100	0
3	100_1	177	100	0_1	100_1	....	0_1	177	11	100	100	0_1	100
4	25	6_1	100	.25	....	100	....	6	6_1	100	25	25	....
5	100	144	100	100	100_1	100	100	4	144	....	....	100	0
6	67	45	25	177	25	100	100	45	137	25_1	67	11	25
7	....	2	....	....	....	....	....	2_1	2	0_1	0_1	0_1	0
8	6	2	25	6	100	25	100	2	2	25	6_1	6	100
9	11	11	11	....	11	....	....	33_1	....	11	11	....	11
10	49	25	25	49	25	25	25	100	25	100	144	19_1	100
11	....	....	....	....	....	....	....	....	....	....	....	....	....
12	16	6	34	45	6	69	25	6	25	34	16	137	6
13	....	....	....	....	....	....	....	....	....	....	....	....	....
14	2	....	....	2	....	....	....	....	....	....	2	2	....
15	20	28	28	11	28	11	11	4	11	4	4	11	4
16	2	....	6	....	25	6	25	....	....	6	2	....	25
17	....	....	....	....	....	....	....	....	....	....	....	....	....
18	3	3	3	21	3	11	11	3	11	3	3	2	3
19	....	....	....	....	....	....	....	....	....	....	....	....	....
20	4	6	20	6	6	20	6	25	6	20	4	6	28
21	2	2	2	....	2	....	....	2	....	2	....	....	2
24	....	....	8	6	20	18	34	1	6	8	1	25	20
Beats.	$\frac{16}{5}$ $\frac{1}{2}$ $\frac{140}{20}$ $\frac{34}{10}$	$\frac{10}{9}$ 1 $\frac{70}{10}$ $\frac{69}{3}$	$\frac{9}{8}, \frac{10}{9}$ $\frac{1}{3}$ $\frac{210}{30}$ $\frac{69}{3}$	$\frac{15}{14}$ $\frac{1}{2}$ $\frac{140}{20}$ $\frac{22}{5}$	$\frac{10}{8}, \frac{16}{15}$ $\frac{1}{3}$ $\frac{210}{30}$ $\frac{69}{3}, \frac{106}{1}$	$\frac{16}{15}$ $\frac{1}{4}$ $\frac{280}{40}$ $\frac{34}{1}$	$\frac{16}{15}$ $\frac{1}{5}$ $\frac{350}{50}$ $\frac{69}{3}$	$\frac{10}{9}$ 1 $\frac{70}{10}$ $\frac{177}{45}$	$\frac{8}{7}, \frac{20}{18}$ 1, 2 $\frac{70}{10}, \frac{35}{5}$ $\frac{7}{1}, \frac{34}{1}$	$\frac{9}{8}, \frac{16}{15}$ $\frac{1}{3}$ $\frac{210}{30}$ $\frac{177}{46}, \frac{20}{1}$	$\frac{10}{9}$ $\frac{1}{2}$ $\frac{140}{20}$ $\frac{234}{76}$	$\frac{7}{6}, \frac{8}{7}$ $\frac{1}{2}$ $\frac{140}{20}$	$\frac{10}{9}$ $\frac{1}{3}$ $\frac{210}{30}$ $\frac{17}{4}$

Concordant Triads. (See p. 399.)

12	13	14	15	16	17	18	19	20	No.
12 5 2	10 8 3	12 5 1	12 5 4	12 8 5	16 10 3	16 5 3	16 6 5	16 12 5	Form.
.... 100 0_1 25 100	.... 0_1 100 .... 0_1	100 25 11 6_1 144	0_1 .... 100 100	.... 0_1 0_1 100	.... 100	.... 0 100 100	0_1 100	.... 0_1 100	Intensity of Joint Harmonics.
11 0_1 6 .... 19_1	25 0_1 100 11 100	3 0_1 2 25	.... 0_1 25 25	.... 0_1 100 25	25 0_1 .... 11 100	25 .... 11 25	100 50	.... 0_1 25	
.... 137 .... 2 11	.... 6 4	0_1 225 11	.... 177 11	.... 100 11	.... 6 0_1 4	0_1 6 0_1 28	0_1 50 33	0_1 100 11	
.... 2 .... 6 25	25 .... 3 .... 25 2 20	.... 6 25	6 .... 20 45	25 .... 6 69	100 .... 3 .... 25 1	100 .... 3 .... 6 1	100 .... 33 .... 6 6	100 .... 25 25	
$\frac{7}{6}, \frac{8}{7}$ $\frac{1}{2}$ $\frac{140}{20}$	$\frac{10}{9}$ $\frac{1}{3}$ $\frac{210}{30}$ $\frac{177}{45}$	$\frac{11}{10}, \frac{12}{11}$ ·1 $\frac{70}{10}$	$\frac{8}{7}, \frac{16}{15}$ $\frac{1}{4}$ $\frac{280}{40}$ .., $\frac{34}{1}$	$\frac{16}{15}$ $\frac{1}{5}$ $\frac{350}{50}$ $\frac{69}{3}$	$\frac{13}{12}$ $\frac{1}{3}$ $\frac{210}{30}$	$\frac{13}{12}$ $\frac{1}{3}$ $\frac{210}{30}$	$\frac{12}{11}$ $\frac{1}{6}$ $\frac{350}{50}$	$\frac{16}{15}$ $\frac{1}{5}$ $\frac{350}{50}$ $\frac{177}{45}$	Interval. Factor. Range. Intensity.

Index.

Form.	No.
1, 3, 5	2
1, 3, 10	8
1, 5, 6	9
1, 5, 12	14
2, 3, 5	1
2, 3, 10	11
2, 5, 6	4
2, 5, 12	12
3, 4, 5	3
3, 4, 10	10
3, 5, 8	5
3, 5, 16	18
3, 8, 10	13
3, 10, 16	17
4, 5, 6	6
4, 5, 12	15
5, 6, 8	7
5, 6, 16	19
5, 8, 12	16
5, 12, 16	20

Table X.—Duplicated Forms of the Concordant Triad. (See p. 400.)

No.	Simple.	Duplicated.			Simple.	Duplicated.		
1	2, 3, 5	2, 3, 4, 5	2, 3, 5, 8	2, 3, 5, 6	C G e	C G c e	C G e c^2	C G e g
2	1, 3, 5	1, 2, 3, 5	1, 3, 4, 5	1, 3, 5, 6	C^4 G e	C^4 C G e	C^4 G c e	C^4 G e g
3	3, 4, 5	3, 4, 5, 6	3, 4, 5, 8	3, 4, 5, 10	G c e	G c e g	G c e c^2	G c e e^2
4	2, 5, 6	2, 4, 5, 6	2, 5, 6, 8		C e g	C c e g	C e g c^2	
5	3, 5, 8	3, 5, 6, 8	3, 5, 8, 10		G e c^2	G e g c^2	G e c^2 e^2	
6	4, 5, 6	4, 5, 6, 8	4, 5, 6, 10	4, 5, 6, 12	c e g	c e g c^2	c e g e^2	c e g g
7	5, 6, 8	5, 6, 8, 10	5, 6, 8, 12		e g c^2	e g c^2 e^2	e g c^2 g^2	
8	1, 3, 10	1, 2, 3, 10	1, 3, 4, 10	1, 3, 6, 10	C^4 G e^2	C^4 C G e^2	C^4 G c e^2	C^4 G g e
9	1, 5, 6	1, 2, 5, 6	1, 4, 5, 6		C^4 e g	C^4 C e g	C^4 c e g	
10	3, 4, 10	3, 4, 6, 10			G c e^2	G c g e^2		
11	2, 3, 10	2, 3, 4, 10	2, 3, 8, 10		C G e^2	C G c e^2	C G c^2 e^2	
12	2, 5, 12	2, 4, 5, 12	2, 5, 8, 12		C e g^2	C c e g^2	C e c^2 g^2	
13	3, 8, 10	3, 6, 8, 10			G c^2 e^2	C g c^2 e^2		
14	1, 5, 12	1, 2, 5, 12	1, 4, 5, 12	1, 5, 8, 12	C^4 e g^2	C^4 C e g^2	C^4 c e g^2	C^4 e c^2 g
15	4, 5, 12	4, 5, 8, 12	4, 5, 10, 12	4, 5, 8, 10	c e g^2	c e c^2 g^2	c e c^2 g^2	c e c^2 e
16	5, 8, 12	5, 8, 10, 12			e c^2 g^2	e c^2 e^2 g^2		
17	3, 10, 16	3, 6, 10, 16	3, 10, 12, 16		G e^2 c^4	G g e^2 c^4	G e^2 g^2 c^4	
18	8, 5, 16	3, 5, 6, 16	3, 5, 10, 16		G e c^4	G e g c^4	G e e^2 c^4	
19	5, 6, 16	5, 6, 10, 16	5, 6, 12, 16		e g c^4	e g e^2 c^4	e g g^2 c^4	
20	5, 12, 16	5, 10, 12, 16			e g^2 c^4	e e^2 g^2 c^4		

Table XI. (See p. 400.)

Forms of the Minor Triad.					Index.	
No.	Form.	Diff. Tones.	Form.	Differential Tones.	Form.	No.
1	3, 5, 15	2, 10, 12	G^4 E b	C^4, e, g	3, 5, 15	1
2	12, 15, 20	3, 5, 8	g b e^2	G^4, E, c	3, 10, 15	8
3	10, 12, 15	2, 3, 5	e g b	C^4, G^4, E	3, 15, 20	12
4	5, 12, 15	3, 7, 10	E g b	G^4, $\flat B\flat$, e	3, 15, 40	18
5	6, 10, 15	4, 5, 9	G e b	C, E, d	5, 6, 15	11
6	15, 20, 24	4, 5, 9	b e^2 g^2	C, E, d	5, 12, 15	4
7	6, 15, 20	5, 9, 14	G b e^2	E, d, $\flat b\flat$	5, 15, 24	16
8	3, 10, 15	5, 7, 12	G^4 e b	E, $\flat B\flat$, g	5, 15, 48	19
9	12, 15, 40	3, 25, 28	g b e^4	G^4, $\sharp g^{2\sharp}$, $\flat b^2\flat$	6, 10, 15	5
10	10, 15, 24	5, 9, 14	e b g^2	E, d, $\flat b\flat$	6, 15, 20	7
11	5, 6, 15	1, 9, 10	E G e	C^8, d, e	6, 15, 40	17
12	3, 15, 20	5, 12, 17	G^4 b e^2	E, g, $\flat d^2\flat$	10, 12, 15	3
13	15, 20, 48	5, 28, 33	b e^2 g^4	E, $\flat b^2\flat$, xj c^4	10, 15, 24	10
14	15, 24, 40	9, 16, 25	b g^2 e^4	d, c^2, $\sharp g^{2\sharp}$	10, 15, 48	20
15	15, 40, 48	8, 25, 33	b e^4 g^4	c, $\sharp g^{2\sharp}$, xj c^4	12, 15, 20	2
16	5, 15, 24	9, 10, 19	E b g^2	d, e, $\frac{12}{12}\frac{16}{15}$ $d^{2\sharp}$	12, 15, 40	9
17	6, 15, 40	9, 25, 34	G b e^4	d, $\sharp g^{2\sharp}$, $\flat d^4\flat$	15, 20, 24	6
18	3, 15, 40	12, 25, 37	G^4 b e^4	g, $\sharp g^{2\sharp}$, $\sharp\sharp e^4\flat$	15, 20, 48	13
19	5, 15, 48	10, 33, 43	E b g^4	e, xj c^4, $\frac{14}{14}\frac{19}{08}$ f^4	15, 24, 40	14
20	10, 15, 48	5, 23, 38	e b g^4	E, $\frac{7}{12}\frac{36}{20}$ $\sharp f^{2\sharp}$, $\frac{12}{12}\frac{16}{15}$ $d^{4\sharp}$	15, 40, 48	15

Table XII.—General Table of Equal Temperament. (See p. 407.)

Just note.	Tempered note.	T, log of tempered pitch.	J, log of just pitch.	ϵ, T − J.	β, beat meter.	Interval.
c	c	·0000000	·0000000	0		I
†c			·0053950	−k		†I
‡c♯	c♯	·0285191 − 7x	·0177287	2k − 7x		‡I♯
d♭	d♭	·0226335 + 5x	·0280285	−k + 5x		2
	c×	·0570382 − 14x				
‡d	d	·0511526 − 2x	·0457574	k − 2x		‡II
d			·0511526	−2x		II
	e♭♭	·0452670 + 10x				
‡d♯	d♯	·0796717 − 9x	·0688813	2k − 9x		‡II♯
†e♭	e♭	·0737861 + 3x	·0791812	−k + 3x	−6k + 18x	3
e	e	·1023052 − 4x	·0969100	k − 4x	5k − 20x	III
	f♭	·0964196 + 8x				
	e♯	·1308243 − 11x				
f	f	·1249387 + x	·1249386	x	4x	4
†f			·1303338	−k + x		†4
‡f♯	f♯	·1534578 − 6x	·1426675	2k − 6x		‡IV
f♯			·1480626	k − 6x		IV
	g♭	·1475722 + 6x				5
	f×	·1819769 − 13x				
‡g	g	·1760913 − x	·1706961	k − x		‡V
g			·1760913	−x	−3x	V
	a♭♭	·1702057 + 11x				
‡g♯	g♯	·2046104 − 8x	·1938200	2k − 8x		‡V♯
†a♭	a♭	·1987248 + 4x	·2041199	−k + 4x	−8k + 32x	6
	g×	·2331295 − 15x				
a	a	·2272439 − 3x	·2218486	k − 3x	5k − 15x	VI
†a			·2272438	−3x		†VI
	b♭♭	·2213583 + 9x				
	a♯	·2557630 − 10x				
b♭	b♭	·2498774 + 2x	·2498773	2x		7
†b♭			·2552725	−k + 2x		†7
‡b	b	·2783965 − 5x	·2676061	2k − 5x		‡VII
b			·2730013	k − 5x		VII
	c♭	·2725109 + 7x				
	b♯	·0058851 − 12x				
c²	c²	·3010300	·3010300	0		VIII
				$\Sigma\epsilon^2$ = 32k² − 212kx + 420x²	$\Sigma\beta^2$ = 150k² − 1078kx + 1998x²	

Where k = ·0053950 and x is arbitrary.

Table XIII.

Wolves of Defective Equal Temperaments. (See p. 415.)

Wolves.	Interval error, ϵ.	Beat meter, β.
Vth wolf = Vw. G♯ e♭ = A♭ e♭ = G♯ d♯	−k−s+11x	−3k−3s+33x
4th wolf = 4w. E♭ G♯ = E♭ A♭ = D♯ G♯	k+s−11x	4k+4s−44x
IIIrd wolf = IIIw. G♯ c = A♭ c B e♭ = B d♯ F♯ B♭ = F♯ A♯ C♯ F = C♯ E♯	−s+8x	−5s+40x
6th wolf = 6w. C G♯ = C A♭ E♭ B = D♯ B B♭ f♯ = A♯ f♯ F c♯ = E♯ c♯	s−8x	8s−64x
3rd wolf = 3w. E♭ F♯ = D♯ F♯ B♭ c♯ = A♯ c♯ F G♯ = E♯ G♯	s−9x	6s−54x
VIth wolf = VIw. F♯ e♭ = F♯ d♯ C♯ B♭ = C♯ A♯ G♯ f = G♯ e♯	−s+9x	−5s+45x

Where $k = \cdot0053950$, $s = \cdot0004901$, and x is arbitrary.

Table XIV. (See p. 419.)

Comparative Table of the Mesotonic and Hemitonic Temperaments.

J. Just Intonation in the keys of *B♭*, *F*, *C*, *G*, *D*, or System of *C*; 33 tones.
M. Mesotonic Temperament in all keys. No. 2 (2); 27 tones.
H. Hemitonic Temperament in all keys. No. 50 (35); 12 tones.

Notes	
J	M
c †c	c
‡c♯ c♯ d♭	c♯ d♭
‡‡d ‡d d	c× d e♭♭
‡d♯ †e♭	d♯ e♭
e †e	e f♭
‡f f †f	e♯ f
‡f♯ f♯ g♭	f♯ g♭
‡g g †g	f× g a♭♭
‡g♯ a♭ †a♭	g♯ a♭
‡a a †a	g× a b♭♭
a♯ b♭ †b♭	a♯ b♭
‡b b	b c♭
‡c² c²	b♯ c²

Table XIV. (*continued*).

Notes.			Logarithms.			Interval Errors.		Beat Factor.		Intervals.
J	M	H	J	M	H	M−J	H−J	M	H	
c	c	c	·00000	·00000	·00000	·00000	·00000			I
†c			·00540			−·00540	−·00540			†I
‡c♯	c♯		·01773	·01908		+·00135	+·00736			‡I♯
c♯		c♯	·02312		·02509	−·00404	+·00197			I♯
d♭	d♭		·02803	·02938		+·00135	−·00294			2
	c×			·03816						
‡‡d			·04036			+·00810	+·00981			‡‡II
‡d	d	d	·04576	·04846	·05017	+·00270	+·00441			‡II
d			·05115			−·00270	−·00098			II
	e♭♭			·05876						
‡d♯	d♯		·06888	·06753	·07526	−·00135	+·00638			‡II♯
†e♭	e♭	d♯	·07918	·07783		−·00135	−·00392	−·018605	−·053965	3
e	e		·09691	·09691		·00000	+·00343	·000000	+·039684	III
†e		e	·10231		·10034	−·00540	−·00197			†III
	f♭			·10721						
	e♯			·11599						
‡f			·11954			+·00675	+·00599			‡4
f	f	f	·12494	·12629	·12543	+·00135	+·00049	+·012440	+·004520	4
†f			·13033			−·00405	−·00491			†4
‡f♯	f♯		·14267	·14537		+·00270	+·00784			‡IV
f♯		f♯	·14806		·15051	−·00270	+·00245			IV
g♭	g♭		·15297	·15567		+·00270	−·00246			5
	f×			·16444						
‡g			·17070			+·00405	+·00490			†V
g	g	g	·17609	·17474	·17560	−·00135	−·00049	−·009304	−·003386	V
†g			·18149			−·00676	−·00589			†V
	a♭♭			·18504						
‡g♯	g♯		·19382	·19382		·00000	+·00686			‡V♯
a♭		g♯	·19873		·20068	+·00539	+·00195			‡6
†a♭	a♭		·20412	·20412		·00000	−·00344	·000000	−·062995	6
	g×			·21290						
‡a			·21645			+·00675	+·00932			‡VI
a	a	a	·22185	·22320	·22577	+·00135	+·00392	+·015553	+·045379	VI
†a			·22724			−·00405	−·00147			†VI
	b♭♭			·23350						
a♯	a♯		·24497	·24228		−·00270	+·00589			VI♯
b♭		a♯	·24988	·25258	·25086	+·00270	+·00098			7
†b♭	b♭		·25527			−·00270	−·00441			†7
‡b			·26761			+·00405	+·00833			‡VII
b	b	b	·27300	·27165	·27594	−·00135	+·00294			VII
	c²♭			·28195						
	b♯			·29073						
‡c²		c²	·29563		·30103	+·00540	+·00540			†VIII
c²	c²		·30103	·30103		·00000	·00000			VIII
Sum of squares.....						·0004735	·0008748	·000829	·010547	

Table XV.—Comparative Table of Equal Te

Systematic No.	Name.	Log *v*.	Error of Vth.		Error of IIIrd.	
			Order.	Error.	Order.	Error.
26	Least Errors	·1747297	32	−·0013616	2	−·0000514
2, 13, 19	Mesotonic	·1747426	31	−·0013488	1	0
11	Equal beats of VI and 4	·1746716	33	−·0014197	6	−·0002838
35	Cycle of 81	·1746703	34	−·0014205	7	−·0002870
30	Least Errors and Beats	·1746439	35	−·0014474	9	−·0003946
28	*Woolhouse's* Equal Harmony	·1746388	36	−·0014525	10	−·0004150
17	Least Beats of V and III	·1747723	30	−·0013190	3	+·0001190
15	Least Beats	·1746359	37	−·0014554	11	−·0004266
34	Cycle of 93	·1747872	29	−·0013039	4	+·0001794
10	Equal and opp. Beats of 3 and 4	·1746200	38	−·0014713	13	−·0004902
47	*Huyghens's* Cycle of 31	·1747900	28	−·0013013	5	+·0001898
43	*Henfling's* Cycle of 50	·1745974	39	−·0014939	14	−·0005806
14, 29	*Drobisch's* Simplest	·1748219	27	−·0012694	8	−·0003174
9	*Dr. Smith's* Equal Harmony	·1745927	40	−·0014986	15	−·0005994
16	Least Beats of 3, III, V	·1744404	41	−·0015309	18	−·0007286
5	Equal Beats of 6 and V	·1748582	26	−·0012331	12	+·0004626
8, 20	Equal Errors of III and 3	·1745199	42	−·0015414	19	−·0007707
21	*Romieu's* Theoretic	·1745163	44	−·0015750	22	−·0009050
7	Equal and opp. Beats of 6 and 4	·1748924	25	−·0011989	15	+·0005994
37	*Drobisch's* Cycle of 74	·1749200	23	−·0011713	17	+·0007108
24, 27	*Drobisch's* least Errors	·1749352	22	−·0011561	19	+·0007706
4	Equal and opp. Beats of III & V	·1749188	24	−·0011725	16	+·0007050
6	Equal Beats of III and 4	·1749674	21	−·0011239	21	+·0008994
38	Cycle of 22	·1745200	43	−·0015713	20	−·0008902
3	Perfect 3rds and VIths	·1742930	45	−·0017983	26	−·0017982
12, 18	Errors of III and V eq. and op.	·1750123	20	−·0010790	23	+·0010790
46	*Sauveur's* Cycle of 43	·1750175	19	−·0010738	24	+·0011008
49	*Woolhouse's* Cycle of 19	·1742807	46	−·0018106	27	−·0018474
23	*Drobisch* after *Delezenne*	·1751830	18	−·0019683	25	+·0015218
41	Cycle of 57	·1742796	47	−·0018117	28	−·0018518
39	Cycle of 67	·1752270	16	−·0008643	29	+·0021378
22	*Romieu's* Anacratic	·1751938	17	−·0008975	32	+·0026056
42	Musicians' Cycle of 55	·1751456	48	−·0019457	31	−·0023878
54	*Marsh's*	·1752790	15	−·0008123	30	+·0021458
31	*Drobisch's* V and III combined	·1754169	14	−·0006744	33	+·0026974
51	Least Wolf Errors	·1755418	13	−·0005495	34	+·0031970
57	No V Wolf	·1755562	12	−·0005351	35	+·0032549
50, 53	Hemitonic	·1756008	11	−·0004905	37	+·0034338
55	Least III and V Wolves	·1757201	10	−·0003712	38	+·0039102
45	Cycle of 45	·1738940	49	−·0021973	36	−·0033942
56	III and V Wolves eq. and opp.	·1757557	9	−·0003356	39	+·0040522
33	*Drobisch's* Cycle of 118	·1760259	6	−·0000654	43	+·0051338
36, 52	Least Wolf Beats	·1759275	8	−·0001638	41	+·0047398
40	Cycle of 65	·1759867	7	−·0001046	42	+·0049766
58	No III Wolf	·1760300	5	−·0000613	44	+·0051498
32	*Drobisch's* Violin	·1760322	4	−·0000591	45	+·0051588
59	No 3rd Wolf	·1760368	3	−·0000545	46	+·0051778
44	*N. Mercator* and *Drobisch*	·1760800	2	−·0000113	47	+·0053498
48	Cycle of 26	·1736700	50	−·0024213	40	−·0042900
1	Pythagorean	·1760913	1	0	48	+·0053958
25	Error of V and 3 eq. and opp.	·1733938	51	−·0026975	48	−·0053958

r of IIIrd.	Error of 3rd.		Melodic Errors.		Harmonic Errors.		Comparison.	
Error.	Order.	Error.	Order.	$\Sigma\epsilon^2$.	Order.	$\Sigma\beta^2$.	Sum of Orders.	Order of sums.
−·0000514	17	−·0013102	1	·0001527	11	·0001513	63	1
0	18	−·0013488	2	·0001528	12	·0001565	64	2
−·0002838	15	−·0011359	4	·0001541	6	·0001363	64	2
−·0002870	14	−·0011335	6	·0001542	5	·0001361	66	3
−·0003946	13	−·0010528	8	·0001559	3	·0001339	68	4
−·0004150	12	−·0010375	9	·0001562	2	·0001338	69	5
+·0001190	19	−·0014380	3	·0001535	15	·0001709	70	6
−·0004266	11	−·0010288	11	·0001564	1	·0001337	71	7
+·0001794	20	−·0014833	5	·0001541	16	·0001718	74	8
−·0004902	10	−·0009811	12	·0001578	4	·0001342	77	9
+·0001898	21	−·0014911	7	·0001543	17	·0001812	78	10
−·0005806	9	−·0009133	14	·0001601	7	·0001367	83	11
−·0003174	22	−·0015868	10	·0001563	18	·0002029	85	12
−·0005994	8	−·0008992	15	·0001606	8	·0001375	86	13
−·0007286	7	−·0008023	18	·0001648	9	·0001451	93	14
+·0004626	24	−·0016957	13	·0001597	19	·0002325	94	15
−·0007707	6	−·0007707	19	·0001664	10	·0001485	96	16
−·0009050	5	−·0006700	16	·0001719	14	·0001623	101	17
+·0005994	25	−·0017983	17	·0001689	20	·0002625	102	18
+·0007108	28	−·0018811	20	·0001679	22	·0002950	110	19
+·0007706	29	−·0019267	21	·0001705	23	·0003127	114	20
+·0007050	27	−·0018775	26	·0001983	21	·0002936	114	20
+·0008994	30	−·0020233	23	·0001764	24	·0003533	119	21
−·0008902	23	−·0016811	22	·0001706	13	·0001589	121	22
−·0017982	1	0	28	·0002329	25	·0003587	125	23
+·0010790	31	−·0020580	24	·0001863	29	·0004168	127	24
+·0011008	32	−·0021736	25	·0001875	30	·0004247	130	25
−·0018474	2	+·0000368	29	·0002375	26	·0003858	130	25
+·0015218	33	−·0026901	27	·0002177	31	·0006077	134	26
−·0018518	3	+·0000401	30	·0002378	27	·0003873	135	27
+·0021378	36	−·0028021	32	·0002563	34	·0008306	147	28
+·0026050	35	−·0027025	31	·0002432	33	·0007556	148	29
−·0023878	4	+·0004421	34	·0002961	32	·0006140	149	30
+·0021458	37	−·0029581	33	·0002794	35	·0009600	150	31
+·0026974	38	−·0033718	35	·0003511	37	·0013524	157	32
+·0031970	39	−·0037465	36	·0004297	38	·0017734	160	33
+·0032546	40	−·0037897	37	·0004397	39	·0018255	163	34
+·0034333	41	−·0039235	39	·0004714	40	·0019940	168	35
+·0039102	42	−·0042814	40	·0005648	42	·0024829	172	36
−·0033942	16	+·0011969	38	·0004461	36	·0012335	175	37
+·0040526	43	−·0043882	41	·0005939	43	·0026391	175	37
+·0051334	48	−·0051988	51	·0009584	28	·0039960	176	38
+·0047398	44	−·0049036	43	·0007554	45	·0034669	181	39
+·0049766	45	−·0050812	44	·0008164	46	·0037795	184	40
+·0051498	46	−·0052111	45	·0008628	47	·0040169	187	41
+·0051586	49	−·0052177	46	·0008651	48	·0040292	192	42
+·0051770	47	−·0052315	47	·0008703	49	·0040549	192	42
+·0053498	50	−·0053611	49	·0009185	50	·0043004	198	43
−·0042902	26	−·0018689	42	·0006244	41	·0019977	199	44
+·0053950	51	−·0053950	50	·0009314	51	·0043659	201	45
−·0053950	34	+·0026975	48	·0009028	44	·0032183	225	46

Error of 3rd.		Melodic Errors.		Harmonic Errors.		Comparison.	
er.	Error.	Order.	$\Sigma\epsilon^2$.	Order.	$\Sigma\beta^2$.	Sum of Orders.	Order of sums.
	−·0013102	1	·0001527	11	·0001513	63	1
	−·0013488	2	·0001528	12	·0001565	64	2
	−·0011359	4	·0001541	6	·0001363	64	2
	−·0011335	6	·0001542	5	·0001361	66	3
	−·0010528	8	·0001559	3	·0001339	68	4
	−·0010375	9	·0001562	2	·0001338	69	5
	−·0014380	3	·0001535	15	·0001709	70	6
	−·0010288	11	·0001564	1	·0001337	71	7
	−·0014833	5	·0001541	16	·0001718	74	8
	−·0009811	12	·0001578	4	·0001342	77	9
	−·0014911	7	·0001543	17	·0001812	78	10
	−·0009133	14	·0001601	7	·0001367	83	11
	−·0015868	10	·0001563	18	·0002029	85	12
	−·0008992	15	·0001606	8	·0001375	86	13
	−·0008023	18	·0001648	9	·0001451	93	14
	−·0016957	13	·0001597	19	·0002325	94	15
	−·0007707	19	·0001664	10	·0001485	96	16
	−·0006700	16	·0001719	14	·0001623	101	17
	−·0017983	17	·0001639	20	·0002625	102	18
	−·0018811	20	·0001679	22	·0002950	110	19
	−·0019267	21	·0001705	23	·0003127	114	20
	−·0018775	26	·0001983	21	·0002936	114	20
	−·0020233	23	·0001764	24	·0003533	119	21
	−·0016811	22	·0001706	13	·0001589	121	22
	0	28	·0002329	25	·0003587	125	23
	−·0020580	24	·0001863	29	·0004168	127	24
	−·0021736	25	·0001875	30	·0004247	130	25
	+·0000368	29	·0002375	26	·0003858	130	25
	−·0026901	27	·0002177	31	·0006077	134	26
	+·0000401	30	·0002378	27	·0003873	135	27
	−·0028021	32	·0002563	34	·0008306	147	28
	−·0027025	31	·0002432	33	·0007556	148	29
	+·0004421	34	·0002961	32	·0006140	149	30
	−·0029581	33	·0002794	35	·0009600	150	31
	−·0033718	35	·0003511	37	·0013524	157	32
	−·0037465	36	·0004297	38	·0017734	160	33
	−·0037897	37	·0004397	39	·0018255	163	34
	−·0039235	39	·0004714	40	·0019940	168	35
	−·0042814	40	·0005648	42	·0024829	172	36
	+·0011969	38	·0004461	36	·0012335	175	37
	−·0043882	41	·0005939	43	·0026391	175	37
	−·0051988	51	·0009584	28	·0039960	176	38
	−·0049036	43	·0007554	45	·0034669	181	39
	−·0050812	44	·0008164	46	·0037795	184	40
	−·0052111	45	·0008628	47	·0040169	187	41
	−·0052177	46	·0008651	48	·0040292	192	42
	−·0052315	47	·0008703	49	·0040549	192	42
	−·0053611	49	·0009185	50	·0043004	198	43
	−·0018689	42	·0006244	41	·0019977	199	44
	−·0053950	50	·0009314	51	·0043659	201	45
	+·0026975	48	·0009028	44	·0032183	225	46

"On the Calculus of Symbols.—Fourth Memoir. With Applications to the Theory of Non-Linear Differential Equations." By W. H. L. Russell, Esq., A.B. Received July 31, 1863*.

In the preceding memoirs on the Calculus of Symbols, systems have been constructed for the multiplication and division of non-commutative symbols subject to certain laws of combination; and these systems suffice for linear differential equations. But when we enter upon the consideration of non-linear equations, we see at once that these methods do not apply. It becomes necessary to invent some fresh mode of calculation, and a new notation, in order to bring non-linear functions into a condition which admits of treatment by symbolical algebra. This is the object of the following memoir. Professor Boole has given, in his 'Treatise on Differential Equations,' a method due to M. Sarrus, by which we ascertain whether a given non-linear function is a complete differential. This method, as will be seen by anyone who will refer to Professor Boole's treatise, is equivalent to finding the conditions that a non-linear function may be externally divisible by the symbol of differentiation. In the following paper I have given a notation by which I obtain the actual expressions for these conditions, and for the symbolical remainders arising in the course of the division, and have extended my investigations to ascertaining the results of the symbolical division of non-linear functions by linear functions of the symbol of differentiation.

Let $F(x, y, y_1, y_2, y_3 \ldots y_n)$ be any non-linear function, in which $y_1, y_2, y_3, \ldots y_n$ denote respectively the first, second, third, ... nth differential of y with respect to (x).

Let U_r denote $\int dy_r$, *i.e.* the integral of a function involving $x, y, y_1, y_2 \ldots$ with reference to y_r alone.

Let V_r in like manner denote $\frac{d}{dy_r}$ when the differentiation is supposed effected with reference to y_r alone, so that $V_r U_r F = F$.

The next definition is the most important, as it is that on which all our subsequent calculations will depend. We may suppose F differentiated (m) times with reference to y_n, y_{n-1}, or y_{n-2}, &c., and y_n, y_{n-1}, or y_{n-2}, &c., as the case may be, afterward equated to zero. We shall denote this entire process by $Z_n^{(m)}$, $Z_{n-1}^{(m)}$, $Z_{n-2}^{(m)}$, &c.

The following definition is also of importance: we shall denote the expression

$$\frac{d}{dx}+y_1\frac{d}{dy}+y_2\frac{d}{dy_1}+y_3\frac{d}{dy_3}+\ldots+y_r\frac{d}{dy_{r-1}}$$

by the symbol Y_r.

* Read Feb. 11, 1864; see Abstract, vol. xiii. p. 126.

Having thus explained the notation I propose to make use of, I proceed to determine the conditions that F may be externally divisible by $\frac{d}{dx}$, or, in other words, that F may be a perfect differential with respect to (x). It will be seen that the above notation will enable us to obtain expressions for the conditions indicated by the process of M. Sarrus.

It is obvious that if we expand F in terms of y_n, in order that the symbolical division with reference $\frac{d}{dx}$ may be possible, the terms involving y_n^2, y_n^3, &c. must vanish.

Hence $V_n^2 F=0$, and consequently

$$F=Z_n^0 F+y_n Z'_n F,$$

where, of course, $Z_n^0 F$, $Z'_n F$ do not contain y_n.

Hence we have

$$\frac{d}{dx}(U_{n-1}Z'_n F)=Y_{n-1}U_{n-1}Z'_n F+y_n Z'_n F,$$

and therefore F becomes

$$\frac{d}{dx}(U_{n-1}Z'_n F)+Z_n^0 F-Y_{n-1}U_{n-1}Z'_n F;$$

and if R_1 be the first remainder,

$$R_1=Z_n^0 F-Y_{n-1}U_{n-1}Z'_n F.$$

The condition that this may be divisible by $\frac{d}{dx}$ will be, as before, $V_{n-1}^2 R_1=0$; hence R_1 becomes

$$Z_{n-1}^0 Z_n^0 F-Z_{n-1}^0 Y_{n-1}U_{n-1}Z'_n F+y_{n-1}(Z'_{1-n}Z_n^0 F-Z'_{n-1}Y_{n-1}U_{n-1}Z'_n F).$$

Now

$$\frac{d}{dx}U_{n-2}(Z'_{n-1}Z_n^0 F-Z'_{n-1}Y_{n-1}U_{n-1}Z'_n F)=$$
$$Y_{n-2}U_{n-2}Z'_{n-1}Z_n^0 F-Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_n F$$
$$+y_{n-1}(Z'_{n-1}Z_n^0 F-Z'_{n-1}Y_{n-1}U_{n-1}Z'_n F);$$

and if R_2 be the second remainder, we find

$$R_2=Z_{n-1}^0 Z_n^0 F-Z_{n-1}^0 Y_{n-1}U_{n-1}Z'_n F$$
$$-Y_{n-2}U_{n-2}Z'_{n-1}Z_n^0 F+Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_n F:$$

the next condition is $V_{n-2}R_2=0$, and therefore

$$R_2=Z_{n-2}^0 Z_{n-1}^0 Z_n^0 F$$
$$-Z_{n-2}^0 Z_{n-1}^0 Y_{n-1}U_{n-1}Z'_n F-Z_{n-2}^0 Y_{n-2}U_{n-2}Z'_{n-1}Z_n^0 F$$
$$+Z_{n-2}^0 Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_n F$$
$$+y_{n-2}(Z'_{n-2}Z_{n-1}^0 Z_n^0 F-Z'_{n-2}Z_{n-1}^0 Y_{n-1}U_{n-1}Z'_n F$$
$$-Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Z_n^0 F+Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_n F).$$

But

$$\frac{d}{dx}\Big\{U_{n-3}(Z'_{n-2}Z^0_{n-1}Z^0_nF-Z'_{n-2}Z^0_{n-1}Y_{n-1}U_{n-1}Z'_nF$$
$$-Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Z^0_nF+Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_nF)\Big\}$$
$$=Y_{n-3}U_{n-3}(Z'_{n-2}Z^0_{n-1}Z^0_nF-Z'_{n-2}Z^0_{n-1}Y_{n-1}U_{n-1}Z'_nF$$
$$-Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Z^0_nF+Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_nF)$$
$$+y_{n-2}(Z'_{n-2}Z^0_{n-1}Z^0_nF-Z'_{n-2}Z^0_{n-1}Y_{n-1}U_{n-1}Z'_nF$$
$$-Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Z^0_nF+Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_nF);$$

whence we find

$$R_3=Z^0_{n-2}Z^0_{n-1}Z^0_nF-Z^0_{n-2}Z^0_{n-1}Y_{n-1}U_{n-1}Z'_nF$$
$$-Z^0_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Z^0_nF+Z^0_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_nF$$
$$-Y_{n-3}U_{n-3}Z'_{n-2}Z^0_{n-1}Z^0_nF+Y_{n-3}U_{n-3}Z'_{n-2}Z^0_{n-1}Y_{n-1}U_{n-1}Z'_nF$$
$$+Y_{n-3}U_{n-3}Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Z^0_nF$$
$$-Y_{n-3}U_{n-3}Z'_{n-2}Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_nF.$$

Hence we infer the following rule for the formation of R_r.

Construct the term

$$Y_{n-r}U_{n-r}Z'_{n-r+1}Y_{n-r+1}U_{n-r+1}Z'_{n-r+2}Y_{n-r+2}U_{n-r+2}Z'_{n-r+2}$$
$$\ldots\ldots Y_{n-2}U_{n-2}Z'_{n-1}Y_{n-1}U_{n-1}Z'_nF.$$

In any symbol Z'_m the accent may be changed into a zero, *i. e.* we may at pleasure substitute Z^0_m anywhere for Z'_m; but in such case the previous symbolical factor $Y_{m-1}U_{m-1}$ must be omitted. This term is positive or negative according as the symbol Z' occurs an even or an odd number of times in it; the aggregate of all the terms thus formed constitute the remainder R_r, and the conditions that F may be externally divisible by $\frac{d}{dx}$ are

$$V_nF=0,\ V_{n-1}R_1=0,\ V_{n-2}R_2=0,\ V_{n-3}R_3=0,\ \&c.$$

We shall now investigate the conditions that $\frac{d}{dx}+P$ may externally divide F where P is a function of (x) and (y).

As before, $V_nF=0$, and in consequence

$$F=Z^0_nF+y_nZ'_nF.$$

Now

$$\left(\frac{d}{dx}+P\right)U_{n-1}Z'_nF=Y_{n-1}U_{n-1}Z'_nF$$
$$+PU_{n-1}Z'_nF+y_nZ'_nF.$$

Hence we shall have

$$R_1=Z^0_nF-Y_{n-1}U_{n-1}Z'_nF-PU_{n-1}Z'_nF.$$

We have $V^2_{n-1}R_1=0$ in order that this remainder may contain only the first power of y_{n-1}, and

$$R_1=Z^0_{n-1}Z^0_nF-Z^0_{n-1}Y_{n-1}U_{n-1}Z'_nF-Z^0_{n-1}PU_{n-1}Z'_nF$$
$$+y_{n-1}(Z'_{n-1}Z^0_nF-Z'_{n-1}Y_{n-1}U_{n-1}Z'_nF-Z'_{n-1}PU_{n-1}Z'_nF),$$

since

$$\left(\frac{d}{dx}+\mathrm{P}\right)\mathrm{U}_{n-2}(\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}-\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F})=$$
$$\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-2}\mathrm{Z}^0_n\mathrm{F}-\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}-$$
$$\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}+y_{n-1}(\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$-\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}-\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F})+$$
$$\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}-\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}.$$

Whence we find that $\frac{d}{dx}+\mathrm{P}$ divides R_1 with a remainder,

$$\mathrm{R}_2=\mathrm{Z}^0_{n-1}\mathrm{Z}^0_n\mathrm{F}-\mathrm{Z}^0_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{Z}^0_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}-\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$+\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}+\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}+\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$+\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}.$$

Putting $\mathrm{Y}^2_{n-2}\mathrm{R}_2=0$, we find in like manner,

$$\mathrm{R}_3=\mathrm{Z}^0_{n-2}\mathrm{Z}^0_{n-1}\mathrm{Z}^0_n\mathrm{F}-\mathrm{Z}^0_{n-2}\mathrm{Z}^0_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{Z}^0_{n-2}\mathrm{Z}^0_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}-\mathrm{Z}^0_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$+\mathrm{Z}^0_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}+\mathrm{Z}^0_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{Z}^0_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}+\mathrm{Z}^0_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$+\mathrm{Z}^0_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}-\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Z}^0_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$+\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Z}^0_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$+\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Z}^0_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$+\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$-\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$+\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{P}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$-\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{Y}_{n-3}\mathrm{U}_{n-3}\mathrm{Z}'_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Z}^0_{n-1}\mathrm{Z}^0_n\mathrm{F}+\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Z}^0_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$+\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Z}^0_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}+\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$-\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{Y}_{n-2}\mathrm{U}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$+\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Z}^0_n\mathrm{F}$$
$$-\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{Y}_{n-1}\mathrm{U}_{n-1}\mathrm{Z}'_n\mathrm{F}$$
$$-\mathrm{PU}_{n-3}\mathrm{Z}'_{n-2}\mathrm{PU}_{n-2}\mathrm{Z}'_{n-1}\mathrm{PU}_{n-1}\mathrm{Z}'_n\mathrm{F}.$$

We see at once that the value of R_r in this case can be formed from that calculated in the last example, by writing P at pleasure for any one or

more of the symbols Y, and taking the aggregate of the terms so formed. The conditions of division will be, as before,

$$V_nF=0,\ V_{n-1}R_1=0,\ V_{n-2}R_2=0\ \ldots.$$

Let us now investigate the conditions that F may be externally divisible by $\frac{d^2}{dx^2}$.

We see at once that F, as before, must take the form $Z_n^0F+y_nZ'_nF$, and also that Z'_nF can contain neither y_n nor y_{n-1}. Hence we shall have

$$V_n^2F=0, \text{ and also } V_{n-1}V_nF=0.$$

Now

$$\frac{d^2}{dx^2}(U_{n-2}Z'_nF)=\frac{d}{dx}\{Y_{n-2}+y_{n-1}V_{n-2}\}U_{n-2}Z'_nF$$
$$=XY_{n-2}U_{n-2}Z'_nF+y_{n-1}XZ'_nF+y_nZ'_nF.$$

Hence we shall have

$$R_1=Z_n^0F-XY_{n-2}U_{n-2}Z'_nF-y_{n-1}XZ'_nF,$$

when we must introduce the conditions

$$V_{n-1}^2R_1=0, \text{ and } V_{n-2}V_{n-1}R_1=0;$$

consequently we shall have

$$R_1=Z_{n-1}^0Z_n^0F-Z_{n-1}^0XY_{n-2}U_{n-2}Z'_nF+$$
$$(Z'_{n-1}Z_n^0F-Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF-Z_{n-1}^0XZ'_nF)y_{n-1}.$$

Now

$$\frac{d^2}{dx^2}U_{n-3}(Z'_{n-1}Z_n^0F-Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF$$
$$-Z_{n-1}^0XZ'_nF)=$$
$$XY_{n-3}U_{n-3}Z'_{n-1}Z_n^0F-XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF$$
$$-XY_{n-3}U_{n-3}Z_{n-1}^0XZ'_nF+(XZ'_{n-1}Z_n^0F-$$
$$XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF-XZ_{n-1}^0XZ'_nF)y_{n-2}+$$
$$(Z'_{n-1}Z_n^0F-Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF-Z_{n-1}^0XZ'_nF)y_{n-1}.$$

Hence

$$R_2=Z_{n-1}^0Z_n^0F-Z_{n-1}^0XY_{n-2}U_{n-2}Z'_nF$$
$$-XY_{n-3}U_{n-3}Z'_{n-1}Z_n^0F+XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF$$
$$+XY_{n-3}U_{n-3}Z_{n-1}^0XZ'_nF+(XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF$$
$$+XZ_{n-1}^0XZ'_nF-XZ'_{n-1}Z_n^0F)y_{n-2}.$$

Introducing the conditions

$$V_{n-2}^2R_2=0,\ V_{n-3}V_{n-2}R_2=0,$$

we find

$$R_2=Z_{n-2}^0Z_{n-1}^0Z_n^0F-$$
$$Z_{n-2}^0Z_{n-1}^0XY_{n-2}U_{n-2}Z'_nF-Z_{n-2}^0XY_{n-3}U_{n-3}Z'_{n-1}Z_n^0F$$
$$+Z_{n-2}^0XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF$$
$$+Z_{n-2}^0XY_{n-3}U_{n-3}Z_{n-1}^0XZ'_nF+(Z'_{n-2}Z_{n-1}^0Z_n^0F$$
$$-Z'_{n-2}Z_{n-1}^0XY_{n-2}U_{n-2}Z'_nF-Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z_n^0F$$
$$+Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF$$
$$+Z'_{n-2}XY_{n-3}U_{n-3}Z_{n-1}^0XZ'_nF+Z_{n-2}^0XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF$$
$$+Z_{n-2}^0XZ_{n-1}^0XZ'_nF-Z_{n-2}^0XZ'_{n-1}Z_n^0F)y_{n-2}.$$

Now

$$\begin{aligned}
&\frac{d^2}{dx^2}U_{n-4}(Z'_{n-2}Z^0_{n-1}Z^0_nF-\\
&Z'_{n-2}Z^0_{n-1}XY_{n-2}U_{n-2}Z'_nF-Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z^0_nF\\
&+Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+Z'_{n-2}XY_{n-3}U_{n-3}Z^0_{n-1}XZ'_nF+Z^0_{n-2}XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+Z^0_{n-2}XZ^0_{n-1}XZ'_nF-Z^0_{n-2}XZ'_{n-1}Z^0_nF)\\
&=XY_{n-4}U_{n-4}Z'_{n-2}Z^0_{n-1}Z^0_nF\\
&-XY_{n-4}U_{n-4}Z'_{n-2}Z^0_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&-XY_{n-4}U_{n-4}Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z^0_nF\\
&+XY_{n-4}U_{n-4}Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+XY_{n-4}U_{n-4}Z'_{n-2}XY_{n-3}U_{n-3}Z^0_{n-1}XZ'_nF\\
&+XY_{n-4}U_{n-4}Z^0_{n-2}XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+XY_{n-4}U_{n-4}Z^0_{n-2}XZ^0_{n-1}XZ'_nF\\
&-XY_{n-4}U_{n-4}Z^0_{n-2}XZ'_{n-1}Z^0_nF\\
&+(XZ'_{n-2}Z^0_{n-1}Z^0_nF-XZ'_{n-2}Z^0_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&-XZ'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z^0_nF\\
&+XZ'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+XZ'_{n-2}XY_{n-3}U_{n-3}Z^0_{n-1}XZ'_nF\\
&+XZ^0_{n-2}XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+XZ^0_{n-2}XZ^0_{n-1}XZ'_nF-XZ^0_{n-2}XZ'_{n-1}Z^0_nF)y_{n-3}\\
&+(Z'_{n-2}Z^0_{n-1}Z^0_nF-Z'_{n-2}Z^0_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&-Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z^0_nF+Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+Z'_{n-2}XY_{n-3}U_{n-3}Z^0_{n-1}XZ'_nF+Z^0_{n-2}XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+Z^0_{n-2}XZ^0_{n-1}XZ'_nF-Z^0_{n-2}XZ'_{n-1}Z^0_nF)y_{n-2}.
\end{aligned}$$

We thus find

$$\begin{aligned}
R_3=&Z^0_{n-2}Z^0_{n-1}Z^0_nF-Z^0_{n-2}Z^0_{n-1}XY_{n-2}U_{n-2}Z'_nF-\\
&Z^0_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z^0_nF+Z^0_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+Z^0_{n-2}XY_{n-3}U_{n-3}Z^0_{n-1}XZ'_nF-XY_{n-4}U_{n-4}Z'_{n-2}Z^0_{n-1}Z^0_nF\\
&+XY_{n-4}U_{n-4}Z'_{n-2}Z^0_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+XY_{n-4}U_{n-4}Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z^0_nF\\
&-XY_{n-4}U_{n-4}Z'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&-XY_{n-4}U_{n-4}Z'_{n-2}XY_{n-3}U_{n-3}Z^0_{n-1}XZ'_nF\\
&-XY_{n-4}U_{n-4}Z^0_{n-2}XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&-XY_{n-4}U_{n-4}Z^0_{n-2}XZ^0_{n-1}XZ'_nF\\
&+XY_{n-4}U_{n-4}Z^0_{n-2}XZ'_{n-1}Z^0_nF\\
&+(XZ'_{n-2}Z^0_{n-1}XY_{n-2}U_{n-2}Z'_nF-XZ'_{n-2}Z^0_{n-1}Z^0_nF\\
&+XZ'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}Z^0_nF\\
&-XZ'_{n-2}XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&-XZ'_{n-2}XY_{n-3}U_{n-3}Z^0_{n-1}XZ'_nF-XZ^0_{n-2}XZ'_{n-1}XY_{n-2}U_{n-2}Z'_nF\\
&+XZ^0_{n-2}XZ^0_{n-1}XZ'_nF+XZ^0_{n-2}XZ'_{n-1}Z^0_nFy_{n-3}).
\end{aligned}$$

Let us now assume

$$R_r = M_r + N_r y_{n-r}.$$

Then M_r is formed according to the following rule:—Form the term

$$XY_{n-r-1}U_{n-r-1}Z'_{n-r+1}XY_{n-r}U_{n-r}Z'_{n-r+2}XY_{n-r+1}U_{n-r+1}Z'_{n-r+3}$$
$$\ldots . XY_{n-3}U_{n-3}Z'_{n-1}XY_{n-2}U_{n-2}Z'_{n}F.$$

Z'_m may in any place be changed into Z^0_m; but in this case either the preceding $XY_{m-2}U_{m-2}$ must be omitted, or the succeeding $XY_{m-1}U_{m-1}$ changed into X. The signs of the terms follow this law. A term not containing X introduced in place of XYU is positive if Z′ occurs in it an even number of times, negative in the contrary case. But every X introduced in place of XYU occasions a change of sign. The aggregate of all the terms thus formed will give M_r.

We form N thus: construct the term

$$XZ'_{n-r+r}XY_{n-r}U_{n-r}Z'_{n-r+2}XY_{n-r+1}U_{n-r+1}\ldots\ldots Z'_{n}F,$$

and a precisely similar rule holds good. R_r is subject to the condition

$$V^2_{n-r}R_r = 0,\ V_{n-r-1}V_{n-r}R_r = 0.$$

Let us now investigate the criterion that F may be divisible by

$$\frac{d^2}{dx^2} + P\frac{d}{dx} + Q,$$

where P and Q are functions of (x) and (y).

Proceeding as before, we have

$$\left(\frac{d^2}{dx^2} + P\frac{d}{dx} + Q\right)U_{n-2}Z'_{n}F =$$
$$\left(\frac{d}{dx} + P\right)(Y_{n-2} + y_{n-1}V_{n-2})U_{n-2}Z'_{n}F + QU_{n-2}Z'_{n}F$$
$$= (XY_{n-2} + PY_{n-2} + Q)U_{n-2}Z_{n}F + y_{n-1}(X+P)Z'_{n}F + y_nZ'_{n}F.$$

The form of this equation gives us the following rule to ascertain the successive remainders. Construct the remainder in the last case as before, and substitute at pleasure Q in any place where XY is found, P in any place where X is found. The aggregate of the term thus formed will give the remainder in this case.

We now investigate the condition that $\frac{d^3}{dx^3}$ may be an external factor of F.

We put, as before, $F = Z^0_nF + y_nZ'_nF$, where Z'_nF must contain neither y_{n-1} nor y_{n-2}, which gives the conditions

$$V^2_nF = 0,\ V_{n-1}V_nF = 0,\ V_{n-2}V_nF = 0.$$

Now we have

$$\frac{d^3}{dx^3}(U_{n-3}Z'_nF) = \frac{d^2}{dx^2}(Y_{n-3} + y_{n-2}V_{n-3})U_{n-3}Z'_nF$$
$$= \frac{d^2}{dx^2}(Y_{n-3}U_{n-3}Z'_nF + y_{n-2}Z'_nF)$$
$$= X^2Y_{n-3}U_{n-3}Z'_nF + y_{n-2}X^2Z'_nF + 2y_{n-1}XZ'_nF + y_nZ'_nF.$$

And we consequently obtain

$$R_1 = Z^0_n F - X^2 Y_{n-3} U_{n-3} Z'_n F - y_{n-2} X^2 Z'_n F - 2y_{n-1} X Z'_n F.$$

Introducing the conditions

$$V^2_{n-1} R_1 = 0,\ V_{n-2} V_{n-1} R_1 = 0,\ V_{n-3} V_{n-1} R_1 = 0,$$

and expanding in terms of y_{n-1}, we have

$$\begin{aligned} R_1 = {} & (Z^0_{n-1} Z^0_n F - Z^0_{n-1} X^2 Y_{n-3} U_{n-3} Z'_n F) + \\ & (Z'_{n-1} Z^0_n F - Z'_{n-1} X^2 Y_{n-3} U_{n-3} Z'_n F) y_{n-1} - Z^0_{n-1} X^2 Z'_n F y_{n-2} \\ & - y_{n-1} y_{n-2} Z'_{n-1} X^2 Z'_n F - 2y_{n-1} Z^0_{n-1} X Z'_n F. \end{aligned}$$

As the coefficient of y_{n-1} in this cannot contain y_{n-2}, we may write this expression,

$$\begin{aligned} R_1 = {} & (Z^0_{n-1} Z^0_n F - Z^0_{n-1} X^2 Y_{n-3} U_{n-3} Z'_n F) - y_{n-2} Z^0_{n-1} X^2 Z'_n F \\ & + (Z^0_{n-2} Z'_{n-1} Z^0_n F - Z^0_{n-2} Z'_{n-1} X^2 Y_{n-3} U_{n-3} Z'_n F - 2 Z^0_{n-2} Z^0_{n-1} X Z'_n F) y_{n-1}. \end{aligned}$$

Let us now assume

$$R_m = L_m + M_m y_{n-m-1} + N_m y_{n-m},$$

where R_m is the mth remainder, and N_m does not contain y_{n-m-1} or y_{n-m-2}.

Hence, expanding in terms of y_{n-m}, we have

$$\begin{aligned} R_m = {} & (Z^0_{n-m} L_m + y_{n-m} Z'_{n-m} L_m) \\ & + (Z^0_{n-m} M_m + y_{n-m} Z'_{n-m} M_m) y_{n-m-1} \\ & + (Z^0_{n-m} N_m + y_{n-m} Z'_{n-m} N_m) y_{n-m} \\ = {} & Z^0_{n-m} L_m + Z^0_{n-m} M_m y_{n-m-1} \\ & + (Z^0_{n-m-1} Z'_{n-m} L_m + Z^0_{n-m-1} Z^0_{n-m} N_m) y_{n-m}. \end{aligned}$$

Now

$$\begin{aligned} & \frac{d^2}{dx^2} \{ U_{n-m-3} (Z^0_{n-m-1} Z'_{n-m} L_m + Z^0_{n-m-1} Z^0_{n-m} N_m) \} \\ & = \frac{d^2}{dx^2} \{ Y_{n-m-3} + y_{n-m-2} V_{n-m-3} \} U_{n-m-3} \\ & \quad \{ Z^0_{n-m-1} Z'_{n-m} L_m + Z^0_{n-m-1} Z^0_{n-m} N_m \} \\ & = X^2 Y_{n-m-3} U_{n-m-3} (Z^0_{n-m-1} Z'_{n-m} L_m + Z^0_{n-m-1} Z^0_{n-m} N_m) \\ & \quad + X^2 (Z^0_{n-m-1} Z'_{n-m} L_m + Z^0_{n-m-1} Z^0_{n-m} N_m) \cdot y_{n-m-2} \\ & \quad + 2X (Z^0_{n-m-1} Z'_{n-m} L_m + Z^0_{n-m-1} Z^0_{n-m} N_m) y_{n-m-1} \\ & \quad + (Z^0_{n-m-1} Z'_{n-m} L_m + Z^0_{n-m-1} Z^0_{n-m} N_m) y_{n-m}. \end{aligned}$$

Hence

$$\begin{aligned} R_{m+1} = {} & (Z^0_{n-m} - X^2 Y_{n-m-3} U_{n-m-3} Z^0_{n-m-1} Z'_{n-m}) L_m \\ & - X^2 Y_{n-m-3} U_{n-m-3} Z^0_{n-m-1} Z^0_{n-m} N_m \\ & - (X^2 Z^0_{n-m-1} Z'_{n-m} L_m + X^2 Z^0_{n-m-1} Z^0_{n-m} N_m) y_{n-m-2} \\ & - (2X Z^0_{n-m-1} Z'_{n-m} L_m - Z^0_{n-m} M_m + 2X Z^0_{n-m-1} Z^0_{n-m} N_m) y_{n-m-1}. \end{aligned}$$

Now consider for a moment the equations

$$\begin{aligned} L_{m+1} &= G_m L_m + H_m M_m + K_m N_m, \\ M_{m+1} &= G'_m L_m + H'_m M_m + K'_m N_m, \\ N_{m+1} &= G''_m L_m + H''_m M_m + K''_m N_m, \end{aligned}$$

and suppose that

$$\begin{aligned} L_{m+1} &= \lambda_1 L_{m-1} + \mu_1 M_{m-1} + \nu_1 N_{n-1} \\ &= \lambda_2 L_{m-2} + \mu_2 M_{m-2} + \nu_2 N_{n-2} = \&c. \\ &= \lambda_r L_{m-r} + \mu_r M_{m-r} + \nu_r N_{n-r} \\ &= \&c. \end{aligned}$$

Then we find

$$\lambda_1 = G_m G_{m-1} + H_m G'_{m-1} + K_m G''_{m-1},$$

$$\begin{aligned} \lambda_2 = {} & G_m G_{m-1} G_{m-2} + H_m G'_{m-1} G_{m-2} + K_m G''_{m-1} G_{m-2} \\ & + G_m H_{m-1} G'_{m-2} + H_m H'_{m-1} G'_{m-2} + K_m H''_{m-2} G'_{m-2} \\ & + G_m K_{m-1} G''_{m-2} + H_m K'_{m-1} G''_{m-2} + K_m K''_{m-1} G''_{m-2}, \end{aligned}$$

$$\begin{aligned} \lambda_3 = {} & G_m G_{m-1} G_{m-2} G_{m-3} + H_m G'_{m-1} G_{m-2} G_{m-3} \\ & + K_m G''_{m-1} G_{m-2} G_{m-3} + G_m H_{m-1} G'_{m-2} G_{m-3} \\ & + H_m H'_{m-1} G'_{m-2} G_{m-3} + K_m H''_{m-2} G'_m G_{m-3} \\ & + G_m K_{m-1} G''_{m-2} G_{m-3} + H_m K'_{m-1} G''_{m-2} G_{m-3} \\ & + K_m K''_{m-1} G''_{m-2} G_{m-3} + G_m G_{m-1} H_{m-2} G'_{m-3} \\ & + H_m G'_{m-1} H_{m-2} G'_{m-3} + K_m G''_{m-1} H_m G'_{m-3} \\ & + G_m H_{m-1} H'_{m-2} G'_{m-3} + H_m H'_{m-1} H'_{m-2} G'_{m-3} \\ & + K_m H''_{m-1} H'_{m-2} G'_{m-3} + K_m H''_{m-1} H'_{m-2} G'_{m-3} \\ & + G_m K_{m-1} H''_{m-2} G'_{m-3} + H_m K'_{m-1} H''_{m-2} G'_{m-3} \\ & + G_m G_{m-1} K_{m-2} G''_{m-3} + H_m G'_{m-1} K_{m-2} G''_{m-3} \\ & + K_m G''_{m-1} K_{m-2} G''_{m-3} + G_m H_{m-1} K'_{m-2} G''_{m-3} \\ & + H_m H'_{m-1} K'_{m-2} G''_{m-3} + K_m H''_{m-1} K'_{m-2} G''_{m-3} \\ & + G_m K_{m-1} K''_{m-2} G''_{m-3} + H_m K'_{m-1} K''_{m-2} G''_{m-3} \\ & + K_m K''_{m-1} K''_{m-2} G''_{m-3}. \end{aligned}$$

Hence we obtain the following rule for the determination of λ_r:—Write down the term $G_m G_{m-1} G_{m-2} \ldots\ldots G_{m-r}$. We may substitute H and K at pleasure for G anywhere except in the last factor, which is always G. Whenever we put H for G, the succeeding letter is to receive a single accent; whenever K for G, the succeeding letter receives a double accent. The aggregate of all the terms thus formed will be λ_r, and we may of course obtain similar expressions for μ_r, &c.

Now if we put

$$\begin{aligned} G_m &= Z^0_{n-m} - X^2 Y_{n-m-3} U_{n-m-3} Z^0_{n-m-1} Z'_{n-m}, \\ H_m &= 0, \\ K_m &= -X^2 Y_{n-m-3} U_{n-m-3} Z^0_{n-m-1} Z^0_{n-m}, \\ G'_m &= -X^2 Z^0_{n-m-1} Z'_{n-m}, \\ H'_m &= 0, \\ K'_m &= -X^2 Z^0_{n-m-1} Z^0_{n-m}, \\ G''_m &= -2X Z^0_{n-m-1} Z'_{n-m}, \\ H''_m &= +Z^0_{n-m} M_m, \\ K''_m &= -2X Z^0_{n-m-1} Z^0_{n-m}, \end{aligned}$$

we shall find the above equations satisfied; and consequently the last investigation gives the law of the formation of the remainders. Each remainder will of course be subject to the three conditions already exhibited.

These results point out the foundations on which symbolical division, as applied to non-linear functions, must rest. We have confined our attention to external division, as more particularly applicable to these functions. When a non-linear equation is proposed for reduction, we must ascertain whether it admits of an external factor by employing the method of division as already explained.

"On the Calculus of Symbols.—Fifth Memoir. With Application to Linear Partial Differential Equations, and the Calculus of Functions." By W. H. L. Russell, Esq., A.B. Communicated by Professor Stokes, Sec. R.S. Received April 7, 1864*.

In applying the calculus of symbols to partial differential equations, we find an extensive class with coefficients involving the independent variables which may in fact, like differential equations with constant coefficients, be solved by the rules which apply to ordinary algebraical equations; for there are certain functions of the symbols of partial differentiation which combine with certain functions of the independent variables according to the laws of combination of common algebraical quantities. In the first part of this memoir I have investigated the nature of these symbols, and applied them to the solution of partial differential equations. In the second part I have applied the calculus of symbols to the solution of functional equations. For this purpose I have given some cases of symbolical division on a modified type, so that the symbols may embrace a greater range. I have then shown how certain functional equations may be expressed in a symbolical form, and have solved them by methods analogous to those already explained.

Since
$$\left(x\frac{d}{dy}-y\frac{d}{dx}\right)(x^2+y^2)=0,$$
we shall have
$$\left(x\frac{d}{dy}-y\frac{d}{dx}\right)(x^2+y^2)u=(x^2+y^2)\left(x\frac{d}{dy}-y\frac{d}{dx}\right)u,$$
or, omitting the subject,
$$\left(x\frac{d}{dy}-y\frac{d}{dx}\right)(x^2+y^2)=(x^2+y^2)\left(x\frac{d}{dy}-y\frac{d}{dx}\right),$$
also
$$x\frac{d}{dy}-y\frac{d}{dx}+x^2+y^2=x^2+y^2+x\frac{d}{dy}-y\frac{d}{dx};$$
therefore the symbols $x\frac{d}{dy}-y\frac{d}{dx}$ and x^2+y^2 combine according to the laws of ordinary algebraical symbols, and consequently partial differential

* Read April 28, 1864. See Abstract, vol. xiii. p. 227.

equations, which can be put in a form involving these functions exclusively, can be solved like algebraical equations. We shall give some instances of this.

Consider, first, the equation

$$x^2\frac{d^2u}{dx^2}-2xy\frac{d^2u}{dx\,dy}+y^2\frac{d^2u}{dy^2}-x\frac{du}{dx}-y\frac{du}{dy}-(x^2+y^2)^2u=f(x,y).$$

This may be put in the form

$$\left\{x\frac{d}{dy}-y\frac{d}{dx}-(x^2+y^2)\right\}\left\{x\frac{d}{dy}-y\frac{d}{dx}+(x^2+y^2)\right\}u=f(x,y);$$

or if $w=\left\{x\frac{d}{dy}-y\frac{d}{dx}+(x^2+y^2)\right\}u,$

we shall have

$$x\frac{dw}{dy}-y\frac{dw}{dx}-(x^2+y^2)w=f(x,y).$$

Lagrange's method will give in this case the equations

$$\frac{dy}{x}=-\frac{dx}{y}=\frac{dw}{f(x,y)+(x^2+y^2)w}.$$

Hence we shall have

$$x^2+y^2=c^2,$$

$$w=-\epsilon^{-c^2\sin^{-1}\frac{x}{c}}\int\frac{dx f(x,\sqrt{c^2-x^2})}{\sqrt{c^2-x^2}}+\beta\epsilon^{-c^2\sin^{-1}\frac{x}{c}}.$$

Let $\displaystyle\int\frac{dx f(x,\sqrt{c^2-x^2})}{\sqrt{c^2-x^2}}=\mathrm{F}(x,c);$

$$\therefore\ w=-\epsilon^{-(x^2+y^2)\sin^{-1}\frac{x}{\sqrt{x^2+y^2}}}\mathrm{F}(x,\sqrt{x^2+y^2})$$

$$+\epsilon^{-(x^2+y^2)\sin^{-1}\frac{x}{\sqrt{x^2+y^2}}}\phi(x^2+y^2),$$

where ϕ is an arbitrary function. We shall denote this expression by $\chi(x,y)$, whence we have for the determination of (u),

$$x\frac{du}{dy}-y\frac{du}{dx}+(x^2+y^2)u=\chi(x,y),$$

which gives

$$u=\epsilon^{(x^2+y^2)\sin^{-1}\frac{x}{\sqrt{x^2+y^2}}}\mathrm{F}_1(x,\sqrt{x^2+y^2})+\epsilon^{(x^2+y^2)\sin^{-1}\frac{x}{\sqrt{x^2+y^2}}}\phi_1(x^2+y^2),$$

where

$$\mathrm{F}_1(x,r)=\int\frac{dx\chi(x,\sqrt{r^2-x^2})}{\sqrt{r^2-x^2}},$$

which completes the solution.

Next let us take the equation

$$x^2\frac{d^2u}{dx^2}-2xy\frac{d^2u}{dx\,dy}+y^2\frac{d^2u}{dy^2}-(2x^3+2xy^2+y)\frac{du}{dy}$$

$$+(2y^3+2x^2y-x)\frac{du}{dx}+(x^2+y^2)^2u=f(x,y).$$

This equation may be written

$$\left(x\frac{d}{dy}-y\frac{d}{dx}\right)^2 u-2(x^2+y^2)\left(x\frac{d}{dy}-y\frac{d}{dx}\right)u+(x^2+y^2)^2u=f(x,y),$$

$$\left\{x\frac{d}{dy}-y\frac{d}{dx}-(x^2+y^2)\right\}\left\{x\frac{d}{dy}-y\frac{d}{dx}+(x^2+y^2)\right\}u=f(x,y),$$

which may be treated as before.

In order to find the most general form of equation to which the symbols $x\frac{d}{dy}-y\frac{d}{dx}$ and x^2+y^2 give rise, we must determine the expansion of $\left(x\frac{d}{dy}-y\frac{d}{dx}\right)^n$. As $x\frac{d}{dy}+y\frac{d}{dx}$ and x^2-y^2 likewise combine according to the laws of algebraical symbols, we shall take $\left(x\frac{d}{dy}+y\frac{d}{dx}\right)^n$ to avoid the negative sign.

Now the expansion of $\left(x\frac{d}{dy}+y\frac{d}{dx}\right)^n$ will consist of all the terms of the form

$$\ldots\left(x\frac{d}{dy}\right)^{\beta}\left(y\frac{d}{dx}\right)^{b}\left(x\frac{d}{dy}\right)^{\alpha}\left(y\frac{d}{dx}\right)^{a},$$

in which

$$a+b+\ldots+\alpha+\beta+\ldots=n.$$

We shall write δ_x for $\frac{d}{dx}$, and δ_y for $\frac{d}{dy}$, where it is to be understood that δ_x and δ_y do not apply to the subject.

Moreover we shall use, as in the third memoir,

$$a_r \text{ for } a\,.\,\frac{a-1}{2}\cdot\frac{a-2}{3}\cdots\frac{a-r+1}{r}.$$

Then we shall have, if $a+\alpha=r$,

$$\left(x\frac{d}{dy}\right)^{\alpha}\left(y\frac{d}{dx}\right)^{a}=x^{\alpha}y^{a}\frac{d^n}{dy^{\alpha}\,dx^{a}}+\alpha x^{\alpha}\delta_y y^{a}\frac{d^{n-1}}{dy^{\alpha-1}\,dx^{a}}+\alpha_2 x^{\alpha}\delta^2{}_y y^{a}\frac{d^{n-2}}{dy^{\alpha-2}\,dx^{a}}$$
$$+\alpha_3 x^{\alpha}\delta^3{}_y y^{a}\frac{d^{n-3}}{dy^{\alpha-3}dx^{a}}+\ldots$$

Again, $a+\alpha+b=n$,

$$\left(y\frac{d}{dx}\right)^{b}\left(x\frac{d}{dy}\right)^{\alpha}\left(y\frac{d}{dx}\right)^{a}$$
$$=y^{a+b}x^{\alpha}\frac{d^n}{dy^{\alpha}\,dx^{a+b}}+b_1 y^{a+b}\delta_x x^{\alpha}\frac{d^{n-1}}{dy^{\alpha}\,dx^{a+b-1}}+b_2 y^{a+b}\delta^2{}_x x^{\alpha}\frac{d^{n-2}}{dy^{\alpha}\,dx^{a+b-2}}$$
$$+b_3 y^{a+b}\delta^3{}_x x^{\alpha}\frac{d^{n-3}}{dy^{\alpha}\,dx^{a+b-3}}\ldots.$$

$$+\alpha_1 x^\alpha y^b \delta_y y^a \frac{d^{n-1}}{dy^{\alpha-1}\,dx^{a+b}} + \alpha_1 b_1 y^b \delta x^\alpha \delta_y y^a \frac{d^{n-2}}{dy^{\alpha-1} dx^{a+b-1}}$$

$$+\alpha_1 b_2 y^b \delta^2{}_x x^\alpha \delta_y y^a \frac{d^{n-3}}{dy^{\alpha-1}\,dx^{a+b-2}}$$

$$+\alpha_2 y^b x^\alpha \delta^2{}_y y^a \frac{d^{n-2}}{dy^{\alpha-2} dx^{a+b}} + \alpha_2 b_1 y^b \delta_x x^\alpha \delta^2{}_y y^a \frac{d^{n-3}}{dy^{\alpha-2} dx^{a+b-1}}$$

$$+\alpha_3 y^b x^\alpha \delta^3{}_y y^a \frac{d^{n-3}}{dy^{\alpha-3}\,dx^{a+b}} +$$

$$+ \&c.$$

And again, if $a+\alpha+b+\beta=n$, we shall have

$$\left(x\frac{d}{dy}\right)^\beta \left(y\frac{d}{dx}\right)^b \left(x\frac{d}{dy}\right)^\alpha \left(y\frac{d}{dx}\right)^a$$

$$= y^{a+b} x^{\alpha+\beta} \frac{d^n}{dy^{\alpha+\beta}\,dx^{a+b}} + b_1 y^{a+b} x^\beta \delta_x x^\alpha \frac{d^{n-1}}{dy^{\alpha+\beta}\,dx^{a+b-1}}$$

$$+ b_2 y^{a+b} x^\beta \delta^2 x^\alpha \frac{d^{n-2}}{dy^{\alpha+\beta}\,dx^{a+b-2}} + b_3 y^{a+b} x^\beta \delta^3 x^\alpha \frac{d^{n-3}}{dy^{\alpha+\beta}\,dx^{a+b-3}} \cdots$$

$$+\alpha_1 x^{\alpha+\beta} y^b \delta_y y^a \frac{d^{n-1}}{dy^{\alpha+\beta-1} dx^{a+b}} + \alpha_1 b_1 x^\beta y^b \delta_x x^\alpha \delta_y y^a \frac{d^{n-2}}{dy^{\alpha+\beta-1}\,dx^{a+b-1}}$$

$$+\alpha_1 b_2 x^\beta y^b \delta^2{}_x x^\alpha \delta_y y^a \frac{d^{n-3}}{dy^{\alpha+\beta-1}\,dx^{a+b-2}} + \cdots$$

$$+\alpha_2 y^b x^{\alpha+\beta} \delta^2{}_y y^a \frac{d^{n-2}}{dy^{\alpha+\beta-2}\,dx^{a+b}} + \alpha_2 b_1 y^b x^\beta \delta_x x^\alpha \delta^2{}_y y^a \frac{d^{n-3}}{dy^{\alpha+\beta-2}\,dx^{a+b-1}} + \cdots$$

$$+\alpha_3 y^b x^{\alpha+\beta} \delta^3{}_y y^a \frac{d^{n-3}}{dy^{\alpha+\beta-3}\,dx^{a+b}}$$

$$+ \cdots$$

$$+\beta_1 x^{\alpha+\beta} \delta_y y^{a+b} \frac{d^{n-1}}{dy^{\alpha+\beta-1}\,dx^{a+b}} + \beta_1 b_1 x^\beta \delta_y y^{a+b} \delta_x x^\alpha \frac{d^{n-2}}{dy^{\alpha+\beta-1}\,dx^{a+b-1}}$$

$$+\beta_1 b_2 x^\beta \delta_y y^{a+b} \delta^2{}_x x^\alpha \frac{d^{n-3}}{dy^{\alpha+\beta-1}\,dx^{a+b-2}}$$

$$+\beta_1 \alpha_1 x^{\alpha+\beta} \delta_y y^b \delta_y y^a \frac{d^{n-2}}{dy^{\alpha+\beta-2}\,dx^{a+b}} + \alpha_1 b_1 \beta_1 \delta_y y^b \delta_x x^\alpha \delta_y y^a \frac{d^{n-3}}{dy^{\alpha+\beta-2} dx^{a+b-1}}$$

$$+\alpha_2 \beta_1 x^{\alpha+\beta} \delta_y y^b \delta^2{}_y y^a \frac{d^{n-3}}{dy^{\alpha+\beta-3}\,dx^{a+b-1}}$$

$$+ \cdots$$

$$+\beta_2 x^{\alpha+\beta}\delta^2_y y^{a+b}\frac{d^{n-2}}{dy^{\alpha+\beta-2}\,dx^{a+b}} + b_1\beta_2 x^{\beta}\delta_x x^{\alpha}\delta^2_y y^{a+b}\frac{d^{n-3}}{dy^{\alpha+\beta-2}\,dx^{a+b-1}}$$

$$+\alpha_1\beta_2\delta^2_y y^b\delta_y y^a\frac{d^{n-3}}{dy^{\alpha+\beta-3}\,dx^{a+b}}$$

$$+\ \ldots$$

$$+\beta_3 x^{\alpha+\beta}\delta^3_y y^{a+b}\frac{d^{n-3}}{dy^{\alpha+\beta-3}\,dx^{a+b}}$$

$$+\ \ldots$$

We are consequently able to see that the general term of

$$\ldots\left(x\frac{d}{dy}\right)^{\beta}\left(y\frac{d}{dx}\right)^{b}\left(x\frac{d}{dy}\right)^{\alpha}\left(y\frac{d}{dx}\right)^{a}$$

is

$$\Sigma_s\Sigma_r\ldots c_{q'}b_{p'}\ldots\beta_q\alpha_p\ldots\delta^q_y y^b\,\delta^p_y y^a\ldots\delta^{q'}_x x^{\beta}\,\delta^{p'}_x x^{\alpha}\cdot\frac{d^{n-m}}{dy^{\alpha+\beta+\ldots-r}\,dx^{a+b+\ldots-s}}$$

where

$$p+q+\ldots=r, \quad\ldots\ldots\ldots\quad (1)$$

$$p'+q'+\ldots=s, \quad\ldots\ldots\ldots\quad (2)$$

and

$$r+s\qquad =m. \quad\ldots\ldots\ldots\quad (3)$$

Hence the general term of $\left(x\frac{d}{dy}+y\frac{d}{dx}\right)^n$ will be

$$\Sigma_n\Sigma_s\Sigma_r\ldots c_{q'}b_{p'}\ldots\beta_q\alpha_p\ldots\delta^q_y y^b\,\delta^p_y y^a\ldots\delta^{q'}_x x^{\beta}\,\delta^{p'}_x x^{\alpha}\frac{d^{n-m}}{dy^{\alpha+\beta+\ldots-r}\,dx^{a+b\ldots-s}}$$

under the conditions (1), (2), (3), and also

$$a+b+\ldots+\alpha+\beta+\ldots=n.$$

Calling the expanded form of $\left(x\frac{d}{dy}+y\frac{d}{dx}\right)^n$, Δ_n, it is easily seen that we can resolve all linear partial differential equations of the form

$$f(x^2-y^2)\Delta_n u+f_1(x^2-y^2)\Delta_{n-1}u+f_2(x^2-y^2)\Delta_{n-1}u+\&c.=\mathrm{F}(x,y).$$

The same property is possessed by a great number of other symbols. Let us examine the condition that

$$(ax+by+c)\frac{d}{dx}-(a'x+b'y+c')\frac{d}{dy},$$

and

$$\mathrm{A}x^2+2\mathrm{B}xy+\mathrm{C}y^2+2\mathrm{E}x+2\mathrm{F}y+\mathrm{H}$$

may combine according to the algebraical law. The condition is easily seen to be

$$(\mathrm{A}x+\mathrm{B}y+\mathrm{E})(ax+by+c)-(\mathrm{C}y+\mathrm{B}x+\mathrm{F})(a'x+b'y+c')=0,$$

from whence we have

$$\mathrm{A}a-\mathrm{B}a'=0,\quad \mathrm{B}b-\mathrm{C}b'=0,$$

$$\mathrm{A}b+\mathrm{B}a=\mathrm{C}a'+\mathrm{B}b',$$

$$\mathrm{B}c+\mathrm{E}b=\mathrm{C}c'+\mathrm{F}b',$$

$$\mathrm{E}a+\mathrm{A}c=\mathrm{F}a'+\mathrm{B}c',$$

and

$$\mathrm{E}c=\mathrm{F}c'.$$

We may consider B=1, which gives the following conditions: $a'b=ab'$, $a=b'$. Also

$$A=\frac{a'}{a},\quad C=\frac{b}{b'},\quad E=\frac{c'}{a},\quad F=\frac{c}{a}.$$

And the symbols may be written

$$b(ax+by+c)\frac{d}{dx}-a(ax+by+c')\frac{d}{dy},$$

and

$$a^2x^2+2ab\,xy+b^2y^2+2ac'x+2cby+H.$$

It follows hence that in order to find the form of the differential equations to which these symbols give rise, we must know the expansion of $\left(X\frac{d}{dx}+Y\frac{d}{dy}\right)^n$, where X and Y are functions of x and y.

The expanded form will be a series of terms like

$$\ldots.\left(Y\frac{d}{dy}\right)^{\beta}\left(X\frac{d}{dx}\right)^{b}\left(Y\frac{d}{dy}\right)^{\alpha}\left(X\frac{d}{dx}\right)^{a}.$$

We must consequently find an expression for $\left(X\frac{d}{dx}\right)^a$ in powers of $\frac{d}{dx}$. It must be remembered that X is a function of x and y, in which (y) during the present process is considered as constant, and therefore X may be looked upon as a function of (x) only.

Now we shall find after a few differentiations, that

$$\begin{aligned}\left(X\frac{d}{dx}\right)^5 &= X^5\frac{d^5}{dx^5}\\ &+(X\delta X^4+X^2\delta X^3+X^3\delta X^2+X^4\delta X)\frac{d^4}{dx^4}\\ &+(X\delta X\delta X^3+X\delta X^2\delta X^2+X^2\delta X\delta X^2\\ &+X\delta X^3\delta X+X^2\delta X^2\delta X+X^3\delta X\delta X)\frac{d^3}{dx^3}\\ &+(X\delta X\delta X\delta X^2+X\delta X\delta X^2\delta X+X\delta X^2\delta X\delta X)\frac{d^2}{dx^2}\\ &+X\delta X\delta X\delta X\delta X\,\frac{d}{dx}.\end{aligned}$$

Now let

$$\left(X\frac{d}{dx}\right)^n=X_n\frac{d^n}{dx^n}+X_n^{(1)}\frac{d^{n-1}}{dx^{n-1}}+X_n^{(2)}\frac{d^{n-2}}{dx^{n-2}}$$
$$+\ldots+X_n^{(r)}\frac{d^{n-r}}{dx^{n-r}}+$$

Then

$$X_n^{(r)}=\Sigma_n X^{\alpha}\delta X^{\beta}\delta X^{\gamma}\ldots$$

where there are r δ's, and $\alpha+\beta+\gamma+\ldots.=n$. Hence we shall have

$$\ldots\left(Y\frac{d}{dy}\right)^{\beta}\left(X\frac{d}{dx}\right)^{b}\left(Y\frac{d}{dy}\right)^{\alpha}\left(X\frac{d}{dx}\right)^{a}$$

$$= \ldots \left\{ Y_\beta \frac{d^\beta}{dy^\beta} + Y_\beta^{(1)} \frac{d^{\beta-1}}{dy^{\beta-1}} + Y_\beta^{(2)} \frac{d^{\beta-2}}{dy^{\beta-2}} + \ldots \right\}$$

$$\left\{ X_b \frac{d^b}{dx^b} + X_b^{(1)} \frac{d^{b-1}}{dx^{b-1}} + X_b^{(2)} \frac{d^{b-2}}{dx^{b-2}} + \ldots \right\}$$

$$\left\{ Y_\alpha \frac{d^\alpha}{dy^\alpha} + Y_\alpha^{(1)} \frac{d^{\alpha-1}}{dy^{\alpha-1}} + Y_\alpha^{(2)} \frac{d^{\alpha-2}}{dy^{\alpha-2}} + \ldots \right\}$$

$$\left\{ X_a \frac{d^a}{dx^a} + X_a^{(1)} \frac{d^{a-1}}{dx^{a-1}} + X_a^{(2)} \frac{d^{a-2}}{dx^{a-2}} + \ldots \right\}.$$

Whence it is obvious that the general expression for the expansion of $\left(X\frac{d}{dx}+Y\frac{d}{dy}\right)^n$ will depend upon principles not materially differing from those already considered.

The symbols we have already considered are only of the first order of differentiation; we shall show that there exist symbols of the second order which combine with certain algebraical quantities as if they were themselves algebraic.

Let us take the symbols

$$a\frac{d^2}{dx^2}+2b\frac{d^2}{dx\,dy}+c\frac{d^2}{dy^2}+2a'\frac{d}{dx}+2b'\frac{d}{dy}+e,$$

and

$$Ax^2+2Bxy+Cy^2+2A'x+2B'y+H.$$

Proceeding as before, we arrive at the following conditions:

$$Ab+Bc=0, \quad \ldots\ldots\ldots \quad (1)$$
$$Bb+Cc=0, \quad \ldots\ldots\ldots \quad (2)$$
$$A'b+B'c=0, \quad \ldots\ldots\ldots \quad (3)$$
$$Aa'+Bb'=0, \quad \ldots\ldots\ldots \quad (4)$$
$$Ba'+Cb'=0, \quad \ldots\ldots\ldots \quad (5)$$
$$Aa+Bb=0, \quad \ldots\ldots\ldots \quad (6)$$
$$Ba+Cb=0, \quad \ldots\ldots\ldots \quad (7)$$
$$A'a+B'b=0, \quad \ldots\ldots\ldots \quad (8)$$
$$2aA+4bB+2cC+4a'A'+4b'B'+e=0. \quad \ldots\ldots \quad (9)$$

Whence we have, putting $B=1$,

$$A=-\frac{c}{b}, \quad C=-\frac{b}{c};$$

and with the following conditions,

$$ac=b^2, \quad a'c-bb'=0$$
$$4a'A'+4b'B'+e=0;$$

the condition $a'c-bb'$ may be otherwise written $a'b-ab'=0$, in consequence of the equation $ac=b^2$.

It will be observed that several of the nine above equations are not independent of the rest; so that the result is much simplified.

I now proceed to apply the calculus of symbols to the solution of functional equations.

Let

$$\int \frac{dx}{\psi(x)} = \chi(x).$$

Then the following formulæ are known:

$$\epsilon^{\psi(x)\frac{d}{dx}} f(x) = f\{\chi^{-1}(\chi x + 1)\},$$

$$(\epsilon^{\psi(x)\frac{d}{dx}})^2 f(x) = f\{\chi^{-1}(\chi x + 2)\}.$$

$$\&c. \qquad = \ldots$$

$$(\epsilon^{\psi(x)\frac{d}{dx}})^r f(x) = f\{\chi^{-1}(\chi(x) + r)\}.$$

These formulæ may be thus expressed in the notation of the calculus of symbols: if $\rho = \epsilon^{\psi(x)\frac{d}{dx}}$, $\pi = x$, θ a functional symbol acting on $f(\pi)$ in such a manner as to convert $f(\pi)$ into $f\chi^{-1}(\chi\pi + 1)$; then

$$\rho f(\pi) = \theta f\pi \,.\, \rho,$$

a general law of symbolical combination due to Professor Boole.

We will now consider two cases of internal and external division in which the symbols combine according to this law. The results, as will appear afterward, will be found useful in the solution of functional equations.

And, first, for internal division. We shall determine the condition that $\rho\psi_1(\pi) + \psi_0(\pi)$ may divide $\rho^n\phi_n(\pi) + \rho^{n-1}\phi_{n-1}(\pi) + \ldots\ldots$ The process will be, *mutatis mutandis*, the same as in my former memoir. The symbolical quotient is

$$\rho^{n-1}\theta\frac{\phi_n\pi}{\psi_1\pi} + \rho^{n-2}\left\{\theta\frac{\phi_{n-1}\pi}{\psi_1\pi} - \theta\frac{\psi_0\pi}{\psi_1\pi}\theta\frac{\phi_n\pi}{\psi_1\pi}\right\} + \ldots;$$

and the required condition is found by equating the symbolical final remainder to zero, and we have

$$\phi_0\pi - \psi_0\pi\theta\frac{\phi_1\pi}{\psi_1\pi} + \psi_0\pi\theta\frac{\psi_0\pi}{\psi_1\pi}\theta\frac{\phi_2\pi}{\psi_1\pi} - \&c. \pm \psi_0\pi\theta\frac{\psi_0\pi}{\psi_1\pi}\theta\frac{\psi_0\pi}{\psi_1\pi}\ldots\theta\frac{\phi_n\pi}{\psi_1\pi} = 0,$$

θ affecting every part of the term which succeeds it.

I shall now give the corresponding condition for an external factor. The symbolical quotient is

$$\rho^{n-2}\phi_n\pi\theta^{-(n-1)}\frac{1}{\psi_1\pi} + \rho^{n-2}\left\{\phi_{n-1}\pi\theta^{-(n-2)}\frac{1}{\psi_1\pi} - \phi_n\pi\theta^{-(n-2)}\frac{1}{\psi_1\pi}\theta^{-1}\frac{\psi_0\pi}{\psi_1\pi}\right\} + \ldots$$

The required condition is found by equating the final remainder to zero; we have

$$\phi_0\pi - \phi_1\pi\frac{\psi_0\pi}{\psi_1\pi} + \phi_2\pi\frac{\psi_0\pi}{\psi_1\pi}\theta^{-1}\frac{\psi_0\pi}{\psi_1\pi} - \&c. \pm \phi_n\pi\frac{\psi_0\pi}{\psi_1\pi}\theta^{-1}\frac{\psi_0\pi}{\psi_1\pi}\theta^{-1}\frac{\psi_0\pi}{\psi_1\pi}\ldots\theta^{-1}\frac{\psi_0\pi}{\psi_1\pi} = 0,$$

θ^{-1} in each term affecting everything which comes after it.

I conclude with some examples of functional equations.

Let the functional equation be

$$f(x) - \alpha f\frac{x}{\sqrt{1+2x^2}} = \mathrm{F}(x);$$

this may be written

$$f(x)-ae^{\frac{d}{dx^{-2}}}f(x)=F(x),$$

or

$$f(x)=\frac{1}{1-ae^{\frac{d}{dx^{-2}}}}F(x),$$

$$=\{1+ae^{\frac{d}{dx^{-2}}}+a^2(e^{\frac{d}{dx^{-2}}})^2+\ldots.\}F(x)$$

$$=F(x)+aF\frac{x}{\sqrt{1+2x^2}}+a^2F\frac{x}{\sqrt{1+4x^2}}+\ldots.$$

To make this solution complete, we must add a complementary function, and we have

$$u=\frac{C}{(\sqrt{a})^{\frac{1}{x^2}}}+\frac{C_1}{(-\sqrt{a})^{\frac{1}{x^2}}}$$

$$+F(x)+aF\frac{x}{\sqrt{1+2x^2}}+a^2F\frac{x}{\sqrt{1+4x^2}}+\ldots..$$

As an example of this, put $F(x)=x$, and the series becomes

$$x+\frac{ax}{\sqrt{1+2x^2}}+\frac{a^2x}{\sqrt{1+4x^2}}+\frac{a^3x}{\sqrt{1+6x^2}}+\ldots$$

$$=\frac{2x}{\sqrt{\pi}}\int_0^\infty dv\,e^{-v^2}(1+ae^{-2x^2v^2}+a^2e^{-4x^2v^2}+\ldots)$$

$$=\frac{2x}{\sqrt{\pi}}\int_0^\infty\frac{e^{-v^2}dv}{1-ae^{-2xv^2}}.$$

As a second example, we will take the equation

$$f\left(\frac{x}{x+2}\right)-3f\left(\frac{x}{x+1}\right)+2f(x)=F(x).$$

This equation may be written

$$(e^{2.\frac{d}{d\frac{1}{x}}}-3e^{\frac{d}{d\frac{1}{x}}}+2)f(x)=F(x),$$

$$(e^{\frac{d}{d\frac{1}{x}}}-1)(e^{\frac{d}{d.\frac{1}{x}}}-2)f(x)=F(x).$$

Now let

$$(e^{\frac{d}{d\frac{1}{x}}}-2)f(x)=\chi(x),$$

and the functional equation resolves itself into the two,

$$f\left(\frac{x}{x+1}\right)-2f(x)=\chi(x),$$

and

$$\chi\left(\frac{x}{x+1}\right)-\chi(x)=F(x),$$

which are known forms.

As a last example, we will take the equation

$$f\left(\frac{3x-2}{2x-1}\right)-\frac{x^2+3x-1}{x}f\frac{2x-1}{x}+x(x+1)f(x)=\mathrm{F}(x);$$

or putting

$$e^{-(x-1)^2\frac{d}{dx}}=\rho, \quad x=\pi,$$

we can write the equation (since $\rho=e^{\frac{d}{d\cdot\frac{x}{x-1}}}$)

$$\left\{\rho^2-\rho\left(\frac{2\pi^2-\pi-3}{\pi-2}\right)+\pi(\pi+1)\right\}u=\mathrm{F}(x).$$

Applying the method of divisors, we see that if the symbolical portion of the first member admit of an internal factor, it must be either $\rho-\pi$ or $\rho-(\pi+1)$.

Now in this case

$$\rho f(\pi)=f\left(\frac{2\pi-1}{\pi}\right)\rho.$$

Hence

$$\theta f(\pi)=f\left(\frac{2\pi-1}{\pi}\right).$$

Wherefore the equation

$$\phi_0\pi-\psi_0\pi\theta\frac{\phi_1\pi}{\psi_1\pi}+\psi_0\pi\theta\frac{\psi_0\pi}{\psi_1\pi}\theta\frac{\phi_2\pi}{\psi_1\pi}-\&c.\pm\psi_0\pi\theta\frac{\psi_0\pi}{\psi_1\pi}\theta\frac{\psi_0\pi}{\psi_1\pi}\ldots\theta\frac{\psi_0\pi}{\psi_1\pi}=0$$

becomes, if we take the factor $\rho-\pi$, and put

$$\psi_1\pi=1, \quad \psi_0\pi=-\pi,$$

$$(\pi+1)-\theta\left(\frac{\pi-3}{\pi-2}\right)-\theta\pi+\theta\pi=0;$$

an identical equation if we put for the symbol θ its equivalent as given above.

Hence $\rho-\pi$ is an internal factor of the symbolical portion of the first member. Effecting the internal division, we have

$$(\rho-(\pi+1))(\rho-\pi)f(x)=\mathrm{F}(x).$$

Let

$$(\rho-\pi)f(x)=\chi(x),$$

and the equation resolves itself into the two,

$$(\rho-(\pi+1))\chi(x)=\mathrm{F}(x)$$

and

$$(\rho-\pi)f(x)=\chi(x);$$

or

$$\chi\left(\frac{2x-1}{x}\right)-(x+1)\chi(x)=\mathrm{F}(x)$$

and

$$f\left(\frac{2x-1}{x}\right)-xf(x)=\chi(x);$$

forms which I have considered in my memoir on the Calculus of Functions published in the Philosophical Transactions for 1862, in which the general solution of the equations

$$\phi(x)-\chi(x)\phi\left\{\frac{a+bx}{c+ex}\right\}=\mathrm{F}(x),$$

where ϕ is the unknown function, has been obtained.

Communications received since the end of the session.

I. "Comparison of Mr. De la Rue's and Padre Secchi's Eclipse Photographs." By Warren De la Rue, F.R.S. Received August 8, 1864.

I have stated, in the Bakerian Lecture read at the Royal Society on April 10, 1862, that the boomerang (prominence E)* was not depicted on Señor Aguilar's photographs. This is true of the prints which came into my hands in England. A visit to Rome in November 1862, however, afforded an opportunity for the examination of the first prints which had been taken in Spain on the day of the eclipse, previous to those printed off for general distribution by Señor Aguilar. I was agreeably surprised to find that the photograph of the first phase of totality showed not only this prominence very distinctly, but also other details, presently to be described, which were quite invisible in Señor Aguilar's copies. I had in fact experienced some difficulty in comparing measurements of my photographs with those of Señor Aguilar's, on account of the indistinctness (woolliness) of the latter, which I have attributed to Padre Secchi's telescope not having followed the sun's motion perfectly. A careful examination of the prints in Padre Secchi's possession has, however, convinced me that this was not the case during the period of exposure of the first negative; for I have been able to identify with a magnifier many minute forms which could only have been depicted by the most perfect following of the sun's apparent motion. For instance, my statement that the prominence H (the fallen tree) was not seen from having been mixed up with the prominence G, is not applicable to Padre Secchi's copy of the first phase of totality, for in it every detail of the fallen tree can be made out.

On expressing to Professor Secchi my surprise at the great discordance between the copy of the first phase of totality sent to me by Señor Aguilar and that of the same phase in his possession, I was informed that after a few positive prints had been taken from the then unvarnished negative, it was strengthened by the usual photographic process with nitrate of silver. This I look upon as an unfortunate mistake, as the images of the prominences were increased and their details hidden, and the beauty of the negative for ever lost.

It occurred to Padre Secchi and myself that although there was no hope

* See Index Map, Plate XV. Phil. Trans. Part I. 1862.

of procuring more satisfactory prints from the original negative of the first phase of totality, yet some advantage would arise from taking an enlarged negative from the positive print in his possession, although it could not be expected to yield as perfect an impression as might have been obtained by enlarging from the original photograph. The enlargement has been successfully accomplished in my presence; and although Professor Secchi will take such means as he may think proper to make known the results of comparisons he may make between my photographs and his own, it will not be out of place for me to add a few remarks by way of appendix to my paper.

Taking the prominences in the order in my index map, Plate XV.:—

Prominence A (the cauliflower or wheatsheaf) has the same form in Padre Secchi's photograph as in mine. It extends considerably less in height above the moon's edge in this copy than in that printed off from the strengthened negative (Señor Aguilar's copy); the bright points of the two branching streams which issue from the summit towards the North are well depicted in the Secchi photograph, but not the fainter parts.

There exists a faint indication of the minute prominence B in the S. photograph.

The convolutions of the prominence C (the floating cloud) are seen in the S. photograph, and its form coincides absolutely with that of mine; it is a little nearer the moon's edge at the point *c*, probably because the telescope was uncovered relatively a little later than at Rivabellosa.

The prominence D cannot be clearly traced in the S. photograph.

The boomerang E is distinctly visible in the S. photograph; the point *e* is apparently prolonged; but this I attribute to an accidental photographic stain, for the bright part *e'* can be well made out.

The long prominence F cannot be made out in the S. photograph, probably from the cause explained in reference to C.

The fallen tree (H in the S. photograph) corresponds in its minutest details with its picture in my own. The articulated extremity *h*, the round points *h' h''*, the point *h'''*, and the connecting branch joining it with the stem are clearly seen.

The prominence G from g'' to g' corresponds precisely in the S. photograph with its image in my own, and a dark marking near *g* also is seen; the narrow portion of this prominence, from *g* to the point immediately below *h*, is not seen in the S. photograph.

The prominence I (the mitre) agrees in form in the S. photograph with its image in my own, even the faint point *i* is there seen. This prominence in the S. photograph extends further from the edge of the moon than in mine; and whereas in my photograph the convex boundary next the moon is cut off by the moon's limb, in Padre Secchi's the convex boundary is complete, and hence in all probability the prominence I presented another case of a floating cloud.

About midway between G and I there is a small round prominence visi-

ble in the S. photograph not seen in mine, which may be accounted for from our different positions in respect to the central line of the eclipse.

Between I and K, at a distance from I equal to about two-thirds the angular interval, there is in the S. photograph a prominence consisting of two round dots, which extend beyond the moon's limb to precisely the same extent as the prominence K protrudes in Professor Secchi's photograph beyond the moon's limb in excess of what it does in my own.

The prominence K has precisely the same form in every respect in the S. photograph as in mine, so far as mine shows it; but on account of parallax, more of it is seen in the S. photograph than in mine.

Beyond K is another prominence, visible in the S. photograph about 17° distant from K, a small round prominence which could not have been visible from my station.

Of the remaining prominences, L, M, N, O, P, Q, R, none were visible at the epoch of the photograph.

In conclusion, the photographic images of the prominences, so far as they are common to the two photographs taken at Miranda and Desierto de las Palmas, accord in their most minute details. The photographs must, from the difference of position of the two stations, have been made at an absolute interval of about seven minutes; and this fact, while it strongly supports the conclusion that the protuberances belong to the sun, at the same time shows that there is no change in their form during an interval much greater than the whole duration of an eclipse.

II. "On Drops." By Frederick Guthrie, Esq., Professor of Chemistry and Physics at the Royal College, Mauritius. Communicated by Professor Stokes, Sec. R.S. Received July 15, 1864.

In the following investigation, the word drop is used in a rather more definite sense than that which is usually attached to it.

In common speech a drop signifies any mass of liquid matter whose form is visibly influenced towards the spherical by the attraction of its parts, and whose sensible motion or tendency is towards the earth. This definition both includes drops with which we are not here concerned, and excludes others which we shall have to consider; for we shall have to measure the size of drops; and it can only be of avail to measure the size of such drops as are formed under fixed and determinable conditions.

How many drops, according to the usual scope of the term, are formed under indefinite conditions. For instance, a rain-drop depends for its size upon such circumstances as the quantity of vapour at the time and place of its formation, the tranquillity and electrical condition of the air, its rate of motion, the number and size of the drops it meets with in its course, &c., all of which are fortuitous, or, at least, immeasurable conditions.

With such drops we have here nothing to do, but only with those which are formed under fixed circumstances. On the other hand, we

shall have to consider drops which move upwards*. Drops of this kind are so seldom met with that no distinguishing name has been given to them. We shall find it convenient to include them in the general term drop, though it may appear at first inapplicable to them.

Without attempting to give an exhaustive definition, it will be sufficient to define a drop as a mass of liquid collected and held together by the attraction of its parts and separated from other matter by the attraction of gravitation. This definition will exclude such drops as those of mist or rain, and will include the upward-moving drops mentioned above.

It follows that the size of a drop may depend upon and be influenced by variation in—

(1) The self-attraction and cohesion of the drop-generating liquid;

(2) Its adhesion to the matter upon which the drop is formed;

(3) The shape of the matter from which the drop moves;

(4) The physical relation of the medium through which the drop moves, on the one hand, to the liquid of which the drop is formed, and on the other, to the matter on which it is formed;

(5) The attraction of the earth, or gravitation, upon the drop-forming liquid and upon the medium, as influenced by their respective and relative densities, and by variation in the attracting power of the earth.

In order to study systematically the influence which each of these factors exerts, each must be varied in succession while the others remain constant.

Denoting the three states of matter, solid, liquid, and gaseous, by the symbols S, L, G respectively, and considering the symbols in the order in which they are written to denote respectively the matter from which the dropping takes place, the drop and the medium, we get a convenient notation.

As we are speaking at present exclusively of liquid drops, L must always hold the middle place in the symbol.

Of the eight symbolically possible variations,

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
SLS,	SLL,	SLG,	LLS,	LLL,	LLG,	GLS,	GLG,

(1), (4), and (7) are physically impossible on account of the superior cohesion of solids over liquids, (6) and (8) are physically impossible on account of the superior density of liquids over gases.

SLL, SLG, and LLL are therefore the only cases we have to consider. That is,

* Owing to the numerical preponderance of downward-moving drops, we are prone to associate the ideas of "drop" and "down." How far I may be justified philologically in using the expression "drop up," must depend upon the relative primitiveness of the noun and verb "drop." Once for all, I beg permission to use the term drop in this more extended sense.

Of course, in the absence of positive levity, an upward drop can only be caused by the downward motion of the medium in which the drop moves.

SLL, from a solid a liquid drops through a liquid.
SLG, from a solid a liquid drops through a gas.
LLL, from a liquid a liquid drops through a liquid.

Of these three cases, two, SLL and LLL, may be inverted; that is, the motion of the drop may be towards or from the earth. The gravitation of the drop may be greater than and overcome the gravitation of the medium, the drop descends; or the gravitation of the medium may overcome that of the drop, the drop ascends. The case SLG cannot be inverted, because, at all events, at the same pressure every known gas is lighter than every known liquid.

It will be convenient to consider the case SLG first, because instances of it come more frequently under our notice than of the other two, and because it will be convenient to consider together those cases which are capable of inversion.

As we are considering the physical aspect of the question, we will only discuss those cases where no chemical action takes place between the terms, and where either no solution takes place, or where it is so small as to be negligible, or of such a kind as to admit of experimental elimination. This limitation of course excludes a vast number of combinations, but it must be made in order to study the purely physical and definite influences which determine the size of a drop.

SLG. *From a Solid a Liquid drops through a Gas.*

The variable factors are

1. The self-attraction and cohesion of the liquid:
   - A. Dependent on its purely chemical constitution.
   - B. Dependent on the proportion and physical relation between its heterogeneous parts, when a mixture.
   - C. Dependent on temperature.
2. The adhesion between the solid and the liquid:
   - A, B, C as in 1.
   - D. Dependent upon the shape of the solid.
3. The adhesion of the gas to the solid.
4. The adhesion of the gas to the liquid.

The factors 3 and 4 may be neglected, as we shall at present only consider the case where the gaseous medium is air at the ordinary barometric pressure.

One of these factors, namely temperature, though varying in different cases, may be supposed in the same case to be the same for the different kinds of matter present. Another factor in the same predicament is the locally constant gravitation at the place where the dropping occurs. Lastly, a condition of great influence is the length of the time-interval between the successive drops. This interval we shall call, for brevity, the growth-time, and denote by gt.

If the above conditions are exhaustive, we may assert that a drop of

liquid will always be of the same size, if it is formed of the same liquid substance and falls from a solid of the same substance, size, and shape, provided that the temperature remain the same, and the growth-time be constant.

The size of the drops may be most conveniently determined by weighing a noted number of them. We are concerned rather with the relative than with the absolute sizes of the drops. The sizes of drops formed of the same liquid are proportional to their weights; of different liquids, to those weights divided by the specific gravities of the liquids.

In the first series of experiments the apparatus, fig. 1, was employed. The globe A, full of the liquid under experiment, is inverted into the cylinder B, containing the same. The mouth of A is supported just in

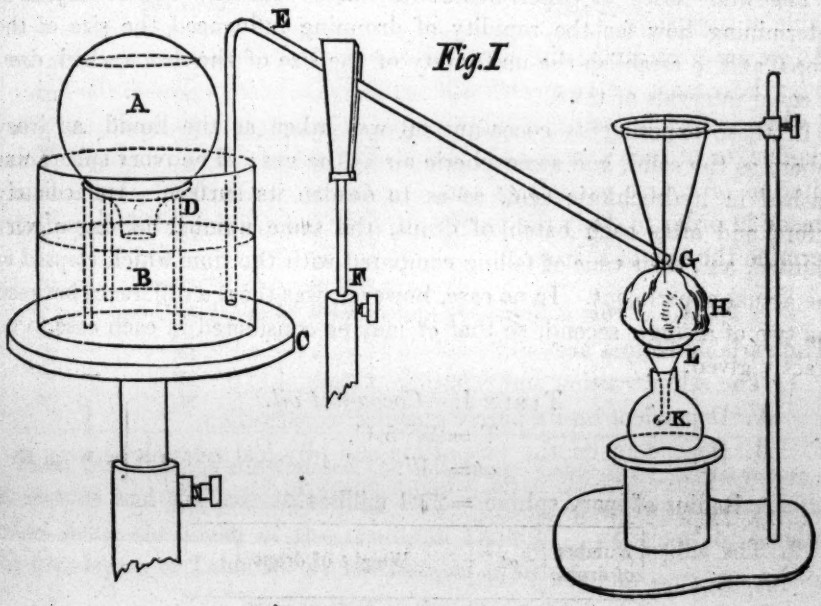


Fig. I

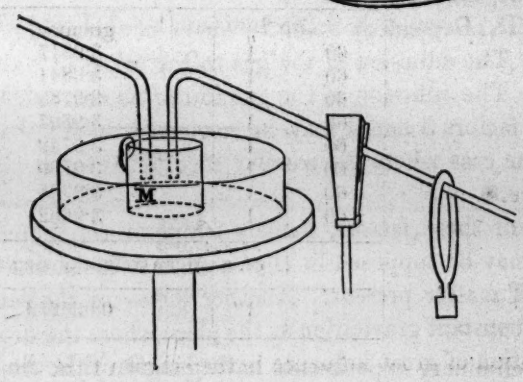


Fig. III.

contact with the surface of the liquid in B, by means of the tripod stand D. A and B are carried on a table, which may be raised or lowered at pleasure. A siphon, E, leads from the reservoir B, and is firmly held by the clamp F. The longer limb of E, from which the liquid flows, is

turned up at the end, and touches a plug of cotton wool at G. The sphere H, from which the dropping takes place, is hung by three thin wires from the ring of a retort-stand. The upper half of the sphere is clothed in cotton wool, which reaches up to the plug at G. The whole arrangement is placed upon a separate table from that which supports the balance, so as to avoid the vibration caused by opening and shutting the balance case. The drops which fall from H enter the funnel L, whose lower end is somewhat bent, so that the drops are thrown out of the vertical, and all upward splashing avoided. The rapidity of the flow through the siphon, and consequent dropping from H, is regulated by raising or lowering the table C. The vessel A acts as a regulator for keeping the level of the liquid in B at a constant height.

The first series of experiments was made with the double object of determining how far the rapidity of dropping influenced the size of the drops, and to establish the uniformity of the size of the drops which drop at equal intervals of time.

In these experiments cocoa-nut oil was taken as the liquid, an ivory sphere as the solid, and atmospheric air as the gas. The ivory sphere was washed in hydrochloric acid, so as to deaden its surface. Immediately before and after each batch of drops, the same number of drops were counted, and their time of falling compared with the time which elapsed in the actual experiment. In no case, however, was there a difference between the two of a single second, so that *gt* may be considered in each case to be exactly given.

TABLE I.—*Cocoa-nut oil.*

T = 28°·5 C.

gt = 1″.

Radius of ivory sphere = 22·1 millims.

Number of drops.	*gt*.	Weight of drops.
	″	gramme.
60	1	3·9817
60	1	3·9841
60	1	3·9784
60	1	3·9807
60	1	3·9742
60	1	3·9730
60	1	3·9735
60	1	3·9682
	gt.	Mean weight of single drop.
	1″	0·066279

Preliminary experiments having shown that the size of a drop is greatly affected by the rate at which the dropping takes place, that is, by the time occupied by the drop in its formation, the following experiments were performed to establish the connexion between the two.

It may be here remarked that with some liquids, of which cocoa-nut oil

is one, a continuous stream of liquid by no means implies a faster delivery of it than may be achieved by a succession of drops. On the contrary, just as by walking more rapid progress may be made than by running, so may dropping deliver more liquid than passes in a stream. A uniformly rapid series of drops may be converted into a stream, and reconverted into drops under certain restrictions, at pleasure, without altering the quantity of liquid delivered. We shall return to this point.

TABLE II.—*Cocoa-nut oil.*

T=28°·7 C.

Radius of ivory sphere =22·1 millims.

Number of drops.	Time between fall of first but one and last drop.	Weight of drops.	Number of drops.	Time between fall of first but one and last drop.	Weight of drops.
	"	gramme.		"	gramme.
60	26	4·5212	60	38	4·3678
60	26	4·5173	60	38	4·3628
60	26	4·5265	60	38	4·3682
60	26	4·5316	120	76	8·7403
60	30	4·3676	60	38	4·3646
60	30	4·3668	60	42	4·2342
60	30	4·3593	60	42	4·2357
60	30	4·3665	60	42	4·2362
60	34	4·4827	60	42	4·2368
60	34	4·4731	60	42	4·2330
60	34	4·4643	60	42	4·2378
60	34	4·4779	60	46	4·1487
60	34	4·4681	60	46	4·1438
60	34	4·4752	60	46	4·1499
60	38	4·3778	60	46	4·1471

From this Table is constructed the following Table III., which shows *gt* in seconds and the corresponding drop-weights in grammes, the latter values being the mean of the results in Table II. *gt* is got by dividing the time-lapses of Table II. by the number of drops.

TABLE III.—*Cocoa-nut oil.*

T=28°·7 C.

Radius of ivory sphere =22·1 millims.

gt.	Mean weight of single drop.
"	grm.
0·433	0·07540
0·500	0·07275
0·567	0·07456
0·633	0·07281
0·700	0·07059
0·767	0·06912
1·000	0·06628*

* Table I. T=28°·5 C.

Hence it appears that, within the above limits, on the whole, the weight or size of a drop diminishes as its growth-time increases. Further, it seems that between the rates $gt=\cdot433$ and $gt=\cdot567$ a minimum occurs, that is, instead of there being a continuous diminution in the weight as the growth-time increases, there is at first a diminution, then an increase, and finally a continuous diminution, so that drops of the rate $gt=\cdot500$ have sensibly the same size as those of the rate $gt=\cdot633$.

In order to establish more precisely the position of this minimum and the general relation between rate and size, the observations must be both more minute and more extended. For this purpose a fresh sample of oil was taken, and the time-intervals extended from 25″ per 60 drops to 240″ per 20 drops; as before, four experiments were made at each time-interval. The mean results are given in Table IV.*, in which the values of gt are obtained by dividing the time-intervals by the number of drops. The mean weights of the single drops are got as in Table III. The weights of oil passing in one second are found by dividing the terms of column 2 by those of column 1, which correspond to them.

TABLE IV.—*Cocoa-nut oil* (specific gravity =0·9195).

T=29° C.—29°·4 C.

Radius of ivory sphere=22·1 millims.

gt.	Mean weight of single drop.	Weight of oil passing per second.
″	grm.	grm.
(0·333)	(0·09264)	(0·27792)
0·417	0·08265	0·19837
0·433	0·08074	0·18631
0·450	0·08185	0·18189
0·467	0·07918	0·16968
0·483	0·07932	0·16412
0·500	0·08017	0·16035
0·517	0·08017	0·15518
0·533	0·07961	0·14927
0·550	0·07693	0·13985
0·567	0·07664	0·13524
0·583	0·07558	0·12957
0·600	0·07334	0·12221
0·617	0·07320	0·11871
0·633	0·07321	0·11560
0·667	0·07260	0·10891
0·750	0·07102	0·09469
0·833	0·06902	0·08283
1·000	0·06605	0·06605
1·500	0·06215	0·04144
2·000	0·05986	0·02993
3·000	0·05710	0·01903
4·000	0·05561	0·01432
5·000	0·05469	0·01094
12·000	0·05201	0·00433

* A Table exhibiting the details is given in the MS., which is preserved for reference in the Archives.

It was found impossible to arrest an exact number of drops when the rate was faster than 60 drops in 25″. A few rather discordant results, got at the rate of 60 drops in 20″, gave a mean of 0·09264 grm. as the weight of a single drop; this tends to show that at this high rate the drops were considerably larger than at any lower rate.

Towards the end of the Table, at the slower rates, the error of time becomes so exaggerated (the least alteration in the adjustment of the instrument makes so sensible a change in the entire time-lapse) that it is nearly impossible to avoid an error of about 0″·5 in the whole time of several minutes. Although the time-error thus becomes palpable, it nevertheless remains, relatively to the whole time-lapse, as immaterially small as the inappreciable errors of the swifter rates of dropping.

The numbers of Table IV. present us with several interesting and important facts.

From gt=·333 to gt=	·433	there is	diminution.
„ „ =·433 „ „ =	·450	„	„ increase.
„ „ =·450 „ „ =	·467	„	„ diminution.
„ „ =·467 „ „ =	·500	„	„ increase.
„ „ =·500 „ „ =12·000		„	„ continual diminution.

The most prominent fact is that, *on the whole*, the drops undergo a continuous diminution in weight or size as gt increases. To such an extent is this the case, that the most rapidly falling drops of the above Table are nearly twice as heavy as the most slowly falling ones. The cause of this is probably to be sought for in the circumstance that when the flowing to the solid is more slow, the latter is covered with a thinner film of liquid, so that, as the drop parts, the solid reclaims by adhesion more of the root of the drop than is the case when the adhesion of the solid to the liquid can satisfy itself from the thicker film which surrounds the drop in the case of a more rapid flow. The influence of rate is seen to extend even to the exceedingly slow rate of gt=12″.

This connexion between rate and weight (or quantity) should not be lost sight of by prescribers and dispensers of medicine. A pharmacist who administers 100 drops of a liquid drug at the rate of three drops per second, may give half as much again, as one who measures the same number at the rate of one drop in two seconds, and so on.

For our present purpose the effect of rate upon the size of a drop is of great moment, because it proves that there is no such thing as a drop of normal size. At no degree of slowness of dropping do drops assume a size unaffected by even a slight change in the rate of their sequence. Hence, whenever a comparison has to be made between the sizes of different drops, we shall have to eliminate this source of difference by taking drops which follow at exactly the same rate.

About the rate at which the diminution of size takes place for equal increments of gt, the Table gives us little information beyond the fact that,

on the whole, the sizes of the drops at the slower rates are less influenced by equal increments of gt than are those of the quicker rates. This, however, only appears distinctly at and below the rate of about $gt=1''{\cdot}00$.

If the connexion between gt and the drop-size be represented by a curve (fig. 2, A), the abscissæ being the values of gt, and the ordinates the corresponding drop-weights, there is apparently no asymptote parallel to the axis of X. The curve presents, however, in its course two secondary maxima and minima:

	Secondary maxima.	Secondary minima.
(1)	$gt = {\cdot}450''$	$gt={\cdot}433''$
(2)	$gt = {\cdot}500''$ or ${\cdot}517''$	$gt={\cdot}467''$

Although at these minima the drops are less than at the immediately succeeding rates, yet the quantity of liquid passed in a given time is, at every rate of dropping, greater than the quantity passed in the same time at every slower rate. The decrease of rate more than counterbalances the temporary increase in the drop-size. This is seen on comparing the numbers of column 3, Table IV., with one another. They are found to decrease continually, though by no means uniformly, as the rate of dropping decreases. The same fact is shown graphically in fig. 2, B.

The second maximum (at $gt={\cdot}500$ and $gt={\cdot}517$) is in remarkable connexion with the rate at which a series of drops may be converted into a continuous stream. At all rates of dropping, from $gt={\cdot}333$ to $gt={\cdot}517$ inclusive, the drops may be converted into a permanent stream by pouring a little additional oil upon the sphere as the drops are falling from it. A stream is thus established which remains for any length of time, if it be protected from all currents of air and vibration. At the rate $gt={\cdot}519$ the stream may be established by the same means for a few seconds (about $30''$), but the continuous part inevitably begins to palpitate, becoming alternately longer and shorter, thinner and thicker, until at last it draws up and is converted into a succession of drops. At the immediately slower rates of dropping the same effect follows, but in each case in a shorter time, so that the slowest rate of dropping, which may be converted into permanent running, coincides with the rate which gives the second maximum size of drops ($gt={\cdot}500$ and $gt={\cdot}517$). The appearance of a drop-convertible stream is peculiar, the narrowing which it undergoes on leaving the solid being remarkably sudden.

In many liquids such secondary maxima are entirely wanting. They appear in liquids of the physical nature of oils, whether those oils be chemically fatty (adipic salt of glycerine), or whether they be miscible with water, as syrups, glycerine itself, &c.

In order to avoid the influence of variations in rate, we shall for the future take the same rate of dropping in all cases, and, unless the contrary be stated, the rate adopted will be $gt=2''$.

The factor, the influence of whose variation on the size of the drop we have next to consider, is the constitution of the liquid of which the drop is formed. For the foregoing experiments concerning the influence of rate, cocoa-nut oil was employed on account of its non-volatility. On allowing a quantity of it, having an exposed surface of about two square inches, to stand for 70 hours, it was found to have increased about 2 milligrammes in weight, probably in consequence of oxidation. Its fixedness, therefore, and its perfect liquidness at the temperature of 28°–30° C., make it well adapted for this special purpose. Chemically and physically, however, it is of little interest for our immediate purpose, because it is a mixture of several substances, the proportion between which is indefinite.

The constitution of a liquid may vary in two ways. A liquid may be a mixture of two or more simple liquids, or a solution of one or more solids in a single or mixed liquid; or secondly, the liquid being single, may vary in the sense of its chemical constitution. It would be clearly impossible to exhaust experimentally the countless variations which might thus arise. We must be satisfied with taking a few simple examples of the two cases.

With the more mobile liquids the apparatus, fig. 1, fails to give a strictly uniform flow. As the liquid descends in B, it adheres by capillary action to the lip of A for some time after the level of B is below the lip. The air at last separates the two, enters the flask A, displaces the liquid there, and restores the level to B, so that although the average height of B is constant, yet it undergoes a series of slight but ceaseless variations. As even such slight irregularities sensibly affect the rate of flow through the siphon, and consequently the rate of dropping from the sphere, the apparatus is slightly modified as follows, fig. 3. Between the reservoir, B, fig. 1, and the dripping sphere, a second reservoir, M, is placed. This is kept in a state of continual overflow. The overflow is regulated by means of a few filaments of cotton wool hanging over the edge of the overflowing vessel, and so fashioned that the end in the overflowing vessel tapers to a point. Finally, the rate of flow is in many instances so sensitive, that it is impossible to procure exactly a predetermined rate by the ordinary screw-adjustment of the holder which carries the siphon. For the final adjustment, it is convenient to depend upon the elasticity of the siphon. A heavy ring is passed over the siphon, which is then firmly fixed so as to deliver the liquid at nearly the required rate. The ring slipped backwards and forwards, bends the siphon more or less, and regulates the flow through it.

Solution of Chloride of Calcium in water.—A solution of chloride of calcium, nearly saturated at 28° C., was taken as the starting-point or solution of maximum saline contents. Half of this solution was mixed with an equal volume of water (solution 2). Half of solution 2 was mixed with its own volume of water, giving solution 3, and so on. In this manner, without knowing the absolute strength of solution 1, we know

that the successive strengths of the saline solutions, whether there be los of volume owing to chemical union or not, are as $s, \frac{s}{2}, \frac{s}{4}, \frac{s}{8}, \ldots\ldots 0$.

These numbers give exactly the relative quantity of solid matter in a unit of volume of the liquid. As, however, solution 1 on dilution evolves heat and therefore probably contracts, the sizes of the drops cannot be derived directly from their weights. The specific gravity of each solution has to be determined experimentally.

TABLE V.—*Solutions of* Ca Cl.

$gt = 2''$.

Radius of ivory sphere = 22·1 millims.

T = 28° C.

Solution of Ca Cl.	Mean weight of single drop*.	Specific gravity.	Relative size of single drop.
Water or $\frac{S}{\infty}$	grm. 0·185166	1·0000	0·18517
$\frac{S}{128}$	0·168137	1·0039	0·16750
$\frac{S}{64}$	0·172907	1·0084	0·17147
$\frac{S}{32}$	0·172593	1·0172	0·16967
$\frac{S}{16}$	0·167222	1·0383	0·16105
$\frac{S}{8}$	0·191008	1·0720	0·17817
$\frac{S}{4}$	0·195839	1·1721	0·16742
$\frac{S}{2}$	0·211396	1·2786	0·16533
S	0·225558	1·4939	0·15098

The column of the relative sizes of the single drops (which is got by dividing the mean weights by the corresponding specific gravities) shows that, under like conditions, a drop of water is larger than a drop of solution of chloride of calcium of any strength whatever. The comparatively small quantity of solid matter in $\frac{s}{128}$ causes the drop to diminish about $\frac{1}{9}$th of its volume.

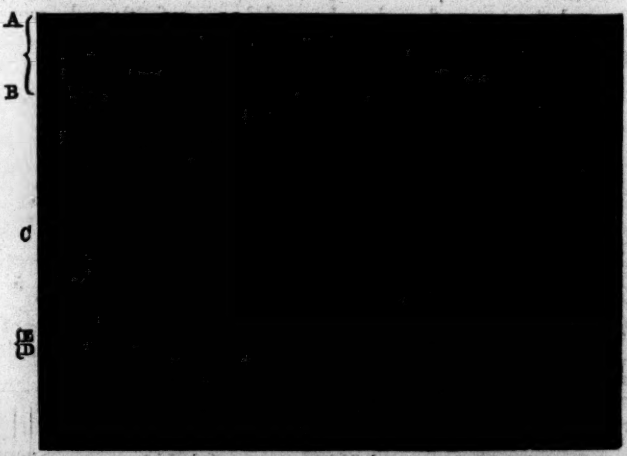
We must bear in mind that the successive increments of solid matter may affect the size of the drop in opposite directions,—by affecting the

* The first number from six, the following numbers from four determinations of the weight of 30 drops.

cohesion of the water, by asserting its own cohesion, by increasing the gravity of the liquid and thereby determining an earlier separation of the drop, and, in this particular case, by the chemical affinity of the solid to the liquid, and the probable formation of hydrates. It is seen that these influences cause an irregularity in the diminution of the size of the drop as it acquires more solid matter. In fact, it is only when the liquid has the considerable strength of $\frac{s}{8}$ that the diminution in drop-size becomes continuous.

In fig. 2, C shows graphically the relation between drop-size and strength. The abscissæ represent the strengths of the solution progressing in geometric

Fig. 2.



ratio; the ordinates show the corresponding comparative drop-sizes. It may be remarked that the curve C bears a striking resemblance to the curve A, *as though increase in solid constituent produced a similar effect upon the drop-size as increase in the time-interval on the drops of a homogeneous liquid.* We may also notice the great difference in size between a drop of water and a drop of oil under the same conditions. From Table IV. we find that a drop of oil of specific gravity ·9195 has the weight ·05986 when $gt=2''$. Hence the comparative sizes of the two are,—

	gt.	Radius of sphere.	T.	Comparative sizes.
Water	$2''$	22·1 mm.	28° C.	0·18517
Oil	$2''$	22·1 mm.	29°–29°·4 C.	0·06510

Or a drop of water is nearly three times as large as the drop of oil, the only difference in the circumstances being that the oil was 1°–1°·4 C. warmer. We shall have to study this point more especially hereafter.

On account of the chemical union which takes place on dissolving Ca Cl in water, it would be useless to give the absolute strengths of the various solutions.

Solution of Nitrate of Potash in water.—Nitrate of potash was the next solid examined, on account of the probable non-existence of hydrates. Seven solutions of nitrate of potash were made of the following strengths by weight:—

(1) 22 of water to 1 of nitrate of potash.
(2) „ „ „ 2 „ „ „
(3) „ „ „ 3 „ „ „
(4) „ „ „ 4 „ „ „
(5) „ „ „ 5 „ „ „
(6) „ „ „ 6 „ „ „
(7) „ „ „ 7 „ „ „

These solutions were made to drop from the ivory sphere at the rate of $gt=2''$. In each instance four batches of drops, of 30 each, were weighed. In the following Table the mean results only are given.

TABLE VI.—*Solutions of Nitrate of Potash.*

$gt=2''$.
T=28° C.
Radius of sphere =22·1 millims.

1. Solution $\frac{HO}{KNO_6}$	2. Mean weight of single drop.	3. Specific gravity by experiment.	4. Relative size of single drop.	5. Weight of KNO_6 in a drop.
Water				
$\frac{22}{0}$	0·18517	1·0000	0·18517	·00000
$\frac{22}{1}$	0·18613	1·0164	0·18314	·00846
$\frac{22}{2}$	0·17908	1·0341	0·17318	·01628
$\frac{22}{3}$	0·17714	1·0511	0·16853	·02411
$\frac{22}{4}$	0·16917	1·0680	0·15840	·03075
$\frac{22}{5}$	0·17805	1·0832	0·16439	·04047
$\frac{22}{6}$	0·18254	1·0987	0·16618	·04978
$\frac{22}{7}$	0·18611	1·1130	0·16723	·05921

Hence it appears that on the addition of the first quantities of nitre $\left(\frac{22}{1}, \frac{22}{2}, \frac{22}{3}, \frac{22}{4}\right)$ the size of the drop is diminished. Afterwards $\left(\frac{22}{5}, \frac{22}{6}, \frac{22}{7}\right)$

the size of the drop is partially recovered. There is a stage of dilution when the specific gravity is 1·0680, where the drop-size is a minimum. Further, it is seen from column 5 that the quantity of nitre in a drop increases continually as the strength of the solution increases, although both the weight and the volume of the drop vary.

Inversely, the regularity of the variation of drop-size, in the case of nitre, points to the absence of hydrates of that body.

It would be delusive to endeavour to construct a formula connecting the specific gravity with the drop-size or drop-weight of the solution; but, as before, a graphic representation serves to show the connexion between the variables. In curve D, fig. 2, the abscissæ represent the quantity of nitrate of potash in solution, the ordinates show the corresponding drop-sizes. As with chloride of calcium, it is seen that the drop-size of water is larger than that of any solution of nitre. Curve E, fig. 2, having the same abscissæ as D, has ordinates which represent the drop-weights.

It is confessedly a matter of great interest, and still greater difficulty, to determine exactly the relation which exists between a dissolved solid and its solvent—that is, to find out whether or when a solid should be viewed as being in combination with a portion of the liquid in which it is dissolved. Such questions may perhaps receive additional light from experiments similar to the above, but more extensive, and performed with this special object in view. Comparing the curves C and D, for instance, there can be little doubt that the secondary maxima and minima of C are owing to the existence of hydrates of chloride of calcium in solution. The only known hydrates of chloride of calcium are Ca Cl, 2 HO and Ca Cl, 6 HO, the latter of which contains 50·7 per cent. of Ca Cl. Solution S contains about 42·5 per cent. It is noteworthy that, while the six-water chloride in the solid state absorbs heat on solution, the solution S evolves heat on dilution, as already mentioned. In the case of nitre we have in the drop-sizes evidence only of the opposite efforts of two cohesions, that of the water and that of the nitre. By pursuing this direction of experimental inquiry, evidence may probably be got concerning the truth of Berthollet's hypothesis of reciprocal recomposition in the case of the mixture of the solutions of two salts, AX and BY, where AY and BX are also soluble in water.

III. "On Drops."—Part II. By Frederick Guthrie, Esq., Professor of Chemistry and Physics at the Royal College, Mauritius. Communicated by Professor Stokes, Sec.R.S. Received October 17, 1864.

We have next to consider the influence which variation in the chemical nature of the drop-forming liquid may exercise upon the drop-size in the case SLG.

The liquids which were selected for this purpose were chosen as being

typical of extensive classes, rather than as being connected with one another in immediate chemical relation. They were—

Water.	Oil of turpentine (turpentol).
Alcohol.	Benzol.
Acetic acid.	Glycerine.
Acetic ether.	Mercury.
Butyric acid.	

These several liquids were allowed to drop under the same conditions, from the bottom of a hemispherical platinum cup. The arrangement of the apparatus was quite similar to that described in Part I., the ivory ball being replaced by the platinum cup, and the overflow of the cup being determined by strips of paper bent over its edge. The case of mercury is the only one which requires some explanation. A few years ago I noticed the fact that mercury which holds even a very little sodium in solution has the power of "wetting" platinum in a very remarkable manner. The appearance of the platinum is quite similar to that presented by amalgamable metals in contact with mercury. But the platinum is in no wise attacked. Further, the amalgam may be washed off by clean mercury, and the latter will also continue to adhere equally closely to the platinum. All the phenomena of capillarity are presented between the two. The surface of the mercury in a platinum cup so prepared is quite concave; and a basin of mercury may be emptied if a few strips of similarly prepared platinum foil be laid over its edge—just as a basin of water may be emptied by strips of paper or cloth, and under the same condition, namely that the external limb of such capillary siphon be longer than the internal one.

I generally use this curious property of sodium-amalgam for cleaning platinum vessels. It enables us now to examine the size of drops of mercury under conditions similar to those which obtain in the case of other liquids*. After the cup had been used for the other liquids, its surface

* In regard to the above-mentioned property of sodium, the following observations may be of interest. At first the explanation naturally suggests itself, that the effect wrought by the sodium may be due to an absorption of oxygen, in consequence of the oxidation of the sodium, the consequent diminution of the gaseous film between the two metals, and the resulting excess in the superior pressure of the air. This, however, cannot be the true explanation, because it is found that the perfect contact between the two, or "wetting," takes place equally well in an atmosphere of nitrogen, carbonic acid, or *in vacuo*. Hence, if I may venture upon a guess, unsupported by experimental evidence, I should be rather disposed to assign the phenomenon to the reducing action of nascent hydrogen derived from the contact of sodium with traces of water. Perhaps even the least oxidizable metals are covered with a thin film of oxide, which is reduced by the nascent hydrogen at the same moment that the mercury is presented to the reduced metal. It is found that iron, copper, bismuth, and antimony are also wetted by mercury if their surfaces are first touched with sodium amalgam. Not only do the latter metals lose this power on being heated (as we might expect, in consequence of their superficial oxidation), but platinum, from which the adhering mercury film has been wiped by the cleanest cloth, or from which it has been driven by heat, also loses the power. It is true that the surface of clean platinum is supposed to condense a

was rubbed with sodium-amalgam and washed with clean mercury. A few strips of similarly prepared platinum foil being bent over the edge and pressed close to the sides of the cup, the mercury could be handled similarly to the other liquids.

The following Table VII. shows,—

1. The liquids examined.
2. The number of drops which were weighed.
3. The weights found.
4. The mean weights of single drops.
5. The observed specific gravity at the given temperature.
6. The relative sizes of single drops.

TABLE VII.

$T=26°$ C.

$gt=2''$.

Radius of curvature of platinum cup = 11·4 millims.

1. Name and formula of liquid.	2. Number of drops.	3. Weight of drops.	4. Mean weight of single drop.	5. Specific gravity.	6. Relative size of single drop.
		grm.	grm.		
Water..... HO.	20	2·9703	0·14828	1·0000	0·14828
	20	2·9923			
	20	2·9472			
	20	2·9603			
	20	2·9533			
Glycerine ..... $C_6 H_8 O_6$.	20	2·5496	0·12804	1·2452	0·10280
	20	2·5576			
	10	1·2877			
Butyric acid ..... $C_8 H_8 O_4$.	20	1·1616	0·05813	1·0017	0·05803
	20	1·1630			
	20	1·1634			
Mercury..... Hg.	20	7·9655	0·78703	13·5728	0·05798
	20	7·8984			
	20	7·7977			
	20	7·8197			
Benzol ..... $C_{12} H_6$.	20	0·9514	0·04778	0·8645	0·05527
	20	0·9488			
	20	0·9579			
	20	0·9644			
Turpentol ..... $C_{12} H_{16}$.	20	0·8675	0·04331	0·8634	0·05016
	20	0·8656			
	20	0·8653			
Alcohol ..... $C_4 H_6 O_2$.	20	0·7890	0·03949	0·8163	0·04960
	20	0·7910			
	20	0·7896			
Acetic ether ..... $C_4 H_5 O, C_4 H_3 O_3$.	20	0·8214	0·04149	0·8930	0·04647
	20	0·8300			
	20	0·8384			
Acetic acid..... $HO\,C_4 H_3 O_3$.	30	1·3636	0·04540	1·0552	0·04302
	20	0·9055			
	20	0·9095			

film of oxygen; and the removal of this might alter the adhesion between the mercury and platinum; but such a film could scarcely exist *in vacuo* or in another gas.

The experimental numbers obtained are given without omission. The liquids are arranged in the order of magnitude of their drop-sizes. It appears from column 5 (of the specific gravities) that some of the liquids employed were not perfectly pure. This, however, is quite immaterial in the present direction of examination, provided that in all cases where the liquids named are in future employed and compared with those of Table VII., identically the same liquids are meant.

The numbers of column 6, with which we are now exclusively concerned, present several points of great interest. In the first place, it appears that the specific gravity of a liquid is not by any means the most powerful determinant of the drop-size. Thus butyric acid, which has sensibly the same specific gravity as water, gives rise to a drop less than half the size of the water-drop; while mercury, of singular specific gravity, has no exceptional drop-size. Lastly, it may be observed how that remarkable body water asserts here again its preeminence. The first impression which these numbers make is, that there are three groups of magnitude, n, $2n$, $3n$. But it is possible that a change in the nature of the solid might throw these drop-sizes into a different order of magnitude; and certainly until a very much greater number of bodies is examined in this sense, it would be premature to attempt to establish anything like a law.

It is sufficient for the present to point out that the drop-size is not directly dependent upon either the specific gravity or boiling-point; nor does it stand in any obvious relation to what is sometimes called the liquidity, mobility, or thinness of a liquid. For we find that glycerine and (from former experiments) cocoa-nut oil both form smaller drops than water, the one being heavier and the other lighter than that body, and both being viscid or sluggish. On the other hand, alcohol and acetic acid, both perfectly mobile liquids, give rise to drops about half as large as those of glycerine*.

Hence it is clear that we are still ignorant of that property of a liquid upon which its drop-size mainly depends. We are not yet in a position to connect the drop-size with any of the known physical or chemical properties of liquids. We approach the solution of the problem by studying the effects of change in some others of the variables.

The adhesion between the liquid which drops and the solid from which it drops is also affected by the curvature and general geometric distribution of the solid at and about its lowest point. And the variation in the adhesion between the solid and liquid, caused by the variation in the geometric distribution of the solid, may and does in its turn affect the size of the drop.

From this aspect, one of the simplest kinds of variation is that offered

* The evaporation of the more volatile of these liquids is a source of slight error; not so much on account of the direct loss in weight of the drop in falling, as by reason of the cooling which it causes, and the consequent variation in density and adhesion. Such source of variation we shall examine in the sequel, and find insignificant.

by a system of spheres of various radii, but made of the same material. And this case is an important one, because it undoubtedly offers the key to all drop-size variation arising from a similar cause. To study this point we may make use of any one convenient liquid, such as water, and cause it to drop at a fixed rate from spheres of various radii, including the extreme case of a horizontal plane. This extreme case, however, presents certain practical difficulties. From a plane it is almost impossible to get a series of drops uniform in growth-time and in position. A ripe drop hanging from a horizontal plane will seek the edge thereof. Several drops may form upon and fall from the same plate at the same time and independently of one another. It is only by employing a plate not absolutely flat, that an approximation to the required conditions can be made. Taking r for the radius of curvature, the first numbers for $r=\infty$ can therefore be considered only as an approximation. The arrangements for the other cases were quite similar to that described in Part I., fig. 3.

No. 1. A glass plate, fastened to and held by a vertical rod.

Nos. 2, 3, 4. Selected globular glass flasks.

Nos. 5, 6, &c. Perfectly spherical glass spheres.

TABLE VIII.—*Water.*

$gt=2''$.

T=22°·5 C.

1.	2. Number of drops.	3. Radius of curvature.	4. Weight of drops.	5. Mean weight and relative size of single drop.
			grm.	
1.	20 20	∞	5·3325 5·2873	0·26549
2.	20 20	mm. 113·1	4·9226 5·0007	0·24808
3.	20 20	70·1	4·5260 4·5218	0·22619
4.	20 20	47·2	4·2781 4·2249	0·21257
5.	20 20	17·5	3·5055 3·4733	0·17497
6.	20 20	15·1	3·3562 3·3500	0·16765
7.	20 20	11·5	3·0281 3·0206	0·15122
8.	20 20	11·2	2·9803 2·9780	0·14896
9.	20 20	10·0	2·8665 2·8619	0·14321
10.	20 20	7·5	2·6765 2·6660	0·13356
11.	20 9	7·1	2·5752 1·1591	0·12877

It appears, therefore, that the drop increases in size according as the radius

of the sphere increases from which the drop falls, and, further, that the difference of drop-size brought about by this cause alone may easily amount to half the largest drop-size. For dispensers of medicine this fact is as important as that pointed out in Part I., where it was shown that the growth-time so materially influenced the drop-size. The lip of a bottle from which a drop falls is usually annuloid. The amount of solid in contact with the dropping liquid is determined by the size of two diameters, one measuring the width of the rim of the neck, the other the thickness of that rim. In most cases the curvature and massing of the solid at the point whence the liquid drops is so irregular as not to admit of any mathematical expression.

The reason why drops which fall from surfaces of greater curvature are larger than those which fall from surfaces of less curvature is surely this:—In the case of a surface of greater curvature the base of the drop has more nearly its maximum size; the centre of gravity of the liquid film from which the drop hangs is nearer to the centre of gravity of the hanging drop; the contact between the two is more extensive and intimate; so that the drop is held for a longer time and therefore grows more.

On comparing columns 3 and 5 of Table VIII., there does not appear to be any obvious law of connexion between the two; nor indeed can the numbers of column 4 pretend to such a degree of accuracy as would justify us in attempting to establish one. This is seen on comparing *inter se* the numbers of column 4. Especially with the spheres of longer radii, there is so much difficulty in getting a uniform wetting of the surface whence the drop falls, and this so materially influences the drop-size, that the numbers found are seen to vary considerably. Greater accord is obtained with spheres of less radii. As we might expect, the same absolute increase in length of radius takes less effect upon the drop-size in the case of longer than in that of shorter radii. The infinite, or at least indefinitely great difference between the radii 1 and 2 produces about the same effect upon the drop-size as the difference of 43 millims. between the radii 2 and 3, and so on.

The following Table of first differences shows this more strikingly:—

$r_{n+1}-r_n$.	$w_{n+1}-w_n$.
∞	0·01854
43·	0·02189
22·9	0·01362
29·7	0·03760
2·4	0·00732
3·6	0·01643
0·3	0·00226
1·2	0·00575
2·5	0·00965
0·4	0·00479

The relation exhibited in this Table supports the supposition that the size of the drop varies inversely as the contents of a figure bounded below by a circular horizontal plane of constant diameter (less than that of the

sphere) tangent to the sphere, laterally by a cylinder of vertical axis standing on the tangent plane and cutting the sphere, and above by the convex surface of the sphere itself (Plate IV. fig. 4).

As the diameter of the sphere still further diminishes, the size of the drop is limited by the possible size of its base, until finally the sphere is completely included in the drop.

It would be interesting, but it would take us too far, to consider the various cases of liquids dropping from cones, edges, solid angles, cylinders, rings, &c. We must content ourselves in this direction with the fact that the size of a drop is greater the more nearly plane is the surface from which the dropping takes place. If it were possible for a drop to fall from a concave surface, we should anticipate a still further increase in its size.

The relation between drop-size and curvature may be more strikingly shown by arranging the spheres one above the other in the order of magnitude.

Plate IV. fig. 5.—Each sphere receives the drops from the higher one. The quantity of water which drops in a given time, from every sphere, is the same. Hence in all cases the number of drops is inversely as the drop-

TABLE IX.—*Water.*

$gt = 2''$.

T = 23° C.

1. Radius of disk.	2. Number of drops.	3. Weight of drops.	4. Mean weight and relative size of single drop.
in.		grm.	
$\frac{5}{20}$	20	3·3682	0·16325
	20	3·1193	
	20	3·2523	
	20	3·3256	
	20	3·2594	
$\frac{4}{20}$	20	2·9693	0·14915
	20	2·9854	
	20	2·9746	
	20	3·0031	
$\frac{3}{20}$	20	1·9333	0·09666
	20	1·9244	
	20	1·9504	
	20	1·9248	
$\frac{2}{20}$	20	1·4618	0·07332
	20	1·4672	
	20	1·4688	
	20	1·4682	
$\frac{1}{20}$	20	0·8250	0·04107
	20	0·8212	
	20	0·8208	
	20	0·8190	

size; so that by counting the number of drops which fall from any two spheres in the same time, we get at once the relative sizes of the respective

drops. For several reasons, this plan of comparison is not sufficiently accurate to measure drop-sizes; but it offers a method of making the difference of drop-size visible to any number of persons at once.

The only other variation in the geometrical relation between the solid and the liquid, which we shall consider, is the variation in the size of a circular horizontal plane from which drops fall.

Five disks of copper foil were cut of the radii $\frac{5}{20}$, $\frac{4}{20}$, $\frac{3}{20}$, $\frac{2}{20}$, $\frac{1}{20}$th of an inch respectively. These were fastened horizontally to vertical wires, and, having been thoroughly cleaned by momentary immersion in nitric acid and washing, water was made to drop from them at the rate $gt=2''$.

Table IX. shows the influence of this kind of variation upon drop-size. The want of accord in the numbers of the largest disk is owing to a peculiar tremor which the drops exhibit at the moment of delivery. The same phenomenon was noticed, but to a less extent, with the next smaller disk. With the remainder it was not noticed.

The curvature and shape of the solid, and its consequent massing towards the liquid, is intimately connected with the next phase of variation which we shall consider, to wit, the variation in the chemical composition of the solid from which the drop falls. The influence of this kind of variation is to be studied by examining the size of drops formed under like circumstances, from spheres of the same size, but made of different material. Since in this case the liquid remains the same, we must limit the solids examined to such as the liquid completely wets. In this case, variation in the drop-size implies a variation in the thickness of the liquid film covering the solid. The latter must be caused by variation in the adhesion between the solid and liquid. Finally, such adhesion can only vary through one or both of two causes—namely, variation in the density of the solid, or in its specific adhesion dependent upon its chemical nature.

The first qualitative experiment was made upon three equal spheres of brass, glass, and cork. They were hung one above the other in the manner before described, so that the drop from one sphere fell upon the lower one*. It was found that, in whatever order the spheres were arranged, when the flow was uniform and not quicker than $gt=2''$, the dropping from the cork took place with the greatest rapidity, that from the glass next, and that from the brass most slowly—showing that the brass gives rise to the largest, the glass to the next largest, and the cork to the least drops. From this it would seem that the drops are in the same order as to size as are the solids as to density. We shall find, however, that this is not always the case, and that some other property as well as density is at work to influence the drop-size. The quantitative experiment, the results of which are given in Table X., confirms the result of the qualitative experiment given above, but shows, at the same time, that the joint influences of den-

* In this kind of experiment there should be a considerable mass of cotton wool on each sphere to receive the drops from the higher one, and, by acting as a reservoir, to regulate the flow.

sity and chemical diversity of the solid have only a small effect upon the drop-size. The conditions of the experiment were similar to those previously described.

TABLE X.—*Water.*

$gt=2''$.
T$=22°{\cdot}9$ C.
$r=7{\cdot}1$ millims.
Number of drops$=20$.

Substance.	Weight of 20 drops.	Mean weight of single drop.
	grms.	grm.
Cork	2·4846 2·4848 2·4832 2·4789 2·4821 2·4877	0·12418
Glass	2·5930 2·5985 2·5989 2·5949 2·5953 2·5900	0·12975
Brass	2·6225 2·6229 2·6260 2·6295 2·6296 2·6116	0·13118

When a liquid drops from a solid it is not always that the adhesion between the solid and liquid is overcome. The phenomenon of "wetting" implies a superiority of the adhesion between the solid and liquid over the cohesion of the liquid; and in all cases where a liquid drops from a solid which it wets, the act of separation is a disruption of the liquid, and not a separation of the liquid from the solid; that is, the separation of the drop is a failure of cohesion and not of adhesion. We are not, however, justified on this account in anticipating that the size of a drop is unaffected by the chemical nature of the solid from which it drops, even in those cases where the adhesion between the solid and liquid is greater than the cohesion of the liquid (that is, where the liquid completely wets the solid), because, although it is the liquid which is broken, yet the size of the broken-off part, or drop, depends in great measure upon the thickness of the residual film, as we have seen in examining the influence of the growth-time (in Part I.) and of the radius of curvature.

Adhesion may also exist between a solid and a liquid which does not wet it, as when a drop of mercury hangs from a glass sphere. But the cohesion of the liquid in such a case, by its effort to bring the liquid to the spherical form, and the weight of the drop so modify the adhesion between

the solid and liquid, by altering the size of the surface of contact between the two, that the size of the drop gives no direct clue to the cohesion of the liquid.

We may now examine a few cases in which, the size of the sphere remaining the same, and its density in some instances nearly so, the matter of the solid varies, but the liquid wets it in all cases. This will show whether the differences of Table X. are due wholly to differences of density of the solid, or also or wholly to differences of chemical constitution.

TABLE XI.—*Water.*

$gt=2$.

T$=23°$ C.

Radius of curvature$=7$ millims.

1. Substance.	2. Weight of 20 drops.	3. Mean weight of single drop.	4. Specific gravity of solid.
	grms.	grm.	
Antimony	2·3905 2·3980 2·3968 2·4016	0·11984	6·80
Sulphur	2·4019 2·4067 2·4046 2·4063 2·4022	0·12021	2·00
Cadmium	2·4362 2·4387 2·4374 2·4358	0·12185	8·70
Zinc	2·4495 2·4481 2·4518 2·4478	0·12246	6·86
Lead	2·4522 2·4525 2·4537 2·4528	0·12264	11·44
Phosphorus ...	2·4532 2·4564	0·12274	2·08
Bismuth.....	2·4528 2·4584 2·4580 2·4589	0·12285	9·90
Tin	2·4843 2·4864 2·4829 2·4861	0·12425	7·29

Equal spheres of the substances were made by casting them in the same bullet-mould. The surfaces of the metals were roughened by momentary immersion in acid; tin and antimony in hydrochloric, the rest in nitric acid. Without this precaution a metallic surface is apt to be wetted only locally, the base edge of the drop is irregular and inconstant, and the drop-

weight varies. Indeed with some metals, such as tin, a smooth and bright surface is scarcely wetted by water.

As the bodies examined have different coefficients of expansion by heat, and one of them expands on solidification, it was necessary to test the equality of their size and remedy any inequality. This was done by arranging three of them, one at each angle of a small equilateral triangle drawn on a large piece of plate glass. Another piece of plate glass was then placed upon the spheres so as to rest on them all three, and slightly loaded. On passing a gauge between the plates, at their edges, the slightest inequality of the spheres could be detected, because the gauge lifted the plate off the smallest of the three balls, which could then be moved. The larger spheres were then reduced in size by brisk agitation in acid. The sulphur and phosphorus were, for the same purpose, washed in ether.

Although there is only a slight difference between the consecutive terms of column 3, yet between the extremes of antimony and tin a well-marked difference exists.

This Table shows that the drop-size stands in no simple relation either to the equivalent density or chemical character of the solid, and establishes the existence of a specific adhesion independent of these. Although the differences of Table IX. may be partly owing to the differences of density of the solids cork, glass, and brass, yet we see from Table X. that there is about half as great a difference between the sizes of drops from antimony and tin as between those from cork and brass, although the difference of density between the first two is small compared with that between the last. Again, sulphur gives rise to drops intermediate between those of antimony and cadmium. Without, therefore, venturing to assert that density is without influence on drop-size, it is clearly proved that it does not exert the most powerful influence.

We have finally to examine the direction and extent of variation in drop-size caused by change of temperature. By altering the density of the liquid, a change in its temperature may affect the drop-weight without altering the drop-size. It may further alter the drop-size by altering the size and therefore the curvature of the solid. Any error introduced by the first of these sources is eliminated by dividing the observed weight by the specific gravity at the proper temperature, as in the case of different liquids at the same temperature. Errors from the second source may be certainly safely neglected, being far within the errors of observation.

In the place where these experiments were made, the range of natural atmospheric temperature is very small. From the coldest to the hottest season the difference scarcely exceeds 10° C. This circumstance made an extended and minute study of the influence of temperature impossible, by preventing more than one observation at each temperature being made.

The liquid taken was water, and the solid was glass. The water was heated to the boiling-point and placed in the apparatus (Part I. fig. 3). The sphere from which the water fell was the bulb of the thermometer which

measured the temperature. Fully the upper half of the sphere was covered with cotton-wool, so that the whole of the sphere was kept wet. The considerable mass of mercury in the bulb of the dropping sphere or thermometer itself served to make more uniform the temperature of the drops; while the actual contact between the drops and the spherical bulb ensured a tolerably close approximation between the actual temperature of the drops and that indicated on the stem of the instrument. Although, therefore, the temperatures observed cannot pretend to any even approximate positive accuracy, yet they are certainly in the actual order of magnitude. The arrangement is seen in Plate IV. fig. 6.

TABLE XII.—*Water*.

$gt=2''$.

$r=7{\cdot}4$ millims.

Number of drops = 20.

Temperature, Centigrade.	Weight of 20 drops.	Weight of single drop.	Relative mean size of single drop (corrected for temperature).
	grms.	grm.	
40·3 (44°·1)	2·5564	0·12782	
40·	2·5795	0·12897	0·12985
37·	2·5826	0·12913	
35·	2·6083	0·13041	
33·9	2·6105	0·13052	
32·6	2·6161	0·13080	
31·2	2·5960	0·12980	
30·6	2·6065	0·13032	0·13066
29·	2·6044	0·13022	
28·2	2·5983	0·12992	
28·	2·6078	0·13029	
27·5	2·6032	0·13016	
20·4 20·4	2·6480	0·13240	0·13262

In the above Table the temperatures are so grouped together that the means of the groups differ from one another by about 10° C. The single drop-weights are correspondingly grouped, and the mean of each group is then divided by the specific gravity of water (0° = 1) at the mean temperature of the group.

It appears then that, for a range of 20° Centigrade, or 36° F., the difference in drop-size effected by change of temperature in the liquid is inappreciably small, not being more than 0·00277, a quantity almost within the limits of experimental error; for on referring to Table X. we find that the greatest difference between the numbers for glass, which should be equal, amounts to 0·00044 grm., or a sixth of the greatest difference due to variation in temperature.

On the whole, then, we may conclude that the temperature has very little influence on the drop-size in the case of water between the above limits. No doubt, near the point of solidification, where liquids have an incipient

structure, the drop-size would be subject to sudden changes of magnitude. A few experiments with other liquids, namely turpentol, acetic acid, and alcohol, showed that with them the drop-size was almost equally insensible to change of temperature; and in all cases, as with water, the lower the temperature, on the whole, the larger the drop.

We have now examined *seriatim* all the chief causes upon which the drop-size depends in the case SLG. They are, 1. Rate of delivery; 2. Solids held in solution; 3. Chemical nature of liquid; 4. Geometric relation between solid and liquid; 5. Density and chemical nature of solid; 6. Temperature.

Our data, however, are still insufficient for us to predict, under all circumstances, the relative sizes of the drops of liquids under known external conditions. Clearly the missing term is closely related to the specific cohesion of the liquid. But what is cohesion? and how can it be measured? It lies perhaps in the nature of things—it seems at least inevitable—that the nomenclature of elementary properties should be vague and unsatisfactory. The properties of solids—hard, soft, brittle, tough, tenacious, elastic, malleable—do not stand in any definite relation to one another. Even the hardness which resists abrasion, the hardness which resists penetration, the hardness which resists crushing are by no means identical; so that one body may possess more of the one sort of hardness than a second body does, while the second body exceeds the first in another sort of hardness. Nor do any of the above-mentioned properties of solids stand in any simple relation to that resistance to the separation of the contiguous parts which is called cohesion. Thus, by no attribution of this single property of cohesion could we define ice or shell-lac, bodies which are at the same time tough, brittle, elastic, and soft.

We are forced to the conception of two distinct kinds of cohesion—*stubborn* and *persistent*. These may coexist, but are not identical. The one is strong to assert, the other pertinacious to maintain. The four following substances may serve to illustrate the possession of these two cohesions in various quantity.

Talc has little stubborn and little persistent cohesion.

Glass has much stubborn and little persistent cohesion.

Gold has little stubborn and much persistent cohesion.

Iron has much stubborn and much persistent cohesion.

The necessity for such a discrimination exists in a yet higher degree in liquids. If we conceive two liquids of different nature dropping from the same substance which they both wet, and if there be only one kind of cohesion, the one which has the greatest cohesion will tend most strongly to assume the spherical form; and this would tend to cause it to drop sooner, or have a smaller drop-size than the other. On the other hand, the liquid of stronger cohesion will cling most strongly to the film of liquid adhering to the solid; this will keep it longer from falling, and thereby increase its drop-size. Hence an increase of cohesion tends to produce two contrary

effects. But if there be a similar distinction between the two kinds of cohesion of liquids, as above pointed out in the case of solids, we have the following consequence. It is the persistent cohesion which causes the assumption of the spherical form, the stubborn which resists the separation of the drop. The former tends to diminish, the latter to increase its size. As one or other predominates, the size of the drop varies.

Accordingly the drop-size is by no means a measure of what is generally called the cohesion of the liquid, but rather a measure of the difference between the two cohesions, stubborn and persistent; and the law is, that the drop-size varies inversely as the persistent, and directly as the stubborn cohesion of the liquid.

In mercury, water, and glycerine the stubborn cohesion is greater in proportion to the persistent cohesion than in the other liquids examined; but it by no means follows that persistent cohesion is wanting in mercury or stubborn in alcohol.

When a drop is in the act of falling its stubborn cohesion is in equilibrium with the resultant of two forces—the one, the persistent cohesion, tending to produce a spherical form, the other the weight of the drop. Since the former of these component forces is, for the same liquid, constant, it seems as though the weight of the drop might be taken as a measure and expression of the stubborn cohesion. But such is not the case, because we have no ground for supposing that the diameter of the drop where the separation occurs is of constant size; on the contrary, it must be conceded that in larger drops this hypothetical surface of stubborn cohesion is larger than in smaller drops. Further, unless we know the exact shape of a drop in all cases, we are not in a position to deduce the size of the surface of cohesion from the drop-size or drop-weight.

In the cases where it has been tried, it has not been found that the nature of the gaseous medium in the case of SLG exerts any appreciable or definite influence upon the drop-size. Taking glass for the solid and water for the liquid, the medium was changed from air to nitrogen, hydrogen, and carbonic acid. The exceedingly slight variation wrought in the drop-size by this change may probably have been due to the different solubility of the gases in water, and the consequent alteration in the cohesion of that liquid.

Having now traced the effect of variation in the conditions which determine the size of a drop in the general case SLG (or where from a solid a liquid drops through a gas), we come to the case SLL (that is, where from a solid a liquid drops through a liquid). As in the cases of SLG, we must here also take the three terms of such chemical nature as to be without action upon one another.

SLL. *From a Solid a Liquid drops through a Liquid.*

A preliminary quantitative experiment was made under the following conditions:—Water was made to drop from a glass sphere at the rate

$gt=5''$. The drops were collected in a tube bearing an arbitrary mark. The number of drops required to fill the tube up to this mark was noted. Then the sphere was surrounded by turpentol, and the rate having been brought again* to $gt=5''$, the number of drops of water necessary to fill the tube up to the same mark was counted. The turpentol being replaced by benzol, the same operation was performed. The entire arrangement of the Stalagmometer† is seen in Plate V. fig. 7.

X, Y are contrivances described in Part I. for giving a uniform flow of water.

The siphon A rests upon the cotton-wool covering half of the dropping sphere and thermometer-bulb G. The sphere is held by its stem B in the clamp H. C is half a globular 1-lb. flask, supported by the filter-stand K. Through the neck of C passes the tube D. C and D are joined liquid-tight by the caoutchouc collar L. A few arbitrary marks are made at E. The lip of C is turned down to a beak at M above the vessel F.

In adjusting the instrument, to get the required value of gt, the holder K is slipped along the table so that the drops from G fall between C and D, and not into D. When the required rate is obtained, it is slipped back again. When such liquids as turpentol are used as media, a little water is poured between D and C to protect the caoutchouc. In all cases where a liquid medium is employed C is filled till it runs over.

In the first experiment, of which the results are given in the following Table XIII., the numbers are subject to two sources of error. The volume filled is rather small, and no allowance is made for meniscus. In this, as in all cases of SLL, great care must be taken not to shake the instrument.

TABLE XIII.—*Water.*

$gt=5''$.

T=22° C.

Radius of glass sphere=7·4 millims.

1. Medium.	2. Number of drops of water required to fill a given volume.	3. Mean of Column 2.	4. Relative size of single drop (through air=1).
Air	58 56	57·0	1·0
Turpentol	28 27 26 26	26·7	2·14
Benzol	7 7	7·0	8·14

There is therefore a greater difference between the drop-sizes of water in benzol and turpentol than between those in turpentol and air. The tur-

* A diminution of gt is observed.

† Σταλαγμὸς, a drop.

pentol and benzol here employed had the specific gravities of 0·863 a 0·864 respectively; they may therefore be considered of equal densi Hence variation in the liquid medium, independent of variation in its de sity, produces an enormous effect upon drop-size. We shall have occas to return to this case.

The influence which the liquid medium exerts on the drop-size, and t share of that influence due to the specific gravity of the medium, will well seen on comparing the drop-sizes of mercury which falls throu various liquid media.

The arrangement of the apparatus for this purpose is seen in Plate fig. 8. As far as A it is similar to fig. 7. The siphon A, fig. 8, is a capill tube; its lower end, which is turned vertically downwards, rests upon a sphe of brass, R, which has been washed with nitric acid and sodium-amalgam, a allowed to soak for some days under mercury. Mercury adheres perfec to such a sphere. In every case the sphere was immersed just halfw in the liquid. A small capsule S is supported in the liquid on a stand about half an inch lower than the bottom of the sphere. As soon as becomes constantly $=5''$, the vessel V is moved so that S comes under Five drops of mercury having been caught, the cup is moved horizontal as before, taken out and replaced by a fresh one, and so on. The batch of five drops are washed, dried, and weighed. The results are given Table XV.

We may, however, previously notice here with advantage a phenomen which attends the separation of drops under several circumstances, b which can be watched most narrowly in the cases of SLL, because in liquid the separation of a drop is less abrupt than in a gas.

When water falls from glass through air, immediately after the dro separates, a very minute drop is frequently projected upwards from th upper surface of the drop*. I have not traced the conditions under whic this supplementary drop is formed; indeed it is sometimes formed, an sometimes not, under apparently similar circumstances. No doubt th proximate cause is that the drop at the instant of separation is not sphe rical; the persistent or retentive cohesion, which brings it almost imme diately to its normal shape, does not allow time for its more excentric part to collect to the main mass; they are therefore by the motion of the mai drop flung off and projected upwards.

The same phenomenon is seen much more distinctly when water drop at this rate ($gt=5''$) through benzol or turpentol. In these cases the per sistent cohesion of the liquid medium comes also into play.

But the most striking example of supplementary drops is seen when

* The secondary drop may be well shown by holding a plate containing anhydrous cupric sulphate about two inches below the dropping solid. The white salt is smoothened by pressure under a plate, and its surface, being porous, absorbs the water-drops instantly and without splashing. The blue spots of hydrated sulphate show where the water has fallen.

Fig. IV.

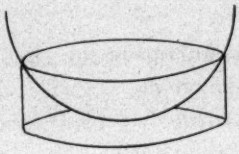


Fig. VI.

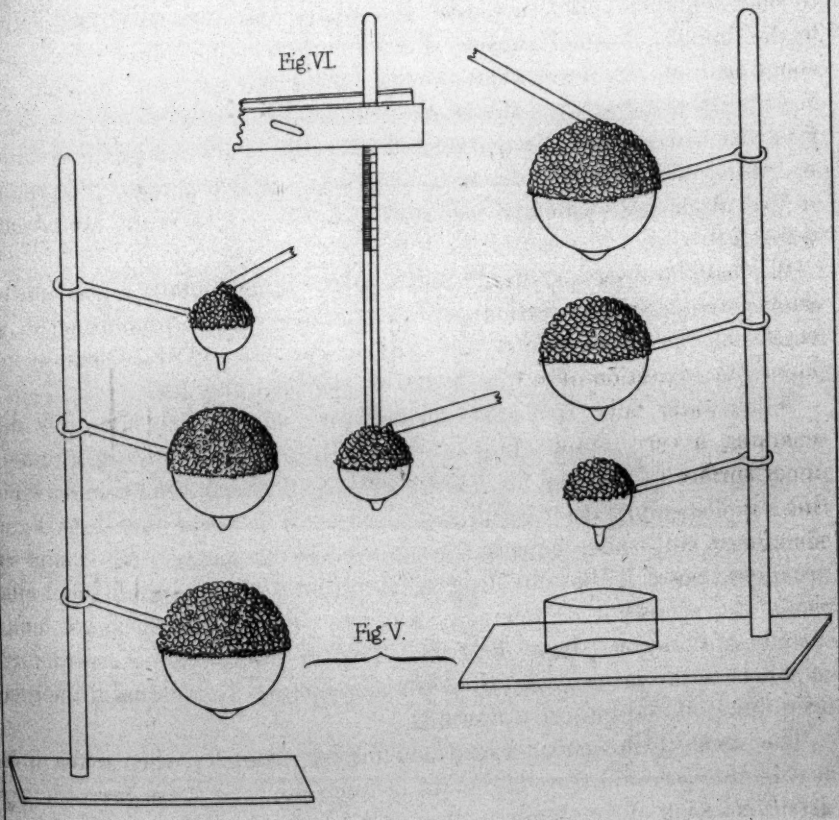
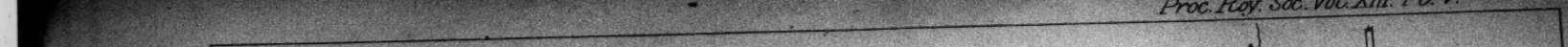
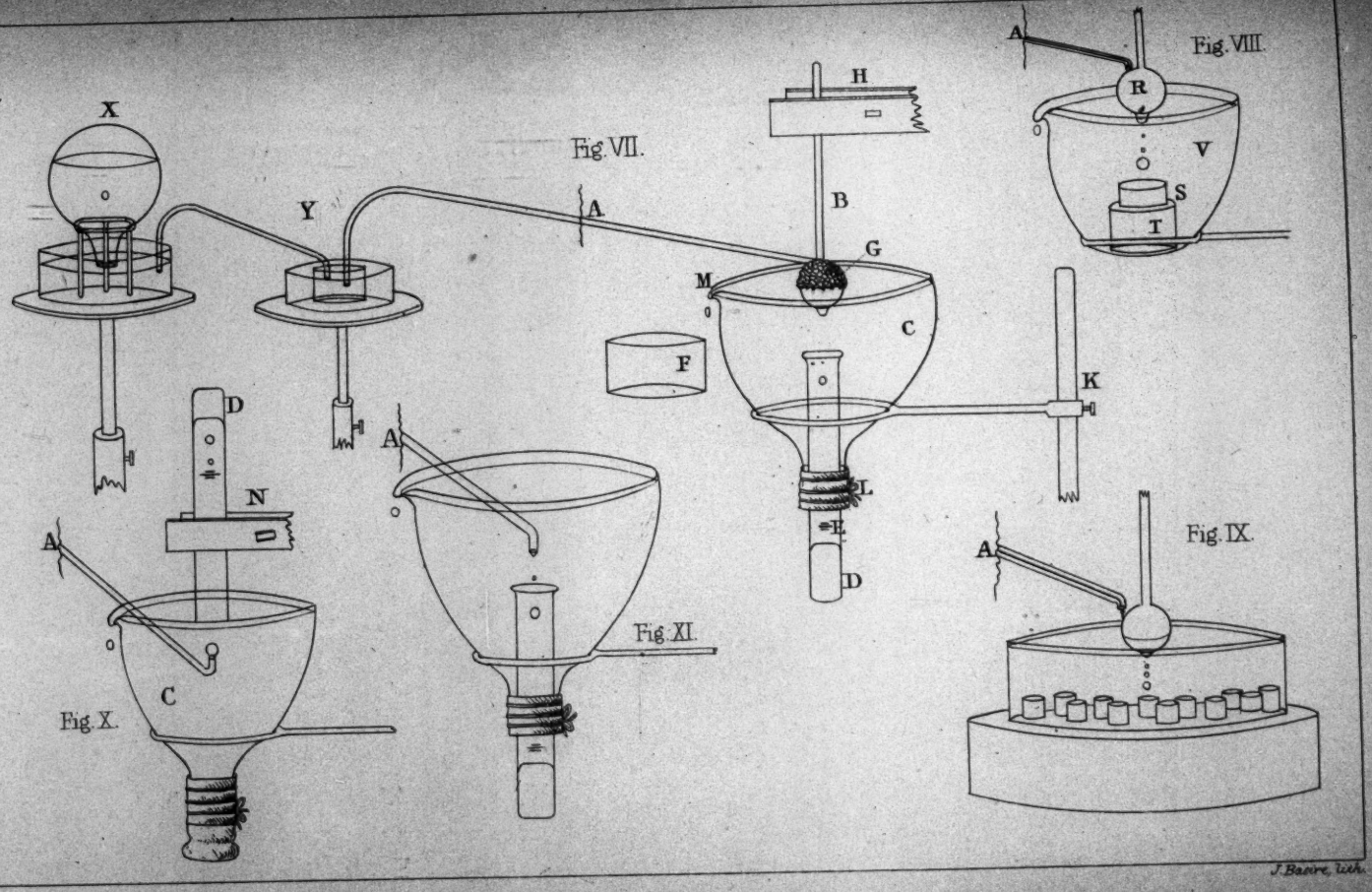


Fig. V.

J. Basire, lith.





J. Basire, lith.

glycerine forms the medium through which mercury drops. In this case, when $gt=5''$, there are always two supplementary drops of mercury formed. It is impossible to determine whether they both have their origin at the same moment and from the same drop. The probability, however, is that they have not, but that one is first separated from the main drop, and the second from the first; for there is always a great disparity between the sizes of the two supplementary drops, whereas, if they were both formed at the same time and for the same reason, we should be justified in expecting greater equality. The drops soon separate in falling, in consequence of the difference of their surfaces. The relative sizes of the main and supplementary drops in the case of mercury falling from copper through glycerine were determined as follows:—A number of porcelain cups (fig. 9) were arranged at the bottom of a shallow dish full of glycerine; when the rate of dropping was uniform at $gt=4''$, the dish was shifted horizontally so that every drop with its two supplements was caught in a separate cup. The globules of mercury in each cup were removed by a little scoop of copper foil. Ten of each kind were collected. After washing and drying, they were weighed, with the following result:—

TABLE XIV.—*Mercury*.

$gt=4''$.
T=21°·3 C.
Radius of sphere=12·8 millims.

	grms.
10 principal drops weighed	6·3447
10 first supplementary drops weighed	0·1242
10 second do. do. do.	0·0229
10 complete drops weighed	6·4918

In all cases of SLL the supplementary drop or drops were collected and weighed or measured with the main drop.

In Table XV.—

Column 1 shows the medium through which the mercury dropped.

Column 2. The number of drops weighed.

Column 3. The weight of the drops. The weight of every batch of drops is given, in order that the approximation between the figures for each liquid may be compared with that between the separate liquids. In two cases only, marked by an asterisk, are the numbers probably erroneous. They are not reckoned in taking the mean.

Column 4. Mean weight of single drop, from column 3.

Column 5. Specific gravity of medium.

Column 6 shows the weight of the drop of mercury *in the liquid*. Since the falling of the drop is determined in part by its weight, and since the weight depends not only upon the size of the drop, but also upon the density of the medium in which it is formed, it is interesting to see how the

size of the drop is affected by the diminution in its weight caused by the density of the medium.

If W_1 = weight of drop of mercury in air,
W_2 = required weight of drop of mercury in liquid,
A = specific gravity of liquid,
B = specific gravity of mercury;

then $$W_2 = W_1 - \frac{A}{B} W_1.$$

The values of W_2 form column 6.

The liquid media are arranged according to the order of magnitude of the numbers of column 4. The salient points of Table XV. are chiefly these:—

1. *The drop-size of a liquid which drops under like conditions through various media does not depend wholly upon the density of the medium and consequent variation in the weight, in the medium, of the dropping liquid.* Thus glycerine, whose density is above that of all the other liquids examined, does not, as a medium, cause the mercurial drop to assume either its minimum or maximum size.

2. The liquids in Table XV. are in the same order as in Table VII. In other words, *if there be two liquids,* A *and* B, *which drop under like conditions through air, and the drop-size of the one,* A, *be greater than that of the other,* B; *then if a third liquid,* C, *be made to drop through* A *and through* B, *the drop-size of* C *through* A *is greater than the drop-size of* C *through* B.

3. Further, on comparing Tables XIII. and XV. it appears that, whether water or mercury drops through turpentol and benzol, the drop through benzol is greater than the drop through turpentol. This we shall afterwards find confirmed in other instances into the law, *If the drop-size of* A *through* B *be greater than the drop-size of* A *through* C, *then the drop-size of* D *through* B *is also greater than the drop-size of* D *through* C.

It is further observed that, while mercury exhibits its largest drop when falling through air, water assumes its smallest drop-size under this condition.

This method of the examination of liquids by drop-size in the case SLL, which brings so prominently forward a comparatively slight difference between similar liquids, may be used, not only to detect commercial adulterations of one liquid by another, but perhaps to distinguish between those remarkably-related isomeric liquid bodies (the number of which is quickly increasing) between whose terms the difference has until lately escaped detection. Of these bodies perhaps the first most remarkable instance was furnished by the two amylic alcohols; but the greatest number at present known is amongst the hydrocarbons.

We may take an example illustrating the use of the stalagmometer in approximately measuring the proportion, in a mixture, of its two chemically and physically similar, but not isomeric constituents.

Suppose we had a liquid which we knew to consist wholly of a mixture of benzol and turpentol, and we wished to find the proportion in which

Table XV.—*Mercury.*

$gt = 5''$.

T = 21°·3 C.

Radius of sphere = 12·8 millims.

1. Medium through which the mercury dropped.	2. Number of drops.	3. Weight of drops.	4. Mean weight in air and relative size of single drop.	5. Specific gravity of medium.	6. Weight of single drop in respective medium.
		grms.	grm.		grm.
Air	5	3·8123	0·76545	0·00	0·76545
	5	3·8138			
	5	3·8169			
	5	3·8389			
	5	3·8141			
	5	3·8131			
	5	3·8182			
	5	3·8557			
	5	3·8430			
	5	3·8105			
	5	3·8535			
Water	5	3·5037	0·69750	1·00	0·64619
	10	6·9384			
	5	3·4534			
	5	3·5047			
	5	3·4918			
	5	3·5015			
	5	3·5066			
Glycerine	5	3·4235	0·61508	1·245	0·55793
	5	3·4215			
	5	3·4235			
	5	3·4285			
	5	3·3627*			
	5	3·4088			
	5	3·4083			
	5	3·4329			
	5	3·4090			
	5	3·3981			
Benzol	5	2·9418	0·59822	0·864	0·56014
	5	2·4875*			
	5	3·0773			
	5	2·9888			
	5	2·9637			
	5	2·9549			
	5	3·0767			
	5	2·9583			
	5	2·9352			
	10	6·0144			
Turpentol	5	2·1427	0·43497	0·863	0·40715
	5	2·1900			
	5	2·1883			
	5	2·1960			
	5	2·1820			
	5	2·1620			
	5	2·1768			
	5	2·1791			
	5	2·1608			
	5	2·1708			

these two ingredients were present. We could scarcely approach to an answer by any of the means hitherto employed. The specific gravities of the two liquids are so close (·864, ·863) that the density of the mixture would give us no substantial aid. Though there is a considerable difference (80° C.) in their boiling-points, no one who is familiar with the difficulties of fractional distillation would place any reliance upon a quantitative separation based upon volatility. Their refractive indices are nearly the same*. Their vapour-densities, 2·77, 4·76, though comparatively different, are not absolutely very wide apart. They are active and passive towards most of the same chemical reagents, and interfere with one another's reactions. If we have recourse to chemical analysis ($C_{12}H_6$, $C_{20}H_{16}$), a very small experimental error would point to a great difference in the proportion of the two.

To find how far the stalagmometer (Plate V. fig. 7) is applicable in this case, it was filled with five liquids in succession:—

1st, with benzol ..... =B.
2nd, with two volumes benzol and one of turpentol =B_2T.
3rd, with one volume benzol and one of turpentol.. =BT.
4th, with one volume benzol and two of turpentol =BT_2.
5th, with turpentol..... =T.

The time-growth being brought in each case to 5″, the number of drops of water required to fill a given volume was counted, allowance being made for the meniscus.

TABLE XVI.

Through.....Air.	T.	BT_2.	BT.	B_2T.	B.
102	51	38	34	31	14
102	51	37	33	31	14
101	50	38	33	31	14
..	49	..	..	..	..
Mean.... 101·7	50·2	37·7	33·3	31	14

Hence a difference of 16·6 per cent. in one of the constituents corresponds to an observed difference, under the most unfavourable conditions, of three drops. In other words, the stalagmometer is sensitive to an alteration of about 6 per cent. By increasing the capacity of the recipient, it is clear that the drop-numbers, and therefore their differences, might be increased at pleasure. Thus by counting the number of drops necessary to fill a volume six times the size, we could tell to within one per cent. how much turpentol and how much benzol were present.

But it is perhaps in the cases of the still more proximate identity of isomeric bodies mentioned above that the stalagmometer may be used rather

* The refractive index of turpentol is 1·476; that of benzol does not appear to have been measured; but that it is almost identical with that of turpentol is seen on mixing the two. In those cases in which I propose chiefly to use the stalagmometer, namely with isomeric liquids, the method of refraction is useless, because isomeric liquids seem always to have the same refractive indices.

as a stalagmoscope, to render evident rather than to measure a difference of drop-size.

From Table XVI. we gather the general law concerning three liquids, two of which are insoluble in the third. *If a liquid,* A, *drop downwards under like conditions in succession through two liquids,* B *and* C, *then its drop-size through any mixture of* B *and* C *is intermediate between its drop-size through* B *and its drop-size through* C; *and the greater the proportion of* $\frac{B}{C}$ *in the mixture, the more nearly does the drop-size of* A *through the mixture approach to the drop-size of* A *through* $\frac{B}{C}$ *alone.*

We have already examined the influences on the drop-size in the case SLG of the density of the dropping liquid, and of its persistent and stubborn cohesions respectively. Increase in the former two tends to diminish the drop-size; increase in the last to increase it. Let us examine in like manner the influence of the similar properties of the medium.

1. *The density of the medium.*—Increase in the density of the medium is equivalent to diminution in the density of the dropping liquid, and must therefore be followed by a tendency to increase in the drop-size.

2. *Stubborn cohesion of medium.*—The resistance to displacement, or stubborn cohesion of the medium, tends to keep back the drop in its place, and makes it necessary for a larger quantity of the dropping liquid to accumulate; that is, it increases the drop-size.

3. *Retentive cohesion of medium.*—The same force of persistent or retentive cohesion which causes a drop of a liquid to take the spherical form, would also cause the liquid to give or tend to give a spherical form to an irregularly-shaped volume of a solid, liquid, gas, or vacuum in it. Thus gas-bubbles in liquids have an approximately spherical form, not by reason of the cohesion of the parts of the gas, but by the persistent cohesion of the liquid medium which moulds the gas into that form by which the cohesion of the liquid is most gratified. Hence increase in the retentive or persistent cohesion of the medium tends to diminish the drop-size of the dropping liquid.

In all cases of SLL we may represent the direction of the influence of the determinants by the following scheme, in which the sign + denotes a tendency to increase, the sign — one to diminish the drop-size:—

SLL.

Dropping Liquid.

+	—
Stubborn cohesion.	Persistent cohesion. Density.

Medium Liquid.

+	—
Stubborn cohesion. Density.	Persistent cohesion.

This scheme is verified by the experimental results obtained. Of all liquids water forms the largest drops in falling through air, because in it the stubborn cohesion prevails to the greatest degree over the joint action of persistent cohesion and weight. In water mercury forms drops greater than in all other liquids, because in water (as a medium) stubborn cohesion and weight prevail to the greatest degree over persistent cohesion.

The case SLL may be inverted if the drop-forming liquid be specifically lighter than the medium liquid. Thus every case of SLL which we have examined in which a liquid, A, drops downwards through a liquid, B, has a counter case in which the liquid B drops upwards* through the liquid A.

In order to measure the size of such ascending drops, the stalagmometer (fig. 7) is modified in form. It is not found possible to cause the dropping liquid to adhere with sufficient completeness and uniformity to a solid sphere immersed in the denser medium, in the cases experimented on. The end of the siphon A was turned upwards, and served as the solid whence the liquid dropped, without the interposition of a sphere or other solid. The measuring-tube D was removed from the neck of the cup C, a stopper being inserted in its place. The cup C was filled with water, and the measuring-tube D, being also filled with water, was inverted into it and supported by the holder H. The modified stalagmometer is seen in Plate V. fig. 10. Care was taken that the end of the siphon A should always be at the same depth beneath the surface of the water in C.

The drop-sizes of the liquids of Table XVI. were first examined by this stalagmometer.

The following Table XVII. shows the number of drops of the various liquids, dropping through water, required to fill the measuring-tube up to the given mark. The measuring-tube employed was different from that used in forming Table XVI. On this account, and because the delivering solid was quite different in shape, and gt only $2''$, no immediate comparison can be made between Tables XVII. and XVI. In Table XVII. correction is made for meniscus.

Table XVII.

$gt = 2''$.

$T = 24°{\cdot}2$ C.

T.	BT_2.	BT.	B_2T.	B.
287	252	229	205	103
287	252	231	205	104
287	251	230	207	104
287	251·7	230	205·7	103·7

We gather from this Table a law quite similar to that deduced from the measurement of the size of the downwards moving drops of water through these same liquids. It is as follows:—

The drop-size of any mixture of two liquids, A *and* B, *dropping up-*

* See Part I. Introduction.

wards through a third liquid C, *is intermediate between the drop-size of* A *through* C *and that of* B *through* C; *and the greater the proportion of* $\frac{A}{B}$ *there is in the mixture, the more nearly does the drop-size of the mixture approach to the drop-size of* $\frac{A}{B}$ *alone.*

It is remarkable that supplementary drops are found in the cases just considered, just as in the case of water dropping through the same liquids. But the supplementary drops of benzol and turpentol through water bear a much smaller ratio to the main drops than do those of water through benzol and turpentol to their main drops. Judging only from the equality in their rate of ascent through the measuring-tube, all these supplementary drops are very exactly of the same size. The supplementary drops were not further examined, but were always collected and measured with the main drops.

Viewed as a means of quantitative chemical analysis, the measurement of the drop-sizes of liquids which drop up through water is yet more sensitive than that of the drop-sizes of water falling downwards through the liquids. Thus, from Table XVII., the least proportional difference of drop-number, caused by an alteration in the proportion of the liquids, is between T and BT_2, where a diminution of 33·33 per cent. in the turpentol and an addition of 33·33 per cent. of benzol causes a difference of 35·3 in the drop-number.

Liquid.....		T.	BT_2.	BT.	B_2T.	B.
Percentage..	B=	0	33·33	50	66·66	100
	T=	100	66·66	50	33·33	0
Difference of percentage..	B	33·33	16·66	16·66	33·33	
	T	33·33	16·66	16·66	33·33	
Difference of drop-number		35·3	21·7	24·3	102·0	

Or this stalagmometer shows the composition of the liquid to within 1 per cent. Further, if the mixture contain less than one-third of benzol, we could determine the proportion, on an average, to within 0·33 per cent.

It may be noticed with regard to SLL that the value of gt is of much less influence upon the drop-size than in the case SLG. It is generally sufficient in the former case that the *average* value of gt should be constant. This is especially the case where the drops are formed from a tube (as the end of a siphon), and not from a convex solid. The reason is obviously that in the former case the thickness of the residual film, upon which we have found the size to depend, is at all rates indefinitely great, while in the latter case it depends upon the rate of supply.

In order to compare the drop-size of A through B with that of B through A under quite similar conditions, the siphon A of fig. 10 was inverted and applied to the cup stalagmometer of fig. 7. The arrangement of the end is seen in fig. 11. In using this form of stalagmometer, the end of the delivery-siphon must be at first wiped dry, so that the water may not

creep back along its outside, and so give rise to an irregular drop-base. Water was made to drop through A, fig. 11, at the same rate, $gt=2''$, and through the same liquids as before, namely T, BT_2, BT, B_2T, B. The same measuring-tube was used as in fig. 10, or Table XVII., and it was filled to the same point. Correction was made for meniscus.

TABLE XVIII.

$gt=2''$.

T=24°·5 C.

	T.	BT_2.	BT.	B_2T.	B.
Number of drops....	256	218	178	162	87
	256	220	177	164	86
	256	..	..	..	86
	..	..	..	..	86
	256	219	177·5	163	86·2

We may now compare Tables XVII. and XVIII., since the conditions of the experiments whence they are got are identical. The drop-sizes are inversely as the drop-numbers. Let us use the symbol X_y to denote the drop-size of the liquid X through medium Y, &c. Comparing, first, the size of a drop of X through medium Y with the size of a drop of Y through medium X, or finding the values of $\frac{X_y}{Y_x}$, we have (putting W for water)

TABLE XIX.

$$\frac{W_B}{B_W}=\frac{103\cdot7}{86\cdot2}=1\cdot203.$$

$$\frac{W_{B_2T}}{B_2T_W}=\frac{205\cdot7}{163}=1\cdot262.$$

$$\frac{W_{BT}}{BT_W}=\frac{230}{177\cdot5}=1\cdot296.$$

$$\frac{W_{BT_2}}{BT_{2_W}}=\frac{251\cdot7}{219}=1\cdot149.$$

$$\frac{W_T}{T_W}=\frac{287}{256}=1\cdot121.$$

Hence in none of these cases is the drop-size of one liquid through another equal to the drop-size of the second through the first. We get the general law, that—

If the liquid X *has a larger drop-size than the liquid* Y *in the liquid* Z, *then the liquid* Z *has a larger drop-size in* X *than it has in* Y. Further,—

If a liquid X *has a larger drop-size than a liquid* Y *in air, then the drop-size of* X *through* Y *is greater than the drop-size of* Y *through* X. Again,—

If the drop-size of X *be greater than the drop-size of* Y, *and the drop-size of* Y *be greater than the drop-size of* Z *in air, then the ratio between the drop-sizes of* X *in any mixture of* Y *and* Z, *and the drop-size of that*

mixture of Y *and* Z *through* X, *is greatest when the ratio between* Y *and* Z *is unity.*

From Tables XVII. and XVIII. we may gather an interesting fact, which illustrates other branches of physics. The drop-numbers of turpentol and benzol through water being relatively 286 and 102, and the drop-numbers of water through benzol and turpentol being relatively 256 and 86·2, we may construct the following Table, in which the theoretical numbers are compared with the experimental ones. The theoretical numbers are got as follows. *Ex.*:—

$$BT_2=\frac{103{\cdot}7+2\times 287}{3}.$$

$$BT=\frac{103{\cdot}7+287}{2}, \text{ \&c.}$$

	T.	BT_2.	BT.	B_2T.	B.
			Theoretical.		
Mixture of benzol and turpentol through water..	287	225·9	195·3	164·8	103·7
			Experimental.		
	287	251·7	230	205·7	103·7
			Theoretical.		
Water through mixtures of benzol and turpentol	256	199·4	171·1	142·8	86·2
			Experimental.		
	256	219	177·5	163	86·2

In all cases, then, the theoretical drop-number is less than the experimental one; or the theoretical drop-size is greater than the experimental one. Mixture impairs cohesion. Generally, when two solids are mixed, the melting-point of the two is lower than the mean of the melting-points of its components; sometimes lower than that of either. When two liquids are mixed, the boiling-point of the mixture (the initial boiling-point) is lower than that of either. The drop-size, which is also a function of the cohesion, we find here in no case to be less than the drop-size of either of the constituents, but in all cases to be less than the theoretical mean. Mixture impairs cohesion.

Further, comparing the drop-sizes of Table XVII. with one another, or all with B_W, we get

TABLE XX.

$$\frac{B_W}{T_W}=\frac{287}{103{\cdot}7}=2{\cdot}767.$$

$$\frac{B_W}{BT_{2_W}}=\frac{251{\cdot}7}{103{\cdot}7}=2{\cdot}427.$$

$$\frac{B_W}{BT_W}=\frac{230}{103{\cdot}7}=2{\cdot}227.$$

$$\frac{B_W}{B_2T_W}=\frac{205{\cdot}7}{103{\cdot}7}=1{\cdot}983.$$

$$\frac{B_W}{B_W}=\frac{103{\cdot}7}{103{\cdot}7}=1{\cdot}000.$$

In like manner, comparing the drop-sizes of Table XX. with one another, or with W_B, we have

TABLE XXI.

$$\frac{W_B}{W_T}=\frac{256}{86{\cdot}2}=2{\cdot}969.$$

$$\frac{W_B}{W_{BT_2}}=\frac{219}{86{\cdot}2}=2{\cdot}541.$$

$$\frac{W_B}{W_{BT}}=\frac{177{\cdot}5}{86{\cdot}2}=2{\cdot}059.$$

$$\frac{W_B}{W_{B_2T}}=\frac{163}{86{\cdot}2}=1{\cdot}890.$$

$$\frac{W_B}{W_B}=\frac{86{\cdot}2}{86{\cdot}2}=1{\cdot}000.$$

Lastly, on comparing these figures with those of Table XX., we get the remarkable law, which it would be difficult to express in words, that

$$\frac{W_B \cdot T_W}{W_T \cdot B_W}=\frac{W_B \cdot BT_{2W}}{W_{BT_2} \cdot B_W}=\frac{W_B \cdot BT_W}{W_{BT} \cdot B_W}=\frac{W_B \cdot B_2T_W}{W_{B_2T} \cdot B_W}=1 \text{ nearly.}$$

The main results with regard to drops may be collected into the following laws:—

SLG.

Law 1.—The drop-size depends upon the rate of dropping. Generally, the quicker the succession of the drops, the greater is the drop; the slower the rate, the more strictly is this the case. This law depends upon the difference, at different rates, of the thickness of the film from which the drop falls.

Law 2.—The drop-size depends upon the nature and quantity of the solid which the dropping liquid holds in solution. If the liquid stands in no chemical relation to the solid, in general the drop-size diminishes as the quantity of solid contained in the liquid increases. The cause of this seems to be that the stubborn cohesion of the liquid is diminished by the solid in solution. Where one or more combinations between the liquid and solid are possible, the drop-size depends upon indeterminate data.

Law 3.—The drop-size depends upon the chemical nature of the dropping liquid, and little or nothing upon its density. Of all liquids examined, water has the greatest, and acetic acid the least drop-size.

Law 4.—The drop-size depends upon the geometric relation between the solid and the liquid. If the solid be spherical, the largest drops fall from the largest spheres. Absolute difference in radii takes a greater effect upon drops formed from smaller than upon those formed from larger spheres. Of circular horizontal planes, within certain limits, the size of the drop varies directly with the size of the plane.

Law 5.—The drop-size depends upon the chemical nature of the solid from which the drop falls, and little or nothing upon its density. Of all

the solids examined, antimony delivers the smallest, and tin the largest drops.

Law 6.—The drop-size depends upon temperature; generally, the higher the temperature the smaller the drop. With water, the effect of a change of temperature of 20° C. about 30° C. is very small.

Law 7.—The nature or tension of the gaseous medium has little or no effect upon drop-size.

SLL.

Law 8.—The drop-size of a liquid which drops under like conditions through various media, does not depend wholly upon the density of the medium and consequent variation in the weight, in the medium, of the dropping liquid.

Law 9.—If there be two liquids, A and B, which drop under like conditions through air, and the drop-size of the one, A, be greater than the drop-size of the other, B, then if a third liquid, C, be made to drop through A and through B, the drop-size of C through A is greater than the drop-size of C through B.

Law 10.—If the drop-size of A through B be greater than the drop-size of A through C, then the drop-size of a fourth liquid, D, through B is also greater than the drop-size of D through C.

Law 11.—If a liquid, A, drop under like conditions in succession through two liquids, B and C, then its drop-size through any mixture of B and C is intermediate between its drop-size through B and its drop-size through C. *Corr.* And the greater the proportion of $\frac{B}{C}$ in the mixture the more nearly does the drop-size of A through the mixture approach to the drop-size of A through $\frac{B}{C}$ alone.

Law 12.—The drop-size of any mixture of two liquids, A and B, dropping through a third liquid, C, is intermediate between the drop-size of A through C and that of B through C; and the greater the proportion of $\frac{A}{B}$ in the mixture, the more nearly does the drop-size of the mixture approach to the drop-size of $\frac{A}{B}$ alone, whether the dropping liquid be heavier or lighter than the liquid medium.

Law 13.—If the liquid X has a larger drop-size than the liquid Y in the liquid Z, then the liquid Z has a larger drop-size in X than it has in Y.

Law 14.—If a liquid, X, has a larger drop-size than a liquid, Y, in air, then the drop-size of X through Y is larger than the drop-size of Y through X.

Law 15.—If the drop-size of X be greater than the drop-size of Y in air, and the drop-size of Y be greater than the drop-size of Z in air, then the ratio between the drop-sizes of X in any mixture of Y and Z, and the drop-size of that mixture of Y and Z through X, is greatest when the ratio between Y and Z is unity.

IV. "On the Chemical Constitution of Reichenbach's Creosote."—Preliminary Notice. By Hugo Müller, Ph.D. Communicated by Warren De la Rue, F.R.S. Received October 1, 1864.

This substance, which has been discovered by Reichenbach amongst the products of destructive distillation of wood, has been repeatedly the subject of chemical investigation, but owing to the difficulty attending its purification, the chemical nature of creosote remained doubtful until 1858, when Hlasiwetz published his elaborate research on this subject.

Up to that time Reichenbach's creosote was frequently confounded with phenol (phenylic alcohol, carbolic acid); and, indeed, the latter had very nearly supplanted the true creosote in its application. Hlasiwetz first prepared the creosote in a chemically pure state, and ascertained its chemical formula to be $C_8 H_{10} O_2$, and showed that this substance, although having some characteristic properties in common with phenol, was a distinct chemical substance, and otherwise in no way related to this body.

At the time of publication of Hlasiwetz's memoir I was myself engaged with the investigation of creosote prepared from wood-tar; and such results as I had then arrived at completely coincided with those obtained by Hlasiwetz.

Having a considerable quantity of pure material at my disposal, I took up this subject again, with the view of obtaining some insight into the chemical constitution of creosote, and I think I am now able to lay before the Society a few results which may serve as a contribution towards the solution of the questions at issue.

I will reserve a full description of my experiments for a future occasion, and confine myself in this communication merely to the description of one reaction, which I consider best calculated to illustrate the results I have obtained.

When pure creosote, boiling constantly (in hydrogen) at 219°C., is brought into contact with concentrated hydriodic acid and heated to boiling, it is acted upon, iodine is set free, and iodide of methyl distils over. As the free iodine interferes with the result of the reaction, I varied the experiment by substituting iodide of phosphorus for hydriodic acid, in the following manner: the creosote is shaken up with a small quantity of water, of which it dissolves a certain portion, then ordinary phosphorus is introduced, and the whole gently warmed. The iodine is now added in small quantities at a time, care being taken that there is always an excess of phosphorus after the iodine has been converted into iodide of phosphorus. If the temperature is now gradually raised to about 95° C., the reaction makes itself manifest by the evolution of vapour of iodide of methyl, which distils over, and which is condensed in an ice refrigerator. As soon as the reaction diminishes, fresh portions of phosphorus and iodine are added, and the experiment so continued until the substance in the retort becomes gradually thicker and thicker, and viscid if allowed to cool.

The distillate collected in the receiver consists mainly of iodide of methyl mixed with some unaltered creosote, from which it is readily liberated by distillation, and agitation with a solution of caustic alkali.

The residue contained in the retort, on being mixed with water, now readily dissolves, with the exception of a small quantity of a heavy brown oil which contains unaltered creosote. The aqueous solution is mixed with a large quantity of water and partly saturated with carbonate of barium, the clear liquid filtered off and precipitated with acetate of lead, the white precipitate well washed and decomposed with sulphuretted hydrogen. The sulphide of lead having been filtered off, the aqueous solution is now carefully evaporated at a low temperature, when a thick heavy liquid is obtained, which in its reactions so closely resembles pyrocatechine or oxyphenic acid, that one would be inclined to consider it identical with this substance if it were not for the apparent impossibility of obtaining it in a crystalline form.

I am still engaged with the determination of the composition of the latter substance; but, from its chemical nature, so far as I have made myself acquainted with it, and from other considerations, I think it more than probable that this substance bears the closest analogy to oxyphenic acid ($C_6 H_6 O_2$), and is in all probability its homologue.

The described decomposition of creosote may be expressed in the following way:—

$$\underbrace{C_8 H_{10} O_2}_{\text{Creosote.}} + HI = \underbrace{CH_3 I}_{\text{Iodide of methyl.}} + C_7 H_8 O_2.$$

According to which creosote may be considered as methylated oxytolylic acid, or oxykressylic acid.

This view gains in probability if we consider the general properties of creosote, and the fact that a lower homologue of creosote, together with free oxyphenic acid, exists amongst the products of distillation of wood. Hlasiwetz has moreover shown that guaiacol is identical with this lower homologue of creosote, which it resembles in every respect.

If the constitution of creosote ($C_8 H_{10} O_2$) turns out to be as stated above, guaiacol ($C_7 H_8 O_2$) may be regarded as methylated oxyphenic acid, and we may therefore expect to obtain by the action of hydriodic acid upon this substance, iodide of methyl and oxyphenic acid.

I am about to carry out the latter experiment.

V. "Researches on the Colouring Matters derived from Coal-tar.—No. IV. Phenyltolylamine." By A. W. Hofmann, LL.D., F.R.S. Received October 19, 1864.

The discovery of diphenylamine among the products of decomposition furnished by the destructive distillation of aniline-blue (triphenylic ros-

aniline) which I have lately communicated to the Royal Society (Proceedings, June 16, 1864), naturally suggested the investigation of analogously constituted bodies in a similar direction. My attention has in the first place been directed to the study of a compound which, from its mode of formation, ought to be designated as toluidine-blue.

When a salt of rosaniline (the acetate for instance) is heated with double its weight of toluidine, phenomena present themselves which are similar to those observed in the analogous experiment with aniline. In the course of a few hours the rosaniline passes through all the different shades of violet, and is ultimately converted into a dark lustrous mass, which dissolves in alcohol with a deep indigo-blue colour. This substance is the acetate of tritolylrosaniline. By treatment with alcoholic ammonia, and subsequently addition of water to the solution, the free base is easily obtained, from which the several salts may be prepared by the usual processes. I have examined only one of these salts, viz. the hydrochlorate. Repeatedly crystallized from boiling alcohol, this salt is obtained in small blue crystals insoluble in water, which at 100° C. contain

$$C_{41}H_{38}N_3Cl = C_{20}\left.\begin{matrix}H_{16}\\(C_7H_7)_3\end{matrix}\right\}N_3, HCl.$$

The formation of toluidine-blue is thus seen to be perfectly analogous to that of aniline-blue.

$$C_{20}H_{19}N_3, \left.\begin{matrix}C_2H_3O\\H\end{matrix}\right\}O + 3\left[\left.\begin{matrix}C_7H_7\\H_2\end{matrix}\right\}N\right] = 3H_3N + C_{20}\left.\begin{matrix}H_{16}\\(C_7H_7)_3\end{matrix}\right\}N_3, \left.\begin{matrix}C_2H_3O\\H\end{matrix}\right\}O$$

I have not examined in detail the properties of this new series of colouring matters. Generally speaking they are more soluble than the corresponding phenyl-compounds, and therefore less easily obtained in a state of purity.

When one of these salts (the acetate for instance) is submitted to dry distillation, water and acetic acid are evolved in the first place; then follow oily products, which, as the temperature rises, become more and more viscid and ultimately solidify into crystalline masses, ammonia being abundantly evolved during the latter stages of the process. Unless the operation has been carried out on rather a large scale, a comparatively small amount of a light porous charcoal remains in the retort. The oily distillate contains several bases. Those boiling at a low temperature are almost exclusively aniline and a little toluidine. The principal portion of the products boiling at a higher temperature is a beautifully crystallized base which is easily purified. By pouring cold spirit upon the interwoven crystals, a brown mother liquor containing other bases is readily separated; the residuary substance has only to be crystallized from boiling alcohol in order to procure a compound of perfect purity.

The chemical deportment of the new substance is very similar to that of diphenylamine. Like the latter it unites with acids, forming salts of very

little stability, splitting up into their constituents under the influence of water, of heat, and even by mere exposure *in vacuo*. In contact with nitric acid the crystals at once assume a blue coloration, with an admixture of green, but nevertheless so similar to the analogous colour-reaction of diphenylamine, that by this test alone the two substances could not possibly be distinguished. The two bases differ, however, essentially in their solubility, their fusing- and boiling-points, and lastly in their composition. The new base is far less soluble in alcohol than diphenylamine; it fuses only at 87° C., while the fusing-point of diphenylamine is 45° C.; the boiling-point of the new base is 334·5° C. (corr.), at which temperature it distils without any decomposition, while diphenylamine boils at 310° (corr.).

The results of analysis lead to the formula

$$C_{13}H_{13}N.$$

A hydrochlorate, crystallizing in little plates, and obtained by the addition of concentrated hydrochloric acid to an alcoholic solution of the base, when dried over lime, was found to contain

$$C_{13}H_{13}N, HCl.$$

Formation and chemical deportment characterize the new base as the mixed secondary monamine of the phenyl- and tolyl-series, as phenyltolylamine*,

$$C_{13}H_{13}N = \left.\begin{matrix} C_6H_5 \\ C_7H_7 \\ H \end{matrix}\right\} N.$$

In consequence of the simultaneous existence in the molecule of the new base of the radicals phenyl and tolyl, its deportment under the influence of dehydrogenating agents became of considerable interest; and indeed, having recognized the nature of the compound, my first experiment consisted in submitting it to the action of corrosive sublimate. Both substances unite to form a dark brown mass which, after having been heated, dissolves in alcohol with a magnificent violet-blue colour. The compound thus produced exhibits the behaviour of the colouring matters generated from rosaniline by substitution. Owing to the peculiar properties of this class of substances, it would be difficult to prepare the new compound in sufficient

* It deserves to be noticed that the percentages of carbon in diphenylamine, phenyltolylamine, and ditolylamine nearly coincide.

	Diphenylamine.	Phenyltolylamine.	Ditolylamine.
Carbon	85·21	85·24	85·28
Hydrogen	6·51	7·10	7·61

The percentages of hydrogen, however, unequivocally distinguish the three compounds. The analysis of phenyltolylamine furnished the following numbers:

	I.	II.
Carbon.....	85·10	85·11
Hydrogen ...	7·30	7·33

quantity for a detailed examination; but, judging from its mode of generation, it will probably be found to be tolyldiphenylrosaniline,

$$\left.\begin{matrix} C_{20}\,H_{16} \\ (C_6\,H_5)_2 \\ C_7\,H_7 \end{matrix}\right\} N_3, H_2\,O.$$

A few additional experiments performed with phenyltolylamine may still briefly be mentioned.

Chloride of benzoyl attacks this substance, especially on application of heat. The product of the reaction remains liquid for a long time, but, when appropriately treated with water, alkali, and spirit, it ultimately solidifies, and separates from boiling alcohol in well-formed crystals,

$$C_{20}\,H_{17}\,NO = \left.\begin{matrix} C_6\,H_5 \\ C_7\,H_7 \\ C_7\,H_5\,O \end{matrix}\right\} N,$$

which are more soluble than the corresponding diphenyl-compound.

The benzoyl derivative is very readily converted into nitro substitutes. In contact with ordinary concentrated nitric acid the crystals are at once liquified, and ultimately entirely dissolve. Addition of water to this solution precipitates a yellow crystalline dinitro-compound,

$$C_{20}\,H_{15}\,N_3\,O_5 = \left.\begin{matrix} C_6\,H_4\,(NO_2) \\ C_7\,H_6\,(NO_2) \\ C_7\,H_5\,O \end{matrix}\right\} N,$$

which is deposited from boiling alcohol in reddish-yellow needles. Perfectly similar treatment converts the diphenyl body into a mononitro substitute. Cold fuming nitric acid, which when acting upon the diphenyl body gives rise to the formation of a dinitro-substitute, transforms the benzoylated phenyltolylamine into a nitro-derivative containing, according to an approximate analysis, not less than 5 atoms of NO_2.

Dinitro-phenyltolylbenzoylamide dissolves in alcoholic soda with a feebly crimson colour. Ebullition of the solution eliminates the benzoyl atom in the form of benzoate, and on cooling small yellowish red crystals,

$$C_{13}\,H_{11}\,N_3\,O_4 = \left.\begin{matrix} C_6\,H_4\,(NO_2) \\ C_7\,H_6\,(NO_2) \\ H \end{matrix}\right\} N,$$

are deposited, which are easily purified by crystallization from alcohol.

Lastly, when treated with reducing agents, the dinitronated phenyltolyl-benzoyl compound is converted into fine white needles of a new base, to which I hope to return as soon as I shall have procured a somewhat larger supply of phenyltolylamine.

It scarcely requires to be mentioned that it is not necessary to prepare the pure toluidine-blue for the purpose of obtaining phenyltolylamine. It suffices to maintain for some hours a solution of ordinary but dried acetate of rosaniline, in its double weight of toluidine, in a flask provided with an upright condensing tube, at a boiling temperature, and to submit the violet-

blue mass produced to destructive distillation over a naked burner. The distillate is treated with hydrochloric acid, and subsequently with water, when aniline and toluidine, together with several other basic substances accompanying the phenyltolylamine, remain as hydrochlorates in solution. The oily layer which separates generally solidifies, or may be purified by rectification. The resulting crystals are crystallized from alcohol.

The same method is also adapted for the preparation of diphenylamine, aniline being substituted for toluidine.

If I have bestowed upon diphenylamine and phenyltolylamine rather more attention than these substances at the first glance appear to claim, I have done so in the hope of gaining additional data for the investigation of the remarkable colouring matters from which these bases are derived. Both constitution and mode of formation of these colouring matters are still involved in darkness. Theory, as it often happens, has not kept pace with practice. The anticipation I expressed in a former note, that the study of the behaviour of the colouring matters under the influence of chemical agents might disclose their true nature, has only very partially been realized. Up to the present moment, chemists have not succeeded in giving a satisfactory account either of the atomic construction of these compounds, or of the mechanism of their formation; and it would therefore scarcely be worth while to return to this question before its definite solution, unless the publication of erroneous statements by M. Schiff had threatened to divert the researches of chemists from this subject.

According to M. Schiff *, the transformation of aniline into aniline-red by means of stannic chloride is represented by the equation

$$20C_6H_7N + 10\,Sn\,Cl_2 = 3(C_{20}H_{19}N_3, HCl) + 6(C_6H_7N, HCl) + H_4NCl + 10\,Sn\,Cl + 4\,C_6H_7N.$$

The formation by means of mercuric nitrate† by the equation

$$20C_6H_7N + 20\,HgNO_3 = 3(C_{20}H_{19}N_3, HNO_3) + 6(C_6H_7N, HNO_3) + H_4N, NO_3 + 10Hg_2NO_3 + 4C_6H_7N.$$

The latter process is accomplished at as low a temperature as 80° C., and, according to M. Schiff, is so elegant that he was enabled to make quantitative experiments. "Within a few hundredths," he says, "we have obtained the requisite quantities of the sought-for materials."

M. Schiff's equations are not conspicuous for elegance and simplicity, but they are absolutely inadmissible for other reasons. These equations utterly ignore the very essence of the process. I have pointed out, some time ago, that the formation of rosaniline involves the presence of *both aniline and toluidine*. Pure aniline furnishes no rosaniline, nor can this body be procured from pure toluidine. This fact I have since further established by many varied experiments, both on the small and on the large scale. The formation of rosaniline thus becomes the means of ascertaining rapidly the presence of toluidine. The amount of the latter

* Compt. Rend. vol. lvi. p. 271. † Ibid. p. 545.

in crude aniline* may become so minute that its presence can no longer be traced by distillation, or by conversion into oxalates. It may be recognized, however, with the utmost facility by submitting the mixture to the action of either corrosive sublimate or arsenic acid; on application of a gentle heat the crimson colour is immediately produced.

In the equations proposed by M. Schiff there figures, moreover, ammonia as an essential term. The existence of ammoniacal salts in the crude rosaniline was pointed out some time ago by Prof. Bolley. But this ammonia (which, as I have satisfied myself, is scarcely ever absent) is, according to my opinion, not an essential product of the reaction that gives rise to the formation of aniline-red. I have established by special and careful experiments that appropriate treatment of a mixture of aniline and toluidine with chloride of mercury at a moderate temperature is capable of producing very considerable quantities of rosaniline without elimination of more than a trace of ammonia. The ammonia generally observed belongs to a different phase of the reaction, being more especially due to the almost invariable production of a small quantity of aniline-blue.

If we wished, even now, to represent in formulæ the relation between rosaniline and the substances which give rise to its formation, the equation

$$C_6 H_7 N + 2C_7 H_9 N = C_{20} H_{19} N_3 + 6H$$

might be looked upon as an expression closely approaching truth. The hydrogen figuring in this equation is eliminated in the form of water, hydrochloric, hydrobromic, hydriodic acids, &c.

But even this equation gives no account of the mechanism of this remarkable process; indeed we cannot hope for the solution of this chemical problem before we shall have succeeded in splitting up the molecule of rosaniline into the atomic groups which enter into its composition. It is true its transformation into aniline- or toluidine-blue, as well as into the several violets which are generated by the substitution of alcohol radicals, prove even now that the rosaniline-molecule still contains three atoms of typical hydrogen, and hence that the complex atom $C_{20} H_{16}$ functions in this triamine with the value of six atoms of hydrogen; but this indeed is the limit of the experimental evidence as yet obtained.

With regard to the number and nature of the simpler radicals into which the carbon and hydrogen atoms of the complex atom $C_{20} H_{16}$ are grouped, we can only speculate. Derived from the radicals *phenyl*, $C_6 H_5$, and *tolyl*, $C_7 H_7$, under the influence of dehydrogenating agents, this complex atom

* Aniline obtained by distillation with potash from certain varieties of indigo, is apt when treated with corrosive sublimate, to furnish traces of rosaniline. I infer from this result that aniline thus produced contains a small proportion of toluidine. The formation of this substance from indigo would be as readily intelligible as the conversion under certain conditions of indigo into salicylic acid, a fact established by Cahours's observations.

Aniline prepared from crystallized isatin does not yield a trace of rosaniline.

may possibly contain the bivalent radicals *phenylene*, C_6H_4, and *tolylene*, C_7H_6,

$$(C_{20}H_{16})'' = \begin{matrix}(C_6H_4)''\\(C_7H_6)''\\(C_7H_6)''\end{matrix},$$

when the molecular construction of the three colouring matters might be represented by the formulæ

Aniline-red.	Aniline-blue.	Aniline-violet.
$\left.\begin{matrix}(C_6H_4)''\\(C_7H_6)_2''\\H_3\end{matrix}\right\}N_3,H_2O$	$\left.\begin{matrix}(C_6H_4)''\\(C_7H_6)_2''\\(C_6H_5)_3\end{matrix}\right\}N_3,H_2O$	$\left.\begin{matrix}(C_6H_4)''\\(C_7H_6)_2''\\(C_2H_5)_3\end{matrix}\right\}N_3,H_2O.$

We must not, however, forget that this is simply an hypothesis, and that the elements in the complex atom $C_{20}H_{16}$ may be associated in a great variety of other groups. An interesting observation quite recently made by Dr. Hugo Müller, and communicated to me by my friend while these pages are passing through the press, may possibly assist in further elucidating the nature of this class of bodies. Dr. Müller has found that rosaniline and its coloured derivatives are instantaneously decolorized by cyanide of potassium, a series of splendidly crystallized, perfectly colourless bases being produced. The composition of these bodies, which will probably be found analogous to a substance similarly obtained from harmaline by Fritzsche, remains to be established.

November 17, 1864.

Major-General SABINE, President, in the Chair.

In accordance with the Statutes, notice of the ensuing Anniversary Meeting for the election of Council and Officers was given from the Chair.

Mr. Gassiot, Dr. J. E. Gray, Dr. Hirst, Mr. Lubbock, and Dr. Odling, having been nominated by the President, were elected by ballot Auditors of the Treasurer's accounts on the part of the Society.

Among the presents announced was a Photograph of the Moon, from Mr. Warren De la Rue, respecting which an extract of a letter from the Donor was read as follows:—"Except to remove white and black spots, the photograph is untouched. The size of the original negative is about one inch, and from this was taken, in the first instance, a positive on glass nine inches in diameter. The glass positive was used for the production of four negatives, each containing a quarter of the disk. The proper distance for viewing the picture is about six feet, or two diameters."

The following communications were read:—

I. "Comparison of Mr. De la Rue's and Padre Secchi's Eclipse-Photographs." By Warren De la Rue, F.R.S. (See page 442.)

II. "On Drops."—Parts I. and II. By Frederick Guthrie, Esq. Communicated by Professor Stokes, Sec. R.S. (See pages 444 and 457.)

III. "On the Chemical Constitution of Reichenbach's Creosote." By Hugo Müller, Ph.D. Communicated by Warren De la Rue, F.R.S. (See page 484.)

IV. "Researches on the Colouring Matters derived from Coal-tar.—No. IV. Phenyltolylamine." By A. W. Hofmann, LL.D., F.R.S. (See page 485.)

V. "On the Spectra of some of the Nebulæ." By W. Huggins, Esq., F.R.A.S.;—a Supplement to the Paper "On the Spectra of some of the Fixed Stars," by W. Huggins, Esq., and W. A. Miller, M.D., Treas. and V.P.R.S. Communicated by the Treasurer. Received September 8, 1864.

(Abstract.)

The author commences by showing the importance of bringing analysis by the prism to bear upon the remarkable class of bodies known as nebulæ, especially since the results obtained by the largest telescopes hitherto constructed appear to show that increase of optical power alone would probably fail to determine the question whether all the nebulæ are clusters of stars too remote to be separately visible.

The little indication of resolvability, the absence of central condensation, the greenish-blue colour, and the intrinsic brightness characterizing many of the nebulæ classed by Sir W. Herschel as planetary, induced the author to select chiefly nebulæ of this class for prismatic observation.

The apparatus employed is that of which a description is given in the paper, "On the Spectra of some of the Fixed Stars," by the author and Dr. W. A. Miller, to which this is a supplement.

No. 4373*, 37 H. IV. Draconis. A bright planetary nebula, with a very small nucleus. The light from this nebula is not composed of light of different refrangibilities, and does not therefore form a continuous spectrum. It consists of light of three definite refrangibilities only, and, after passing through the prisms, remains concentrated in three bright lines. The strongest of these occupies a position in the spectrum about mid-

* These numbers refer to the last catalogue of Sir J. F. W. Herschel, Phil. Trans. Part I. 1864, pp. 1–138.

way between *b* and *F*, and was found, by the method of simultaneous observation, to be coincident with the brightest of the lines of nitrogen.

A little more refrangible, a second line is seen. At about three times the distance of the second line, a third, very faint line occurs; this coincides in position with Fraunhofer's *F*, and one of the lines of hydrogen. Besides the three bright lines, an exceedingly faint continuous spectrum of the central bright point was perceived.

The planetary nebula, 4390, Σ 6, Tauri Poniatawskii; 4514, 73 H. IV. Cygni; 4510, 51 H. IV. Sagittarii; 4628, 1 H. IV. Aquarii; 4964, 18 H. IV., the annular nebula in Lyra 4447, 57 M., and the Dumb-bell in Vulpecula 4532, 27 M., gave spectra identical with the spectrum of 37 H. IV., except that in the case of some of these the strongest only of the three bright lines was seen.

It is obvious that these nebulæ can no longer be regarded as clusters of stars. In place of an incandescent solid or liquid body transmitting light of all refrangibilities through an atmosphere which intercepts by absorption some of them, such as our sun and the fixed stars appear to be, these nebulæ, or at least their photosurfaces, must be regarded as enormous masses of luminous gas or vapour.

On this supposition the absence of central condensation admits of explanation; for even if the whole mass of the gas is luminous, the light emitted by the portion of gas beyond the surface visible to us would be in great measure absorbed by the portion of gas through which it would have to pass, and for this reason there would be presented a luminous surface only. The small brilliancy of the nebulæ, notwithstanding the considerable angle which in most cases they subtend, is in accordance with the very inferior splendour of glowing gas as compared with incandescent solid or liquid matter.

The extreme simplicity of constitution which the three bright lines suggest, whether or not we regard them as indicating the presence of nitrogen, hydrogen, and a substance unknown, is opposed to the opinion that they are clusters of stars.

The following nebulæ and resolvable clusters gave a continuous spectrum:—4294, 92 M. Herculis; 4244, 50 H. IV. Herculis; 116, 31 M., the Great Nebula in Andromeda; 117, 32 M. Andromedæ; 428, 55, Andromedæ; 826, 26 H. IV. Eridani.

In the spectrum of 31 M., the nebulæ in Andromeda, and in that of the companion nebula, 32 M., the red and part of the orange are wanting.

VI. "On the Composition of Sea Water in different Parts of the Ocean." By Dr. George Forchhammer, Professor in the University of Copenhagen. Communicated by the President. Received July 28, 1864.

This Paper was in part read.

November 24, 1864.

Major-General SABINE, President, in the Chair.

In accordance with the Statutes, notice was given from the Chair of the ensuing Anniversary Meeting, and the list of Council and Officers proposed for election was read as follows:—

President.—Major-General Edward Sabine, R.A., D.C.L., LL.D.

Treasurer.—William Allen Miller, M.D., LL.D.

Secretaries.— { William Sharpey, M.D., LL.D.
George Gabriel Stokes, Esq., M.A., D.C.L.

Foreign Secretary.—Professor William Hallows Miller, M.A.

Other Members of the Council.—Professor John Couch Adams, M.A.; James Alderson, M.D.; George Busk, Esq., Sec. L.S.; Col. Sir George Everest, C.B.; Hugh Falconer, M.A., M.D.; John Peter Gassiot, Esq.; John Edward Gray, Ph.D.; Thomas Archer Hirst, Ph.D.; Sir Henry Holland, Bart., M.D., D.C.L.; Henry Bence Jones, M.A., M.D.; Sir Roderick Impey Murchison, K.C.B.; William Odling, M.B.; Professor William Pole, C.E.; Rev. Bartholomew Price, M.A.; Sir John Rennie, Knt.; The Lord Stanley.

The reading of Professor Forchhammmer's Paper was resumed and concluded.

The present memoir contains the full exposition of the author's researches, of which he gave a preliminary notice on the 22nd of May, 1862. An abstract is given under that date, vol. xii. p. 129.

November 30, 1864.

ANNIVERSARY MEETING.

Major-General SABINE, President, in the Chair.

Dr. Gray, on the part of the Auditors of the Treasurer's Accounts, reported that the total receipts during the past year, including a balance of £658 0*s.* 1*d.* carried from the preceding year, amount to £4926 1*s.* 4*d.*, and that the total expenditure in the same period amounts to £4242 7*s.* 3*d.*, leaving a balance at the Bank of £679 6*s.* 7*d.*, and in the hands of the Treasurer of £4 7*s.* 6*d.*

On the motion of Mr. Currey, the thanks of the Society were voted to the Treasurer and Auditors.

The Secretary read the following Lists:—

Fellows deceased since the last Anniversary.

On the Home List.

William Allen, Capt. R.N.
William Bingham Baring, Lord Ashburton, D.C.L.
The Ven. Charles Parr Burney, D.D., Archdeacon of Colchester.
Samuel Cartwright, Esq.
The Rev. William Cureton, D.D., Canon of Westminster.
Admiral the Hon. George Elliot.
The Rev. Josiah Forshall, M.A.
James William Freshfield, Esq.
Joseph Henry Green, Esq., D.C.L.
Hudson Gurney, Esq.
Leonard Horner, Esq.
Luke Howard, Esq.
William Chadwell Mylne, Esq.
The Rev. Frederick Nolan, LL.D.
Joseph Ellison Portlock, Major-General, R.E.
Archibald Robertson, M.D.
Robert Dundas Thomson, M.D.
Robert Torrens, Lieut.-Colonel.

On the Foreign List.

Baron Giovanni Plana. | Heinrich Rose.
Friedrich Georg Wilhelm Struve.

Fellows elected since the last Anniversary.

On the Home List.

Sir Henry Barkly, K.C.B.
William Brinton, M.D.
T. Spencer Cobbold, M.D.
Alexander John Ellis, Esq.
John Evans, Esq.
William Henry Flower, Esq.
Thomas Grubb, Esq.
Sir John Charles Dalrymple Hay, Bart.
William Jenner, M.D.
Sir Charles Locock, Bart., M.D.
William Sanders, Esq.
Col. William James Smythe, R.A.
Lieut.-Col. Alexander Strange.
Robert Warington, Esq.
Nicholas Wood, Esq.

On the Foreign List.

Claude Bernard. | Jean Bernard Léon Foucault.
Adolph Charles Wurtz.

The first Report of the Scientific Relief Committee was read to the Meeting by the Treasurer, as follows:—

"In presenting this, the first Report of the proceedings of the Royal Society Relief Fund Committee, it may be advisable very briefly to explain the circumstances under which the Fund originated, as well as the distribution of the Income arising therefrom.

"On the 26th of May, 1859, a statement was presented to the Council,

intimating a desire on the part of several of the Fellows of the Royal Society, to promote the establishment of a permanent fund for the aid of such scientific men, or their families, as may from time to time require and deserve assistance. A scheme having been suggested, it was after some modification approved, and adopted by the Council of the Royal Society, and copies thereof were forwarded to the Fellows of the Chartered Societies.

"It formed no part of the scheme to attempt the grant of annuities; it was rather intended to afford *prompt* relief of the immediate wants of those upon whom sudden affliction had fallen; although at the same time, it in no way debarred a continuation of such relief being given should the funds admit thereof.

"The Committee, as originally named by Council Minutes of 22nd December, 1859, consisted of Mr. Busk, Dr. W. A. Miller, Mr. Wheatstone, Dr. Tyndall, and Mr. Gassiot. Subsequently Dr. Carpenter, Mr. Sylvester, and Mr. Huxley were appointed to succeed those Members who retired by rotation; and this year Dr. Miller and Mr. Gassiot will also retire; it being a regulation that no member can remain for a longer period than five consecutive years on the Committee.

"The first Meeting took place on the 19th March 1860, Mr. Wheatstone in the Chair. At this time £3204 14*s.* had been received, and invested in £3351 7*s.* 6*d.* New 3 per Cents. In 1860 there was only one applicant for relief; a sum was voted, but in consequence of the applicant's sudden decease, the cheque was subsequently returned to the Treasurer and cancelled.

"In 1861 there were six applicants, to five of whom relief was afforded.

"In 1862 there were four applicants, of whom three were relieved.

"In 1863 there were five applicants, of whom four were relieved.

"In 1864 there were two applicants—both were relieved.

"The total amount thus expended has been £460; and there still remains £233 1*s.* 11*d.* applicable for relief, exclusive of £17 2*s.*, subscriptions to be invested.

"The amount of the Fund, as invested, consists of £5300 New 3 per cent. Consols, every shilling (with the above exception of £17 2*s.*) which has been received having been invested, without any deduction for expenses; the economical principle on which this Fund was originally founded has been thus strictly carried out. The Council of the Royal Society grants the use of a room, as well as the little stationery that is from time to time required, and permits the Assistant-Secretary to summon a meeting of the Members as soon as any application for relief is received. One of the members of the Committee undertakes the duties of a Secretary, and in this simple manner *all* expense is avoided, while the relief, if granted, is immediate. The amount at disposal for distribution is necessarily limited to the accumulation of interest, the gross amount of subscriptions being invested.

"The peculiar and distinctive character of the Royal Society Relief Fund is, that although relief has invariably been granted with the greatest promptitude, the claims have nevertheless been most carefully examined by those whose pursuits in life enable them to form the most correct conclusion as to the character of the claimant. The Committee can most conscientiously state that, in no instance, has any applicant having the slightest claim on the Fund been refused assistance. Sometimes the payments to recipients have been the first information they have received of such a fund being in existence; the Committee may have on some occasions regretted that it had not a larger sum at command, but the relief, whatever its amount, has always been received with gratitude for the substantial aid it afforded, and with pleasure as a recognition on the part of the Council of the Royal Society of good service rendered to science.

"The Committee is naturally desirous to retain sufficient funds in hand to meet any sudden emergency that may arise; at this time there appears a balance somewhat larger than usual, but this arises from there having been only two applicants this year instead of five, as in 1863. It is very probable that if the *existence* of such a fund were more generally known, especially among the members of the Chartered Scientific Societies, further subscriptions would be tendered."

(Signed) W. A. Miller.
W. B. Carpenter.
J. J. Sylvester.
T. H. Huxley.
J. P. Gassiot.

Burlington House,
24th November, 1864.

On the motion of Mr. Busk, seconded by Dr. Gray, the thanks of the Society were voted to Mr. Gassiot for his services in originating and conducting the Scientific Relief Fund.

The President then addressed the Society as follows:—

Gentlemen,

When I had the honour of addressing you at the Anniversary Meeting in 1862, I informed you of the steps which had been taken by the Council towards the formation of an Index to the Scientific Periodical Literature of the Nineteenth Century, or, more precisely, a Catalogue of the Titles of Scientific Memoirs contained in the Scientific Periodicals, in all languages, from the commencement of the present century to the year 1860. The preparation of this work has progressed steadily; and, so far as regards a Manuscript Catalogue to be kept in our own library for the use of the Fellows generally, it may be considered to be very nearly complete. The limit in point of time to which the entries of the Titles should be restricted has been extended to 1863 inclusive. A circular which has been addressed by the Foreign Secretary to seventy

Academies with which the Royal Society is more or less in communication, as well as to several other institutions and individuals, to ascertain whether they possessed any works suitable for cataloguing which were not in our library (enclosing at the same time a list of the works already indexed), has been largely replied to; and we have thus been enabled to considerably extend our Catalogue. The number of titles which it at present includes exceeds 180,000, entered in manuscript volumes, available for reference in our library. Our communications have been responded to in several instances with offers of assistance, and in general with strong expressions of interest in the progress of the work, and of anticipated advantage from its publication.

After a full consideration of the different modes in which such a publication might be effected, the Council decided that the manuscript should in the first instance be offered to Her Majesty's Government, to be printed at the public charge, at Her Majesty's Stationery Office or otherwise, as might be deemed expedient; and that a certain number of copies should be presented to Scientific Institutions at home and abroad, in the name of the British Government and of the Royal Society; the remainder of the impression being offered for sale at the cost of paper and printing only, and the proceeds applied towards the discharge of the expense incurred in the printing, no pecuniary return being looked for on the part of the Society.

The Council have availed themselves of the good offices of a valued Fellow of the Society, Sir Henry Holland, who had been a member of the Council whilst the Catalogue was in progress, to bring the subject of the publication under the consideration of Her Majesty's Government, whose favourable reply has been this morning received.

From the best judgment which the Council can form, there is reason to believe that by the time the Government may be ready to commence the printing, the Catalogue will have been completed with all the titles that can be obtained. The manuscript, as at present prepared, contains the Titles in the order in which they stand in each of the periodical works indexed; but for publication it will be arranged according to authors' names, and be followed by an Alphabetical Index according to subjects. The expense hitherto incurred by the Society in its preparation amounts to about £1400; and possibly £300 or £400 more will be required in the preparation of the Index of Subjects, and in the general revision of the press.

It is with great satisfaction that we see the value and importance of the Philosophical Transactions recognized increasingly from year to year both at home and abroad. Besides the contributions from our own members and countrymen in the past year, to some of which I may briefly advert in the sequel, we have received a very valuable paper from Professor George Forchhammer, of Copenhagen, to a brief

notice of which, as being presented to us from abroad, I will give the first place.

This communication forms a valuable contribution to a great subject —the history of the Sea. It contains a full and compendious inquiry into the constituents of the water of the ocean, divided into seventeen geographical regions, each of which is studied separately from samples taken both at the surface and at various depths. An accurate view is thus gained for the first time of the sea as a whole, and conclusions of great generality are obtained. The minute analytical processes followed in several hundred analyses were so conducted as, in the opinion of competent judges, to inspire entire confidence. They confirm the presence in sea-water of the twenty-five elements already reported by other chemists, and add two others, boron and aluminium, to the number. But it is chiefly by the application of the data thus obtained to the elucidation of various geographical problems of great and general interest that we are led to recognize the full importance of this memoir. I may permit myself to notice one or two of the most remarkable of the conclusions established by it.

In the Atlantic the saline ingredients in the sea-water (the samples being taken at proper distances from the land) decrease with increasing depth. This is found to hold good even to extreme depths. The existence of a Polar current in the depths of the Atlantic is hence inferred, since it is a well-established fact that the equatorial seas are richer, and the polar seas poorer, in saline ingredients.

The large amount of saline contents found by analysis of the water of the well-known current flowing from north-east to south-west, between Iceland and the east coast of Greenland, shows it to be, not as heretofore supposed, a polar current, but one of equatorial origin. The inference is, that it is a returning branch of the great Gulf-stream which we have recently had reason to recognize as extending to the shores of Nova Zembla and to the north coasts of Spitzbergen, carrying to Nova Zembla the floats of the Norwegian fishermen, and to Spitzbergen the same floats, mingled with wood from Siberia. May it not be possible that the "iceless sea, teeming with animal life," described by the adventurous American explorer Dr. Kane as viewed from the promontory which formed the northern limit of his research, is, as he himself surmised, but an extension of the same equatorial stream which produces correspondingly abnormal effects at Spitzbergen, as well as at every other point to which its course has been traced?

When physical researches shall be resumed within the circle which surrounds the Pole, this perhaps will be one of the earliest problems to receive solution—a solution rendered now so simple by the method of inquiry which Professor Forchhammer has made known to us.

In Astronomy the most important results in the year are Sir John

Herschel's Catalogue of *Nebulæ*, which forms Part I. of the Transactions for 1864; and Mr. Huggins's application of the spectroscope to study those mysterious forms, and to examine the chemical constitution of stars and planets. The nebulæ are so widely different from the other heavenly bodies, that ever since their first discovery they have been objects of wonder and speculation. For a long time only three or four of them were known, till about a century ago Messier added about 100 more. His labours, however, were completely eclipsed by the elder Herschel, who gave unprecedented power to the telescope, and used it with skill and sagacity which have never been surpassed. He was rewarded by discovering 2500 of these objects, and finding in them varieties of figure of the most unexpected character.

After such a harvest it might have been thought that little was left to be gleaned in this field; but his accomplished son was not less successful in the northern hemisphere; and when his zeal and energy led him to the Cape of Good Hope, he found in that almost unexplored sky the opportunity of even surpassing his father, and connecting more than 1000 other nebulæ with his own name. Since then the great Achromatics of Poulkova and Harvard, and the gigantic reflectors of Rosse and Lassell have revealed many strange arrangements in them, some of which are scarcely reconcileable with ordinary dynamics; and it is evident that the study of their nature is likely to occupy much of the attention of astronomers for years to come.

At such a crisis no more acceptable gift could be made to Nebular Astronomy than a work which should embody in a practical manual the sum of our actual knowledge of this department, and do for it what the Catalogue of the British Association and others are performing for the stellar branch. And none was so fit for this task as Sir John Herschel himself. Besides his own unequalled familiarity with the subject, and thorough cognizance of the labours of others, he had an advantage in his acquaintance with his father's manuscripts and the labours of his aunt, a person not less an honour to her name, such as none other could possess. And yet more, he had an interest in this pursuit such as can be expected in very few, but without which the labour that must have been expended on this catalogue would have been insupportable. It contains, arranged in Right Ascension, the places of 5079 nebulæ for 1860 with the precessions so chosen that they will be available for at least sixty years to come. Each nebula has its discoverer indicated, the number of observations, and a description of its form and character, composed of a very few letters, but to any one who will learn their symbolic meaning, of marvellous fullness and precision. Copious notes and references, and an introduction which gives full historical information, complete all that can be required by observers.

And that observers will be numerous as soon as Mr. Huggins's researches become generally known, must be anticipated. At our last

Anniversary I noticed briefly that he and Dr. Miller were examining the spectra of the fixed stars; their results have appeared in our Transactions for this year, and can scarcely be estimated too highly. Before them Fraunhofer, and recently Donati, Secchi and some others had examined a few stellar spectra with more or less success; but with inferior apparatus, and far less extensively. Their superiority is mainly due to their referring the spectral lines to no mere instrumental scale (which, from the inevitable variations of the spectroscope-prisms, must be liable to error), but to standard spectra of known elements which are formed in juxtaposition with that to be examined, and to which its lines can be compared with extreme precision. As the stars are but points in a good telescope, it is necessary to expand these images into lines that the details of their spectra may be perceptible. This weakens their light, already feeble; and therefore the telescope to which the spectroscope is applied must be of considerable power; in this instance it has 8 inches aperture. They have examined more than fifty stars, and find all to have a constitution analogous to that of our own sun, and like it to show the presence of several terrestrial elements.

The lines due to sodium seem universal; so also those of magnesium and iron. Hydrogen is also frequent; in two stars it is wanting. The absence of nitrogen from all is very singular, unless it be present under conditions which alter its relation to light. It is to be regretted that the spectra were not compared with nickel, chromium, and the other elements that have been found in aërolites; this, however, we hope will be done for those elements, and indeed for all the other terrestrial ones. The explanation given of the different colours of the stars is doubtless the true one; it arises from the existence of different bands of absorption in particular parts of the spectrum. Thus in the beautiful star β Cygni, these bands are crowded in the blue end of the spectrum of the orange star, and in the red end of that of the blue star. Possibly further investigation of elements may enable us to conjecture why in dichromic double stars the small one is always blue or green. In the planets a closer conformity to the spectrum of the sun, by whose light they shine, might be expected—and is found. In the moon there are no lines whatever due to an atmosphere, nor in Venus. In Jupiter and Saturn, besides the solar lines, there are some identical with those produced by our own atmosphere, and one which must be due to the presence of some peculiar gas or vapour. Mars is still more peculiar; but the appearances give no countenance to the notion that his red colour is due to the chemical composition of his soil. Mr. Huggins has gone yet further and attacked the nebulæ, though their extreme faintness might seem to elude all the power of the spectroscope. The result is at once a strong proof of the soundness of Bacon's precept in doubtful cases, "*Fiat experimentum*," and a demolition of more than one accepted hypothesis. The notion of the elder Herschel,

that these strange bodies were masses of vapour, gradually condensing by radiation of heat and the action of gravity, and that (according to Laplace) in condensing they threw off rings which shrunk up into planets, satellites, and comets, was long in favour with Cosmogonists. Lord Rosse's discovery, that many of them which were cited as evidences of this, were in truth mere clusters of stars; that many more, though not totally resolvable, were yet dotted over with lucid points, produced a reaction which led to doubt the existence of any but stellar materials in the remote heavens. But here we have a totally different view opened. Mr. Huggins began with the planetary nebulæ, so called from presenting well-defined luminous disks (which, however, Lord Rosse has found in all that he examined to be complicated ring and spiral formations). To Mr. Huggins's great surprise he found them to be totally different from the stars; instead of continuous spectra interrupted by a multitude of dark lines, he found (which made it possible to observe them at all) that their light consisted entirely of three bright lines; the faintest indicating the presence of hydrogen, the next some unknown element, and the third a line which coincides with the brightest of nitrogen, but which he hesitates to refer to that body because no others of its bright lines are present. In some of them a very faint star-spectrum is also visible, due of course to the small stars which are found in many nebulæ, often in such numbers that their presence can scarcely be merely optical. Some nebulæ, and all the clusters which in ordinary telescopes look like them, do not show the distinctive three lines, but merely star-spectra. The importance of this new view, this new "Explorator Abyssi," to borrow a phrase of Hooke, cannot be rated too highly; and you will I am sure join me in expressing our hopes that these gentlemen will press onward in the bright path which they have opened, and perhaps with even higher instrumental powers.

One who has not taken part in this kind of observation must speak with reserve; but I cannot refrain from expressing a wish that their telescope were of much greater aperture—as two or three feet. The object of the telescope is in this case mere concentration of light, and it will probably be attained without anything of that perfect finish which is required for exact definition. If so, a mirror of silvered glass, or even copper, and of comparatively short focus, or one of Fresnel's Light-house lenses, might be a powerful auxiliary.

The remarks which I ventured to make in my Address at the last Anniversary, on the expediency of combining Pendulum Experiments with the Astronomical and Geodesical operations of the Great Indian Arc, have given occasion to a correspondence between Colonel Walker, the Superintendent of the Indian Trigonometrical Survey, and myself, which has been subsequently augmented by letters from several Fellows of the Royal Society conversant with such subjects, written in reply to

a Circular which, with the concurrence of the Council, I addressed to them. This correspondence having been submitted in a printed form through the proper channel to the Secretary of State for India, the proceedings proposed by Colonel Walker for carrying out the experiments have been officially sanctioned. An application to the Royal Society for a loan of two of the pendulums which had been previously employed in similar experiments, with their attendant clock, has been acceded to; and a Vacuum Apparatus convenient for transport, in which the pendulums may be vibrated at the Indian stations, has been constructed at the Kew Observatory. Captain Basevi, of the Royal Engineers, appointed to conduct the experiments in India, has resided for some weeks at Kew for the purpose of making himself practically acquainted with the instruments, and the mode of experimenting with them. Circumstances having obliged Captain Basevi to proceed to India before the base observations could be made at Kew, these have been undertaken by the Kew Observing Staff, and will be carried on in an apartment arranged for the purpose, which will be hereafter available for the verification of the same pendulums on their return from India at the close of the operations; and will, moreover, supply a convenient locality for future occasions of a similar kind, as well as for the comparison of the pendulums of other countries. We may not unreasonably anticipate that such experiments may henceforward be regarded as an appropriate accompaniment to the measurement of arcs in all parts of the globe.

I am glad to be able to add that Colonel Walker has also directed that the Indian Survey should be provided with instruments for determining the absolute values of the three magnetic elements at the Indian stations. We may thus hope to obtain a further investigation of the (apparent) systematic anomaly in the direction of the lines of magnetic force in the central parts of India, which is so remarkable a feature in the admirable magnetic survey of that portion of the British dominions executed by the Messrs. de Schlagintweit.

Whilst on the subject of terrestrial magnetism, I may be permitted to notice that, in a paper presented in the last session, I have called the attention of the Society to a very remarkable feature, made known by the comparison of the disturbances of the magnetic declination in England, and at stations nearly in the same latitude in the eastern parts of Asia. The days in which such phenomena occur are almost without exception the same in both localities; the hours (of absolute time) at which the maxima and minima of disturbances characterized by a common type take place are, within very small limits, also the same; but the *direction* in which the magnet is at the same moment deflected in these otherwise most accordant phenomena is systematically *opposite* in England and in eastern Siberia. I attach of course far more importance to the fact itself than to the hypothesis which guided me to its anticipation, and thence to its discovery; still, an hypothesis which has

led to the knowledge of a fact of so much theoretical importance entitles itself to some consideration; whilst no one can doubt that a knowledge of the fact itself strengthens the desire for the multiplication of stations in distant parts of the globe, at which these phenomena are systematically observed; and in this view the Society will hear with pleasure that, by intelligence very recently received, we learn that the authorities in the colonies of Mauritius and Victoria have decided on the establishment of magnetical observatories supported by colonial funds, to be equipped with similar instruments to those at Kew, and to be conducted on the system which is there exemplified. We are led in these two instances to recognize the advantage which a colony derives from having a Governor whose education has fitted him to appreciate the importance of cultivating the physical sciences.

We have learnt with satisfaction from Stockholm that the Swedish Expedition to Spitzbergen has returned from the second year of a survey preliminary to the measurement of an arc of the meridian; and that the result has been that no doubt is entertained of the practicability of the measurement of an arc of at least 3°, with a possibility of further extension. The report of the completion of the preliminary survey is be published in the early part of the winter; and the summer of 1865 is looked forward to for the commencement of the arc itself.

In Meteorology, we have, in the 2nd Part of the Philosophical Transactions for 1863, the first of a series of papers by Dr. Hermann de Schlagintweit on the "Numerical Elements of Indian Meteorology," based on observations made chiefly by officers in the late East India Company's Service at 207 stations, the original records of which were placed by the Indian Government in the author's hands for reduction and coordination. The present paper discusses the temperatures and the isothermal lines over India, both for the entire year and for each of the meteorological seasons. The differences between the forms of the isothermals in the different seasons are excessive, and full of instruction to climatologists who will study their causes. For the full elucidation of the subject, however, we must await a similar coordination of the barometric and hygrometric relations; there is probably no portion of the globe where the connexion of the periodic variations of the barometer, thermometer, and hygrometer manifests itself so clearly and so instructively as in Hindostan. There the "Trades" become "Monsoons": the aërial pressure describes from winter to summer a regularly inflected hollow curve; whilst the aqueous vapour inversely increases its tension; and the temperature attains its maximum before the sun has reached its greatest altitude*.

* See in illustration the annual variations in the plate of the "Report on the Meteorology of Bombay" in the British Association volume for 1845.

I proceed to announce the awards which the Council has made of the Medals in the present year; and to state the grounds on which those awards have been made.

The Copley Medal has been awarded to Charles Darwin, Esq., F.R.S., for his important researches in Geology, Zoology, and Botanical Physiology.

In 1832 Captain FitzRoy, commissioned by the Admiralty to proceed in command of the 'Beagle,' on a Voyage of Survey to the Southern Hemisphere, liberally offered, in the interest of science, to give up half his cabin to any qualified naturalist who would volunteer to serve on the Expedition, no remuneration being attached to the duty. Mr. Charles Darwin, then a ripe student at Cambridge, ardently devoted to the study of natural history, having heard of the offer, like Sir Joseph Banks, in the earlier of Cook's Voyages, eagerly came forward as a volunteer. The voyage of the 'Beagle' extended over the consecutive years from 1832 to 1836, and embraced regions presenting such fertile fields for research in the Volcanic, Coral, and other Islands of the Atlantic and Pacific Oceans, that the results of his observations actively occupied, after his return, ten years of Mr. Darwin's time in publication; and have since mainly suggested and determined the most prominent of his latest labours. His scientific works and memoirs have included a very wide range of subjects, which may be classified under the heads of Geology, Zoology, Physiological Botany, Physical Geography, and Genetic Biology, each of which he has enriched with important original contributions. The award of the Copley Medal has been founded on Mr. Darwin's researches in the three first-named branches of science.

Geology.—Mr. Darwin has been preeminently successful in the solution of a great problem in physical geography, and in applying it to the explanation of geological phenomena, by his important work on the Structure and Distribution of Coral Reefs, which appeared in 1842. The successive voyages of many eminent navigators had shown that vast tracts in the deepest parts of the Pacific and Indian Oceans were studded with circular groups of Coral Islets enclosing lagoons; and that long detached reefs of the same formation flanked lines of coast in a nearly unbroken stretch of about 1000 miles. The reefs and islands have been carefully mapped and surveyed; and the different forms exhibited by them had been accurately classified under the names of "Atoll Islands," "Encircling Reefs," "Barrier Reefs," and "Fringing or Shore Reefs." Eminent naturalists had observed the habits and mode of growth of the Zoophytes in the most favourable localities; and the comparatively shallow depths at which the reef-building species live had been determined. But no satisfactory explanation of the phenomena was arrived at—why Atolls assumed their peculiar form, and why Barrier Reefs included broad lagoon channels between them and the

contiguous coasts. Some explanations were indeed propounded, but were manifestly inadequate to meet all the conditions.

The subject was in this state when Mr. Darwin took it up. Combining careful observation upon Coral Reefs and Atolls with reflections upon the range and distribution of Volcanic Islands, he divided the area of the Pacific and Indian Oceans into tracts of elevation and depression. All the Coral Phenomena of Atolls, Encircling Reefs, and Barrier Reefs were accounted for upon the supposition of a long-protracted but gradual subsidence under the ocean of large areas of land, admitting the Coral Formation to grow up whilst its foundation sunk down; while Fringing Reefs were explained as appertaining to areas of elevation where the land had risen up or was progressively rising. On this view the apparently discordant facts ranged themselves in perfect harmony. Fringing Reefs were thus converted into Barrier Reefs, and Encircling Reefs into Atolls, "the instant the last particle of land sinks beneath the ocean." Representing on a map the two classes of facts by different colours, it was shown that active volcanos are absent in the region of Atolls and Barrier Reefs, while they abound in areas of Coral Fringes and of elevation.

Mr. Darwin's researches were received by naturalists with lively satisfaction as fixing an era in the history of the investigation, and as contributing one of the most important illustrations which geology had received since it had been shaped into a science. A flood of light was thus thrown upon the ancient calcareous formations which enter so largely into the composition of the superficial crust of the earth; and something like a definite idea was arrived at respecting areas of depression and elevation.

In addition to his researches upon coral reefs, Mr. Darwin has made numerous contributions to Geology, both in the descriptive and theoretical divisions of the science. As belonging to the former class may be cited his admirable 'Journal of Researches,' containing observations on the Geology of the various countries visited during the voyage of the 'Beagle'; notes during a survey of the east and west coasts of South America, with a transverse section of the Cordilleras between Valparaiso and Mendoza; Geological observations on South America, published as a separate work in 1846; Geological observations on the Volcanic Islands visited during the voyage of the 'Beagle,' with brief notices of the Geology of Australia, New Zealand, and the Cape of Good Hope, also published as a separate work; on a remarkable Bar of Sandstone off Pernambuco; on the Deposits containing extinct Mammalia in the neighbourhood of the Plata; on the Geology of the Falkland Isles; on the Distribution of Erratic Blocks in South America.

In theoretical or speculative geology may be cited his memoir on the connexion of certain Volcanic Phenomena in South America, and

on the Formation of Mountain Chains and Volcanos as the effect of Continental Elevations; his memoirs on the Parallel Roads in Glenroy; on the Effects produced by the ancient Glaciers of Carmarthenshire; on the Transport of Erratic Boulders from a lower to a higher level; and on the Origin of Saliferous deposits. From the Ossiferous Superficial deposits in the neighbourhood of the Plata Mr. Darwin brought home an important collection of fossil mammalian remains, which formed the subject of a separate volume by Professor Owen. In his memoir 'On the Formation of Mould,' as the result of the digestive process of the common earthworm, he furnished a fresh and instructive illustration of the large effects which are produced in the organic kingdom by the continued agency of apparently insignificant instruments.

The present occasion admits of little more than a bare enumeration of these labours, which are stamped throughout with the impress of the closest attention to minute details and accuracy of observation, combined with large powers of generalization. The Geological Society of London signalized its estimate of their importance by the award of a Wollaston Medal.

Zoology.—In zoological science Mr. Darwin's eminent merits were to some extent acknowledged ten years ago by the award of a Royal Medal. On that occasion the zoological work that was most particularly distinguished was his Monograph on the Cirripeds, a class of animals whose life, history, structure, and classification had previously been involved in the greatest obscurity and confusion. Notwithstanding the difficulties attending the study of these animals, and the extraordinary anomalies presented in their structure, habits, and affinities, Mr. Darwin was successful, as the result of unwearied labour and patience, and of the exercise of the most acute and accurate observation, in clearing up all that was obscure, and in disclosing for the first time numerous facts of the utmost interest and importance. But since the principal points contained in this monograph have been already detailed in the Proceedings of the Royal Society on the occasion referred to, it is needless here to recapitulate them. It will be sufficient to remark that the justness of the estimation then placed upon Mr. Darwin's labours has since been completely confirmed by the concurrent voice of all Zoologists, and that the Monograph on the Cirripeds is universally acknowledged to be a model of what such a work should be, and as fully entitling its author to a place in the foremost rank of zoological observers and authors. His labours in the same department were completed by the publication, about the same time, of two monographs on the Fossil Cirripeds of Great Britain, published by the Palæontological Society. This subject, which had before received scarcely any attention, and was left in the greatest obscurity, is in these monographs treated in the most complete and exhaustive manner; and all its difficulties were removed for future observers by the lucid and admirable definition of the various

parts, and the light thrown upon the structure and relations of the animals.

In other departments of zoology Mr. Darwin's labours, though not given in the same complete form as in the Monographs, have been numerous and important. They are distinguished by the same extent and variety of knowledge, the same scrupulous fidelity, accuracy, and minuteness of observation, and by the sagacity with which the most important generalizations have been drawn. Amongst these contributions should more especially be noticed the observations on the distribution and habits of the animals described in the 'Zoology of the Voyage of the Beagle,' and in the 'Journal of Researches' in the course of that voyage, in which most interesting and justly popular work we find first distinctly enunciated the important "law of the succession of Types," or the law that existing animals have a close relation with the extinct species found in the same regions. Nor, amongst the many other weighty and interesting remarks contained in this volume, should notice be omitted of those which have reference to the common assumption that the presence of the remains of large animals necessarily implies that the country inhabited by them must have possessed a luxuriant vegetation. The fallacy of this assumption is plainly shown by Mr. Darwin, and the importance of his correction of a prevalent error of this kind can hardly be overrated.

In his most recent work 'On the Origin of Species,' although opinions may be divided or undecided with respect to its merits in some respects, all will allow that it contains a mass of observation bearing upon the habits, structure, affinities, and distribution of animals, perhaps unrivalled for interest, minuteness, and patience of observation. Some amongst us may perhaps incline to accept the theory indicated by the title of this work, while others may perhaps incline to refuse, or at least to remit it to a future time, when increased knowledge shall afford stronger grounds for its ultimate acceptance or rejection. Speaking generally and collectively, we have not included it in our award. This on the one hand; on the other hand, I believe that, both collectively and individually, we agree in regarding every real *bonâ fide* inquiry into the truths of nature as in itself essentially legitimate; and we also know that in the history of science it has happened more than once that hypotheses or theories, which have afterwards been found true or untrue, being entertained by men of powerful minds, have stimulated them to explore new paths of research, from which, to whatever issue they may ultimately have conducted, the explorer has meanwhile brought back rich and fresh spoils of knowledge.

Botanical Physiology.—Mr. Darwin's first botanical work, 'On the various contrivances by which British or Foreign Orchids are fertilized by Insects, and on the effects of Intercrossing,' marks an epoch in the history of physiological botany, and taken in all its bearings, is perhaps

the most masterly treatise on any branch of vegetable physiology that has ever appeared. The objects which the author had in view in entering on a comprehensive study of the sexual system of Orchideæ, were to show that the contrivances by which they are fertilized are as varied and almost as perfect as any of the most beautiful adaptations of the animal kingdom, and that these contrivances have for their main object the fertilization of one flower by the pollen of another. In pursuance of this object Mr. Darwin set himself to investigate, first, the structure and development of the flower of living specimens of nearly every British species; secondly, to observe how impregnation was naturally effected in each; thirdly, to make a similar structural investigation of the principal exotic forms; and, fourthly, to ascertain by experiment the method by which these also are in all probability fertilized. To these investigations Mr. Darwin brought all the resources of a most skilful microscopic dissector, of an unwearied and exact observer, of a sagacious experimentalist fertile in resources, of an entomologist versed in the structure and habits of insects, and of an excellent judgment in interpreting obscure phenomena, and drawing from them correct conclusions.

The result is a work no less remarkable for the novelty of its facts, and for the importance of their bearing, than for its being the first which correlates the structure with the functions of the floral organs of one of the largest and most conspicuous of the families of plants. It would not be difficult to justify this strong encomium by examples of great interest taken from the work itself, but it would be incompatible with the limits of this Address; suffice it therefore to say, that the general conclusion to which Mr. Darwin arrives is, that all the forms, even the most grotesque, which the floral organs of Orchids possess, are directly and obviously of use, and that every structural and physiological modification, however minute, tends, with scarcely an exception, to ensure the fertilization of the ovules of one plant by the pollen of another.

Mr. Darwin's next contribution to physiological botany is entitled "On the two forms, or dimorphic condition, in the species of *Primula*, and on their remarkable sexual relations." The phenomenon of there being two distinct forms of flower in the genus *Primula* has long been familiar to naturalists, but the real nature of the difference between them, and of their respective functions, had not occurred to anyone. Mr. Darwin first suspected that the relations between the forms might be sexual, and he has since, with consummate skill, incontrovertibly proved this to be the case. By a most searching examination of a vast number of specimens of each form, in cultivated varieties as well as in species, he found that, in all, the two forms presented in their stigmatic surfaces, ovules, and pollen, *constant* differences, unbroken by a single instance of transition between the distinct forms. By experiments continued for several years, he proved that in this genus complete fer-

tility is only obtained by impregnating one form by the pollen of the other, each species of *Primula* being divided into two sets or bodies, which cannot be called distinct sexes, for both are hermaphrodite; yet they are so in a certain sense, for they require reciprocal union to effect perfect fertility. This remarkable fact has as yet no known parallel. The cross impregnation is effected by insects, the structure of insect and flower being such that one form cannot by this means be impregnated either by its own pollen or by that of a flower of its own form.

In a subsequent paper Mr. Darwin has shown that in a species of the genus *Linum*, which is also dimorphic, whilst the pollen in the two forms is absolutely undistinguishable microscopically, and the stigmas differ but slightly and not in any apparently important respect, the pollen of one form is ineffectual when placed on the stigma of its own flower, but acts immediately upon that of the other form. The generally received idea that the impregnation of plants may be effected indifferently by wind or by insects, is shown to be fallacious; plants being structurally adapted for the one or the other, and not indifferently for either.

He has similarly investigated a still more complicated case in the common *Lythrum salicaria* of our ditches, which, as regards its flowers, is trimorphous, there being three instead of two sexual forms, differing in the relative lengths of their two series of stamens and of their styles. In estimating the novelty and value of Mr. Darwin's botanical discoveries, we should not overlook that they have all been obtained by the study of some of the most familiar and conspicuous of our native plants, and some of the best-known and easily procured cultivated exotics.

Mr. Busk,

I will request you to present this Medal in the name of the Society to your friend Mr. Darwin, and with it the expression of our deep regret that the state of his health prevents the gratification which it would have been to us all to have welcomed him here on this day.

And you will be able to tell him, from your own observation, the hearty satisfaction with which the Society regards the bestowal of this, its highest mark of esteem, in evidence of its appreciation of labours almost incessantly pursued for now between thirty and forty years.

The Council has awarded a Royal Medal to Warren De la Rue, Esq., F.R.S., for his observations on the Total Eclipse of the Sun in 1860, and for his improvements in Astronomical Photography.

The advantages which photography affords in the delineation of celestial objects and phenomena are now generally recognized. This art has been already successfully applied to depict the infinitely diversified surface of the moon; the sun with its spots and faculæ, and those mysterious appendages which are only to be seen during a total eclipse;

as well as the markings on the surfaces of some of the principal planets. It has also been employed to measure the apparent distances and angles of position of double stars, and to record the time of the sun's passage over the wires of a transit instrument with much greater accuracy than could be done by an actual observer; and it is probable that the number of the useful applications of the art to astronomy has by no means reached its limit.

Among those who have successfully cultivated this comparatively new department of astronomy, it may perhaps be fairly said that Mr. De la Rue stands preeminent. It is not that he claims any priority in making this application of the photographic art. Several astronomers have made experiments in celestial photography, and some of these earlier than Mr. De la Rue, but no one, I believe, has devoted himself so systematically and assiduously to overcome the many difficulties which are met with in the process, and no one has been so successful in the results which he has obtained, particularly in regard to the sun and moon, his photographic pictures of which bodies are sufficiently delicate in their details to admit of the most precise measurement. No one who has not seen Mr. De la Rue's pictures of the moon can form an idea of their exquisite sharpness and beauty of definition. No doubt part of the superiority of these pictures is due to his employment of a *reflecting* telescope of exquisite defining power, the large mirror of which was figured by his own hands, and by peculiar machinery of his own contrivance. Thus he entirely avoided those imperfections of the actinic image which arise from outstanding chromatic dispersion in the very best refractors, especially when, as usual, they are achromatized for the luminous and not for the chemical rays of the spectrum.

The late Professor Bond, of Cambridge, in the United States, with the assistance of Messrs. Whipple and Black, of Boston, was the first to make a photographic picture of any heavenly body. In the year 1845 he obtained good pictures of α Lyræ and of Castor, and in 1850, by means of the great reflector of the Harvard Observatory, he obtained a daguerreotype of the moon, which was placed in the Exhibition of 1851. It was the sight of this which first gave the impulse to Mr. De la Rue's labours in this direction.

At the latter end of 1852, Mr. De la Rue took some successful positive lunar photographs on collodion by means of an equatorially mounted reflecting telescope of 13 inches aperture, made by himself; and he was probably the first to employ in celestial photography the then recently discovered collodion process, which is that now exclusively adopted. At that period he had not applied any driving motion to the telescope, and was therefore obliged to follow the apparent motion of the moon by means of a sliding frame fixed in the eyepiece holder, and moveable by the hand; but in 1857 he applied a driving clock to the telescope, the rate of which could be adapted to

follow the moon in right ascension, and thus of course a much greater approach to perfection was attainable.

In the department of solar photography, Mr. De la Rue stands almost alone. Assisted by the suggestions of the late Mr. John Welsh, he devised the Kew photoheliograph, which is now regarded as a model instrument for taking instantaneous sun-pictures. This instrument was taken to Spain by Mr. De la Rue in 1860 in order to observe the total solar eclipse, and the undertaking was perfectly successful. Numerous photographs were taken during the partial phase, and two during the totality, that showed the forms and positions of the red protuberances much more perfectly than had ever been done before. The experience gained respecting the photographic energy of the light of the prominences is also very valuable; for it is shown that a much shorter time of exposure than Mr. De la Rue ventured upon would have been sufficient to obtain good pictures, so that it may be expected that in future eclipses astronomers will be able to get a greater number of photographs during the totality, while the photographs themselves will be sharper and better defined. The photographs obtained in the solar eclipse of 1860 have been discussed by Mr. De la Rue in a most elaborate paper in the Philosophical Transactions for 1862, forming the Bakerian Lecture. In this memoir, and in his reports on the progress of Celestial Photography in the volumes of the British Association for 1859 and 1861, Mr. De la Rue has fully described his processes and instruments, and has thus deepened the feelings of obligation to him, by giving others the benefit of his long experience in the art, though even with this guidance no one can hope to attain to great proficiency in its practice without time, patience, and perseverance.

Mr. De la Rue,

I have the gratification of presenting you with this Medal, which the Council has awarded to you for your most valuable labours in the application of Photography to Astronomy.

You, Sir, are in the enjoyment of the fullest health and strength, and may, we hope, look forward to many years of active and useful occupation in departments which you may be said to have created for yourself.

You may be well assured, Sir, that the sympathy and interest of the Royal Society will continue to attend you in this work, while it will count on many valuable contributions from you to its Transactions in future years.

The Council has awarded a Royal Medal to Jacob Lockhart Clarke, Esq., F.R.S., for his researches on the intimate structure of the Spinal Cord and Brain, and on the development of the Spinal Cord, published in five memoirs in the Philosophical Transactions, and in other writings.

On the occasion of conferring on one of our Fellows a well-earned

reward for his enlightened and persevering labour in investigating the structure of the central organs of the nervous system, we may not inappropriately recal to mind that one of those who, in former times, most signally advanced this department of anatomical knowledge was an early Fellow of this Society, Dr. Thomas Willis, and that his celebrated treatise on the Anatomy of the Brain was illustrated by drawings from the pencil of no less eminent an associate than Sir Christopher Wren, also a Fellow, and afterwards President of the Society. But in that day, and down to a recent period, the investigation was carried on with the naked eye, and accordingly the more intimate structure and organization were hidden from view until brought to light in our own time by the aid of the microscope. It is true that the tissue of the brain and nerves did not escape the notice of the earlier microscopic observers; of this, indeed, there is well-known evidence in the early volumes of our own Transactions; still their inquiries were almost entirely confined to the microscopic characters of the fibres and other constructive elements of the cerebral and nervous tissues, whereas the application of the microscope, in tracing the arrangement of these elements, and their combination into an organized structure, dates little further back than the last twenty years.

In this field of inquiry Mr. Clarke has been one of the most able and successful workers. No sooner had he entered upon it than he introduced an important improvement into the method of investigation. The observations had previously been made on opaque sections of the parts examined, on which only the coarser features of the structure could be discerned; but Mr. Clarke devised a process for rendering them transparent, whereby it became possible to trace the finer and more intimate arrangement; and this method has not only, in his own hands, proved fruitful in valuable results, but, having been adopted by his fellow-labourers in the same pursuit, has been most influential on the general progress of the inquiry.

The investigation itself, even with every available aid, is singularly difficult and laborious. Section after section of the spinal cord throughout its whole length, and of its complex cranial prolongation, must be carefully scrutinized, thoughtfully compared, and, for the most part, minutely delineated; and sometimes many different specimens must be prepared and examined in order to make out a single point. In this exhausting work Mr. Clarke has long and meritoriously persevered amid the calls of an active professional life. The results of his labours are made known in his various papers, published chiefly in the Philosophical Transactions, and in the elaborate delineations with which they are illustrated. And it is pleasing to know that, in all essential points, his observations have been confirmed by the most accurate and trustworthy of his contemporaries.

After what has been said of the difficulties of the inquiry, it will be

readily conceived that Mr. Clarke's investigations, successful though they have been, have left not a few intricacies of structure still to be unravelled; and, in speaking of the advances actually made, it will be sufficient here to indicate the chief points which have either been made known for the first time, or more exactly determined and placed in a clearer light through Mr. Clarke's labours.

One of the principal subjects of investigation was the grey substance which forms the interior part of the spinal cord. The figure which this part assumes in different regions of the cord has been more exactly described and delineated, and the nature and arrangement of its constituent elements more fully examined and more clearly exposed than heretofore. Two columns or tracts, composed of nerve-cells, and previously undescribed, have been shown to exist in the grey substance through nearly the whole length of the cord, and two others in a shorter extent. Moreover Mr. Clarke was, as we believe, the first to point out that the central canal of the spinal cord is lined with epithelium, and he certainly first explained the true nature of the tissue immediately surrounding the canal, which had previously been mistaken for nervous substance.

The course and connexions of the fibres of the nerve-roots after they enter the substance of the spinal cord have, as yet, been by no means fully made out; but Mr. Clarke's investigations have shed considerable light on that obscure point of anatomy, and, amongst other observations of moment, he has shown that a part of the posterior or sentient roots take, in the first instance, a downward direction—an unlooked-for anatomical fact, which was afterwards strikingly shown by Brown-Sequard to be in harmony with physiological experiment.

The structure of the medulla oblongata, and the relation of its several tracts or divisions to the columns of the spinal cord, as well as the intimate nature of the grey masses which are there superadded, and their connexion with special sets of fibres and nerve-roots, are questions which have long tried the skill and patience of anatomists, and which have received fresh elucidation from the keen scrutiny and sagacious interpretation of Mr. Clarke; and in this branch of his inquiry he has arrived at new facts, and has been able to correct serious errors which had been introduced on respected authority.

But the researches of Mr. Clarke on the spinal cord have not been confined to its perfected structure; he has investigated the mode of its development in the fœtus; and one of his papers in the Philosophical Transactions contains a minute account of the changes observable in the form and structure of both the white and grey substance at successive stages of development, in man, mammalia, and birds; also, a description of the intimate structure of the intervertebral ganglia, and of the mode of development of the cells and fibres which enter into the formation of these different parts.

Finally, although it must be confessed that the knowledge acquired of the intimate structure of the nervous centres has as yet afforded but little direct insight into their functional mechanism, it cannot be doubted that approved and trustworthy investigations of structure tend powerfully to promote physiological truth, by enabling us to distinguish between true and false anatomical data when used as a basis of physiological reasoning, and by sweeping away the imaginary groundwork of much vain speculation and erroneous doctrine.

Mr. Clarke,

I beg leave to present you with this Medal, which has been awarded to you, on the recommendation of those in the Council most competent to judge of the subjects on which you are engaged, as a well-earned reward of your persevering labours, which the Council is well aware have demanded from you the devotion of all the leisure remaining to you amidst the duties of an active professional career. I need scarcely add that the Council hopes that this mark of approbation of services already performed will also be an incitement to further labours in the same or in kindred fields of research.

The Council has awarded the Rumford Medal to Professor John Tyndall, F.R.S., for his researches on the Absorption and Radiation of Heat by Gases and Vapours.

Previously to the researches of Professor Tyndall, hardly anything had been done in the way of an experimental determination of the absorption of radiant heat by gases and vapours. Melloni had inferred from his experiments that atmospheric air is sensibly diathermanous in a length such as that of an ordinary room, while Dr. Franz came to the conclusion that a column of air only 3 feet long absorbed more than $3\frac{1}{2}$ per cent. of the heat-rays from an argand lamp. The discrepancy of these results gives some view of the difficulty of the experiments; but it is only by the perusal of the earlier part of Professor Tyndall's first memoir on the subject, that the skill and patience can be appreciated with which the various sources of error were one by one detected and eliminated by him.

In his first memoir he shows that the elementary gases, hydrogen, oxygen, nitrogen, as well as air freed from moisture and carbonic acid, when examined in a length of about 4 feet, exert on the heat radiated from lampblack at 212°, an absorption not exceeding about $3\frac{1}{2}$ per cent., and liable to be altered by the slightest impurity present in the gases. The results obtained with compound gases and with vapours, on the other hand, were very different,—olefiant gas, for instance, in the same length absorbing 81 per cent. of the incident heat. About twenty gases and vapours were examined, and that not only at atmospheric pressure (or, in the case of vapours, in a state of saturation), but also at a variety

of inferior pressures. Among these substances may particularly be mentioned, as being constituents of our atmosphere, carbonic acid and aqueous vapour, both of which, but more especially the latter, were found to exert a considerable absorptive action, so that Professor Tyndall concludes that it is to its aqueous vapour that the absorption of radiant heat by our atmosphere is mainly due. He also shows that the gases operated on when heated, radiate heat in an order corresponding to that of their absorption.

In his second memoir, besides continuing the same subject, he has struck out a new and remarkable method of determining the absorption and radiation of heat by gases and vapours, depending on what he calls *dynamic* radiation and absorption. In this method the results are arrived at without the use of any extraneous source of heat or cold whatsoever, but simply by the heat or cold produced by the condensation or rarefaction of the gas itself, or of a mixture of the vapour with air or a diathermanous gas.

It may serve to show the difficulties which beset the inquiry, arising from the interference of disturbing causes, to state that two such experienced physicists as Professor Tyndall and Professor Magnus of Berlin should have arrived at, and long maintained, opposite conclusions respecting the absorption of radiant heat by air, and the influence of aqueous vapour. This led Professor Tyndall in a third memoir to consider more especially the case of aqueous vapour, which he had already treated in his two former papers. The result is that his conclusions have been so confirmed by a system of checks and counterchecks, and by the complete harmony which they present with what we know to be true in other cases, that it seems impossible to doubt their correctness.

The conclusion that the chief absorbing action of the atmosphere on non-luminous heat is due to the aqueous vapour which it contains, has numerous and important bearings on meteorology, and has been applied by Professor Tyndall to the explanation of some phenomena which appear hitherto to have been imperfectly understood.

In his fourth memoir, besides various other subjects of investigation, he compares the absorption of heat by vapours with that produced by the same substances in the liquid state, and concludes that for non-luminous radiant heat (in accordance with what we know for light) the general character of the absorption is the same, in whichever of the two states the substance may be found.

In a fifth memoir, just published, he examines among other things the penetrative power of the heat radiated from various flames, and shows that such heat is absorbed with especial facility by the gases which result from the combustion.

Professor Tyndall concludes from his researches that, as a general rule, the opacity of a substance with respect to radiant heat from a source of comparatively low temperature increases with the chemical

complexity of its molecule; and he has given some remarkable instances in which the law is found to be true. Whatever may be thought of our ability to explain the law in the present state of our knowledge respecting the molecular constitution of bodies, the law itself is in any case highly remarkable.

Professor Tyndall,

In presenting you with this Medal I will acknowledge that I am more embarrassed in the choice of the expressions which should accompany it than in the three preceding instances, arising from the warm feelings of personal regard and admiration which I have felt for you, I may permit myself to say, from the very commencement of your scientific career in this country. It has been your fate, Sir, that each last achievement may almost be said to have dimmed the lustre of those which preceded it. I will only add the expression of my heartfelt hope that your health, so valuable to us all, and which ought not therefore to be lightly compromised, will be preserved to enable you to accomplish fresh labours which shall still surpass those which have gone before.

On the motion of Sir Henry Holland, seconded by the Master of the Mint, it was resolved—"That the thanks of the Society be returned to the President for his Address, and that he be requested to allow it to be printed."

The Statutes for the election of Council and Officers having been read, and Mr. John Hogg and Professor Huxley having been, with the consent of the Society, nominated Scrutators, the votes of the Fellows present were collected, and the following were declared duly elected as Council and Officers for the ensuing year:—

President.—Major-General Edward Sabine, R.A., D.C.L., LL.D.

Treasurer.—William Allen Miller, M.D., LL.D.

Secretaries.—{ William Sharpey, M.D., LL.D.
George Gabriel Stokes, Esq., M.A., D.C.L.

Foreign Secretary.—Professor William Hallows Miller, M.A.

Other Members of the Council.—Professor John Couch Adams, M.A.; James Alderson, M.D.; George Busk, Esq., Sec. L.S.; Colonel Sir George Everest, C.B.; Hugh Falconer, M.A., M.D.; John Peter Gassiot, Esq.; John Edward Gray, Ph.D.; Thomas Archer Hirst, Ph.D.; Sir Henry Holland, Bart., M.D., D.C.L.; Henry Bence Jones, M.A., M.D.; Sir Roderick Impey Murchison, K.C.B.; William Odling, M.B.; Professor William Pole, C.E.; Rev. Bartholomew Price, M.A.; Sir John Rennie, Knt.; The Lord Stanley.

On the motion of Mr. Arnott, seconded by Mr. Bowman, the thanks of the Society were voted to the Scrutators. The Society then adjourned.

Receipts and Payments of the Royal Society between December 1, 1863, and November 30, 1864.

	£	s.	d.
Balance at Bank, and on hand	658	0	1
Annual Subscriptions and Compositions	1623	16	0
Rents	254	12	6
Dividends on Stock	928	5	2
Ditto, Ditto, Trust Funds	280	0	7
Ditto, Ditto, Stevenson Bequest	523	7	0
Sale of Transactions, Proceedings, &c.	403	15	3
Chemical Society, Proceedings, 1863–64	50	0	0
Miss Burdett Coutts—Half cost of Plates to Dr. Hear's and Mr. Pengelly's Papers	90	0	0
B. Botfield, F.R.S., Bequest	70	0	0
Tea Expenses and Gas, repaid	38	9	6
Parcel Charges recovered	5	15	3

Estates and Property of the Royal Society, including Trust Funds.

Estate at Mablethorpe, Lincolnshire (55 A. 2 R. 2 P.), £126 0*s.* 0*d.* per annum.

Estate at Acton, Middlesex (34 A. 3 R. 11 P.), £110 0*s.* 0*d.* per annum.

Fee-farm rent in Sussex, £19 4*s.* per annum.

One-fifth of the clear rent of an estate at Lambeth Hill, from the College of Physicians, £3 per annum.

£14,000 Reduced 3 per Cent. Annuities.

£27,969 15*s.* 7*d.* Consolidated Bank Annuities.

£516 9*s.* 8*d.* New 2½ per Cent. Stock.

	£	s.	d.
	£4926	1	4

	£	s.	d.
Salaries, Wages, and Pension	1018	11	6
£1000 Consolidated Bank Annuities, bought at 89¾	898	15	0
The Scientific Catalogue	203	13	7
Books for the Library and Binding	171	8	3
Printing Transactions and Proceedings, Paper, Binding, Engraving, and Lithography	1385	18	3
Upholstery, Painting, and Miscellaneous Expenses	80	0	11
Coal and Lighting	115	1	0
Tea Expenses	44	19	10
Fire Insurance	42	1	6
Shipping Expenses	10	17	5
Taxes	13	7	6
Law Expenses	22	2	0
Stationery	7	16	6
Postage, Parcels, and Petty Charges	30	19	6
Subscriptions:—Rebuilding Acton Church	25	0	0
" Mablethorpe Schools	2	2	0
J. Oakley, Acton Inclosure Expenses	39	3	3
Smith and Oakley, Survey of Mablethorpe Estate	20	17	6
Donation Fund	60	0	0
Wintringham Fund	34	16	0
Copley Medal Fund	4	19	5
Prof. Tyndall, Bakerian Lecture	4	0	0
Prof. Helmholtz, Croonian Lecture	2	18	3
Rev. Dr. Stebbing, Fairchild Lecture	2	18	1
	4242	7	3
Balance at Bank	679	6	7
" Catalogue Account	1	7	1
" Petty Cash "	3	0	5
	£4926	1	4

WILLIAM ALLEN MILLER,
Treasurer.

Investments up to July 1863, New 3 per Cent. Annuities £5300 0 0

Treasurer.

Investments up to July 1863, New 3 per Cent. Annuities £5300 0 0

£5300 0 0

Dr. *Cr.*

	£	s.	d.		£	s.	d.
To Balance, Subscriptions and Dividends	425	3	11	By Grants	125	0	0
				Balance	300	3	11
	£425	3	11		£425	3	11

Statement of Income and Expenditure (apart from Trust Funds) during the Year ending November 30, 1864.

	£	s.	d.		£	s.	d.
Annual Subscriptions	1131	16	0	Salaries, Wages, and Pension	1018	11	6
Admission Fees	150	0	0	The Scientific Catalogue	203	13	7
Compositions	342	0	0	Books for the Library £86 9 11	171	8	3
Rents	254	12	6	Binding ditto 84 18 4			
Dividends on Stock (exclusive of Trust Funds)	928	5	2	Printing Transactions, Part II. 1863, and Parts I. & II. 1864 ..... 457 12 11			
,, on Stevenson Bequest	523	7	0	Ditto Proceedings, Nos. 58–67 ..... 244 15 9			
Sale of Transactions, Proceedings, &c.	403	15	3	Ditto Miscellaneous 142 6 0	1385	18	3
B. Botfield, F.R.S.—Bequest	70	0	0	Paper for Transactions and Proceedings ... 188 8 0			
Chemical Society, for Proceedings, 1863–64	50	0	0	Binding and Stitching ditto 96 7 6			
Chemical Society, Tea Expenses £14 19 5				Engraving and Lithography 256 8 1			
Linnean Society, Tea Expenses 14 19 5				Painting and Cleaning	15	3	0
Geographical Society, Gas at Evening Meetings 8 4 8	38	9	6	Upholstery, Repairs, and Miscellaneous Expenses	64	17	11
St. George's Rifles, ditto 0 6 0				Coal and Lighting	115	1	0
Parcel Charges recovered	5	15	3	Tea Expenses	44	19	10
				Fire Insurance	42	1	6
Income available for the Year ending Nov. 30, 1864	3898	0	8	Subscriptions:—Rebuilding Acton Church	25	0	0
Expenditure in the Year ending Nov. 30, 1864	3234	0	6	,, Mablethorpe Schools	2	2	0
				J. Oakley, Acton Inclosure Expenses	39	3	3
				Smith and Oakley, Survey of Mablethorpe Estate	20	17	6
				Shipping Expenses	10	17	5
				Taxes	13	7	6
				Law Expenses	22	2	0
				Stationery	7	16	6
				Postage, Parcels, and Petty Charges	30	19	6
Excess of Income over Expenditure in the Year ending Nov. 30, 1864	£664	0	2	WILLIAM ALLEN MILLER, *Treasurer.*	£3234	0	6

The following Table shows the progress and present state of the Society with respect to the number of Fellows:—

	Patron and Honorary.	Foreign.	Having compounded.	Paying £2 12s. annually.	Paying £4 annually.	Total.
November 30, 1863.	6	49	324	4	274	657
Since compounded..	.....	.....	+3	.....	−3	
Since elected	.....	+3	+4	.....	+11	+18
Since deceased	.....	−2	−11	−1	−6	−20
November 30, 1864.	6	50	320	3	276	655

December 8, 1864.

Dr. WILLIAM ALLEN MILLER, Treasurer and Vice-President, in the Chair.

It was announced from the Chair that the President had appointed the following Members of the Council to be Vice-Presidents:—

The Treasurer.
Mr. Gassiot.
Sir Henry Holland.

The following communications were read:—

I. "Researches on certain Ethylphosphates." By ARTHUR HERBERT CHURCH, M.A. Oxon., Professor of Chemistry, Royal Agricultural College, Cirencester. Communicated by A. W. HOFMANN, Ph.D., LL.D. Received October 21, 1864.

The constitution, properties, and derivatives of the so-called conjugated sulphurous and sulphuric acids have been made the subject of numerous researches, and have led, in the hands of Gerhardt and others, to very interesting results. I have examined at different times* several members

* On the Benzole Series, Parts 1 & 2. Phil. Mag. April and June 1855.
On the Spontaneous Decomposition of certain Sulphomethylates. Phil. Mag. July 1855.
On the Action of Water upon certain Sulphomethylates. Phil. Mag. Jan. 1856.
On Parabenzole, Parts 1 & 2. Phil. Mag. June 1857, and Dec. 1859.

of the methylsulphuric, phenylsulphurous, nitrophenylsulphous and other series, but have lately turned my attention to the analogous compounds of the phosphoric series. Some remarkable substances have been thus made, their constitution seeming to have a direct bearing upon the important question of the atomicity and equivalency of certain of the metallic elements.

Several substances might have served as starting-points for these new inquiries. A curious compound, phenylphosphoric acid, $C_6H_5H_2PO_4$, was prepared; but its instability, and the oxidation to which it and its salts are liable, rendered it unsuited for the present purpose. I intend to describe in the present paper but one series of salts, formed from Pelouze's ethylphosphoric acid, $C_2H_5H_2PO_4$. This compound, containing two atoms of easily replaceable hydrogen, appeared admirably adapted for the purpose in view. It is readily prepared by digesting (for 48 hours) finely crushed glacial phosphoric acid with alcohol of 90 per cent.:—

$$^{*}\left.\begin{matrix}C_2H_5\\H\end{matrix}\right\}O+HPO_3=\left.\begin{matrix}C_2H_5\\H\\H\end{matrix}\right\}PO_4.$$

From the ethylphosphoric acid thus formed the barium salt was prepared in large quantity and of perfect purity. This compound, $C_2H_5, Ba_2, PO_4 + 6H_2O$, is remarkable for being less soluble in boiling water than in water at 70° C., or even at 15°—a characteristic property of several other ethylphosphates. Boiling water, in fact, affects this barium salt in a peculiar manner. If to its boiling saturated solution a quantity of the ordinary crystallized salt be added, the crystals instantly assume a pearly aspect, and are found, after having been filtered off, washed once with boiling water, and dried *in vacuo* over sulphuric acid, to have lost $5H_2O$, and thus to have the formula

$$C_2H_5, Ba_2, PO_4 + Aq.$$

They thus contain the proportion of water found in the majority of the ethylphosphates. When, on the other hand, cold water is poured on these crystals, or on the salt dried at 100°, the lost water is regained, the nacreous aspect of the dried salt disappears, while a great increase in its bulk occurs.

From the barium salt the lead and silver compounds are readily made. To a solution of ethylphosphate of barium nearly saturated at 70°, acetate of lead or nitrate of silver is added in slight excess, the liquid allowed to cool and then filtered. The collected precipitate is to be washed with cold water. The lead salt may be obtained anhydrous by heating it to 130°–150°: it is almost insoluble in cold water, but is slightly soluble in hot water, from which it may be crystallized. The silver salt dissolves to some extent in hot water, and separates in pearly plates as the liquid cools. It is blackened, especially when moist, on exposure to daylight. Dried in

* The following are the atomic weights adopted:—C=12, O=16, Hg=200; I have provisionally retained for Ag, Ba, Pb, &c. the lower atomic weights till lately in general use.

the water-oven, it retains one atom of water of crystallization, and has the formula $C_2 H_5 Ag_2 PO_4 + Aq$. It was chiefly by double decomposition with the barium, lead, and silver salts that the compounds presently to be described were formed. The perfect purity of the substances used was established by rigorous experiment; a silver-determination, for instance, in the argentic ethylphosphate used in many of the reactions to be detailed further on, gave the following numbers:—

10·27 grains of the salt dried at 100° C. gave 8·22 grains of Ag Cl. This result corresponds to 60·24 per cent of Ag, while the percentage required by the formula $C_2 H_5, Ag_2, PO_4 + Aq$ is 60·33. The other analyses were equally satisfactory.

I give, in the present communication, a selection of the most interesting of the numerous results obtained during the course of my experimental inquiry. Many points of departure for other researches have occurred—the investigation, for example, of the products, volatile and fixed, of the destructive distillation of the ethylphosphates, and the determination of the varying amount of water of crystallization in several of the salts prepared. I may cite the barium salts as illustrations. Not only do the salts already mentioned exist, namely one containing 6 Aq and the other 1 Aq, but a third compound may be obtained by evaporating at about 50° or 60° C. a saturated solution of the ordinary barium salt, and filtering off the deposited crystals rapidly. The slender pearly plates thus formed are perfectly definite and constant in composition; they probably consist of equal equivalents of the two former salts. Analysis gave the following numbers:—

7·04 grains gave 5·01 grains of Ba, SO_4.

12·88 grains lost at 130° C. 2·5 grains of $H_2 O$.

These results correspond to 41·85 per cent. of Ba, and 19·407 per cent. of $H_2 O$; the formula $2 (C_2 H_5 Ba_2 PO_4) + 7$ Aq demands 42·28 per cent. of Ba, and 19·44 per cent. of $H_2 O$.

Ferric Ethylphosphate.—Equivalent proportions of argentic ethylphosphate and pure crystallized anhydrous ferric chloride were weighed out. The silver salt was mixed with some quantity of hot water, and the ferric chloride, previously dissolved in hot water, then added, the liquid being kept warm for some time. On filtering, a pale yellow liquid was obtained which contained no silver, and the merest trace of chlorine. On heating this liquid to the boiling-point, pale straw-yellow films separated from it: a quantity of these was collected, washed with cold water and with alcohol, and, after having been dried in the water-oven, analyzed with the following results:—

I. ·6115 grm. of the ferric salt gave on combustion with chromate of lead ·296 grm. of CO_2 and ·22 grm. of $H_2 O$.

II. ·475 grm. gave ·142 grm. of $Fe_4 O_3$.

III. ·393 grm. gave ·115 grm. of $Fe_4 O_3$.

IV. ·475 grm. gave ·294 grm. of $Mg_4 P_2 O_7$.

V. 1·317 grm. lost at 150° C. ·134 grm. of $H_2 O$.

These numbers correspond to the formula $(C_2H_5)_3Fe_4 3PO_4+3Aq$, as may be seen in the comparison given below of the theoretical and experimental percentages.

	Theory.		Experiment.			
			I.	II.	III.	IV.
C_6	72	13·38	13·19	—	—	—
H_{21}	21	3·91	3·92	—	—	—
Fe_4	112	20·82	—	20·82	20·56	—
P_3	93	17·28	—	—	—	17·28
O_{15}	240	44·61	—	—	—	—
	538	100·00				

In analysis V. 10·17 per cent. of water were lost by drying the air-dried salt at 150°: the formula above given requires 10·03 per cent. If we allow the formula fe Cl to express the molecule of ferric chloride, giving to the iron in it the atomic weight $\frac{28\times2}{3}=18{\cdot}67$, then the ferric ethylphosphate may be written

$$\left.\begin{matrix}C_2H_5\\ fe'\\ fe'\end{matrix}\right\}PO_4+Aq.$$

Few chemists would now admit such an expression to be anything more than what may be termed an *equivalent* formula, comparable with that of ethylphosphoric acid itself, yet representing one-third only of the true atomic weight of the iron compound. It may, however, be worth while to consider whether there be any mode of arriving at a decision concerning the formula of the ferric ethylphosphate—whether the above simple expression be admissible, or the more complex form

$$\left.\begin{matrix}(C_2H_5)_3\\ Fe^{I.V}\\ Fe^{I.V}\\ Fe^{I.V}\\ Fe^{I.V}\end{matrix}\right\}3PO_4+3Aq.$$

It appeared to me that, if the latter expression be the true one, we ought to be able to replace $\frac{1}{4}$th or $\frac{3}{4}$ths of the iron present by another metal: if the simpler formula be correct, then any other replacement but that of $\frac{1}{2}$ would be impossible, unless indeed we suppose that the very chemical process made use of to effect the replacement causes a coalescence of three atoms of the original salt, in order that one more complex atom of the new mixed compound may be thereby constructed.

With this object in view, several experiments were devised. A solution saturated at 60° and containing a known proportion of ferric and aluminic ethylphosphates was brought to the boiling-point, and the salt thus separated removed by filtration. In other cases absolute alcohol was added to the warm concentrated solution till a part only, often but a small part, of the salt was precipitated. By these methods, and by the action of

mixed ferric and aluminic chlorides upon argentic ethylphosphate, several salts were obtained of constant as well as of definite composition. For in the majority of experiments where the same or slightly varied proportions were employed, the same compound was obtained, even where but a very small portion of the new compound, compared with the quantity formed, was allowed to separate or be precipitated. This observation applies to the salts obtained by ebullition and evaporation, by precipitation with alcohol, and by the action of the mixed ferric and aluminic chlorides upon the argentic ethylphosphate.

When the proportion of aluminium to iron in the mixed ethylphosphates ranged near the ratio 13·7 : 84, the salt first formed gave on analysis results corresponding to the formula

$$\left.\begin{matrix}(C_2H_5)_3\\ Fe_3\\ Al\end{matrix}\right\} 3PO_4 + 3\,Aq.$$

In an analysis where both Al and Fe were determined, the following results were obtained :—

1·056 grm. gave ·241 grm. Fe_4O_3, and ·0515 grm. Al_4O_3.

A comparison of these results with theory gives—

	Theory.	Experiment.
Percentage of Fe	16·04	15·97
Percentage of Al	2·62	2·51

When the ratio 27·4 : 56 was attained, there was no difficulty in obtaining a salt having the formula

$$\left.\begin{matrix}(C_2H_5)_3\\ Fe_2\\ Al_2\end{matrix}\right\} 3PO_4 + 3\,Aq.$$

In one analysis

·83 grm. gave ·1305 grm. Fe_4O_3, and ·081 grm. Al_4O_3.

A comparison of these results with theory gives—

	Theory.	Experiment.
Percentage of Fe	11·18	11·01
Percentage of Al	5·38	5·21

But on further increasing the proportion of aluminium to iron up to 41·1 : 28, no other definite compound could be obtained, though the salt

$$\left.\begin{matrix}(C_2H_5)_3\\ Fe\\ Al_3\end{matrix}\right\} 3PO_4 + 3\,Aq$$

might be reasonably supposed to exist. Yet the two compounds obtained, if from their constancy of composition when prepared in diverse manners (volumetric determinations of iron in both salts were made with nearly the same results as those just given) I am justified in deeming them definite salts, not mixtures, may lead one to conclude the formula for the ferric ethylphosphate

$$\left.\begin{matrix} C_2H_5 \\ fe \\ fe \end{matrix}\right\} PO_4+Aq$$

inadmissible, and the expression

$$\left.\begin{matrix} (C_2H_5)_3 \\ Fe_4 \end{matrix}\right\} 3PO_4+3Aq$$

correct, since we can replace not half only of its iron, but one-fourth also by aluminium—a replacement manifestly impossible with the simpler expression. I am submitting this matter to further scrutiny by an investigation of the mineral phosphates containing not only aluminium and iron, but also calcium &c.

The higher atomic weights of iron still remain to be considered in connexion with these complex salts. If the atom of ferricum be triatomic and therefore $=56$, then the normal ferric ethylphosphate already described is readily represented thus,

$$\left.\begin{matrix} (C_2H_5)_3 \\ Fe''' \\ Fe''' \end{matrix}\right\} 3PO_4+3Aq,$$

a compound which equally well admits the expression

$$\left.\begin{matrix} (C_2H_5)_3 \\ Ffe^{VI} \end{matrix}\right\} 3PO_4+3Aq,$$

where Ffe=112. But with the mixed ferric-aluminic ethylphosphates the case is altered. The compound $(C_2H_5)_3\,Al_2\,Fe_2\,3PO_4+3Aq$ allows us indeed to assume the triatomicity of ferricum and aluminium,

$$\left.\begin{matrix} (C_2H_5)_3 \\ Fe''' \\ Al''' \end{matrix}\right\} 3PO_4+3Aq,$$

though excluding the supposed hexatomic value of these metals; while the other salt described, $(C_2H_5)_3$, Fe_3, Al, $3PO_4+3Aq$, does not allow their triatomicity even; we return in fact to the oldest view, where Fe=28, and is sesquiatomic.

Ferroso-ferric Ethylphosphate.—By acting upon a warm saturated solution of basic ethylphosphate with a solution of mixed ferrous and ferric sulphates, filtering rapidly, and adding to the filtrate strong alcohol till a precipitate begins to separate, a solution is obtained which, after filtration and standing, soon deposits a greenish-white precipitate, slightly crystalline. This salt is constant in composition when prepared under rather widely varied conditions; but if in its preparation the ferric salt preponderate, the normal ferric ethylphosphate will be first precipitated. Perhaps a better way of preparing the new compound than that above given consists in warming a strong solution of ethylphosphoric acid with ferrous hydrate, filtering and adding strong alcohol. The precipitate produced by either method is to be washed with weak alcohol, and dried as rapidly as possible *in vacuo* over sulphuric acid. It contains iron in both conditions, and

gave results, according to the subjoined analytical details, agreeing with the formula

$$\left.\begin{matrix}(C_2H_5)_3\\ Fe^{I.V}\\ Fe^{I.V}\\ Fe'_3\end{matrix}\right\} 3PO_4+3Aq.$$

An identical compound may also be obtained by following the several plans adopted in preparing the ferric ethylphosphate; it is, however, very difficult to prevent a partial oxidation of the ferrosum in the salt. It will be noted that the atomic weight 28 is indicated by the constitution of this compound.

In order to analyze the ferroso-ferric ethylphosphate, the following methods were adopted. In a preliminary examination of the salt it was found that strong mineral acids did not effect the separation of phosphoric acid from it: it was also seen that its acid solution gave the ordinary reactions of both conditions of iron. For analysis a weighed amount of the salt was dissolved in dilute sulphuric acid, and the amount of standard permanganate solution decolorized by it ascertained; this gave the amount of ferrosum in the salt taken. A second experiment was then made, in which the total amount of iron in both forms was determined by permanganate after reduction of the sulphuric solution with pure zinc; the difference between the two percentage results gave the percentage of iron existing as ferricum in the original compound. The numbers thus obtained were on the whole satisfactory; the total amount of iron agreed nearly with that demanded by theory, though the amount of ferrosum in the salt was never less than ·5 per cent. below the required proportion. It was in fact impossible entirely to prevent oxidation of the salt; but it will be perceived that the ferric oxide thus produced, not being lost, introduced but an inconsiderable error into the determinations. Nor did any inconvenience arise from the presence of phosphovinic acid, which, curiously enough, was found to be without reducing action on the permanganate, even in the presence of sulphuric acid. The following results were obtained in the analysis of the ferroso-ferric ethylphosphate.

The ferroso-ferric ethylphosphate was dried *in vacuo* over sulphuric acid. In each experiment with the permanganate solution ·5 gramme of the iron salt was taken. Each cubic centimetre of the permanganate solution corresponded to ·00492 grm. of Fe. The following are the results obtained by this method of analysis, three different preparations of the ethylphosphate being used:—

Before reduction of the ethylphosphate with zinc.	I.	14·4 cub. centims. permang. = ·070848 Fe
	II.	14·2 cub. centims. permang. = ·069864 Fe
	III.	14·0 cub. centims. permang. = ·06888 Fe
	IV.	14·4 cub. centims. permang. = ·070848 Fe
After reduction of the ethylphosphate	V.	25·0 cub. centims. permang. = ·123 Fe
	VI.	24·6 cub. centims. permang. = ·121032 Fe

The mean of the first four experiments gives 14·022 as the percentage amount of ferrosum in the compound. The theory $(C_2H_5)_3(Fe'''')_2(Fe')_3 3PO_4+3Aq$ demands 14·841.

The mean of the last two experiments gives 24·402 as the total percentage of iron in the compound. The theoretical percentage is 24·745. If the experimental mean percentage of ferrosum be deducted from the total mean percentage of iron arrived at by experiment, thus, 24·402−14·022=10·38, the number arrived at gives the percentage of ferricum in the compound; the formula above given requires 9·894 per cent.: thus the experimental percentage exceeds the theoretical by ·486 per cent.—a small error, considering the very great difficulties attending the manipulation of this easily oxidized salt.

Uranylic Ethylphosphate.—Some pure uranic oxide, Ur_4O_3, was prepared by repeatedly acting on uranic nitrate with alcohol, the pasty mixture being heated on a water-bath. When the separation of the oxide was complete, it was mixed at a temperature of about 60° or 70° with a weak solution of ethylphosphoric acid. After dilution with hot water and digestion, the solution was filtered and evaporated. As soon as the boiling-point was attained, the solution almost solidified from the separation of clear yellow gelatinous masses of the new salt. These were collected by filtration of the boiling liquid after a portion of the water had evaporated. Submitted to analysis, they gave results leading to the formula C_2H_5, U_4O_3, PO_4+Aq. This uranic salt thus agrees in constitution with the inorganic uranic phosphates already known, and lends additional support to Péligot's Uranyle theory. The following list gives the formulæ of various uranylic phosphates, uranyle being represented by the expression U_2O:—

U_2O, H_2, PO_4+Aq	Monuranic phosphate.
$2U_2O, H, PO_4+3Aq$ / $2U_2O, H, PO_4+4Aq$	Diuranic phosphate.
$3U_2O, PO_4$ (?)	Triuranic phosphate.
$2U_2O, Ca, PO_4+4Aq$	Diurano-calcic phosphate (lime-uranite).
$C_2H_5, 2U_2O, PO_4+Aq$	Diurano-ethylphosphate.

This new uranic salt, like many other ethylphosphates, is less soluble in water at 100° than in water at 60° or 70°. It separates from its solutions in pale yellow flocks, which dry up in the water-oven into amorphous brittle masses of a bright lemon-yellow colour. Attempts to replace a portion of the uranyle in this salt by calcium and by silver led to no definite results. The following are the analyses of the uranylic ethylphosphate dried at 100°:

I. ·517 grm. gave ·106 grm. of Co_2 and ·0835 grm. of H_2O.
II. ·375 grm. gave ·254 grm. of protosesquioxide of uranium, U_6O_4.
III. ·3335 grm. gave ·226 grm. of U_6O_4.
IV. ·6645 grm. gave ·1735 grm. of $Mg_4P_2O_7$.
V. ·1 grm dried at 100° lost at 150° ·044 grm. of H_2O.

These results correspond to the following percentages:—

	Theory.		Experiment.			
			I.	II.	III.	IV.
C_2	24	5·79	5·59	—	—	—
H_7	7	1·69	1·79	—	—	—
U_4	240	57·97	—	57·33	57·54	—
P.....	31	7·54	—	—	—	7·29
O_7	112	27·01	—	—	—	—
	414	100·00				

The loss of water in analysis V. amounted to 4·4 per cent.; the theory C_2H_5, $2U_2O$, PO_4 + Aq requires 4·35 per cent.

Arsenious Ethylphosphate.—The replacement of the basic hydrogen of ethylphosphoric acid by such an element as arsenic appeared to present some features of interest. The experiment was thus made. To a weighed quantity of pure arsenious chloride in a small flask, an equivalent quantity of anhydrous ethylphosphate of lead was added (in one experiment ethylphosphate of silver). The mixture became warm, and after moderate heating solidified. It was extracted with warm water, and the filtered extract evaporated. Beautiful feathery crystals separated in considerable quantity. Once crystallized from a solution, they appeared to dissolve less readily a second time in water. The cause of this phenomenon was soon discovered. Water gradually decomposes this salt, giving arsenious anhydride and ethylphosphoric acid. Although the analysis of the first crop of crystals was tolerably satisfactory, the original method of preparing the compound was abandoned, and another plan adopted. It was found that ethylphosphoric acid readily dissolves arsenious acid at the boiling-point, and that on heating and evaporating the solution, beautiful crystals of the arsenious ethylphosphate separate. In order to study this reaction more closely, the experiment was repeated, substituting, however, common orthophosphoric for the ethylphosphoric. The arsenious anhydride readily dissolved in considerable quantity on ebullition; and after filtration and cooling, an abundant crop of brilliant crystals was deposited from the filtrate. These crystals were not perceptibly affected by washing with cold water, and proved to be completely volatile when heated in a test-tube over a spirit-lamp. In fact they were nothing but octahedra of arsenious anhydride. Further experiments showed that it was not possible in this manner to form an arsenious phosphate; so that the statement in Gmelin's Handbook, referring to this salt as probably obtainable by the process above given, would seem to require correction. The normal arsenious phosphate, $As PO_4$, remains to be discovered; a peculiar interest consequently attaches to the salt now under review, as the only arsenious phosphate known. Prepared by either of the processes above given, pressed between folds of filtered paper, and dried *in vacuo*, it gave on analysis numbers very nearly agreeing with the expression

$$\left.\begin{matrix}(C_2H_5)_3\\ As^{III}{}_2\end{matrix}\right\} 3\,PO_4.$$

The arsenic in the arsenious ethylphosphate was determined as sulphide, the precipitation being effected according to the directions given by Fresenius. The salt was dried by pressure between folds of filter-paper, it was then placed *in vacuo* over sulphuric acid, and finally heated for a short time in the water-oven. The arsenious sulphide obtained on its analysis was dried at 100° C.

I. 1·347 grm. gave ·639 grm. of As_2S_3.

II. ·591 grm. gave ·29 grm. of As_2S_3.

The formula $(C_2H_5)_3As_2\,3PO_4$ requires 28·74 per cent. of arsenic: analysis I. gave 28·95, while II. gave 29·58, the specimen of salt submitted to analysis in the latter case having been partially decomposed by washing, ethylphosphoric acid being thus removed, and consequently an excess of arsenious anhydride remaining in the residual salt.

The ease with which the arsenious ethylphosphate is formed induced me to hope that similar success would attend experiments made with another triatomic element, phosphorus. When an action is established between terchloride of phosphorus and ethylphosphate of silver, an oily product may be removed from the mass by means of anhydrous ether, but it yields on analysis results less definite than could be wished. Yet the reaction is a promising one: I hope to recur to it shortly, and to experiment in a similar manner with antimony and bismuth compounds.

Ditetrethyliac Ethylphosphate.—The ordinary ethylphosphate of ammonium is very readily made; its aqueous solution becomes acid on evaporation, but the salt may be obtained in a semicrystalline form by drying its saturated solution *in vacuo* over sulphuric acid. The salt heated carefully in an oil-bath for some time loses ammonia as well as water, but yet appears to yield, among other products, ethylphosphamic acid. A different and much more definite kind of decomposition takes place with the compound next to be described.

When a hot solution of argentic ethylphosphate is mixed with a solution of iodide of tetrethylium, a change occurs expressed by the equation

$$C_2H_5,Ag_2,PO_4+2\,[(C_2H_5)_4NI]=C_2H_5,[(C_2H_5)_4N]_2,PO_4+2AgI.$$

If the two salts be employed in the exact proportions indicated by this equation, it will be found that after boiling them together the new compound is contained in the filtrate. On evaporating this liquid first of all at 100°, and then *in vacuo* over sulphuric acid, a syrup, and finally a mass of confused crystals will be obtained; by long drying, these crystals lose their transparency, most probably because they have thus parted with some of their water of crystallization. The salt is intensely soluble in cold water, and deliquescent. The analyses of this salt were not exact, but corresponded sufficiently with the formula of an ethylphosphoric acid in which two atoms of hydrogen had been replaced by two atoms of tetrethylium. This view of its constitution is amply confirmed, not only by the mode in which the salt is prepared, but also by a singular decomposition

which it undergoes when heated. It begins indeed to decompose, though very slightly, at 100°, even when water is present, giving off a distinct odour of triethylamine. But on heating the salt itself to a temperature exceeding 100°, decomposition becomes more rapid, and the substance is finally resolved into triethylamine and triethylic phosphate,

$$C_2H_5,[(C_2H_5)_4N]_2,PO_4=(C_2H_5)_3PO_4+2(C_2H_5)_3N.$$

The triethylamine was analytically identified by a platinum-determination in the double chloride made from it by addition of hydrochloric acid and platinic chloride. The metamorphosis of this ethylphosphate is perhaps more easily seen by means of the following arrangement of its formula:—

$$\left.\begin{array}{l} C_2H_5 \\ \left.\begin{array}{l} C_2H_5 \\ C_2H_5 \\ C_2H_5 \end{array}\right\}N \\ C_2H_5 \\ \left.\begin{array}{l} C_2H_5 \\ C_2H_5 \\ C_2H_5 \end{array}\right\}N \\ C_2H_5 \end{array}\right\}PO_4.$$

Several ethylphosphates have been prepared besides those described in the present paper; most of these salts, however, presented no marked features of interest. The ammonium, nickel, chromic, mercurous, and platinic compounds were investigated more particularly. The mercurous ethylphosphate is somewhat difficult to prepare; it is best made by adding a few drops of mercurous nitrate to a strong solution of potassic ethylphosphate, filtering off the grey precipitate first formed, and then adding a further quantity of the mercurous nitrate in solution; if the solutions are not too concentrated the salt gradually separates in pearly plates. Hot water partially dissolves this salt, the residue becoming yellow, and the solution acquiring a distinct acid reaction. It is slightly soluble in cold water, though not altogether without decomposition; it is insoluble in alcohol. Dissolved in dilute nitric acid and precipitated by chloride of sodium, the air-dried crystals of this salt gave the following result:—

·236 grm. gave ·191 grm. of Hg″ Cl.

This corresponds to 71·09 per cent. of mercury; the formula $C_2H_5Hg''_2PO_4+2$ Aq requires 71·45 per cent. The more probable formula, $C_2H_5Hg''_2PO_4+$Aq, requires 73·82 per cent.

In offering the foregoing results to the Society, I do not wish it to be supposed that I consider them conclusive so far as regards the theoretical considerations introduced into the present paper. It is possible that the various aluminium and iron salts described may be mixtures only, in spite of their apparent constancy of composition; or, again, it may be that their formulæ ought to be doubled or quadrupled. Moreover the constitution of ethylphosphoric acid itself has not been made out: I trust that the study of diethyl-, ethylpyro-, and ethylmeta-phosphoric acids, and of the pro-

ducts of the action of heat on the ethylphosphates, may aid in determining this question. Meanwhile the observation, already recorded, as to the stability of ordinary ethylphosphoric acid and its salts in the presence of permanganate of potassium requires a word or two of further comment. When argentic diethylphosphate is acted upon with iodine, the silver and one atom of ethyl may be removed, and after treatment with finely divided silver and a little oxide of silver to remove any iodine and hydriodic acid, and then with excess of carbonate of barium, an ethylphosphate of barium is obtained, which, unlike the ordinary salt, immediately reduces permanganate of potassium; perhaps the ethyl in this salt exists in a different and less intimate form of combination. I am inclined to think that the permanganates will afford, in some cases, criteria for the detection of slight differences in isomeric compounds, although it would be premature at present to hazard an exact interpretation of the phenomena to which they give rise. I may add that treatment of an ethylphosphate with strong nitric acid fails to decompose the ethylphosphoric acid; so that phosphoric acid cannot thus be separated from this remarkably stable body.

II. "A Dynamical Theory of the Electromagnetic Field." By Professor J. Clerk Maxwell, F.R.S. Received October 27, 1864.

(Abstract.)

The proposed Theory seeks for the origin of electromagnetic effects in the medium surrounding the electric or magnetic bodies, and assumes that they act on each other not immediately at a distance, but through the intervention of this medium.

The existence of the medium is assumed as probable, since the investigations of Optics have led philosophers to believe that in such a medium the propagation of light takes place.

The properties attributed to the medium in order to explain the propagation of light are—

1st. That the motion of one part communicates motion to the parts in its neighbourhood.

2nd. That this communication is not instantaneous but progressive, and depends on the elasticity of the medium as compared with its density.

The kind of motion attributed to the medium when transmitting light is that called transverse vibration.

An elastic medium capable of such motions must be also capable of a vast variety of other motions, and its elasticity may be called into play in other ways, some of which may be discoverable by their effects.

One phenomenon which seems to indicate the existence of other motions than those of light in the medium, is that discovered by Faraday, in which the plane of polarization of a ray of light is caused to rotate by the action

of magnetic force. Professor W. Thomson* has shown that this phenomenon cannot be explained without admitting that there is motion of the luminiferous medium in the neighbourhood of magnets and currents.

The phenomena of electromotive force seem also to indicate the elasticity or tenacity of the medium. When the state of the field is being altered by the introduction or motion of currents or magnets, every part of the field experiences a force, which, if the medium in that part of the field is a conductor, produces a current. If the medium is an electrolyte, and the electromotive force is strong enough, the components of the electrolyte are separated in spite of their chemical affinity, and carried in opposite directions. If the medium is a dielectric, all its parts are put into a state of electric polarization, a state in which the opposite sides of every such part are oppositely electrified, and this to an extent proportioned to the intensity of the electromotive force which causes the polarization. If the intensity of this polarization is increased beyond a certain limit, the electric tenacity of the medium gives way, and there is a spark or "disruptive discharge."

Thus the action of electromotive force on a dielectric produces an electric displacement within it, and in this way stores up energy which will reappear when the dielectric is relieved from this state of constraint.

A dynamical theory of the Electromagnetic Field must therefore assume that, wherever magnetic effects occur, there is matter in motion, and that, wherever electromotive force is exerted, there is a medium in a state of constraint; so that the medium must be regarded as the recipient of two kinds of energy—the actual energy of the magnetic motion, and the potential energy of the electric displacement. According to this theory we look for the explanation of electric and magnetic phenomena to the mutual actions between the medium and the electrified or magnetic bodies, and not to any direct action between those bodies themselves.

In the case of an electric current flowing in a circuit A, we know that the magnetic action at every point of the field depends on its position relative to A, and is proportional to the strength of the current. If there is another circuit B in the field, the magnetic effects due to B are simply added to those due to A, according to the well-known law of composition of forces, velocities, &c. According to our theory, the motion of every part of the medium depends partly on the strength of the current in A, and partly on that in B, and when these are given the whole is determined. The mechanical conditions therefore are those of a system of bodies connected with two driving-points A and B, in which we may determine the relation between the motions of A and B, and the forces acting on them, by purely dynamical principles. It is shown that in this case we may find two quantities, namely, the "reduced momentum" of the system referred to A and to B, each of which is a linear function of the velocities of A and B. The effect of the force on A is to increase the momentum of the system

* Proceedings of the Royal Society June 1856 and June 1861.

referred to A, and the effect of the force on B is to increase the momentum referred to B. The simplest mechanical example is that of a rod acted on by two forces perpendicular to its direction at A and at B. Then any change of velocity of A will produce a force at B, unless A and B are mutually centres of suspension and oscillation.

Assuming that the motion of every part of the electromagnetic field is determined by the values of the currents in A and B, it is shown—

1st. That any variation in the strength of A will produce an electromotive force in B.

2nd. That any alteration in the relative position of A and B will produce an electromotive force in B.

3rd. That if currents are maintained in A and B, there will be a mechanical force tending to alter their position relative to each other.

4th. That these electromotive and mechanical forces depend on the value of a single function M, which may be deduced from the form and relative position of A and B, and is of one dimension in space; that is to say, it is a certain number of feet or metres.

The existence of electromotive forces between the circuits A and B was first deduced from the fact of electromagnetic attraction, by Professor Helmholtz* and Professor W. Thomson†, by the principle of the Conservation of Energy. Here the electromagnetic attractions, as well as the forces of induction, are deduced from the fact that every current when established in a circuit has a certain persistency or momentum—that is, it requires the continued action of an unresisted electromotive force in order to alter its value, and that this "momentum" depends, as in various mechanical problems, on the value of other currents as well as itself. This momentum is what Faraday has called the Electrotonic State of the circuit.

It may be shown from these results, that at every point in the field there is a certain direction possessing the following properties:—

A conductor moved in that direction experiences no electromotive force.

A conductor carrying a current experiences a force in a direction perpendicular to this line and to itself.

A circuit of small area carrying a current tends to place itself with its plane perpendicular to this direction.

A system of lines drawn so as everywhere to coincide with the direction having these properties is a system of lines of magnetic force; and if the lines in any one part of their course are so distributed that the number of lines enclosed by any closed curve is proportional to the "electric momentum" of the field referred to that curve, then the electromagnetic phenomena may be thus stated:—

The electric momentum of any closed curve whatever is measured by the number of lines of force which pass through it.

* Conservation of Force. Berlin, 1847: translated in Taylor's Scientific Memoirs, Feb. 1853, p. 114.

† Reports of British Association, 1848. Phil. Mag. Dec. 1851.

If this number is altered, either by motion of the curve, or motion of the inducing current, or variation in its strength, an electromotive force acts round the curve and is measured by the decrease of the number of lines passing through it in unit of time.

If the curve itself carries a current, then mechanical forces act on it tending to increase the number of lines passing through it, and the work done by these forces is measured by the increase of the number of lines multiplied by the strength of the current.

A method is then given by which the coefficient of self-induction of any circuit can be determined by means of Wheatstone's electric balance.

The next part of the paper is devoted to the mathematical expression of the electromagnetic quantities referred to each point in the field, and to the establishment of the general equations of the electromagnetic field, which express the relations among these quantities.

The quantities which enter into these equations are :—Electric currents by conduction, electric displacements, and Total Currents; Magnetic forces, Electromotive forces, and Electromagnetic Momenta. Each of these quantities being a directed quantity, has three components; and besides these we have two others, the Free Electricity and the Electric Potential, making twenty quantities in all.

There are twenty equations between these quantities, namely Equations of Total Currents, of Magnetic Force, of Electric Currents, of Electromotive Force, of Electric Elasticity, and of Electric Resistance, making six sets of three equations, together with one equation of Free Electricity, and another of Electric Continuity.

These equations are founded on the facts of the induction of currents as investigated by Faraday, Felici, &c., on the action of currents on a magnet as discovered by Oersted, and on the polarization of dielectrics by electromotive force as discovered by Faraday and mathematically developed by Mossotti.

An expression is then found for the intrinsic energy of any part of the field, depending partly on its magnetic, and partly on its electric polarization.

From this the laws of the forces acting between magnetic poles and between electrified bodies are deduced, and it is shown that the state of constraint due to the polarization of the field is such as to act on the bodies according to the well-known experimental laws.

It is also shown in a note that, if we look for the explanation of the force of gravitation in the action of a surrounding medium, the constitution of the medium must be such that, when far from the presence of gross matter, it has immense intrinsic energy, part of which is removed from it wherever we find the signs of gravitating force. This result does not encourage us to look in this direction for the explanation of the force of gravity.

The relation which subsists between the electromagnetic and the electrostatic system of units is then investigated, and shown to depend upon what

we have called the Electric Elasticity of the medium in which the experiments are made (*i. e.* common air). Other media, as glass, shellac, and sulphur have different powers as dielectrics; and some of them exhibit the phenomena of electric absorption and residual discharge.

It is then shown how a compound condenser of different materials may be constructed which shall exhibit these phenomena, and it is proved that the result will be the same though the different substances were so intimately intermingled that the want of uniformity could not be detected.

The general equations are then applied to the foundation of the Electromagnetic Theory of Light.

Faraday, in his "Thoughts on Ray Vibrations"*, has described the effect of the sudden movement of a magnetic or electric body, and the propagation of the disturbance through the field, and has stated his opinion that such a disturbance must be entirely transverse to the direction of propagation. In 1846 there were no data to calculate the mathematical laws of such propagation, or to determine the velocity.

The equations of this paper, however, show that transverse disturbances, and transverse disturbances only, will be propagated through the field, and that the number which expresses the velocity of propagation must be the same as that which expresses the number of electrostatic units of electricity in one electromagnetic unit, the standards of space and time being the same.

The first of these results agrees, as is well known, with the undulatory theory of light as deduced from optical experiments. The second may be judged of by a comparison of the electromagnetical experiments of Weber and Kohlrausch with the velocity of light as determined by astronomers in the heavenly spaces, and by M. Foucault in the air of his laboratory.

Electrostatic units in an electromagnetic unit	310,740,000 metres per second.
Velocity of light as found by M. Fizeau	314,858,000.
Velocity of light by M. Foucault.....	298,000,000.
Velocity of light deduced from aberration	308,000,000.

At the outset of the paper, the dynamical theory of the electromagnetic field borrowed from the undulatory theory of light the use of its luminiferous medium. It now restores the medium, after having tested its powers of transmitting undulations, and the character of those undulations, and certifies that the vibrations are transverse, and that the velocity is that of light. With regard to normal vibrations, the electromagnetic theory does not allow of their transmission.

What, then, is light according to the electromagnetic theory? It consists of alternate and opposite rapidly recurring transverse magnetic disturbances, accompanied with electric displacements, the direction of the electric displacement being at right angles to the magnetic disturbance, and both at right angles to the direction of the ray.

* Phil. Mag. 1846. Experimental Researches, vol. iii. p. 447.

The theory does not attempt to give a mechanical explanation of the nature of magnetic disturbance or of electric displacement, it only asserts the identity of these phenomena, as observed at our leisure in magnetic and electric experiments, with what occurs in the rapid vibrations of light, in a portion of time inconceivably minute.

This paper is already too long to follow out the application of the electromagnetic theory to the different phenomena already explained by the undulatory theory. It discloses a relation between the inductive capacity of a dielectric and its index of refraction. The theory of double refraction in crystals is expressed very simply in terms of the electromagnetic theory. The non-existence of normal vibrations and the ordinary refraction of rays polarized in a principal plane are shown to be capable of explanation; but the verification of the theory is difficult at present, for want of accurate data concerning the dielectric capacity of crystals in different directions.

The propagation of vibrations in a conducting medium is then considered, and it is shown that the light is absorbed at a rate depending on the conducting-power of the medium. This result is so far confirmed by the opacity of all good conductors, but the transparency of electrolytes shows that in certain cases vibrations of short period and amplitude are not absorbed as those of long period would be.

The transparency of thin leaves of gold, silver, and platinum cannot be explained without some such hypothesis.

The actual value of the maximum electromotive force which is called into play during the vibrations of strong sunlight is calculated from Pouillet's data, and found to be about 60,000,000, or about 600 Daniell's cells per metre.

The maximum magnetic force during such vibrations is ·193, or about $\frac{1}{10}$ of the horizontal magnetic force at London.

Methods are then given for applying the general equations to the calculation of the coefficient of mutual induction of two circuits, and in particular of two circles the distance of whose circumferences is small compared with the radius of either.

The coefficient of self-reduction of a coil of rectangular section is found and applied to the case of the coil used by the Committee of the British Association on Electrical Standards. The results of calculation are compared with the value deduced from a comparison of experiments in which this coefficient enters as a correction, and also with the results of direct experiments with the electric balance.

December 15, 1864.

J. P. GASSIOT, Esq., Vice-President, in the Chair.

A letter addressed to the President by Dr. William Farr, F.R.S., was read, as follows:—

General Register Office, Somerset House,
Dec. 2, 1864.

My dear Sir,—The Registrar-General requests that you will do him the favour to present the accompanying copy of the English Life Table to the Royal Society.

It contains some work by Scheutz's Machine, on which a Committee of the Royal Society reported; and the Table is the first national Table which has been constructed, except one for Sweden.

The method I employed I described in the paper which you did me the honour to print in the Transactions. I have extended the method, and have described its application to life and other risks.

I am, &c.,
W. Farr.

The President of the Royal Society.

The following communications were read:—

I. "On the production of Diabetes artificially in animals by the external use of Cold." By Henry Bence Jones, M.D., F.R.S. Received November 16, 1864.

In 1789 Lavoisier wrote:—"La respiration n'est qu'une combustion lente de carbone et d'hydrogène qui est semblable en tout à celle qui s'opère dans une lampe ou dans une bougie allumée; et sous ce point de vue les animaux qui respirent sont des véritables corps combustibles qui brûlent et se consument."

The different degrees of oxidation of different substances in the different parts of the body at different times, forms still, and will long continue to form, one of the largest and most important parts of the animal chemistry of health and of disease.

Notwithstanding all that Professor Liebig has done, the knowledge of the phenomena of oxidation in the body is only at its commencement. Take, for example, a grain of starch. It enters into the body, becomes sugar, is acted on by oxygen, and ultimately passes out as carbonic acid and water. This is the final result of the perfect combustion; but what are the different stages through which the starch passes? what happens if the oxidation stops at any of these stages—that is, when imperfect combustion occurs?

The combustion may be made imperfect in at least three different ways: —First, by insufficient oxygen. Secondly, by overwhelming fuel. Thirdly, by reducing the temperature so low that chemical action is checked.

From each of these causes the following scale of the combustion of starch in the body may be formed.

When there is perfect combustion, then carbonic acid and water are produced. With less perfect combustion, oxalic and other vegetable acids are formed. With the least possible combustion sugar results.

Between perfect combustion and the most imperfect combustion (that is, between carbonic acid and sugar) there are probably many steps, formed by many different acids; and as in a furnace one portion of the coal may be fully burnt, whilst other portions are passing through much less perfect combustions, or are not burnt at all, so different portions of starch may reach different steps in the scale of combustion, and sugar, acetic acid, oxalic acid, carbonic acid, and many other acids between acetic and oxalic acid may be simultaneously produced.

From this account of the oxidation of starch, it follows that sugar should always be found in the urine whenever any of the three causes mentioned reduce the oxidation in the system to its minimum. In other words, by stopping the combustion that occurs in the body, diabetes should be produced artificially.

It has long been known that an excess of sugar taken into the blood by injection causes temporary diabetes. This is imperfect combustion from excess of the combustible substance.

The diabetes of old age, of pregnancy, and after the inhalation of chloroform, may be considered as arising from imperfect combustion in consequence of a deficiency of oxygen. Bernard's diabetes from injury of the floor of the fourth ventricle probably belongs to this cause.

The third mode of checking the chemical actions in the body is by reducing the temperature. This has not yet been proved to cause diabetes, though it ought as surely to stop oxidation as excess of fuel or insufficiency of oxygen.

The simplest experiment consists in placing an animal in ice. The cold soon deprives it of feeling, and perfect insensibility is produced. My friend Dr. Dickinson undertook to give me the urine of rabbits before they were placed in ice, and after they had died from the effect of the cold.

Experiment 1.—This lasted one hour and twenty-three minutes. The cold was very carefully applied; fresh ice was added from time to time. The temperature in the rectum fell from 103° F. to 73° F. In the liver immediately after death the temperature was 76° F. The urine made immediately before the application of cold gave no perceptible trace of sugar with sulphate of copper and liquor potassæ. The urine collected after death gave marked reduction with these reagents, and when boiled with liquor potassæ alone it deepened in colour. The acid reaction also was distinctly more

marked in this urine than in that made before the application of cold. The total quantity of urine obtained after death was between two and three drachms.

Experiment 2.—This lasted seven hours and a half, in consequence of an interruption caused by the melting of nearly all the ice surrounding the rabbit. The temperature fell from 101° F. in the mouth to 69° F. after death. The urine made before the application of cold contained no sugar. The quantity of urine obtained after death was so small that I was unable to prove to my own satisfaction that sugar was present in it.

Experiment 3.—This lasted four hours and five minutes. The temperature at the commencement was 101° F. in the mouth. The urine made before the icing was alkaline from fixed alkali. It did not give any trace of sugar, and when mixed with yeast and put in a warm place it rapidly putrefied. Its specific gravity was 1014. The urine obtained after death was strongly acid, and contained crystals of oxalate of lime. It gave a plentiful reduction of oxide of copper when boiled with sulphate of copper and liquor potassæ. When boiled with liquor potassæ alone, it deepened markedly in colour. When mixed with yeast it quickly fermented most distinctly. The specific gravity was 1020.

I sent my results to Professor Brücke, and I asked him to repeat my experiments, and I have received from him the following account written in June 1864.

"The urine of rabbits always shows small quantities of sugar. No regimen could make it disappear entirely. Hay, grass, carrots; hay and grass mixed, and absolute fasting were tried, but all showed traces of sugar in the urine.

"The quantity of urine obtained was too little to allow of the fermentation test being successful, but by using the lead process, the potash, the bismuth, and the copper test gave very feeble evidence of sugar. The experiments agreed so well, that I can have no doubt that sugar is always present. The rabbit proved to be pregnant, and as I had found that in the last stage of pregnancy sugar is often increased in the urine, I thought this might account for the sugar in the urine; so I took a strong male rabbit, and then again I found that traces of sugar occur normally in the urine of rabbits as in the urine of man.

"Then I repeated your experiment. Notwithstanding what I had found, the results were striking. The quantity of sugar in the urine after freezing was incomparably greater (war ganz unvergleichlich gross).

"The urine after freezing differed also from the urine of health in the fact that, after precipitating with acetate of lead and filtering, the clear fluid gave a very small precipitate with basic acetate of lead; but on the addition of ammonia, a plentiful white precipitate fell and deposited itself, leaving the fluid clear.

"I have not made any quantitative experiments; they are not much to be

trusted, and the difference between the urine of rabbits killed by cold and all other rabbit's urine is so great that it is most distinctly perceptible (auf das Entschiedenste fühlbar) without any quantitative experiments."

II. "On the Action of Chloride of Iodine upon Organic Bodies." By Maxwell Simpson, M.B., F.R.S. Received November 22, 1864.

In a former communication * I stated to the Society that chloride of iodine combines directly with ethylene and propylene gases. I have since ascertained that it also directly combines with those radicals which are at the same time both mono- and tri-atomic. Iodide of allyle and bromide of aldehydene, having the desired atomicity, were the bodies I selected for my experiments.

Action of Chloride of Iodine on Iodide of Allyle.—In order to determine the union of these bodies, it was simply necessary to mix and agitate them. The chloride of iodine used was in the form of a watery solution. During the agitation the mixture became warm, and assumed a dark colour from the liberation of iodine. To complete the reaction, it was gently heated over the lamp for a short time. By these means a dark oily liquid was obtained, which was separated from the excess of chloride of iodine, washed with dilute potash, then with pure water, and distilled. Almost the entire liquid passed over between 190° and 215° Cent. The fraction distilling between 205 and 210 being very considerable in quantity, was collected separately and analyzed, having been previously decolorized by agitation with mercury. The numbers obtained correspond sufficiently well with the formula $C_6 H_5 Cl_2 I$, as will be seen from the following Table :—

	Theory.		Experiment.
		Per cent.	
C_6	36·00	15·06	15·49
H_5	5·00	2·09	2·25
Cl_2	71·00	29·70	..
I	127·00	53·15	..
	239·00	100·00	

The excess of carbon is accounted for by the slight decomposition suffered by the liquid during distillation, which was evidenced by the liberation of iodine.

This body I propose to call iodo-bichlor-allyle. It is a colourless oil, is insoluble in water, and has a sweet and biting taste. The following equation will explain the reaction which generates this body; it is a case of addition and double decomposition, $C_6 H_5 I + 2\,Cl\,I = C_6 H_5 Cl, Cl\,I + I\,I$. One equivalent of chloride of iodine converts, by double decomposition, the

* Proceedings, vol. xii. p. 278.

iodide into the chloride of allyle; and with this the other equivalent then directly combines.

Action of Chloride of Iodine on Bromide of Aldehydene.—A mixture of 1 volume of bromide of aldehydene and 2 volumes of chloride of iodine was exposed in a sealed tube to the temperature of 100° Cent. for about two hours. The tube was then opened, the excess of chloride of iodine removed, and the dark-coloured oil which was formed washed with dilute potash and distilled. The entire liquid passed over between 175° and 200° Cent., suffering at the same time slight decomposition with liberation of iodine. The portion distilling between 190° and 200° was collected apart, deprived of free iodine by agitation with mercury, and analyzed. The numbers obtained approach sufficiently near the formula $C_4 H_3 Br Cl I$ to leave no doubt as to its being the true one:—

	Theory.	Per cent.	Experiment.
C_4	24·0	8·90	9·47
H_3	3·0	1·10	1·43
Br	80·0		
Cl	35·5	90·00	91·00
I	127·0		
	269·5	100·00	

This body I suppose I must call iodo-chlor-brom-aldehydene. It is a colourless oil, is insoluble in water, and, like the former body, has a sweet and biting taste. It is formed by the direct addition of one atom of chloride of iodine to one atom of bromide of aldehydene.

I have also tried the action of chloride of iodine on cyanide of allyle, in the expectation of forming the body $C_8 H_5 Cy Cl I$, and from this, by the action of potash, the acid $C_8 H_8 O_8$. My expectations, however, were not realized. On heating these substances together as in the former cases, all the iodine of the chloride of iodine was set free; and I obtained on evaporating the liquor, instead of an oil, a mixture of chloride of ammonium and an organic acid, probably crotonic.

The solution of chloride of iodine employed in these and my former experiments on ethylene and propylene gases, was made by conducting a stream of chlorine into 1600 grains of water holding in suspension 700 grains of iodine. The mixture was kept cold and agitated repeatedly during the passage of the gas, which was interrupted when all the iodine, with the exception of a trace, had been dissolved.

It is highly probable that bromide of acetylene ($C_4 H_2 Br_2$) and similar non-saturated bodies may be also made to combine directly with chloride of iodine.

III. On Fermat's Theorem of the Polygonal Numbers, with Supplement. By the Right Hon. Sir Frederick Pollock, F.R.S. Received December 5, 1864.

(Abstract.)

This paper (with its Supplement) proposes a proof of the first two theorems of Fermat, relating to the polygonal numbers, viz. that every number is composed of not exceeding three triangular numbers, and not exceeding four square numbers. And this is done by a method entirely new, founded on the properties of the triangular numbers and the square numbers, and the relation they bear to each other, and on the expansion of an algebraical expression of three members into a *line*, a *square*, and a *cube*, so as to obtain every possible value of the whole expression; and throughout the proof every number or term in a series (except in the Table) is expressed by the roots of the squares that compose it, and the *roots only* are dealt with, and *not* the *numbers* or the *squares* that compose them; a Table is constructed from the triangular numbers, *thus* (see opposite page).

Mode of constructing the Table.

The series of triangular numbers is in the centre of the Table. Below that series the adjoining terms are united, and they form the square numbers 1, 4, 9, &c.; the next adjoining terms are united, and they form the next row, and so on.

Above the triangular numbers each term is doubled, forming the series above, and then the adjacent terms are added together and form the next row, and so on; the differences above are 1, 3, 5, 7, 9, &c. $(2n+1)$, and below are 2, 4, 6, 8, 10, &c. $(2n)$.

From the examination of which Table it appears that the sum of any two triangular numbers, however remote from each other in the series, is always a square number plus a double triangular number; that is,

$$\frac{a^2+a}{2}+\frac{b^2+b}{2}=c^2+c+d^2;$$

and the difference between the sum of two triangular numbers and the sum of some other two triangular numbers may be any number whatever, odd or even, positive or negative. The first of these propositions is mentioned and proved algebraically in the Philosophical Transactions of the year 1861, p. 412; the result is perhaps more clear when presented in a tabular form as above (but more extensively and at large in the paper); it is obviously capable of strict algebraical proof. These two propositions, and the expansion of an algebraical expression into a line, a square, and a cube (exhausting every possible value of the expression), are the foundation of the whole proof, which, in addition to proving the first and second theorems of Fermat, proves also that every odd number has in some form or other

		27 9	31 9	37 9	45 9	55 9	67 9	81 9	97 9	115 9	135 9
6. And so on	....	18 7	22 7	28 7	36 7	46 7	58 7	72 7	88 7	106 7	126 7
5. Sum of the next	....	11 5	15 5	21 5	29 5	39 5	51 5	65 5	81 5	99 5	119 5
4. Sum of those next to the others	....	6 3	10 3	16 3	24 3	34 3	46 3	60 3	76 3	94 3	114 3
3. Sum of the next adjoining triangular numbers	..	3 1	7 1	13 1	21 1	31 1	43 1	57 1	73 1	91 1	111 1
2. Double of the triangular numbers ..	0	2	6	12	20	30	42	56	72	90	110
**1. Series of *Triangular Numbers*	**0**	**1**	**3**	**6**	**10**	**15**	**21**	**28**	**36**	**45**	**55**

2. Square numbers (sum of adjoining triangular numbers).....	..	1 2	4 2	9 2	16 2	25 2	36 2	49 2	64 2	81 2	100 2
3. Sum of the next adjoining numbers..	..	3 4	6 4	11 4	18 4	27 4	38 4	51 4	66 4	83 4	102 4
4. Sum of the next	..	7 6	10 6	15 6	22 6	31 6	42 6	55 6	70 6	87 6	106 6
5. Sum of the next	..	13 8	16 8	21 8	28 8	37 8	48 8	61 8	76 8	93 8	112 8
6. And so on.....	..	21 10	24 10	29 10	36 10	45 10	56 10	69 10	84 10	101 10	120 10
		31	34	39	46	55	66	79	94	111	130

of the roots, two roots equal, and also in some form two roots differing by 1; also that in some form of the roots the algebraic sum of the roots will be equal to 1.

If $a^2+a+b^2-(m^2+m+n^2)$ be equal to any number whatever, odd or even, it must equal a number represented by $p^2-(c^2+c)$; and as

$$a^2+a+b^2-(m^2+m+n^2)=p^2-(c^2+c),$$
$$\therefore\ a^2+a+b^2+c^2+c=m^2+m+n^2+p^2.$$

These two expressions are equivalent to each other, and any number which is of the one form is also of the other form; and if they be doubled and 1 be added to each, they will become

$$2a^2+2a+1+2b^2+2c^2+2c,\ 2m^2+2m+1+2n^2+2p^2,$$

and either of them will represent any odd number whatever. For $a^2+a+b^2-(m^2+m+n^2)$ not only equals $p^2-(c^2+c)$, but it also equals $p^2-(c^2+c)+q$; and if both be doubled and 1 be added,

$$2m^2+2m+1+2n^2+2p^2+2q=2a^2+2a+1+2b^2+2c^2+2c;$$

if therefore to either form any even number $(2q)$ be added, it is still of the form of the other, and therefore still of its own form, that is,

$$2m^2+2m+1+2n^2+2p^2+2q$$

is still of the form $2m^2+2m+1+2n^2+2p^2$, and that form therefore represents any odd number.

It is shown in the paper that when $2a^2+2a+1$, $2b^2$, $2c^2+2c$ is expanded, $2a^2+2a+1$ becomes a series (by the addition of 4, 8, 12, &c.) whose terms will be 0, 0, 0, 1; 0, 0, 1, 2; 0, 0, 2, 3, &c., and may be considered as a line whose general expression is $0, 0, a, (a+1)$.

When $2b^2$ is added to each term by the addition of 2, 6, 10, 14, &c. it becomes a square whose general term is $b, b, a, a+1$ (these being roots whose squares added together form the term in the square). Lastly, when $2c^2+2c$ is added (by decreasing a and increasing $a+1$, 1 each time) and the square becomes a cube, every term has two roots equal, but is composed of not exceeding 4 square numbers; and as on the addition to any term of any even number $(2q)$ the term so increased will still be within the cube (extended indefinitely), the cube will contain every odd number; but if $2m^2+2m+1+2n^2$ be formed into a square, and then by the addition of $2p^2$ be raised into a cube (the terms n, n in each term being one increased, and the other diminished by 1), every term in the cube will have two roots differing by 1, and will be composed of not exceeding 4 square numbers; and this cube also will contain every odd number for the same reason that the other will.

Supplement.

Lastly, $a^2+a+b^2-(m^2+m+n^2)$ will (as it equals any number) equal either $p^2-\frac{c^2+c}{2}$ or $p^2-\frac{c^2+c}{2}+q$, and therefore $2a^2+2a+1$, $+2b^2+c^2+c$ will equal $2m^2+2m+1+2n^2+2p^2$ with or without $2q$.

In raising $a, a+1, b, b$ to a cube by adding c^2+c, it must be by the

addition of 2, 4, 6, 8, 10, &c., which must be added alternately to each; 2, 6, 10, 14, &c. to b, b, and 4, 8, 12, 16, &c. to $a, a+1$; but the effect of this alternate addition of 2, 6, 10, &c. to b, b, by increasing one of them by 1 and diminishing the other, and of 4, 8, 12, &c. to $a, a+1$ by decreasing each time by 1 and increasing $a+1$ by 1, is to make the algebraic sum of the four roots at all times equal to 1, as is distinctly shown in the paper; and if $2a^2+2a+1+2b^2+c^2+c$ will represent any odd number, then $2a^2+2a+1+2b^2+c^2+c=2n+1$, deducting 1 and dividing by 2.

$a^2+a+b^2+\frac{c^2+c}{2}=n$, and as a^2+a+b^2 equals the sum of 2 triangular numbers and $\left(\frac{c^2+c}{2}\right.$ is a triangular number$\left.\right)$, therefore every number is composed of not exceeding three triangular numbers.

IV. "On the Structure and Affinities of *Eozoon Canadense*." In a Letter to the President. By W. B. Carpenter, M.D., F.R.S. Received December 14, 1864.

I cannot doubt that your attention has been drawn to the discovery announced by Sir Charles Lyell in his Presidential Address at the late Meeting of the British Association, of large masses of a fossil organism referable to the Foraminiferous type, near the base of the Laurentian series of rocks in Canada. The geological position of this fossil (almost 40,000 feet beneath the base of the Silurian system) is scarcely more remarkable than its zoological relations; for there is found in it the evidence of a most extraordinary development of that Rhizopod type of animal life which at the present time presents itself only in forms of comparative insignificance —a development which enabled it to separate carbonate of lime from the ocean-waters in quantity sufficient to produce masses rivalling in bulk and solidity those of the stony corals of later epochs, and thus to furnish (as there seems good reason to believe) the materials of those calcareous strata which occur in the higher parts of the Laurentian series.

Although a detailed account of this discovery, including the results of the microscopic examinations into the structure of the fossil which have been made by Dr. Dawson and myself, has been already communicated to the Geological Society by Sir William E. Logan, I venture to believe that the Fellows of the Royal Society may be glad to be more directly made acquainted with my view of its relations to the types of Foraminifera which I have already described in the Philosophical Transactions.

The massive skeletons of the Rhizopod to which the name *Eozoon Canadense* has been given, seem to have extended themselves over the surface of submarine rocks, their base frequently reaching a diameter of 12 inches, and their thickness being usually from 4 to 6 inches. A vertical section of one of these masses exhibits a more or less regular alternation of calcareous and

siliceous layers, these being most distinct in the basal portion. The specimens which the kindness of Sir William E. Logan has given me the opportunity of examining, are composed of carbonate of lime alternating with serpentine—the calcareous layers being formed by the original skeleton of the animal, whilst the serpentine has filled up the cavities once occupied by its sarcode-body. In other specimens the carbonate of lime is replaced by dolomite, and the serpentine by pyroxene, Loganite, or some other mineral of which silex is a principal constituent. The regular alternation of calcareous and siliceous layers which is characteristic of the basal portion of these masses, frequently gives place in the more superficial parts to a mutual interpenetration of these minerals, the green spots of the serpentine being scattered over the surface of the section, instead of being collected in continuous bands, so as to give it a granular instead of a striated aspect. This difference we shall find to depend upon a departure from the typical plan of growth, which often occurs (as in other Foraminifera) in the later stages—the minute chambers being no longer arranged in continuous tiers, but being piled together irregularly, in a manner resembling that in which the *cancelli* are disposed at the extremities of a long bone.

The minute structure of this organism may be determined by the microscopic examination either of thin transparent sections, or of portions which have been submitted to the action of dilute acid, so as to remove the calcareous shell, leaving only the siliceous *casts* of the chambers and other cavities originally occupied by the substance of the animal. Each of these modes of examination, as I have shown on a former occasion*, has its peculiar advantages; and the combination of both, here permitted by the peculiar mode in which the *Eozoon* has become fossilized, gives us a most complete representation not only of the skeleton of the animal, but of its soft sarcode-body, and its minute pseudopodial extensions as they existed during life. In well-preserved specimens of *Eozoon*, the shelly substance often retains its characters so distinctly, that the details of its structure can be even more satisfactorily made out than can those of most of the comparatively modern Nummulites. And even the hue of the original sarcode seems traceable in the canal-system; so exactly does its aspect, as shown in transparent sections, correspond with that of similar canals in recent specimens of *Polystomella*, *Calcarina*, &c. in which the sarcode-body has been dried.

This last circumstance appears to me to afford a remarkable confirmation of the opinion formed by Mr. Sterry Hunt upon mineralogical grounds—that the siliceous infiltration of the cavities of the *Eozoon* was the result of changes occurring *before* the decomposition of the animal. And the extraordinary completeness of this infiltration may be the result (as was suggested by Professor Milne-Edwards with regard to the infiltration of fossil bones and teeth, in the course of the discussion which took place last year on the Abbeville jaw) of the superiority of the process of

* Memoir on *Polystomella* in Phil. Trans. for 1860, pp. 538, 540.

substitution, in which the animal matter is replaced (particle by particle) by some mineral substance, over that of mere *penetration*.

The *Eozoon* in its living state might be likened to an extensive range of building made up of successive tiers of chambers, the chambers of each tier for the most part communicating very freely with each other (like the secondary chambers of *Carpenteria**, so that the segments of the sarcodic layer which occupied them were intimately connected, as is shown by the continuity of their siliceous models. The proper walls of these chambers are everywhere formed of a pellucid vitreous shell-substance minutely perforated with parallel tubuli, so as exactly to correspond with that of *Nummulites*, *Cycloclypeus*, and *Operculina*†; and even these minute tubuli are so penetrated by siliceous infiltration, that when the calcareous shell has been removed by acid, the internal casts of their cavities remain in the form of most delicate needles, standing parallel to one another on the solid mould of the cavity of the chamber, over which they form a delicate filmy layer.

But, between the proper walls of the successive tiers of chambers, there usually intervene layers of very variable thickness, composed of a homogeneous shell-substance; and these layers represent the "intermediate" or "supplemental" skeleton which I have described in several of the larger Foraminifera, and which attains a peculiar development in *Calcarina*‡. And, as in *Calcarina* and other recent and fossil Foraminifera, this "intermediate skeleton" is traversed by a "canal-system"§ that gave passage to the prolongations of the sarcode-body, by the agency of which the calcareous substance of this intermediate skeleton seems to have been deposited. The distribution of this canal-system, although often well displayed in transparent sections, is most beautifully shown (as in *Polystomella* ‖) by the siliceous *casts* which are left after the solution of the shell, these casts being the exact models of the extensions of the sarcode-body that originally occupied its passages.

In those portions of the organism in which the chambers, instead of being regularly arranged in floors, are piled together in an "acervuline" manner, there is little trace either of "intermediate skeleton" or of "canal-system"; but the characteristic structure of their proper walls is still unmistakeably exhibited.

Whilst, therefore, I most fully accord with Dr. Dawson in referring the *Eozoon Canadense*, notwithstanding its massive dimensions and its zoophytic mode of growth, to the group of Foraminifera, I am led to regard its immediate affinity as being rather with the *Nummuline* than with the *Rotaline* series—that affinity being marked by the structure of the proper wall of the chambers, which, as I have elsewhere endeavoured to show ¶,

* Phil. Trans. 1860, p. 566.
† Ibid. 1856, p. 558, and pl. xxxi. figs. 9 & 10.
‡ Ibid. 1860, p. 553.
§ Ibid. 1860, p. 554, plate xx. fig. 3.
‖ Ibid. 1860, plate xviii. fig. 12.
¶ Introduction to the Study of the Foraminifera, chap. iii.

is a character of primary importance in this group, the plan of growth and the mode of communication of the chambers being of secondary value, and the disposition of the "intermediate skeleton" and its "canal-system" being of yet lower account.

I cannot refrain from stopping to draw your attention to the fact, that the organic structure and the zoological affinities of this body, which was at first supposed to be a product of purely physical operations, are thus determinable by the microscopic examination of an area no larger than a pin-hole—and that we are thus enabled to predicate the nature of the living action by which it was produced, at a geological epoch whose remoteness in *time* carries us even beyond the range of the imagination, with no less certainty than the astronomer can now, by the aid of "spectrum analysis," determine the chemical and physical constitution of bodies whose remoteness in *space* alike transcends our power to conceive.

The only objections which are likely to be raised by palæontologists to such a determination of the nature of *Eozoon*, would be suggested by its zoophytic mode of growth, and by its gigantic size. The first objection, however, is readily disposed of, since I have elsewhere shown * that a minute organism long ranked as zoophytic, and described by Lamarck under the designation *Millepora rubra*, is really but an aberrant form of the *Rotaline* family of Foraminifera, its peculiarity consisting only in the mode of increase of its body, every segment of which has the characteristic structure of the *Rotaline*; and thus, so far from presenting a difficulty, the zoophytic character of *Eozoon* leads us to assign it a place in the *Nummuline* series exactly corresponding to that of *Polytrema* in the *Rotaline*. And the objection arising from the size and massiveness of *Eozoon* loses all its force when we bear in mind that the increase of Foraminifera generally takes place by gemmation, and that the size which any individual may attain mainly depends (as in the Vegetable kingdom) upon the number of segments which bud *continuously* from the original stock, instead of detaching themselves to form independent organisms; so that there is no essential difference save that of continuity, between the largest mass of *Eozoon* and an equal mass made up of a multitude of *Nummulites*. Moreover there is other evidence that very early in the Palæozoic age the Foraminiferous type attained a development to which we have nothing comparable at any later epoch; for it has been shown by Mr. J. W. Salter † that the structure of the supposed coral of the Silurian series to which the name *Receptaculites* has been given, so closely corresponds with that which I have demonstrated in certain forms of the *Orbitolite* type ‡, as to leave no doubt of their intimate relationship, although the disks of *Receptaculites* sometimes attain a diameter of 12 inches, whilst that of the largest *Orbitolite* I have seen does not reach

* Introduction to the Study of the Foraminifera, p. 235.

† Canadian Organic Remains. Decade i. ‡ Phil. Trans. 1855.

$\frac{8}{10}$ths of an inch. And it is further remarkable in this instance, that the gigantic size attained by *Receptaculites* proceeds less from an extraordinary multiplication of segments, than from such an enormous development of the individual segments as naturally to suggest grave doubts of the character of this fossil, until the exactness of its structural conformity to its comparatively minute recent representative had been worked out.

In a private communication to myself, Dr. Dawson has expressed the belief that *Stromatopora* and several other reputed corals of the Palæozoic series will prove in reality to be gigantic Zoophytic Rhizopods, like *Eozoon* and *Receptaculites*; and I have little doubt that further inquiry will justify this anticipation. Should it prove correct, our ideas of the importance of the Rhizopod type in the earlier periods of geological history will undergo a vast extension; and many questions will arise in regard to its relations to those higher types which it would seem to have anticipated.

In the present state of our knowledge, however, or rather of our ignorance, I think it better to leave all such questions undiscussed, limiting myself to the special object of this communication—the application of my former Researches into the Minute Structure of the Foraminifera, to the determination of the nature and affinities of the oldest type of Organic Life yet known to the geologist.

December 22, 1864.

Dr. WILLIAM ALLEN MILLER, Treasurer and Vice-President, in the Chair.

The following communications were read :—

I. "On the Functions of the Fœtal Liver and Intestines." By Robert James Lee, B.A. Cantab., Fellow of the Cambridge Philosophical Society. Communicated by Robert Lee, M.D. Received November 1, 1864.

(Abstract.)

After some introductory remarks, the author observes that the intestines of the fœtus, between the fifth and ninth months of intra-uterine life, "contain a flocculent substance of orange-pink colour in the duodenal portion, a slimy green substance (the meconium) in their lower portion.

"The nature of the former was ascertained by Dr. Prout to be highly albuminous and nutritious. The nature of the latter, of which Dr. John Davy has given a complete analysis (Trans. Med. Chir. Soc.), is simply excrementitious. In the intermediate portion of the intestine the substance is of intermediate character; the more nutritive its property, the higher its position in the intestine."

He next points out that the mesenteric glands which belong to the

duodenum are most numerous, and that they diminish towards the lower portion of the intestine; so that they are in greatest number where the intestine contains most nutritive substance.

"From this examination," the author continues, "no further proof is required that digestion and absorption are performed, as Harvey believed, during fœtal life.

"The origin of the albuminous substance in the intestine was supposed by Harvey to be the liquor amnii, which he attempts to prove is swallowed by the fœtus *in utero*.

"In the Bird, as will be seen, the origin of this albuminous substance was ascribed by John Hunter to the yelk-sac.

"In the year 1829 it was shown by Dr. Robert Lee, in a paper published in the Philosophical Transactions, 'On the Functions of the Fœtal Liver and Intestines,' that Harvey's explanation was not correct, and that there is satisfactory evidence to prove that the *Liver* is the source of this albuminous substance."

In the fœtal bird on the twelfth day of incubation, or later, "the liver is seen to occupy both sides of the abdomen, as in the human fœtus. The yelk-sac is seen fixed to the small intestine; the white more than half absorbed. The umbilical vein receives blood from the chorion membrane, in which it has been exposed to the influence of the oxygen of the atmosphere; it receives blood also from the yelk-sac and from the white. So that the nature of the blood in the portal vein of the fœtal bird is both highly nutritious and arterial in character.

"The intestines are in the same condition as in the human fœtus.

"The origin of the albuminous substance may be ascertained to be from the same organ, namely, from the liver.

"That John Hunter was mistaken in supposing that albumen passed through the vitelline duct (that part of the yelk-sac which is connected with the intestine), is generally allowed; and his supposition may be almost disproved by the fact that it is not possible to inject any fluid into the yelk-sac from the intestine. Besides, Hunter states that it passes through only at the time of hatching, which is not the case, as the intestines are full long before the bird is hatched. The lacteals of the fœtal bird cannot be seen.

"To take another class of animals, the Marsupialia. The liver in the fœtal kangaroo at the time of birth (that is, in the sixth week of utero-gestation), in the words of Professor Owen, 'consisted of two equal and symmetrically diposed lobes' (Art. 'Marsupialia,' Cyclopæd. Anat. and Physiology). As soon, however, as the mode of life is changed and the umbilical vein closed, the liver begins to diminish in size. Yet there is this resemblance between a Marsupial animal five or six months old, and a human fœtus of the same age *in utero*, that, although the source of nutrition is different, yet the intestines are supplied with nutritive substance, and digestion proceeds in both cases alike, the nutritive substance

in one case being derived from the placenta, in the other from the mamma of the mother kangaroo.

"From the foregoing facts certain conclusions may be drawn.

"1. With regard to the placenta.

"Since the organs of the fœtal bird are in the same condition as in the human fœtus, the nature of the blood supplied to them is probably the same. If so, the umbilical vein of the human fœtus contains blood highly nutritious and arterial in character, and therefore the function of the placenta corresponds to that of the chorion membrane, yelk, and white combined; it is in fact the means of absorption, as the veins absorb the yelk and white, and the substitute of the lung in adult life. There is no need of lymphatic vessels in the placenta.

"2. With regard to the liver.

"That the function of this organ is to separate a highly nutritious substance from the blood of the portal vein; and this is true both of the liver of the fœtal bird and of the human fœtus.

"3. That this albuminous substance is not in a condition to be directly absorbed from the umbilical vein, but is elaborated and separated for absorption by the lacteal vessels.

"4. That there is reason to believe that this function of the liver continues to a great extent during adult life; for the portal vein in that state receives veins which correspond to the umbilical vein in the fact that they proceed from the source of nutrition. That the liver must be actively engaged after the introduction of food into the intestinal canal, and its secretion then more plentiful than at other times."

II. "Completion of the Preliminary Survey of Spitzbergen, undertaken by the Swedish Government with the view of ascertaining the practicability of the Measurement of an Arc of the Meridian." In a letter addressed to Major-General Sabine by Captain C. Skogman, of the Royal Swedish Navy: dated Stockholm, Nov. 21, 1864. Communicated by the President. Received December 15, 1864.

"On the receipt of your letter of the 12th of November, I started immediately in quest of Professor Nordenskjöld, to obtain from him the materials for the fulfilment of your wishes in respect to the Spitzbergen Expedition. The Professor, with his usual obliging frankness, at once complied with my request, and communicated to me the Minutes from which I have compiled the subjoined brief Report of his proceedings. You must excuse the hasty manner in which the Report itself, as well as the accompanying map, has been put together, as time presses if my letter has to reach you before your Anniversary on the 30th. The map has no pretensions to exactness, but must be viewed merely in the light of a dia-

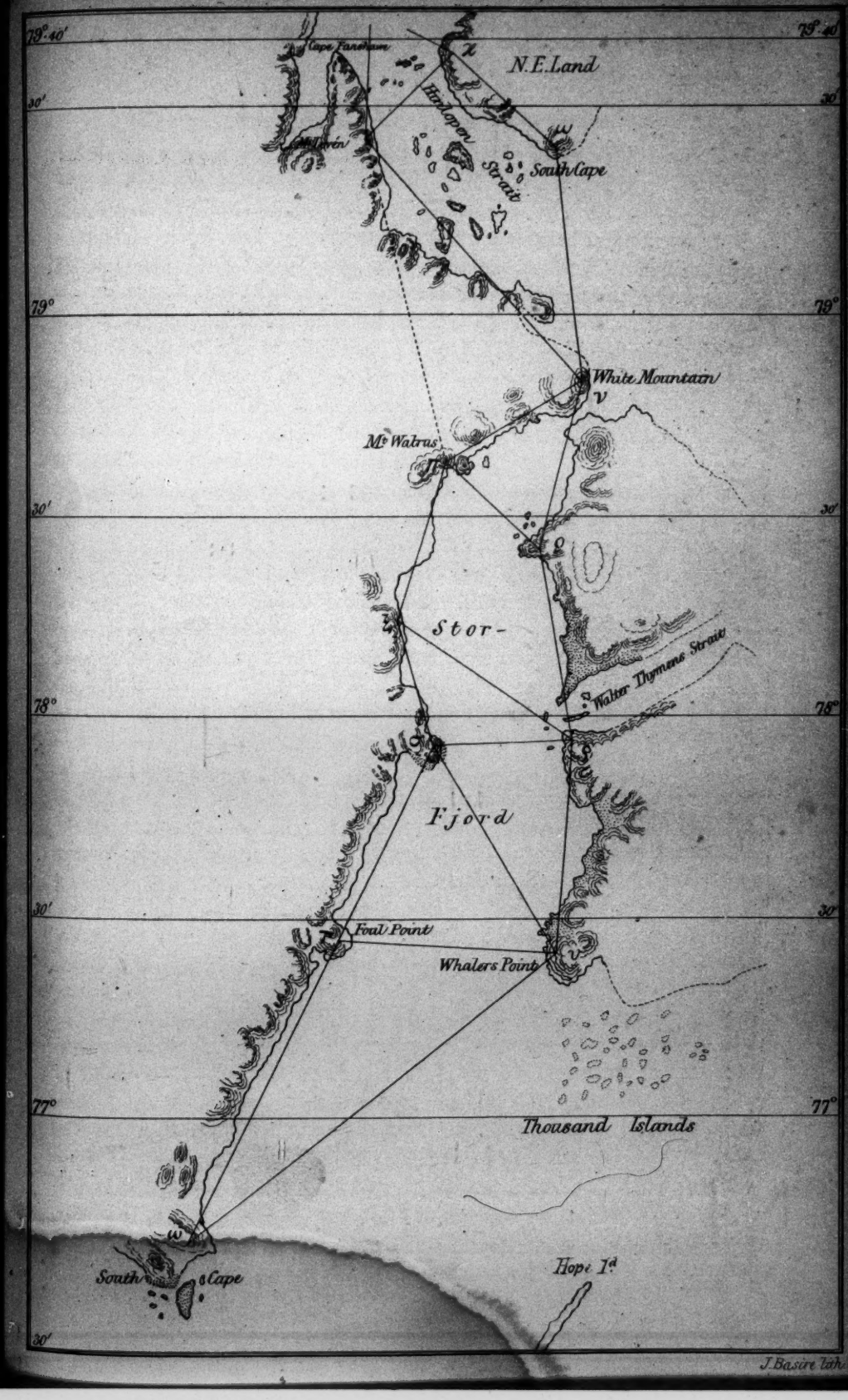
gram to show the extent and shape of the triangles, which may also have to undergo future minor modifications.

"*Report on the Swedish Expedition to Spitzbergen in* 1864.

"During the expedition of 1861 several attempts were made to penetrate into the Storfjord, or Wide Jaws Water; but from ice and calms (the Expedition not being provided with a steamer) they all proved ineffectual. As it was evident, however, that the firth in question is, beyond comparison, the best locality in the island for carrying on the measurement of an arc of the meridian, provided only that it is accessible to vessels, it was resolved that a fresh attempt should be made; and the Estates of the Kingdom having liberally granted the necessary means, another Expedition was fitted out, though on a smaller scale than that of 1861. Mr. Chydenius, who in 1861 had been particularly occupied in selecting and determining the stations for the Survey, unfortunately died in the beginning of 1864. His place has been supplied by Professor Nordenskjöld of the Academy of Sciences at Stockholm, and Mr. Dunér, Professor of Astronomy in the University of Lund, both having been in the Expedition of 1861.

"A small vessel having been chartered at Tromsoe in Norway, they started in the first days of June, and made Bear Island on the 17th, having been detained by gales and adverse winds. Shortly afterwards they reached the opening of the Storfjord; and there appeared to be a good chance of getting in; but the ice soon packed, and, after several ineffectual attempts to force the vessel through, they had to bear up to the western side of Spitzbergen. On June 23rd they were off Bell Sound, but ice and calms prevented their getting in. On June the 25th they anchored at Safe Haven in Ice Sound. Here they remained shut in by the ice until July 16th, making the best use they could of their time by examining the greater part of the Sound, which was found to be considerably larger in extent than is laid down in the charts. Having got out, and returning to the southward, they were met by a heavy southerly gale, which obliged them to run for Bell Sound, where they were detained until July 27th, meanwhile completing the survey of the coasts of that Sound. Being again delayed by head-winds and calms, they did not reach South Cape until August the 7th, and on the 9th had succeeded in getting past the Thousand Islands to Whalers' Point, close to which is one of the southernmost stations within the firth* [marked v on the Map, from which, at a height of 1200 feet, the summits of the three stations, ω, τ, and ρ, were seen against the sky]. On the 10th they reached Foul Point, on the opposite land [where a mountain, 1600 feet high (τ), was ascended, from which the summits v, ρ, ξ, and o were seen projected against the sky, with the exception of o, which was backed by land]. On the 16th the third

* The sentences within brackets are supplied from a letter of a still more recent date, from Dr. Otto Torell and Professor Dunér, written from Lund.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station, ρ, was visited [and, from a height of 1100 feet, the summits υ, τ, σ, ξ, and π were observed projected against the sky, and o against other mountains]. On the 21st, after having ridden out a heavy gale, they succeeded in climbing Mount Walrus [marked π], a mountain 1100 feet high, surrounded by glaciers, and laid down as an island on the existing charts. [From this mountain the station marked λ in Mr. Chydenius's map (Royal Society Proceedings, vol. xii. Plate IV.) was seen.]

"Proceeding in the boats they reached, on the 22nd, and ascended a mountain 2500 feet high, situated near the channel which joins the Storfjord with the southern opening of Hinlopen Straits. This was named White Mountain [and is marked ν on the Map]. From this summit they saw on a clear bright day the South Cape of North-east Land (μ), Mount Löven about the middle of Hinlopen Straits on the west shore, and the station marked κ on the eastern shore. Having thus ascertained satisfactory points in the Storfjord, they proceeded again to the west coast of Spitzbergen, with the intention of pushing to the northward as far as possible, but had not proceeded far when they fell in with several boats with the crews of wrecked sealing vessels. Of course they were obliged to take these men on board; and being short of provisions for the increased number of hands, and the season drawing towards its close, they put back to Tromsoe. The sealing vessels had been wrecked on the east side of North-East Land, having got there by the north of the island. The men had afterwards made their way in the boats through Hinlopen Straits, having thus circumnavigated North-East Land—a feat said never to have been accomplished before.

"The shores of the Storfjord are mountainous. The glens and valleys between the ridges are for the most part filled by glaciers, especially on the western shore. The mountains average from 1000 to 1500 feet in height, and belong in general to the Jura formation, which is here and there broken through by basaltic rocks (hyperite). In the Jura have been found skeletons, though not complete, of an Ichthyosaurus, closely resembling the species found in Arctic America by Sir Edward Belcher's Expedition. Mr. Malmgren, of the University of Helsingfors in Finland, accompanied the Expedition in the capacity of zoologist."

III. "On the Sextactic Points of a Plane Curve." By A. Cayley, F.R.S., Sadlerian Professor of Mathematics, Cambridge. Received November 5, 1864.

(Abstract.)

It is, in my memoir "On the Conic of Five-pointic Contact at any Point of a Plane Curve" (Phil. Trans. vol. cxlix. (1859) pp. 371–400), remarked that as in a plane curve there are certain singular points, viz. the points of inflexion, where three consecutive points lie in a line, so there are singular

points where six consecutive points of the curve lie in a conic; and such a singular point is there termed a "sextactic point." The memoir in question (here cited as "former memoir") contains the theory of the sextactic points of a cubic curve; but it is only recently that I have succeeded in establishing the theory for a curve of the order m. The result arrived at is that the number of sextactic points is $=m(12m-27)$, the points in question being the intersections of the curve m with a curve of the order $12m-27$, the equation of which is

$$\begin{aligned}&(12m^2-54m+57)\ \mathrm{H}\ \mathrm{Jac.}\ (\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{H}}})\\ &+(m-2)\ (12m-27)\ \mathrm{H}\ \mathrm{Jac.}\ (\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}})\\ &+40\,(m-2)^2\qquad\qquad \mathrm{Jac.}\ (\mathrm{U},\mathrm{H},\Psi)=0,\end{aligned}$$

where $\mathrm{U}=0$ is the equation of the given curve m, H is the Hessian or determinant formed with the second differential coefficients (a, b, c, f, g, h) of U, and, $(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})$ being the inverse coefficients ($\mathfrak{A}=bc-f^2$, &c.), then

$$\begin{aligned}\Omega&=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H}\,\}\,(\partial_x,\partial_y,\partial_z)^2\,\mathrm{H},\\ \Psi&=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H}\,\}\,(\partial_x\mathrm{H},\partial_y\mathrm{H},\partial_z\mathrm{H})^2;\end{aligned}$$

and Jac. denotes the Jacobian or functional determinant, viz.

$$\mathrm{Jac.}\ (\mathrm{U},\mathrm{H},\Psi)=\left|\begin{matrix}\partial_x\mathrm{U},&\partial_y\mathrm{U},&\partial_z\mathrm{U}\\ \partial_x\mathrm{H},&\partial_y\mathrm{H},&\partial_z\mathrm{H}\\ \partial_x\Psi,&\partial_y\Psi,&\partial_z\Psi\end{matrix}\right|$$

and Jac. (U, H, Ω) would of course denote the like derivative of (U, H, Ω); the subscripts ($\bar{\mathrm{H}}$, $\bar{\mathrm{U}}$) of Ω denote restrictions in regard to the differentiation of this function, viz. treating Ω as a function of U and H,

$$\Omega=(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H}\,\}\,(a',b',c',2f',2g',2h');$$

if (a', b', c', f', g', h') are the second differential coefficients of H, then we have

$$\begin{aligned}\partial_x\Omega&=(\partial_x\mathfrak{A},\ldots\}\,(a',\ldots)\qquad(=\partial_x\Omega_{\bar{\mathrm{H}}})\\ &\quad+(\mathfrak{A},\ldots\}\,(\partial_x a',\ldots)\qquad(=\partial_x\Omega_{\bar{\mathrm{U}}});\end{aligned}$$

viz. in $\partial_x\Omega_{\bar{\mathrm{H}}}$ we consider as exempt from differentiation (a', b', c', f', g', h') which depend upon H, and in $\partial_x\Omega_{\bar{\mathrm{U}}}$ we consider as exempt from differentiation $(\mathfrak{A},\mathfrak{B},\mathfrak{C},\mathfrak{F},\mathfrak{G},\mathfrak{H})$ which depend upon U. We have similarly $\partial_y\Omega=\partial_y\Omega_{\bar{\mathrm{H}}}+\partial_y\Omega_{\bar{\mathrm{U}}}$, and $\partial_z\Omega=\partial_z\Omega_{\bar{\mathrm{H}}}+\partial_z\Omega_{\bar{\mathrm{U}}}$; and in like manner

$$\mathrm{Jac.}\ (\mathrm{U},\mathrm{H},\Omega)=\mathrm{Jac.}\ (\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{H}}})+\mathrm{Jac.}\ (\mathrm{U},\mathrm{H},\Omega_{\bar{\mathrm{U}}}),$$

which explains the signification of the notations Jac. (U, H, $\Omega_{\bar{\mathrm{H}}}$), Jac. (U, H, $\Omega_{\bar{\mathrm{U}}}$).

The condition for a sextactic point is in the first instance obtained in a form involving the arbitrary coefficients (λ, μ, ν); viz. we have an equation of the order 5 in (λ, μ, ν) and of the order $12m-22$ in the coordinates (x, y, z). But writing $\mathfrak{H}=\lambda x+\mu y+\nu z$, by successive transformations we

throw out the factors $\vartheta^2, \vartheta, \vartheta, \vartheta$, thus arriving at a result independent of (λ, μ, ν); viz. this is the before-mentioned equation of the order $12m-27$. The difficulty of the investigation consists in obtaining the transformations by means of which the equation in its original form is thus divested of these irrelevant factors.

IV. "On a Method of Meteorological Registration of the Chemical Action of Total Daylight." By Henry E. Roscoe, B.A., F.R.S. Received November 8, 1864.

(Abstract.)

The aim of the present communication is to describe a simple mode of measuring the chemical action of total daylight, adapted to the purpose of regular meteorological registration. This method is founded upon that described by Prof. Bunsen and the author in their last Memoir* on Photochemical Measurements, depending upon the law that equal products of the intensity of the acting light into the times of insolation correspond within very wide limits to equal shades of tints produced upon chloride-of-silver paper of uniform sensitiveness—light of the intensity 50, acting for the time 1, thus producing the same blackening effect as light of the intensity 1 acting for the time 50. For the purpose of exposing this paper to light for a known but very short length of time, a pendulum photometer was constructed; and by means of this instrument a strip of paper is so exposed that the different times of insolation for all points along the length of the strip can be calculated to within small fractions of a second, when the duration and amplitude of vibration of the pendulum are known. The strip of sensitive paper insolated during the oscillation of the pendulum exhibits throughout its length a regularly diminishing shade from dark to white; and by reference to a Table, the time needed to produce any one of these shades can be ascertained. The unit of photo-chemical action is assumed to be that intensity of light which produces in the unit of time (one second) a given but arbitrary degree of shade termed the standard tint. The reciprocals of the times during which the points on the strip have to be exposed in order to attain the standard tint, give the intensities of the acting light expressed in terms of the above unit.

By means of this method a regular series of daily observations can be kept up without difficulty; the whole apparatus needed can be packed up into small space; the observations can be carried on without regard to wind or weather; and no less than forty-five separate determinations can be made upon 36 square centimetres of sensitive paper. Strips of the standard chloride-of-silver paper tinted in the pendulum photometer remain as the basis of the new mode of measurement. Two strips of this paper are exposed as usual in the pendulum photometer: one of these strips is fixed

* Phil. Trans. 1863, p. 139.

in hyposulphite-of-sodium solution, washed, dried, and pasted upon a board furnished with a millimetre-scale. This fixed strip is now graduated in terms of the unfixed pendulum strip by reading off, by the light of a soda-flame, the position of those points on each strip which possess equal degrees of tint, the position of the standard tint upon the unfixed strip being ascertained for the purpose of the graduation. Upon this comparison with the unfixed pendulum strip depends the subsequent use of the fixed strip. A detailed description of the methods of preparing and graduating the strips, and of the apparatus for exposure and reading, is next given. The following conditions must be fulfilled in order that the method may be adopted as a trustworthy mode of measuring the chemical action of light:—

1st. The tint of the standard strips fixed in hyposulphite must remain perfectly unalterable during a considerable length of time.

2nd. The tints upon these fixed strips must shade regularly into each other, so as to render possible an accurate comparison with, and graduation in terms of, the unfixed pendulum strips.

3rd. Simultaneous measurements made with different strips thus graduated must show close agreement amongst themselves, and they must give the same results as determinations made by means of the pendulum photometer, according to the method described in the last memoir.

The fixed strips are prepared in the pendulum apparatus, and afterwards fixed in hyposulphite of sodium. A series of experiments is next detailed, carried out for the purpose of ascertaining whether these fixed strips undergo any alteration by exposure to light, or when preserved in the dark. Two consecutive strips were cut off from a large number of different sheets, and the point upon each at which the shade was equal to that of the standard tint was determined. One half of these strips were carefully preserved in the dark, the other half exposed to direct and diffuse sunlight for periods varying from fourteen days to six months, and the position of equality of tint with the standard tint from time to time determined. It appears, from a large number of such comparisons, that in almost all cases an irregular, and in some cases a rapid fading takes place immediately after the strips have been prepared, and that this fading continues for about six to eight weeks from the date of the preparation. It was, however, found that, after this length of time has elapsed, neither exposure to sunlight nor preservation in the dark produces the slightest change of tint, and that, for many months from this time, the tint of the strips may be considered as perfectly unalterable.

The value of the proposed method of measurement entirely depends upon the possibility of accurately determining the intensities of the various shades of the fixed strip in terms of the known intensities of the standard strips prepared in the pendulum photometer. The author examines this question at length, and details two methods of graduating the fixed strips,

giving the results obtained in several series of determinations, in order that the amount of experimental error may be estimated. Curves exhibiting the graduation of several strips are also given; and from these the author concludes that the determinations agree as well as can be expected from such photometric experiments, the mean error between the positions 40 and 80 min. on the strip in one series of graduations not exceeding 1 per cent. of the measured intensity. To each fixed strip a Table is attached, giving the intensity of the light which must act for 1 second upon the standard paper, in order to produce the tints at each millimetre of the length of the strip.

The methods of exposure and reading are next described. The exposure of the paper is effected very simply by pasting pieces of standard sensitive paper upon an insolation band, and inserting the band into a thin metal slide having a small opening at the top and furnished with a cover, which can be made instantly to open or close the hole under which the sensitive paper is placed. When one observation has thus been made, and the time and duration of the insolation noted, the remaining papers can be similarly exposed at any required time; and thus the determinations can be very easily carried on at short intervals throughout the day.

The reading-instrument consists of a small metallic drum, furnished with a millimetre scale, and upon which the graduated strip is fastened. The drum turns upon a horizontal axis, and the insolation band, with the exposed papers upon it, is held against the graduated strip, so that by moving the drum on its horizontal axis the various shades of the strip are made to pass and repass each of the papers on the insolation band, and the points of coincidence of tint on the strip and on each of the exposed papers can be easily ascertained by reading off with the monochromatic soda-flame.

In the next section of the paper the author investigates the accuracy and trustworthiness of the method. This is tested in the first place by making simultaneous measurements of the chemical action of daylight by the new method and by means of the pendulum photometer, according to the mode described in the last memoir, upon which the new method is founded. Duplicate determinations of the varying chemical intensity thus made every half-hour on four separate days give results which agree closely with each other, as is seen by reference to the Tables and figures showing the curves of daily chemical intensity which are given in the paper. Hence the author concludes that the unavoidable experimental errors arising from graduation, exposure, and reading are not of sufficient magnitude materially to affect the accuracy of the measurement. As a second test of the trustworthiness and availability of the method for actual measurement, the author gives results of determinations made with two instruments independently by two observers at the same time, and on the same spot. The tabulated results thus obtained serve as a fair sample of the accuracy with which the actual measurement can be carried out; and

the curves given represent graphically the results of these double observations. From the close agreement of these curves, it is seen that the method is available for practical measurement.

In order to show that the method can be applied to the purposes of actual registration, the author gives the results of determinations of the varying intensity of the chemical action of total daylight at Manchester on more than forty days, made at the most widely differing seasons of the year. These measurements reveal some of the interesting results to which a wide series of such measurements must lead. They extend from August 1863 to September 1864; and Tables are given in which the details of observations are found, whilst the varying chemical intensity for each day is expressed graphically by a curve.

As a rule, one observation was made every half-hour; frequently, however, when the object was either to control the accuracy of the measurement or to record the great changes which suddenly occur when the sun is obscured or appears from behind a cloud, the determinations were made at intervals of a few minutes or even seconds.

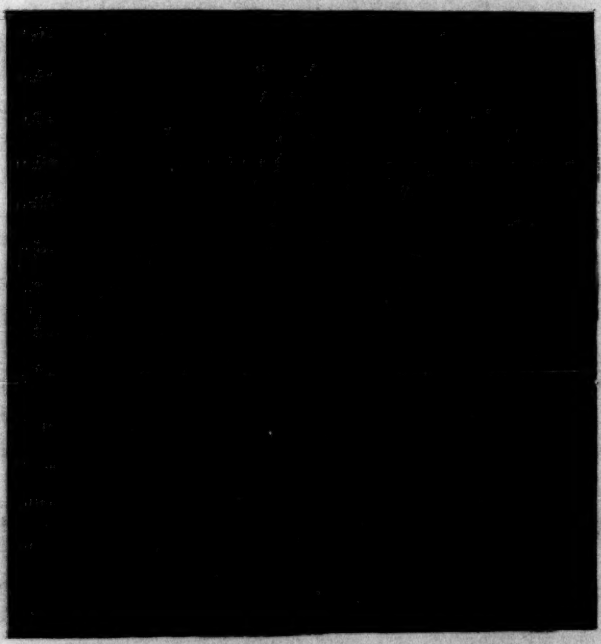
Consecutive observations were carried on for each day for nearly a month, from June 16th to July 9th, 1864; the labour of carrying out these was not found to be very great, and the results obtained are of great interest. By reference to the Tables, it is seen that the amount of chemical action generally corresponds to the amount of cloud or sunshine as noted in the observation; sometimes, however, a considerable and sudden alteration in the chemical intensity occurred when no apparent change in the amount of light could be noticed by the eye. The remarkable absorptive action exerted upon the chemically active rays by small quantities of suspended particles of water in the shape of mist, or haze, is also clearly shown. For the purpose of expressing the relation of the sums of all the various hourly intensities, giving the daily mean chemical intensities of the place, a rough method of integration is resorted to: this consists in determining the weights of the areas of paper inscribed between the base-line and the curve of daily intensity, that chemical action being taken as 1000 which the unit of intensity would produce if acting continuously for twenty-four hours. The remarkable differences observed in the chemical intensity on two neighbouring days is noticed on the curves for the 20th and 22nd of June 1864: the integrals for these days are 50·9 and 119, or the chemical actions on these days are in the ratio of 1 to 2·34.

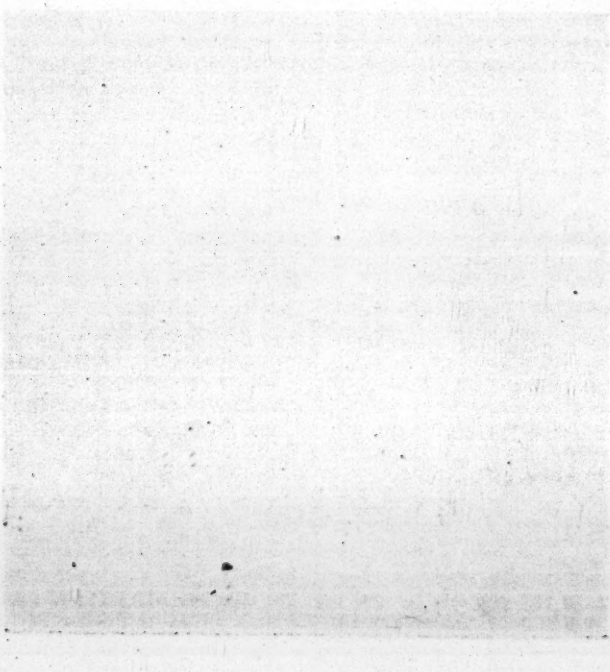
The chemical action of light at Manchester was determined at the winter and summer solstices, and the vernal and autumnal equinoxes: the results of these measurements are seen by reference to the accompanying curves. The integral for the winter solstice is 4·7, that of the vernal equinox 36·8; that of the summer solstice 119, and that of the autumnal equinox 29·1. Hence, if the chemical action on the shortest day be taken as the unit, that upon the equinox will be represented by 7, and that upon the longest day by 25.

The results of simultaneous measurements made at Heidelberg and Manchester, and Dingwall and Manchester, are next detailed.

From the integrals of daily intensity the mean monthly and annual chemical intensity can be ascertained, and thus we may obtain a knowledge of the distribution of the chemically acting rays upon the earth's surface, such as we possess for the heating rays.

Figure showing curves of daily chemical intensity at Manchester, in spring, summer, autumn, and winter.





OBITUARY NOTICES OF FELLOWS DECEASED

Between 30th Nov. 1862 and 30th Nov. 1863.

Arthur Connell was the eldest son of Sir John Connell, Judge of the Admiralty Court of Scotland and author of a well-known work on the law of Scotland respecting tithes. He was born in Edinburgh on the 30th of November 1794. Having received his early education at the High School of that city, he (in 1808) entered the University, where he studied under Playfair, Leslie, Dugald Stewart, and Hope. Mr. Connell then became for a time a student in the University of Glasgow, and having there obtained a Snell Exhibition, he went to Balliol College, Oxford, in 1812. In 1817 he passed Advocate at the Scotch Bar, but never practised; following a decided turn for science which had early shown itself, he devoted himself to the pursuit of chemistry, which became his main occupation, and in 1840 he was appointed to the professorship of that science in the University of St. Andrews. In St. Andrews Mr. Connell continued to study and teach his favourite science till 1856, when the fracture of a limb, and its effects upon a constitution already long enfeebled, completely incapacitated him for active duty.

Mr. Connell was chosen a Fellow of this Society at the annual election in 1855; from 1829 he had been a Fellow of the Royal Society of Edinburgh, in whose 'Transactions' or in the 'Edinburgh Philosophical Journal' his various scientific memoirs and communications have appeared. Most of his published researches belong to the province of mineral analysis, in which he justly attained a high reputation for skill and accuracy. To him is due the merit of establishing several new mineral species, and of showing that in certain minerals baryta exists in combination with silicic acid; and as an example of his nicety as an analyst, we may refer to his determination of the constitution of Greenockite from a single grain of that mineral. But Mr. Connell's labours as a chemist were not confined to a single field of inquiry; he was the author of valuable researches on voltaic decompositions, published in the Transactions of the Royal Society of Edinburgh; and he contrived an instrument for ascertaining the dew-point, which is superior in some respects to that generally used.

Mr. Connell died on the 31st of October 1863. He was of a modest, retiring nature and gentle disposition. To those who enjoyed his private friendship, it was well known that the merit he evinced as an earnest and faithful worker in science was but the manifestation, in that special direction, of the excellent qualities which belonged to his natural character.

Edward Joshua Cooper, one of our most distinguished amateur astronomers, received the first impulses which made him pursue that science from his mother's early teaching, and from his visits to the Armagh Observatory while at the endowed school of that city. Thence he passed to Eton, and from it to Christ Church, Oxford, where it may be feared that in those days he met little encouragement in his favourite pursuit.

After leaving Oxford he travelled extensively, with a sextant, chronometer, and telescope as his inseparable companions. While at Naples in 1820 he met Sir William Drummond, some of whose wild inferences from the Dendera Zodiac he showed to be inconsistent with sound astronomy. Sir William replied that these objections were based on the inaccuracy of the existing drawings; and Mr. Cooper met that by going to Egypt, securing the services of an accomplished Italian artist, and bringing home correct plans of the Dendera ceiling, which, with many other drawings, he printed for private distribution under the title 'Egyptian Scenery.' He thence visited Persia, Turkey, Germany, and Scandinavia as far as the North Cape, accumulating a great mass of observations of latitude and longitude, which unfortunately remain unpublished. Shortly after his return to Ireland the death of his uncle placed him in possession of a large estate, and enabled him to carry out on a great scale the plan which he had formed of determining some portion of the small stars which were in general neglected by the great observatories. In the year 1831 he purchased, from Cauchoix of Paris, an object-glass of 13·3 inches aperture and 25 feet focus, the largest then existing, which in 1834 was mounted with perfect success at his magnificent mansion of Markree, on an equatorial constructed by Mr. Grubb of Dublin. It is of cast iron (the first application of that material to astronomical instruments), and stands in the open air, encircled by the buildings of a first-rate observatory, which contain, among other *chefs-d'œuvres*, a fine meridian-circle by Ertel, with eight microscopes, and an 8-inch object-glass. These instruments were at once applied to active work, in which he had a most able cooperator, his first assistant, Mr. A. Graham; and the Memoirs of the Royal Astronomical Society and of the Royal Irish Academy (of both which he was a member) bear ample testimony of their diligence. One result of their labours was the discovery of the planet Metis; but his greatest work is his 'Catalogue of Ecliptic Stars.' This (which was published by aid from the Government grant placed at the disposal of the Royal Society, and which the Royal Irish Academy honoured with their Cunningham Medal) contains upwards of 60,000 stars down to the 12th magnitude, of which very few had been previously observed. The places are reduced to 1850, and though only approximate, possess a sufficient degree of precision for the use to which they were destined. Their probable error is $\pm 2''{\cdot}6$, both in right ascension and in declination. The value of this Catalogue to future astronomers can scarcely be overrated, for many facts tend to show that much is to be learned by studying these minute stars. As an instance, it may be stated that fifty stars of his own Catalogue and twenty-seven of others' were found to have disappeared during the progress of the observations. Many of these, no doubt, are variable; many probably are unknown planets; some perhaps have great proper motion. But when shall we have such a survey of the whole sky as that of this comparatively small zone!

He promised a fifth volume, the materials for which are nearly complete,

as is also the case with a set of star-charts, in which, besides all the stars actually observed, as many are inserted as could be interpolated by the eye. It is much to be desired that these should not be lost to astronomy; and all who love that science will express a hope that his representatives will complete a work which has done such honour to their name.

Mr. Cooper continued to observe almost up to his death, which occurred on April 23, 1863, shortly after that of his wife, to whom he was deeply attached. The date of his election into the Royal Society is June 2, 1853. He represented the county of Sligo in Parliament for many years, and was a kind and good landlord, making great exertions to educate and improve his numerous tenantry. His personal qualities were of a high order, blameless and fascinating in private life, a sincere Christian, no mean poet, an accomplished painter and linguist, an exquisite musician, and possessing a wide and varied range of general information.

Joshua Field was a member of a well-known firm of Civil Engineers at Lambeth. The chief occupation of his life was mechanical engineering, and he had an important share in the improvements which have been made during his time in the construction of marine steam-engines. Mr. Field was one of the founders of the Institution of Civil Engineers, and one of its earliest Vice-Presidents. In 1848 he was elected President, on which occasion he, in his inaugural address, gave a sketch of the progress of improvement in steam navigation. He became a Member of the Royal Society in 1836. His death took place on the 11th of August 1863, at the age of 76.

Richard Fowler, M.D., the oldest Fellow of the Royal Society, died at his residence, Milford, near Salisbury, April 13, 1863, in his 98th year. He was born in London, November 28, 1765, and at an early age was so feeble in health, that it was thought necessary to send him to reside with a relation in Staffordshire. He was thence sent to Edinburgh for his education. While yet a youth he was entered for the medical profession, and while pursuing his studies he eagerly embraced the opportunity of attending the lectures on Moral and Political Philosophy of Dugald Stewart, to whose influence he ever afterwards referred with gratitude. From Edinburgh he proceeded to Paris, at a time when Louis XVI., Marie Antoinette, and the Dauphin were yet to be seen in their regal state; and Dr. Fowler was fond of telling how he saw them thus, and also of having exchanged greetings with Talleyrand while yet the young and courtly Bishop of Autun. He remained long enough in Paris to witness much of the strife of the first French Revolution. He was personally acquainted with Mirabeau, and often listened to his eloquence in the National Assembly. On his return to Edinburgh he was admitted, in Nov. 1790, a Member of the Speculative Society of that city, which had been founded in 1764, and has numbered among its members some of the most eminent

men of Great Britain. Dr. Fowler was an active member, and brought forward questions on politics and social economy for discussion. He early adopted the Liberal side in politics, and that of complete toleration in religion, and throughout his long career he was a steady supporter of Liberal principles. In 1793 he published a work entitled 'Experiments and Observations relative to the influence lately discovered by M. Galvani, and commonly called Animal Electricity.' The work consists of 176 pages, and contains an account of numerous experiments with different metals on frogs, the earthworm, the hearts of cats and rabbits, &c. The work also contains a letter from Professor Robison giving an account of some experiments with dissimilar metals. Dr. Fowler took his medical degree at the University of Edinburgh, Sept. 12, 1793; he was admitted Licentiate of the Royal College of Physicians of London in March 1796, about which time he settled at Salisbury, and was elected Physician to the Infirmary of that city, an office which he filled during forty years; after which he continued to be one of its consulting physicians to the time of his death. In 1802 he was elected a Fellow of the Royal Society. In 1805 he married the daughter of William Bowles, Esq., of Heale House. He had an extensive medical practice during many years, and a still more extensive acquaintance with the leading men of the day, for which he was partly indebted to his early friendship with the Marquis of Lansdowne and Lord Holland, and also to his own social qualities and conversational powers, the latter being enlivened by anecdote, apt quotation, and varied knowledge, which enabled him to say something agreeably and well on almost every subject; at the same time his kindly nature mellowed and improved everything he said and did. In 1831 he became a Member of the British Association, and during a number of years made frequent communications to it on subjects in mental philosophy and their relation to physiology. The last communication was in 1859, when Dr. Fowler, then in his 94th year, made the journey to Aberdeen for the purpose of being present at the Congress. He took great interest in the mental condition of the deaf, dumb, and blind, and was fond of inquiring how it was that persons so afflicted displayed a higher intelligence than that of the most sagacious of the lower animals, and how by touch alone the meaning of others can be communicated to the blind and deaf, and instantly interpreted. During the later years of his life, Dr. Fowler was himself afflicted with loss of sight. Indeed, in his work published in 1793, he complains (page 76) that the weak state of his eyes did not permit him to look intently at minute objects. Nevertheless his mental activity was so great, that when he could no longer see to read he kept employed two men and two boys in reading to him, and writing down from his dictation memoranda for future papers. One of his latest acts, in conjunction with Mrs. Fowler, was to purchase and endow a suitable home for the Salisbury and South Wiltshire Museum, in which he took a great interest, and bestowed on it a large portion of his books and collections. Dr. Fowler was also a

Fellow of the Antiquarian Society, and a member of the Zoological Society, and of the Edinburgh Medical and Speculative Societies.

Peter Hardy (born Dec. 17, 1813, died April 23, 1863) was the son of an officer in the artillery, and the brother of Mr. T. Duffus Hardy of the Record Office and Mr. William Hardy of the Duchy of Lancaster. He was educated for an actuary, was placed in the Equitable Office in 1829, was appointed actuary of the Mutual Assurance Office in 1837, and of the London Assurance Office in 1850. The details of his professional life, active and well employed as it was, offer nothing to record. He distinguished himself in 1839 by the publication of a sound mathematical work on interest, containing tables subdivided into quarter rates per cent. In 1840 he published a system of notation for life contingencies, which, besides its ingenuity, was of a practicable character. In 1848 he was one of the founders of the Institute of Actuaries. He was elected Fellow of the Royal Society in 1839, immediately after the publication of his first work. Mr. Peter Hardy possessed that talent for research which has contributed to render both his brothers eminent in their several walks of life. His walk, indeed, did not offer any opportunities; but the taste found vent in the collection of old books and the study of early printing. In this he would have been eminent, had it been anything but an employment of leisure. His library, very large for that of an actuary, contained rare specimens, of which he knew the history and the literary value.

John Taylor was born at Norwich, on the 22nd of August 1779, and died in London on the 5th of April 1863. He was the eldest of five brothers belonging to a well-known and respected family in his native city. In early youth Mr. Taylor showed a natural turn for mechanical contrivance, and a decided bent towards the profession of an engineer; but as his native district afforded but little scope for engineering work, he was brought up to land-surveying, as the employment within reach which seemed to fall in most nearly with his natural taste. From this occupation, however, he was called off at the early age of nineteen, to take the management of the Wheal Friendship Mine in Cornwall. Inexperienced as he then was in all that specially concerned mining work, he was, nevertheless, appointed to that charge, in reliance on his ability to master the task he was to undertake, and on his diligence and integrity in the performance of it.

Soon after he became thus engaged, Mr. Taylor published, in the 'Philosophical Magazine' (for 1800), an article "On the History of Mining in Devon and Cornwall;" and in succeeding years he communicated to the world, in that and other journals, the results of his experience concerning the operations and processes followed in mining, and the means of improving them, and on the general economy of mines.

But whilst his daily occupation was in directing practical work, it was Mr. Taylor's constant aim to elevate the art of mining and place it on a

scientific basis. With this purpose he (in 1829) undertook the editorship, and, to a considerable extent, the cost of a publication which he hoped would conduce to the end in view. This work, which was entitled 'Records of Mining,' did not meet with the support he had looked for, and was not continued; but the part published contains four contributions from his own pen. One of these, which deserves especial notice, is an excellent and well-digested paper, in which he advocates the establishment of a School of Mines in Cornwall, and presents a "Prospectus," setting forth the details of a plan for the foundation, government, and maintenance of an institution of that kind, and for regulation of the instruction it should afford. This proposal did not meet with encouragement at the time, but the establishment of the present School of Mines, although he had no active share in it, may be traced to Mr. Taylor's earnest and judicious representations.

In 1812 Mr. Taylor settled in London, and engaged with his brother in a chemical manufactory at Stratford in Essex; but although he had long applied himself to chemistry as a scientific pursuit, and attained a high reputation as a metallurgic chemist, he was compelled to relinquish it as a profession in consequence of the increased extent and importance of his mining business.

Throughout his eminently practical life Mr. Taylor strove constantly to maintain a healthy reciprocity between science and practice; and while availing himself of his scientific knowledge for the improvement of technical processes, he was no less earnest in turning to account his experience and opportunities as a mining engineer and metallurgist for the promotion of the sciences of geology, mineralogy, and chemistry. We accordingly find his name enrolled as a member of various scientific bodies, both British and Foreign. He was one of the earliest Fellows of the Geological Society, and from 1816 to 1844 filled the office of its Treasurer. In 1825 he was elected a Fellow of the Royal Society, and repeatedly served on the Council. But of the several Societies to which he belonged, the British Association for the Advancement of Science has been the most indebted to Mr. Taylor's useful cooperation, both scientific and administrative. He was, indeed, present at its birth, and the first meeting of its first Council was held at his house. This was on the 26th of June 1832. He was also the first Treasurer, and held that office till September 1861, when the infirmities of age constrained him to retire from it. On relieving him from the duties he had so long and so well discharged, the Council of the Association joined in a unanimous expression of respect for his character and gratitude for his long and valuable services to the cause of science. The Council of University College, London, expressed themselves in no less respectful and grateful terms when, for the same reason, he retired from the Treasurership of that Institution, which he had held for many years.

William Tooke was born on the 22nd of November 1777, at St. Peters-

burg, where his father was then Chaplain to the factory of the Russian Company. Mr. Tooke was a solicitor by profession, but amidst the calls of business he was able to devote much time and attention to public affairs, and took part in the formation of various public institutions which were established during the active period of his life. He was long a leading member and Vice-President of the Society of Arts, and was elected President on the vacancy caused by the death of His Royal Highness the Prince Consort. His election to the Royal Society was in 1818. From 1832 to 1837 he sat in Parliament for the borough of Truro. Besides a compilation on French history, in two volumes, which appeared in 1855, and occasional contributions to magazines, Mr. Tooke, when a young man, published, anonymously, an edition of Churchill's Poems, with notes and a Life of the Author, which was favourably reviewed by Southey. He died in London on the 20th September 1863.

Rear-Admiral JOHN WASHINGTON was born on the 1st of January 1801. He entered the Navy in 1812, on board the 'Juno,' fitting for the American station, where he served for the next two years, and took part in various active and successful operations against the enemy. In 1814 he returned to England, and entered the Royal Naval College at Portsmouth. During the two years he spent as a student in that Institution he applied himself diligently to the scientific study of his profession, and laid the foundation of that skill and accomplishment which he afterwards attained in nautical surveying and hydrography, and through which in the after course of his life he was able to render much valuable service to the maritime interests of this and other countries.

After serving at sea for some years, and rising to the rank of Commander, he returned home for a time, but in 1841 he was appointed to continue the Survey of the North Sea, in which duty he was employed until the close of 1844. In 1842 he was promoted to the rank of Post Captain.

This Survey was Captain Washington's last service afloat; but his experience and judgment were turned to public account at different times, when he acted as a Royal Commissioner on important questions affecting the interests of navigation and of our maritime industry. In 1853 he visited some of the Russian fortresses in the Baltic, and the results of his observations proved of the greatest value in the conduct of the warlike operations which soon followed.

On the retirement of Sir Francis Beaufort, Captain Washington was appointed Hydrographer to the Admiralty, and was promoted to the rank of Rear-Admiral in 1862. His anxious and unremitting application to the various duties of his office is believed by his friends to have shortened his valuable life, which was closed at Frascati, near Havre, on the 16th of September 1863. He was a member of various learned Societies at home and abroad, and was Secretary of the Royal Geographical Society from

1835 to 1841. His election into the Royal Society is dated February 13, 1845.

César Mansuète Despretz was born at Lessines in Belgium, on the 13th of May 1789. At an early age he came to Paris for the purpose of devoting himself to the study of chemistry and physics. His intelligence and industry soon attracted the attention of Gay-Lussac, who appointed him répétiteur of his course of lectures on Chemistry at the École Polytechnique. He became the Professor of Physics at the Sorbonne in 1837, having previously held a similar office in the École Polytechnique and the Collége Henri IV. In 1822 the Academy awarded him the prize for the best memoir on the causes of animal heat. In 1825 he published an elementary treatise on Physics, which in 1836 reached a fourth edition; and in 1830 the Elements of Theoretical and Practical Chemistry. He was the author of numerous memoirs published in the 'Annales de Chimie' and in the 'Comptes Rendus,' dating from 1817 up to 1858. These memoirs give an account of researches on the specific heat and conductivity of metals and various mineral substances; on the propagation of heat in liquids; on the transmission of heat from one solid body to another; on the heat absorbed in fusion; on the elastic force of vapours; on the density and latent heat of vapours; on the compressibility of liquids; on the density of gas under different pressures; on the displacement and oscillations of the freezing-point of the mercurial thermometer; on the heat developed during combustion; on the expansion of water, and the temperature at which water and saline solutions attain a maximum density; on the modifications which metals undergo under the joint action of heat and ammoniacal gas; on the chemical action of voltaic electricity, the light and heat of the voltaic arc, and the intensity of the voltaic current; on the electricity developed by muscular contraction; on chloride of boron; on the decomposition of water, carbonic acid, and acetic acid; on the decomposition of salts of lead; on the limits of high and low musical notes; on the fusion and volatilization of some refractory substances under the triple action of the voltaic battery, the sun, and the oxyhydrogen blowpipe.

Though not successful in making any brilliant theoretical discoveries, the important scientific facts he has observed and arranged bring his name perpetually before the reader of any modern treatise on Physics. He laboured hard to fulfil to the utmost his duties as a Professor at the Sorbonne; and his lectures, being carefully prepared and well illustrated by experiments, attracted a numerous auditory. He was elected a Foreign Member of this Society in 1862.

His character was upright and benevolent, his tastes simple, and his habits regular in the extreme. It was his custom every year to make a long excursion in England, Germany, or Italy, by himself, and without letting any one know the day of his departure from Paris.

His last illness was preceded by several slight attacks of cerebral con-

gestion. These were followed by congestion of the lungs, of which he died on the 15th of March 1863.

Eilhardt Mitscherlich was born on the 7th of January 1794, in the village of Jever in Oldenburg. His father was pastor of Neuende; his uncle, the well-known philologer, was Professor in Göttingen. He was educated at the Gymnasium of Jever, under the historian Schlosser. Following the example of his uncle, and encouraged by Schlosser, he devoted himself to the study of history, philology, and especially the Persian language. In order to prosecute these studies, he went in 1811 to the University of Heidelberg, and in 1813 to Paris. He had hoped to be allowed to accompany an embassy to Persia, but was prevented by the fall of Napoleon. In 1814, on his return to Germany, he commenced writing a history of the Ghurides and Kara-Chitayens, compiled from manuscripts in the Göttingen Library, and of which a specimen was published in 1815 under the title "Mirchondi historia Thaheridarum." Unwilling to renounce his favourite project of travelling in Persia, he determined to accomplish it without any extraneous assistance. The only way in which it appeared possible to travel was in the character of a physician; accordingly he resolved to study medicine. He went to Göttingen for this purpose, and first applied himself to the introductory sciences, especially to chemistry, which so fascinated him that he gave up philology and his intention to visit Persia. In 1818 he went to Berlin for the purpose of obtaining license to lecture. Link allowed him to carry on his researches in the laboratory of the University. Here he undertook the examination of the phosphates and arseniates, and confirmed the accuracy of the latest conclusion arrived at by Berzelius, viz. that phosphoric and arsenic acid contain each five equivalents of oxygen, while phosphorous and arsenious acid contain three equivalents. He noticed at the same time that the similarly constituted phosphates and arseniates crystallized in similar forms. Up to this period he had never paid any especial attention to crystallography, but the conviction that he was on the eve of a great discovery allowed him no rest; he studied the laws of crystallography, learned the method of measuring the angles of crystals, and soon satisfied himself that the phosphates and arseniates are not merely similar but identical in form, and that, consequently, bodies exist of dissimilar composition having the same crystalline form, and that these bodies are compounds containing respectively the same number of equivalents. Many minerals appeared to confirm this law, viz. the carbonates, dolomite, chalybite, diallogite and calcite, and the sulphates, baryte, celestine, and Anglesite. In confirmation, however, of this discovery he considered it necessary to appeal to artificial salts which crystallize readily and distinctly, and are easily obtained of sufficient purity, so that his conclusions might be confirmed by any one without difficulty. The neutral sulphates of protoxide of iron, oxide of copper, oxide of zinc, and magnesia, which all con-

tain water, mostly in different proportions, appeared peculiarly well fitted for this purpose. He found that the following were similar in form:—(1) sulphate of copper and sulphate of protoxide of manganese; (2) sulphate of protoxide of iron and sulphate of oxide of cobalt; (3) sulphate of magnesia, sulphate of oxide of zinc, and sulphate of oxide of nickel. He also found that the salts which had dissimilar forms contained a different number of equivalents of water, and that those which had similar forms contained the same number. He then mixed the solutions of the different sulphates, and found that the resulting crystals had the form and the same number of equivalents of water as some one of the unmixed sulphates. Lastly, he examined the combinations of these sulphates with sulphate of potash, and showed that the double salts had all similar forms belonging to the oblique system, and that they were composed of one equivalent of the earthy or metallic sulphate, one equivalent of sulphate of potash, and six equivalents of water. The memoir in which these observations are recorded was presented to the Berlin Academy on the 9th of December 1819. In the course of the preceding August Berzelius came to Berlin, on his way from Paris to Stockholm. He became acquainted with Mitscherlich, and conceived such an opinion of his talents, that he suggested him to the Minister Altenstein as the most fitting successor to Klaproth in the chair of Chemistry in the University of Berlin. Altenstein did not at the moment act upon this suggestion, but consented to the proposal that Mitscherlich should perfect his chemical education by working for some time under the guidance of Berzelius. In Stockholm he continued and extended his researches on the phosphates and arseniates, and wrote a memoir on the subject, which appeared in the Transactions of the Swedish Academy. In it he described with great care the forms of the acid and neutral phosphates and arseniates of potash, soda, and ammonia, the neutral double salts of potash and soda, and of ammonia and soda, and the phosphates and arseniates of oxide of lead. He showed in every case that the phosphates and arseniates have similar forms and analogous compositions. Urged by Berzelius to give a name to this newly detected property of the chemical elements, he designated it by the term *isomorphism*. This discovery was of the highest importance to the theory of chemical equivalents, inasmuch as it explained the exceptions to the law of definite proportions in the mineral system of Berzelius. It appeared moreover, from the crystallization of the mixtures of the different sulphates, that isomorphous substances combine in all proportions; and that they replace one another in indefinite proportions in the composition of minerals was proved by Mitscherlich's fellow-students, Heinrich Rose and Bonsdorff, in the cases of augite and amphibole.

The doctrine of isomorphism, moreover, was an admirable test of the determination of the equivalents of the different elements, whilst the smallness of the number of changes in the equivalents of the simple substances that followed the discovery of isomorphism, is an indication of the

admirable sagacity with which they had been determined by Berzelius. Mineralogists and chemists had long been occupied with researches on the relation between chemical composition and crystalline form; they had discovered a number of important facts bearing upon the subject, but no one had discovered the basis upon which the phenomena rested. Fuchs had already observed that some of the constituents of a mineral might be replaced by others without any change of form, and had called these constituents *vicarious*, but by adducing the sesquioxide of iron and lime as vicarious constituents in Gehlenite, he showed that the true explanation had eluded his grasp. Fuchs had moreover remarked the close resemblance of the mineral sulphates to one another, as well as that of the rhombohedral carbonates. He also showed that strontianite was not rhombohedral as Hauy supposed, but prismatic, and that it resembled Aragonite in form. The small percentage of strontian detected in Aragonite by Stromeyer was regarded by Fuchs as the cause of the resemblance of the forms of the two minerals, as the very small quantity of carbonate of lime in chalybite had been supposed the cause of its resemblance to calcite. The only conclusion which Fuchs drew from the resemblances of these minerals was, that certain substances possess such an overpowering force of crystallization, that, even when present in small quantity, they constrain other substances to assume their form.

In November 1821 Mitscherlich returned to Berlin, was elected a Member of the Academy of Sciences and appointed Professor extraordinary in the University, and remained in that position till 1825, when he became Professor in ordinary. In the summer of 1822 he gave his first lecture on Chemistry to a large audience. He also continued his researches on isomorphism, and those which he had commenced in Stockholm, especially those which bore upon the artificial formation of minerals. He exhibited to the Academy a collection of about forty crystallized substances, which he had found in the slag-heaps surrounding the copper-smelting furnaces of Fahlun during a visit he paid to that place in 1820, in the company of Berzelius. Of these, however, he described only two, a silicate of protoxide of iron isomorphous with olivine, and a mica, the composition of which approximates closely to that of a black mica of Siberia. He resumed these researches along with Berthier in the winter of 1823 and 1824, which he passed in Paris, and by fusing the mineral constituents together in proper proportions, succeeded in producing diopside, idocrase, and garnet.

In the course of his examination of the phosphates and arseniates he had observed that the acid phosphate of soda crystallizes in two totally different forms, both of which belong to the prismatic system, but cannot be referred to the same parameters. From this he inferred that the ultimate atoms of crystallized bodies by change of circumstances may admit of a change in their arrangement, and hazarded the opinion that, as Aragonite resembles strontianite and cerussite in form, and calcite re-

sembles dolomite, chalybite, and diallogite, it is possible for the substances isomorphous with Aragonite to crystallize in the form of calcite, and the substances isomorphous with calcite to crystallize in the form of Aragonite, and so greatly enlarge each group of isomorphous bodies. This opinion was looked upon with great distrust by chemists and mineralogists. All the examples he had brought forward were taken from compound bodies, which possibly might have contained admixtures which analysis had failed to detect, and the substances assumed to have the same composition might after all be different. These doubts were suggested by the analyses of Aragonite, which had been pronounced by some of the most eminent chemists of the time to be pure carbonate of lime; then Stromeyer detected strontia in it, which, notwithstanding that its amount was very small, and different in Aragonite from different localities, was immediately regarded as the cause of the difference of its form from that of calcite; lastly, Buchholtz proved the existence of a variety of Aragonite absolutely free from any admixture of strontia, to which, therefore, the difference of form could not by any possibility be due. At this conjuncture Mitscherlich made the remarkable discovery that sulphur also takes different forms under different circumstances. The crystals obtained from solutions belong to the prismatic system, and are identical in form with those which occur in nature; but when sulphur is fused and allowed to cool, with proper management distinct crystals are obtained, but they are entirely different from the former, inasmuch as they belong to the oblique system. This observation was of great importance, because sulphur being a simple substance crystallizable at pleasure in either of its two forms, the difference of form could not be attributed to a difference of composition. He had already proved that the acid phosphate of soda and carbonate of lime possessed the same property of crystallization in two different forms, which he now considered as appertaining to all simple substances and their chemical combinations, and to which he gave the name of *dimorphism*. He regarded it, moreover, as affording an explanation of the fact that bodies possessing analogous chemical constitutions are not always isomorphous. The memoir on the dimorphism of sulphur was presented to the Academy on the 26th of July 1826.

It was found that the forms of isomorphous substances are not absolutely identical, except, of course, when they belong to the cubic system, but exhibit some differences, showing that the chemical nature of the substance is not altogether without influence on the form. In order to determine the difference between the angles of isomorphous bodies with greater accuracy than was attainable by the use of the ordinary Wollaston's goniometer, he caused a goniometer to be constructed by Pistor, provided with four verniers, each reading to 10″, and with a telescope magnifying twenty times for viewing the reflexions of the signal in the faces of the crystal. With this instrument, in the summer of 1823 he began to measure the angles of calcite from Iceland, and was surprised to find differences in the

angle between the same pair of cleavages amounting to 20″, a difference which, though small, was too large to be attributed to errors of pointing or reading. The observations were made in the morning and in the afternoon in a room facing the south. The morning observations differed from those made in the afternoon, but the observations made at the same period of the day agreed well with one another; also the temperature of the room in the afternoon was nearly 4° C. higher than in the morning. He therefore concluded that the variation of the angle could only be due to the unequal expansion of the crystal in different directions. He increased the difference of temperature by immersing the crystal in a bath of heated mercury, and found that the cleavages became more nearly at right angles to one another, by 8′ 34″, for an increase of temperature of 100° C. In dolomite from Traversella, Breunnerite from Pfitsch, chalybite from Ehrenfriedersdorff similar changes occurred amounting to 4′ 6″, 3′ 29″, and 2′ 22″ respectively, for a change of temperature of 100° C. A large number of other crystals examined by him afforded like results. In the winter of 1823–1824, during his stay in Paris, he measured the expansion of calcite in volume by Dulong's method, and found it equal to 0·001961 for 100° C. Hence it appears that by an increase of temperature of 100° C. the crystal expands 0·00288 in the direction of its axis, and contracts 0·00056 in a direction at right angles to its axis. He confirmed the accuracy of this most unexpected result by comparing, at different temperatures, the thicknesses of two plates of calcite of nearly equal thickness, bounded by planes parallel and at right angles to the axis respectively, and the thickness of a plate bounded by planes parallel to the axis with that of a plate of glass of nearly the same thickness, the expansion of which was known. His memoir on this important discovery was presented to the Academy on the 10th of March 1825.

The large goniometer which he employed in these observations being too cumbersome, and also too costly to be used by mineralogists in measuring the angles of crystals, he contrived an instrument more convenient for ordinary use, reading to half a minute, and provided with a telescope having a magnifying power of not more than three. The signal consists of cross wires in the focus of a collimator, as in the goniometers of Rudberg and Babinet. The adjustment of the crystal is effected by a very ingenious contrivance due to M. Oertling, by whom many of these instruments have been constructed. By the invention of this goniometer, which has come into general use under the name of Mitscherlich's goniometer, he conferred a great boon on mineralogists. A minute description of it appeared in the Memoirs of the Berlin Academy for 1843, a considerable time after it was originally contrived, and not till its value had been tested by long use.

Of his observations on the effect of heat on the double refraction of crystals, little is known beyond a notice in Poggendorff's 'Annalen' of the remarkable changes which occur in gypsum when heated. At the ordinary temperature of the atmosphere the optic axes lie in a plane at right angles

to the plane of symmetry, and make angles of about 60° with a normal to the plane of symmetry. On warming the crystal the optic axes approach the plane of symmetry, and at about 92° C. they coincide, exhibiting the phenomena of a uniaxal crystal, and on further increasing the temperature they open out in the plane of symmetry.

In 1827 Mitscherlich discovered selenic acid, and the isomorphism of seleniate of potash with sulphate of potash, and afterwards of other seleniates with the corresponding sulphates. In 1830 he observed the isomorphism of manganate of potash with sulphate of potash. This led him to a further examination of manganese, and to the discovery of the isomorphism of the permanganates with the perchlorates, and to the isolation of the hydrate of permanganic acid. At a later period (1860) he repeated, by new and more accurate methods, the analysis of permanganate of potash, which had been called in question, confirming the exactness of the earlier analysis; he succeeded at the same time in isolating the anhydrous permanganic acid.

The crystallographic researches he carried on about the time of the discovery of the new acids were extremely numerous, yet very little has been made known respecting them. He prepared a large number of salts in his laboratory, determined the systems to which they belonged, measured some of the angles, and drew by hand the figures of their principal combinations. But this, though it satisfied his own curiosity, was manifestly insufficient for publication, and the new discoveries that presented themselves were much more attractive than the wearisome and time-consuming task of preparing his researches for the press. He made, however, an attempt to carry out his intention of describing the forms of the most important simple and compound bodies. He commenced with the sulphates, seleniates, and chromates, because these salts present almost all the phenomena on which the laws of crystalline form and chemical composition are founded. He described the sulphates and seleniates of soda and of oxide of silver; the sulphate, seleniate, and chromate of oxide of silver and ammonia; the sulphate and seleniate of oxide of nickel, and the seleniate of oxide of zinc; the anhydrous and hydrous chloride of sodium; iodide of sodium and bromide of sodium; sulphate, seleniate, and chromate of potash, and sulphate of ammonia. Unfortunately these were his last regular contributions to crystallographic chemistry. Long afterwards he described the forms of the chloride and iodide of mercury, the latter of which is dimorphous, and the forms of phosphorus, iodine, and selenium crystallized from solution in bisulphide of carbon, which proved to be in an isomeric state differing in density from fused selenium.

In 1833 his crystallographic labours were interrupted by the publication of his 'Treatise on Chemistry.' For this work he had been long preparing himself by original researches, by associating with the most eminent chemists of Europe, by visiting their laboratories, and the most important chemical manufactures and smelting-furnaces. A large number of original

observations of his own are embodied in this work, which had never appeared in any scientific journal. A fifth edition was commenced in 1855, but left unfinished. In this year he commenced his important labours on the density of the vapour of bromine, sulphur, phosphorus, arsenic, and mercury, nitrous acid, nitric acid, sulphuric acid, &c., and on the relation of the density of vapours to their chemical equivalents. In the same year he commenced his researches on benzoyl, which suggested to him a simple theory of the constitution of those organic combinations in which compound radicals are assumed to exist. His experiments on the formation of ether led him to the doctrine of chemical combinations and decompositions by contact, whereby dormant affinities in mixtures, or compounds held together by feeble affinities, become active by mere contact with a substance chemically inactive. These labours in the domain of organic chemistry wholly occupied him for nearly twelve years. At the conclusion of this period he turned his attention to geology. Indeed, ever since he had engaged in researches on the artificial production of minerals, he used to theorize on the formation of rocks, and on the existence of mineral springs and volcanos. In his earlier travels, while his main object was the examination of chemical manufactures and smelting-furnaces, his attention was also directed to the geology of the countries through which he passed. He frequently devoted the concluding lectures of each half-year's course to a sketch of the geological structure of the earth, and the changes which its surface had undergone. Year after year he made systematic journeys in the Eifel, with the intention of publishing a complete description of the extinct volcanos of that district, and connecting it with a theory of volcanic action. And, as the study of this region made a comparison with the volcanos of other countries desirable, he visited in succession the principal volcanic districts of Italy, France, and Germany. But, notwithstanding all this preparation, the description of the Eifel was never printed, with the exception of some pages distributed among the hearers of lectures of a popular character given by him in the winter of 1838 and 1839. In these he states the views of the nature of volcanic processes which he then entertained. They appear to have been founded on a very careful study of volcanic phenomena. He supposes the explosive action to be caused by the vapour of water. The only hypothesis, however, by which the presence of water in an active volcano could at that time be accounted for, was beset by serious difficulties. These have since been removed by the beautiful experiment made by Daubrée, which shows that when one side of a stratum of porous rock is heated, water in contact with the opposite side makes its way through it, in the direction of the heated part, notwithstanding the high pressure of the vapour generated on that side.

During the autumnal vacation of 1861 he made his last geological excursion in the Eifel; in December of that year he began to suffer from disease of the heart, the complaint increased in severity in the summer of

1862, and he had much difficulty in completing his course of lectures. In the autumn of this year he went again to the Rhine, but only to stay in a country-house near Bonn, the home of his son-in-law, Professor Busch. Here his health appeared to revive, and he returned to Berlin feeling so much better that he commenced his winter lectures; a fortnight before Christmas, however, he was obliged to give them up, never again to be resumed. In the spring of 1863 he retired to a country-house at Schöneberg, near Berlin, and here, on the morning of the 28th of August, his valuable life was closed by a painless death. His name will ever be cherished in the annals of that science which he had so greatly enriched. Few philosophers have ever united such a versatility of genius with a mind so severely disciplined, or who, possessing such a talent for observing, were able to deduce such important results from their observations.

He was member of probably every Academy in Europe. He was elected Foreign Member of the Royal Society in 1828; the Royal Medal was awarded to him in 1829, "for his Discoveries relating to the Laws of Crystallization, and the Properties of Crystals."

In 1852 he was elected Foreign Associate of the French Institute, in the place of Œrsted.

The greater part of the preceding notice is extracted from an Address to the German Geological Society by Professor G. Rose, Mitscherlich's successor as President of the Society.

Carl Ludwig Christian Rümker was born on the 28th of May 1788, at Neubrandenburg in Mecklenburg-Strelitz, in the service of which State his father held an important position. After a careful preparatory education at home, he was sent to the Graue Kloster at Berlin, and later to the Engineering Academy of that place. In 1807 he passed the Government examination for qualification as an engineer and architect.

In consequence of the gloomy aspect of affairs in Prussia after the peace of Tilsit, he endeavoured to establish himself in Hamburg; but here also, finding no prospect of occupation in the profession he had adopted, he resolved in 1808 to go to England with the intention of devoting himself to a seafaring life. Accordingly, in the 21st year of his age, he began the world anew, under the most unfavourable circumstances, in a strange country, without friends, and entirely cut off from his home by the continental blockade. With an energy and strength of character peculiarly his own, he overcame the difficulties of his situation, and obtained an appointment as Midshipman in the Navy of the East India Company. Feeling dissatisfied with this service, he entered into that of the German house of Rücken in London, and visited many parts of the world in their ships. In 1811 or 1812 he obtained admission into the Royal Navy, and served during the latter part of the war on board various ships of the Mediterranean fleet. He was first appointed to the 'Benbow'; afterwards he became Naval Instructor on board the 'Montague,' Captain Peter Heywood (formerly of the

'Bounty'), of whom he used to speak in terms of the greatest regard, as the most kind-hearted and excellent man he had ever known. He was then transferred to the 'Albion,' and on his passage out from England to join Sir Charles Penrose, fought at the battle of Algiers.

During a visit to Genoa Rümker became acquainted with the Baron v. Zach, to whom he submitted the results of various astronomical observations in order to obtain his opinion of their value. The Baron soon discovered his talent for astronomy, encouraged him to cultivate that science, and aided him with his advice and the use of his astronomical library. Rümker's first observations, occultations, and the determination of the latitude and longitude of Malta, where he was stationed for a considerable time, were published in v. Zach's 'Correspondance Astronomique.'

In 1817, when the Fleet returned to England, he quitted the Naval Service and went to Hamburg, carrying with him the friendship and esteem of his comrades of all ranks, which he had won by his ability and energy, combined with a peculiar suavity of manner. Here he was appointed Principal of the School of Navigation. In the society of Schumacher, the Director of the Observatory of Altona, Repsold, and Woltmann, his taste for astronomy was strengthened, and he found many opportunities of extending his knowledge of the subject. As at that time Hamburg did not possess an Observatory, he built one at his own expense on the Stintfang.

In 1821 he resigned his post at the School of Navigation in order to accompany Sir Thomas Brisbane, Governor of New South Wales, to whom he had been introduced by Captain Heywood, and to take charge of the Observatory which Sir Thomas purposed founding in that colony.

Rümker's labours in the Observatory of Paramatta are well known to astronomers. In 1822 he observed the first calculated reappearance of Encke's comet, which was invisible in Europe, and thereby first confirmed the shortness of its periodic time. He afterwards observed and discovered many other comets, some of which were not seen in Europe. He observed the sun in the solstices, made many observations with Kater's pendulum, and determined the magnetic declination and inclination. These and other observations were published in a separate volume of the Transactions of the Royal Society for 1830. His observations of the stars of the southern hemisphere are in part contained in the 'Brisbane Catalogue,' and in the 'Preliminary Catalogue of Fixed Stars in the Southern Hemisphere,' published by himself at Hamburg in 1832. In after years, however, he was never able to find leisure for continuing the work, and the greater part of the observations remain still unpublished.

In 1829 he returned to Europe to resume his post as Principal of the School of Navigation, and to undertake the Direction of the New Observatory built by the Senate of Hamburg. He devoted himself with the most unwearied diligence to the duties of these two offices. After nights passed in observing, he made his appearance at the School of Navigation at

eight both in summer and winter, and remained there teaching for five and even seven hours, regardless of his failing health, which was unable to sustain so severe a trial.

Under his care the school attained an unexpected prosperity. It produced the most distinguished sailors of the German merchant navy, and the teachers of almost all the Schools of Navigation on the coast of the North Sea have been his pupils. The number of students, which at the time of his appointment to the school was only 60, amounted to 250 in 1857. He possessed in an unusual degree the art of teaching. By the clearness of his methods, and a singular patience and mildness which encouraged the self-respect of his pupils and gained their confidence, and especially by his power of adapting his teaching to the comprehension of each individual, he succeeded in preparing the most uncultivated sailor for the examination in navigation often in a surprisingly short time, so as to enable him to pass it with credit.

His 'Handbook of Navigation,' which appeared in 1843, and has gone through three large editions, is used as a text-book in most of the Schools of Navigation on the shores of the North Sea, in Austria, and in Russia.

He devoted himself with equal or even still greater energy to his duties in the Observatory. The principal instruments consisted of an equatorially mounted refractor of 5-feet focal length by Fraunhofer, and a meridian circle constructed by the brothers Repsold, which was mounted in 1836. The observations made with the refractor are published in Schumacher's 'Astronomische Nachrichten,' and in the Monthly Notices of the Astronomical Society. With the meridian circle he undertook the determination of the places of all the fixed stars visible through its telescope,—a work of many years' duration, the results of which he published in the years 1843–59 under the title of a Catalogue of 12,000 fixed stars, but in reality containing upwards of 15,000.

In speaking of the observations made with the refractor, at the Anniversary Meeting of the Astronomical Society in 1854, when the medal of the Society was awarded to M. Rümker, the Astronomer Royal, President of the Society, expressed himself in the following terms:—

"For a very long time M. Rümker has been known as furnishing extra-meridional observations of comets and newly discovered planets, possessing the highest degree of accuracy, and extending to times when the objects which *he* could successfully observe were lost to other astronomers furnished apparently with much more powerful means. I have myself visited the observatory and inspected the instruments which have been devoted to these observations, and I have inquired, How is it that with instruments so insignificant you have been able to see so much more than others could see who are so much better equipped? The answer was very simple. Energy, care, patience,—in these, I believe, is contained the whole secret. M. Rümker perhaps possesses in perfection the sensibility of eye and the acuteness of ear which are required for the most delicate observations; but

these powers, which might seem at first to be original gifts of nature, have, I do not doubt, acquired very much of their activity from their careful and energetic use."

Adverting to the Catalogue of fixed stars, for which more especially the medal was bestowed, Mr. Airy observes,—

"Had this Catalogue proceeded from an observatory of which the personal establishment was charged with no other labours, we should have considered it as a highly meritorious work. . . . . What, then, shall we say to this work in the circumstances under which it has reached us? It has come, the voluntary enterprise of an individual, who could not, by any construction of his connexion with the Hamburgh Observatory, be supposed to owe to the world a hundredth part of the labour which it has cost. It is the fruit of observations made in the watches of the night, and calculations made in the leisure hours of the day, by a person who would seem, to vulgar eyes, to be engrossed to the limits of human endurance by an onerous professional office. Well may we consider it as a remarkable instance of voluntary labour, undertaken under difficult circumstances, not for public display, but as an aid to science, and skilfully and steadily directed to that purpose alone."

M. Rümker was a Member of the Royal Academies of Münich and Göttingen, the Batavian Society of Rotterdam, the Royal Astronomical and many other English and Foreign learned Societies. He was elected Foreign Member of the Royal Society in 1855.

After having laboured long and profitably, repeated attacks of illness, accompanied by an asthmatic cough which increased in severity at each relapse, forced him at length (in 1857) to rest from his labours, and to seek a milder climate for the benefit of his shattered health.

At his suggestion the care of the Navigation School was entrusted to M. Niebour, who had been his assistant for many years, and that of the Observatory to his only son, George Rümker, at that time the Astronomer of the Durham Observatory, and now his successor as Director of the Observatory of Hamburg.

He had visited and been pleased with Lisbon during his earlier voyages, and was induced to select that place for his retreat. There, after a residence of six years, tenderly watched over by his wife, a lady of English birth, and the discoverer of the comet VI of 1847, retaining full possession of his faculties, he died on the 21st of December 1862.

Having served in the British Navy, and received the medal given to those who shared in the battle of Algiers, he was followed to the grave by officers of the British fleet in the Tagus, and by his German friends. He lies buried in a spot chosen by himself, close to Fielding's grave in the cemetery of the church of Estrella.

INDEX TO VOL. XIII.

ABEL (F. A.) on some phenomena exhibited by gun-cotton and gunpowder under special conditions of exposure to heat, 204.

Acid, liquid stannic and metastannic, on the preparation of, 340.

——, liquid titanic, on the preparation of, 340.

——, liquid tungstic, dialysis of, 340.

——, molybdic, decomposition of, 341.

——, on a colloid, a normal constituent of human urine, 314.

——, silicic, and other analogous colloidal substances, on the properties of, 335.

Acids derivable from the cyanides of the oxy-radicals of the di- and tri-atomic alcohols, 44.

—— of the lactic series, notes of researches on, 140.

Admission of Fellows, 276, 278.

Aërial tides, 329.

Air-pump, description of a new mercurial, 321.

Airy (G. B.), first analysis of 177 magnetic storms, registered by the magnetic instruments in the Royal Observatory, Greenwich, from 1841 to 1857, 48.

Albumen, on organic substances artificially formed from, 350.

Alcogel, 337.

Alcosol, 337.

Algebraic equations, on the differential equations which determine the form of the roots of, 245.

Alizarine, 87, 150.

Alkaloids, researches on isomeric, 303.

Amyloid substance met with in the animal economy, further observations on, 317.

Anæsthetics, effect of, upon blood, 159.

Aniline-blue, on, 9.

Aniline-yellow, on, 6.

Anniversary Meeting, November 30, 1863, 21; November 30, 1864, 494.

Annual Meeting for election of Fellows, June 2, 1864, 276.

Arc of meridian, at Spitzbergen, on the measurement of, 83.

——, preliminary survey for the measurement of an, at Spitzbergen by the Swedish Government, 551.

Attraction, local, uncertainty occasioned by, in the map of a country, 18, 253.

Auckland (Earl of), letter from, concerning Sir J. South's experiments on the vibrations occasioned by railway trains, 68.

Barometer, description of an improved mercurial, 160.

Bayma (Rev. J.) on molecular mechanics, 126.

Beale (L. S.), new observations upon the minute anatomy of the papillæ of the frog's tongue, 384.

——, indications of the paths taken by the nerve-currents as they traverse the caudate nerve-cells of the spinal cord and encephalon, 386.

Berkeley (Rev. M. J.), Royal Medal awarded to, 35.

Bernard (C.), elected foreign member, 277.

Biliverdin, on the supposed identity of, with chlorophyll, 144.

Binocular vision, on the normal motions of the human eye in relation to, 186.

Blood, on the influence of physical and chemical agents on, 157.

——, on the reduction and oxidation of the colouring-matter of the, 355.

Blood-vessels, on the distal communication of the, with the lymphatics, 327.

Boole (G.) on the differential equations which determine the form of the roots of algebraic equations, 245.

Bruniquel, description of the cavern of, and its organic contents (Part I.), 277.

Calculus of symbols, on the, (fourth memoir) 126, 428; fifth memoir, 227, 432.

Candidates for election, list of, March 3, 1864, 153.

——, list of selected, May 12, 1864, 228.

Capello (Senhor) and Stewart (B.), results of a comparison of certain traces produced simultaneously by the self-recording magnetographs at Kew and at Lisbon; especially of those which record the magnetic disturbance of July 15, 1863, 111.

Carpenter (W. B.) on the structure and affinities of *Eozoon Canadense*, 545.
Carter (T. A.) on the distal communication of the blood-vessels with the lymphatics; and on a diaplasmatic system of vessels, 327.
Cayley (A.), a second memoir on skew surfaces, otherwise scrolls, 244.
—— on the sextactic points of a plane curve, 553.
Cerebellum, on the functions of the, 177, 334.
Chase (P. E.), aërial tides, 329.
Child (G. W.), experimental researches on spontaneous generation, 313.
Chinoline, on the higher homologues of, 311.
Chlorine, action of, on methyl, 225.
Chlorophyll, remarks on the constitution of, 144.
Chrysaniline, 7.
Church (A. H.), researches on certain ethylphosphates, 520.
Clarke (J. L.), Royal Medal awarded to, 512.
Coal-tar, colouring-matters derived from, 6, 9, 341, 485.
Cold, its effect in producing diabetes, 537.
Connell (A.), obituary notice of, i.
Cooper (E. J.), obituary notice of, i.
Copley Medal awarded to Rev. A. Sedgwick, 31; to C. Darwin, 505.
Creosote, on the chemical constitution of Reichenbach's, 484.
Croonian Lecture, April 14, 1864.—On the normal motions of the human eye in relation to binocular vision, 186.
Curve-plane, on the sextactic points of, 553.

Darwin (C.), Copley Medal awarded to, 505.
Daylight, total, on a method of meteorological registration of the chemical action of, 555.
De la Rue (W.), comparison of Mr. De la Rue's and Padre Secchi's eclipse photographs, 442.
——, his photograph of the moon, 491.
——, Royal Medal awarded to, 510.
Density, variations of, in mineral substances produced by heat, 240.
Despretz (C. M.), obituary notice of, viii.
Diabetes, on the production of, artificially in animals by the external use of cold, 537.
Diazobenzol, compounds of, 378.
Dickinson (W. H.) on the functions of the cerebellum, 177, 334.
Dietary, inquiries into the national, 298.
Diphenylamine, 341.
Drops, on, 444, 457.

Earth's crust, speculations on the constitution of the, 18, 253.
Eclipse photographs, comparison of Mr. De la Rue's and Padre Secchi's, 442.
Electrical force, further inquiries concerning the laws and operation of, 364.
Electromagnetic field, a dynamical theory of, 531.
Ellis (A. J.) on the conditions, extent, and realization of a perfect musical scale on instruments with fixed tones, 93.
—— on the physical constitution and relations of musical chords, 392.
—— on the temperament of musical instruments with fixed tones, 404.
Eozoon Canadense, on the structure and affinities of, 545.
Equations of rotation of a solid body about a fixed point, 52.
Erman (A.) on the magnetic elements and their secular variations at Berlin, 218.
Etherogel, 338.
Ethylphosphates, researches on certain, 520.
Eye, on the normal motions of the human, in relation to binocular vision, 186.

Fairbairn (W.), experiments to determine the effects of impact, vibratory action, and a long-continued change of load on wrought-iron girders, 121.
Farr (W.), life tables printed by Scheutz's calculating machine, 537.
Field (J.), obituary notice of, iii.
Figure of the earth, a method of obtaining, free from ambiguity, 18, 253.
Fœtus, description of the pneumogastric and great sympathetic nerves in an acephalous, 90.
Foraminifera, on some, from the North Atlantic and Arctic Oceans, 239.
Forchhammer (G.) on the composition of sea-water in different parts of the ocean, 493.
Foreign members elected: Bernard (C.), Foucault (J. B. L.), Wurtz (A. C.), 277.
Foucault (J. B. L.), elected foreign member, 277.
Fowler (R.), obituary notice of, iii.
Frankland (R.) and Duppa (B. F.), notes of researches on the acids of the lactic series: No. I. action of zinc upon a mixture of the iodide and oxalate of methyl, 140.
Frog, tongue of, new observations upon the minute anatomy of the papillæ of, 384.

Gases, respiratory, mutual action of the blood and the, 157.
—— and vapours, on the spectra of ignited, 153.
Gasometer, description of a new mercurial, 321.

Gassiot (J. P.), description of a train of eleven sulphide-of-carbon prisms arranged for spectrum analysis, 183.
——, Royal Medal awarded to, 36.
Glaisher (J.) on the meteorological results shown by the self-registering instruments at Greenwich, during the extraordinary storm of October 30, 1863, 19.
Glycerogel, 338.
Glycerosol, 338.
Glyptodon, on the osteology of the genus, 108.
Gompertz (B.), second part of the supplement to the two papers on mortality published in the Philosophical Transactions in 1820 and 1825, 228.
Graham (T.) on the properties of silicic acid and other analogous colloidal substances, 335.
Greenwich, magnetic variations observed at, 87.
Griess (P.) on a new class of compounds in which nitrogen is substituted for hydrogen, 375.
Gun-cotton, on some phenomena exhibited by, under special conditions, 204.
——, remarks on, 29.
Gunpowder, on some phenomena exhibited by, under special conditions, 204.
Guthrie (F.) on drops, 444; Part II. 457.

Haig (Capt. R. W.), account of magnetic observations made between the years 1858 and 1861 inclusive in British Columbia, Washington Territory, and Vancouver Island, 15.
Hardy (P.), obituary notice of, v.
Harley (G.) on the influence of physical and chemical agents upon blood; with special reference to the mutual action of the blood and the respiratory gases, 157.
Harris (Sir W. S.), further inquiries concerning the laws and operation of electrical force, 364.
Hartnup (J.) on the great storm of December 3, 1863, as recorded by the self-registering instruments at the Liverpool Observatory, 109.
Haughton (Rev. S.) on the joint-systems of Ireland and Cornwall, and their mechanical origin, 142.
Heat, specific, of solid and liquid bodies, 229.
——, variations of density produced by, in mineral substances, 240.
Helmholtz (H.) on the normal motions of the human eye in relation to binocular vision, 186.
Hennessy (H. G.) on the synchronous distribution of temperature over the earth's surface, 312.
Herschel (Sir J.), a general catalogue of nebulæ and clusters of stars for the year 1860·0, with precessions for 1880·0, 2.
Hicks (J.), description of an improved mercurial barometer, 169.
Hofmann (A. W.), note on kinone, 4.
——, researches on the colouring-matters derived from coal-tar: No. I. on aniline-yellow, 6; No. II. on aniline-blue, 9; No. III. diphenylamine, 341; No. IV. phenyltolylamine, 485.
Huggins (W.) on the spectra of some of the chemical elements, 43.
—— on the spectra of some of the nebulæ, 492.
Huggins (W.) and Miller (W. A.) on the spectra of some of the fixed stars, 242.
Hulke (J. W.), a contribution to the minute anatomy of the retina of amphibia and reptiles, 138.
Huxley (T. H.) on the osteology of the genus *Glyptodon*, 108.
Hydrogel, 337.
Hydrosol, 337.

Ibbetson (L. L. B.), readmitted, 15.
Imaginary roots, an inquiry into Newton's rule for the discovery of, 179.
Integral numbers of the indeterminate equation, on the criterion of resolubility in, 110.
Intestines, fœtal, on the functions of the, 549.
Iodine, action of chloride of, upon organic bodies, 540.
Ireland and Cornwall, mechanical origin of the joint-systems of, 142.

Joint-systems of Ireland and Cornwall, on the, 142.
Jones (H. Bence) on the production of diabetes artificially in animals by the external use of cold, 537.

Kew curves, comparison of, 111, 115.
Kinone, note on, 4.
Kopp (H.), investigations of the specific heat of solid and liquid bodies, 229.

Lee (R. J.), a description of the pneumogastric and great sympathetic nerves in an acephalous fœtus, 90.
—— on the functions of the fœtal liver and intestines, 549.
Life tables, printed by Scheutz's calculating machine, 537.
Lisbon curves, comparison of, 113, 115.
Liver, fœtal, on the functions of the, 549.
Lutidine, physical properties of, 305.

M'Donnell (R.), further observations on the amyloid substance met with in the animal economy, 317.

Madder, *Rubia munjista*, examination of, 86, 145.
Magnesium, on, 217.
Magnet, table of the mean declination of the, at Lisbon, from January 1858 to December 1863, 347.
Magnetic elements and their secular variations at Berlin, 218.
—— declination, comparison of the most notable disturbances of, at Kew and Nertschinsk, 247.
—— ——, results of hourly observations made at Port Kennedy, 84; comparison of, with observations made at Point Barrow, 85.
—— ——, semiannual inequality of, 347.
—— disturbance, comparison of certain traces produced simultaneously by the self-recording magnetographs at Kew and at Lisbon, July 15, 1863, 111.
—— disturbances, retrospective view of the progress of the investigation into the laws and causes of, 247.
—— observations made in British Columbia, Washington Territory, and Vancouver Island, 15.
—— storms, first analysis of 177 registered by the magnetic instruments in the Royal Observatory, Greenwich, 48.
—— variations observed at Greenwich, 87.
Marcet (W.) on a colloid acid, a normal constituent of human urine, 314.
Mauve, or aniline-purple, on, 170.
Maxwell (J. C.), a dynamical theory of the electromagnetic field, 531.
Meteorites, on the microscopical structure of, 333.
Meteorological registration of the chemical action of total daylight, 555.
Methyl, action of chlorine on, 225.
Mitscherlich (E.), obituary notice of, ix.
Molecular mechanics, on, 126; principles of, 127; mathematical evolution of principles of, 131; application of the principles of, 135.
—— physics, contributions to, fifth memoir, 160.
Mortality, second part of supplement to two papers on, 228.
Müller (H.) on the chemical constitution of Reichenbach's creosote, 484.
Munjistine, 86, 147; tinctorial power of, 148.
Musical currents, on the physical constitution and relations of, 392.
—— instruments with fixed tones, on the temperament of, 404.
—— scale, on the conditions, extent, and realization of a perfect, 93.
Myology, on some varieties in human, 299.
Nebulæ and clusters of stars, catalogue of, 2.
——, on the spectra of some of the, 492.
Nerve-currents, indications of the paths taken by, through the caudate nerve-cells of the spinal cord and encephalon, 386.
Nerves, pneumogastric and great sympathetic, in an acephalous fœtus, description of, 90.
Nitrogen substituted for hydrogen in a new class of compounds, 375.
Nitropurpureine, 150.

Obituary Notices of deceased Fellows:—
Arthur Connell, i.
Edward Joshua Cooper, i.
Joshua Field, iii.
Richard Fowler, M.D., iii.
Peter Hardy, v.
John Taylor, v.
William Tooke, vi.
Rear-Admiral John Washington, vii.
César Mansuète Despretz, viii.
Eilhardt Mitscherlich, ix.
Carl Ludwig Christian Rümker, xvi.
Owen (R.), description of the cavern of Bruniquel, and its organic contents.—Part I. Human remains, 277.

Parker (W. K.) and Jones (T. R.) on some foraminifera from the North Atlantic and Arctic Oceans, including Davis Strait and Baffin Bay, 239.
Perkin (W. H.) on mauve or aniline-purple, 170.
Phenyltolylamine, 485.
Phipson (T. L.) on magnesium, 217.
——, note on the variations of density produced by heat in mineral substances, 240.
Plane water-lines, on, 15.
Plücker (J.) and Hittorf (J. W.) on the spectra of ignited gases and vapours, with especial regard to the different spectra of the same elementary gaseous substance, 158.
Point Barrow, magnetic observations made at, 85.
Pollock (Sir F.) on Fermat's theorem of the polygonal numbers, 542.
Polygonal numbers, on Fermat's theorem of the, 542.
Port Kennedy, observations of the magnetic declination made at, 84.
Pratt (Ven. J. H.) on the degree of uncertainty which local attraction, if not allowed for, occasions in the map of a country, and in the mean figure of the earth as determined by Geodesy; a method of obtaining the mean figure free from ambiguity, from a comparison of the Anglo-Gallic, Russian, and In-

dian arcs; and speculations on the constitution of the earth's crust, 18, 253.
Prestwich (J.) on some further evidence bearing on the excavation of the valley of the Somme by river-action, as exhibited in a section at Drucat near Abbeville, 135.
Purpureine, 87, 148.
Purpurine, action of bromine on, 152.

Quadratic forms containing more than three indeterminates, on the orders and genera of, 199.
—— forms, on complex binary, 278.

Radiant heat, researches on, (fifth memoir) 160.
Railway trains, experiments on the vibrations occasioned by, in a tunnel, 65.
Rankine (W. J. M.) on plane water-lines, 15.
Retina of amphibia and reptiles, contribution to the minute anatomy of, 138.
Robinson (Rev. T. R.), description of a new mercurial gasometer and air-pump, 321.
Rosaniline, methylic, ethylic, and amylic derivatives of, 13.
Roscoe (H. E.) on a method of meteorological registration of the chemical action of total daylight, 555.
Royal Medal awarded to Rev. M. J. Berkeley, 35; to J. P. Gassiot, 36; W. De la Rue, 510; J. L. Clarke, 512.
Rubia munjista, the East-Indian madder, or munjeet of commerce, examination of, 86, 145.
Rumford Medal awarded to J. Tyndall, 515.
Rümker (C. L. C.), obituary notice of, xvi.
Russell (W. H. L.) on the calculus of symbols (fourth memoir). With applications to the theory of non-linear differential equations, 126, 423.
—— —— (fifth memoir), with applications to linear partial differential equations and the calculus of functions, 227, 432.
Russian arc, remarks on, 24.

Sabine (Gen. E.), anniversary address, 497.
——, a comparison of the most notable disturbances of the magnetic declination in 1858 and 1859 at Kew and Nertschinsk, preceded by a brief retrospective view of the progress of the investigation into the laws and causes of the magnetic disturbances, 247.
——, results of hourly observations of the magnetic declination made by Sir Francis Leopold M'Clintock, R.N., and the officers of the yacht 'Fox,' at Port Kennedy, in the Arctic Sea, in the winter of 1858–59; and a comparison of these results with those obtained by Captain Maguire, R.N., and the officers of H.M.S. 'Plover,' in 1852, 1853, and 1854, at Point Barrow, 84.
Schorlemmer (C.), on the action of chlorine upon methyl, 225.
Scientific relief fund, first report of, 495.
Sea-water, composition of, in different parts of the ocean, 493.
Sedgwick (Rev. A.), Copley Medal awarded to, 31.
Silveira (Senhor da), a table of the mean declination of the magnet in each decade from January 1858 to December 1863, derived from the observations made at the magnetic observatory at Lisbon; showing the annual variation or semiannual inequality to which that element is subject, 347.
Simpson (M.) on the acids derivable from the cyanides of the oxy-radicals of the di- and tri-atomic alcohols, 44.
—— on the action of chloride of iodine upon organic bodies, 540.
Skew surfaces, otherwise scrolls, second memoir on, 244.
Skogman (Capt. C.), completion of the preliminary survey of Spitzbergen, undertaken by the Swedish Government with a view of ascertaining the practicability of the measurement of an arc of the meridian, 551.
Smee (A. H.) on organic substances artificially formed from albumen, 350.
Smith (E.), inquiries into the national dietary, 298.
Smith (H. J. S.) on complex binary quadratic forms, 278.
—— on the criterion of resolubility in integral numbers of the indeterminate equation $f=ax^2+a'x'^2+a''x''^2+2bx'x''+2b'xx''+2b''x'x=0$, 110.
—— on the orders and genera of quadratic forms containing more than three indeterminates, 199.
Somme, valley of the, on the excavation of, by river-action, 135.
Sorby (H. C.) on the microscopical structure of meteorites, 333.
South (Sir J.), experiments, made at Watford, on the vibrations occasioned by railway trains passing through a tunnel, 65.
Spectra of ignited gases and vapours, on, 153.
—— of some of the chemical elements, 43.
—— of some of the fixed stars, 242.
Spectrum analysis, description of a train of eleven sulphide-of-carbon prisms arranged for, 183.
Spitzbergen, preliminary survey by the

UNIVERSITY OF OKLAHOMA

Swedish Government for the measurement of an arc of meridian at, 551.
Spontaneous generation, experimental researches on, 313.
Spottiswoode (W.) on the equations of rotation of a solid body about a fixed point, 52.
Squalls, sudden, of 30th October and November 21, 1863, on, 51.
Stars, clusters of, catalogue of, 2.
——, spectra of some of the fixed, 242.
Stenhouse (J.), examination of *Rubia munjista*, the East-Indian madder or munjeet of commerce, 86, 145.
Stewart (B.) on the sudden squalls of 30th October and 21st November 1863, 51.
——, remarks on sun-spots, 168.
Stokes (G. G.) on the supposed identity of biliverdin with chlorophyll, with remarks on the constitution of chlorophyll, 144.
——, on the reduction and oxidation of the colouring-matter of the blood, 355.
Storm, on the great, of Dec. 3, 1863, 109.
Storms, magnetic, first analysis of 177, 48.
Sulphagel, 338.
Sun-spots, remarks on, 168.
Sylvester (J. J.), an inquiry into Newton's rule for the discovery of imaginary roots, 179.

Taylor (J.), obituary notice of, v.
Telescope, southern, remarks on, 23.
Temperature, on the synchronous distribution of, over the earth's surface, 312.
Tetrazodiphenyl, 382.
Tides, aërial, 329.
Tooke (W.), obituary notice of, vi.
Torell (Dr. Otto), extract of a letter to General Sabine from, dated Copenhagen, Dec. 12, 1863, 83.
Triangulation, on the middle-European, 84.
Triphenylic leucaniline, 12.
—— rosaniline, 12.
Tyndall (J.), researches on radiant heat.—Fifth memoir. Contributions to molecular physics, 160.
——, Rumford Medal awarded to, 515.

Urine, physiological relations of the colloid acid of, 316.

Vessels, on a diaplasmatic system of, 327.

Washington (J.), obituary notice of, vii.
Williams (C. G.), researches on isomeric alkaloids, 303.
Wolf (Prof.) on the magnetic variations observed at Greenwich, 87.
Wood (J.) on some varieties in human myology, 299.
Wrought-iron girders, experiments to determine the effects of impact, vibratory action, and a long-continued change of load on, 121.
Wurtz (A. C.), elected foreign member, 277.

Zinc, action of, upon a mixture of the iodide and oxalate of methyl, 140.

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